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Electromagnetic generation of ultrasound in metals*

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The electromagnetic generation of transverse acoustic waves in metals in the presence of a static magnetic field normal to the surface is discussed with reference to an isotropic effective mass m^* of the conduction electrons. From a semiclassical argument, it is shown that in addition to the direct Lorentz force and the collision-drag force, each lattice ion experiences a Bragg-reflection force proportional to m/m^*-1. In the nonlocal limit, when the ratio m/m^* is greater than unity, this force causes the generated acoustic amplitude as a function of magnetic field to deviate significantly from the monotonic dependence that is expected from the free-electron theory of metals. However, this force does not provide significant modification to the free-electron theory for predicting the rotation of the plane of polarization, the attenuation coefficient of shear acoustic waves, and the properties of the helicon-phonon interaction.

I. INTRODUCTION

The free-electron theory for the electromagnetic generation of transverse ultrasound in metals was first studied by Quinn[1] and has since been discussed by a number of other authors.[2-5] The mechanism of the process may be summarized as follows. The metal consists of conduction electrons and a lattice of positively charged ions. Electrons move freely in the background of these positive ions except for infrequent collisions with impurities in the lattice, characterized by a constant relaxation time τ. When an electromagnetic wave is incident on the metal surface (Fig. 1), eddy currents in the skin layer allow the electric field to penetrate. The conduction electrons in the skin layer are accelerated and transfer their excess momentum to the lattice through collisions. The resulting force on the ions is called the collision-drag force. The lattice ions also experience a direct Lorentz force due to the electric field in the skin layer. The Lorentz force due to the ac magnetic field in the electromagnetic wave is negligible. If the conduction electrons are completely free, as is approximately the case in metals like Na or K, these two are the only forces acting on the lattice of the metal.

If these two forces are dynamically unbalanced, they will excite propagating transverse acoustic waves that can be detected at the opposite surface of the sample. In the absence of a static magnetic field this happens only if the electronic mean free path l is larger than the skin depth δ. The collision-drag force is then spatially separated from the direct Lorentz force, and the two produce a shear on the lattice. The shear wave thus generated in the nonlocal limit is polarized parallel to the electric field in the skin layer. The corresponding amplitude of the wave is known as the nonmagnetic-direct-generation (NMDG) amplitude. If, however, $l<\delta$ the two forces locally cancel each other and there is no acoustic generation in this local limit.

If there is a static magnetic field, H_0 present, the electrons experience a Lorentz force,

$$(-e\langle\vec{v}_k\rangle\times\vec{H}_0)/c\equiv(\vec{j}_e\times\vec{H}_0)/nc\,, \tag{1}$$

where $\vec{j}_e$ is the electronic current density, n is the density of electrons, and c is the velocity of light. The momentum acquired is given up to the lattice in the process of collisions. The result is generation of an acoustic wave with amplitude linear in H_0 and polarized in the direction perpendicular to the electric field. This amplitude has been known as the magnetic-direct-generation (MDG) amplitude. Actually Eq. (1) represents the total force per lattice ion in the local limit, because only in this limit can one assume that the magnetic Lorentz force on the electron is transmitted directly to the lattice ion.

Quinn's[1] free-electron theory based on the above ideas covers both local and nonlocal conditions and, as reported by Turner *et al.*,[2] explains the generation data in potassium samples at large magnetic fields. However, Gaerttner[5] and others[6,7] in their experiments on potassium and aluminum samples, have found considerable disagreement with the theory in the region of small magnetic fields. The discrepancies are shown in Figs. 2 and 3, and are discussed below.

The magnitude of the generated acoustic-wave amplitude $|\xi|$ as a function of the magnetic field has a large dip near $H_0\sim 1$ kG (Fig. 2), compared to the smooth rise predicted by theory. This is probably due to two underlying discrepancies (Fig. 3): (i) The NMDG component of the amplitude is much larger at $H_0=0$ and, with the increase of H_0, falls off much faster than what theory predicts.

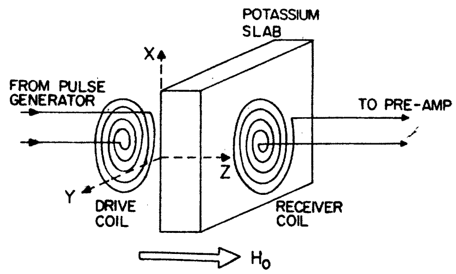

FIG. 1. Geometry of the coil-coil experimental used by Chimenti *et al.* (Ref. 7), and definition of the coordinate system (broken lines) used for calculating (Sec. III) the amplitude of electromagnetic generation.

(ii) The MDG component is nonlinear for small values of H_0, whereas simple theory predicts only a slight departure from linearity.

In a recent paper Kaner and Falco[8] have introduced another force on the lattice in addition to the magnetic Lorentz force, as shown in Eq. (1). Although this force has been shown to produce some qualitative features of the experimental data, we do not understand its origin and the degree of its relevance to simple metals. The force does not contain any parameter characterizing a departure from the free-electron model. On the other hand, there is definitely the need of such a parameter in any model, because, as can be seen in Ref. 5, the amount of discrepancy between Quinn's theory and experiment is different for different metals.

In the present paper, we introduce real-metal effects into the theory in terms of an isotropic effective mass m^* of conduction electrons. In real metals there is a nonzero, periodic crystal potential which manifests itself in giving rise to electronic band structures and hence in modifying the dynamics of otherwise free conduction electrons. The introduction of an effective-mass tensor for conduction electrons is a simple way of

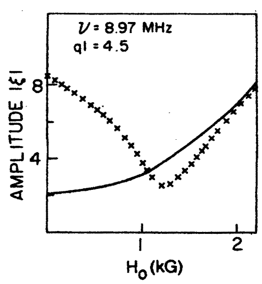

FIG. 2. Discrepancies between experimental results and free-electron theory for electromagnetic generation of ultrasound in potassium ($\vec{q} \parallel [100]$, $T = 4.2$ °K) after Chimenti *et al.* (Ref. 7). The generation amplitude has been plotted as a function of static magnetic field. The free-electron theory (solid line) has been normalized at high field.

taking these modified dynamics into account.

For simplicity we assume that the constant-energy surfaces of electrons in wave-vector space are spherical. In this case the electron under the influence of an external force will move in the periodic lattice as if endowed with an effective mass m^* that is different from the free-electron mass m. The fact that the conduction electron does not respond to an external field in the way it should if it were free causes a partial transfer of its momentum to the lattice. This is analogous to the momentum transfer to the lattice by an x-ray photon undergoing a Bragg reflection. The force on the lattice arising from this momentum transfer is what has been called the Bragg-reflection force by Kittel.[9] Thus the Bragg-reflection force is a natural consequence of the existence of a nonzero crystal potential. Although the effective mass m^* is treated as a parameter in the theory, its value should be consistent with the known band structure of the metal in question.

In phenomena concerning acoustic waves in metals, the Bragg-reflection force should play an important role, especially when the direct Lorentz force on the lattice is small. It is, therefore, of interest to obtain an expression for this force and study its effects on the phenomena which might be affected by its presence. For this purpose, we start with Jones and Zener's relation,[10]

$$\frac{d(\hbar\vec{k})}{dt} = -e\left(\vec{E} + \frac{\langle\vec{v}_k\rangle \times \vec{H}_0}{c}\right) . \tag{2}$$

Then from a semiclassical argument, consistent with momentum balance between electrons and ions, we show that each lattice ion experiences a Bragg-reflection force proportional to $m/m^* - 1$. A microscopic derivation of this force is presented in the Appendix. In the case of generation of sound, this force is shown to have substantial effects only at small values of H_0 and in the nonlocal limit. In the local limit or/and at large magnetic fields, we obtain the results of Quinn. We also show that for values of m/m^* appreciably larger than unity, features similar to those found in the data for potassium and aluminum can be produced. However, the value of m/m^* needed to fit the potassium data is intolerably larger than the known band-structure value. In the case of attenuation of shear waves, we find that in the Kjeldaas approximation[11] there is no significant difference in the results for the attenuation coefficient and the rotation of the plane of polarization compared to those obtained from free-electron theory. There is no modification whatsoever in the phenomena of helicon-phonon interaction.

The plan of the paper is as follows. In Sec. II we present a simple derivation of the Bragg-reflection force in the presence of external electric and magnetic fields and thus obtain a modified expression for the net force on the lattice ion. The amplitude of the shear acoustic wave is calculated in Sec. III following the same method as suggested by Quinn.[1] We also present our results for various values of m/m^* and compare them with the experimental data. Section IV contains a brief study of the effect of the Bragg-reflection force on the phenomena of ultrasonic attenuation and the helicon-phonon interaction. In Sec. V we summarize our results and discuss the possibility of extending the theory to metals that have complicated band structures.

II. FORCE ON LATTICE IONS

In a completely-free-electron model, as we already mentioned in the Introduction, there are two types of forces acting on the lattice ions: (i) the direct Lorentz force and (ii) the collision-drag force. The direct Lorentz force $\vec{F}_d$ for a monovalent metal is given by

$$\vec{F}_d = e[\vec{E} + (\vec{u} \times \vec{H}_0)/c] \tag{3}$$

per ion, where $\vec{u}$ is the local velocity of the ion, $\vec{E}$ is the electric field arising from the electromagnetic wave that is incident normally on the metal surface, and H_0 is the static magnetic field applied normal to the surface. In Eq. (3) we have neglected the Lorentz force due to the magnetic field associated with the electromagnetic wave. The collision-drag force $\vec{F}_c$, in the approximation used by Rodriguez[12] and justified by Holstein[12] from a microscopic viewpoint, can be written as

$$\vec{F}_c = m(\langle\vec{v}_k\rangle - \vec{u})/\tau , \tag{4}$$

where τ is a constant relaxation time and $\langle\vec{v}_k\rangle$ is the electron velocity averaged over all states $\vec{k}$ in the Fermi sphere and weighted by the zero-temperature Fermi distribution. Although these are the only forces present in the free-electron model, in the present model an additional force on each lattice ion will arise in order to preserve momentum balance.

Consider the energy $\mathcal{E}_k$ versus the wave vector $\vec{k}$ relationship for an electron with an isotropic effective mass m^*,

$$\mathcal{E}_k = \hbar^2 k^2/2m^* . \tag{5}$$

Then the group velocity of an electron in the state $\vec{k}$ is

$$\vec{v}_k = \hbar\vec{k}/m^* . \tag{6}$$

Thus the true momentum of the electron is

$$\vec{p}_k = (m/m^*)\hbar\vec{k} , \tag{7}$$

different from $\hbar\vec{k}$. Also consider the Jones and Zener relation,[10]

$$\frac{d(\hbar\vec{k})}{dt} = -e\left(\vec{E} + \frac{\langle\vec{v}_k\rangle \times \vec{H}_0}{c}\right) \equiv -\vec{\Gamma} . \tag{8}$$

Equation (8), together with Eq. (6) implies that in the presence of external fields, a conduction electron acquires an acceleration $\langle\vec{a}_k\rangle$ which can be written as

$$\langle\vec{a}_k\rangle = -\vec{\Gamma}/m^* . \tag{9}$$

Since the true mass of an electron is m, the Lorentz force $\vec{F}_L$ on the electron with effective mass m^* is, therefore,

$$\vec{F}_L = (-m/m^*)\vec{\Gamma} . \tag{10}$$

$\vec{F}_L$, as given by Eq. (10) represents the true Lorentz force experienced by a conduction electron in the crystal. Obviously, $\vec{F}_L$ is different from $\hbar\vec{k}$, the Lorentz force on a free electron. As the result of a nonzero crystal potential, created by lattice ions in the metal, there is thus an extra force on the electron. Consequently the lattice ion itself must experience an equal and opposite reaction force $\vec{F}_B$, which is called the Bragg-reflection force and is given by

$$\vec{F}_B = (m/m^* - 1)\vec{\Gamma} . \tag{11}$$

The Bragg-reflection force $\vec{F}_B$ is a logical consequence of having a nonzero crystal potential. Note that in the limit $m^* \to m$, i.e., in the free-electron limit, $\vec{F}_B \to 0$.

We consider a monovalent metal with n conduction electrons per unit volume. Thus the number of ions per unit volume is also n. The total force per lattice ion can then be written as

$$\begin{aligned}\vec{F}_t &= \vec{F}_d + \vec{F}_c + \vec{F}_B \\ &= e\frac{m}{m^*}\vec{E} - \left(\frac{m}{m^*} - 1\right)\frac{\vec{j}_e \times \vec{H}_0}{nc} - \frac{e\vec{j}_e}{\sigma_0}\frac{m}{m^*} - \frac{m\vec{u}}{\tau} \\ &\quad + e\frac{\vec{u} \times \vec{H}_0}{c} ,\end{aligned} \tag{12}$$

with $\sigma_0 = ne^2\tau/m^*$ and $\vec{j}_e = -ne\langle\vec{v}_k\rangle$. From Eq. (12), one can easily show that in the local limit and in the absence of a magnetic field, there is no net force on the lattice ion. This is a requirement for electrical neutrality of metals and cannot be met rigorously without considering the Bragg-reflection force.

III. GENERATED SOUND-WAVE AMPLITUDE

For the geometry of the experiment, as shown in Fig. 1, we assume that a plane-polarized electromagnetic (em) wave is incident on the metal surface with the magnetic field vector $\vec{H}$ along the y direction and the propagation vector $\vec{q}$ in the z

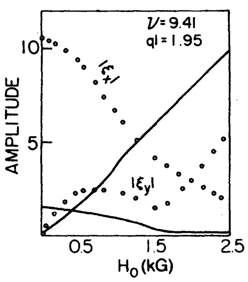

FIG. 2. Discrepancies between experimental results and free-electron theory in the magnetic field dependence of NMDG and MDG amplitudes in potassium (fast shear wave, $\vec{q}\|[110]$, $T = 4.2\,°K$) after Gaerttner *et al.* (Ref. 6). Free-electron theory (solid lines) is for $ql = 1.95$ and has been normalized at high field. Small circles represent experimental points.

direction. The surface of incidence is at $z = 0$ while the surface of detection is at $z = \infty$. A static magnetic field $\vec{H}_0$ is applied normal to the surface and is parallel to $\vec{q}$. We consider a monovalent, cubic metal and assume that $\vec{q}$ is parallel to [100] direction. In this configuration, the two possible shear acoustic modes have equal velocities. Although the degeneracy in the velocity will be lifted due to the presence of the static magnetic field, this effect will be negligibly small in the range of magnetic fields that we shall consider here. Extensions of the following calculations to other orientations of $\vec{q}$ will not yield any new information.

Given $F_i(z)$, the long-range force on the lattice ion, as a function of z, it is straightforward to evaluate the amplitude of generated acoustic waves from the wave equation,

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} = \rho s^2 \frac{\partial^2 \vec{\xi}}{\partial z^2} + \vec{f}, \tag{13}$$

where $\vec{f}$ is the force on ions per unit volume and is given by

$$\vec{f} = n\vec{F}_i. \tag{14}$$

Here $\vec{\xi}$ is the displacement of the ion from its equilibrium position, s is the transverse speed of sound, ρ is the density of the metal and n is the number of ions per unit volume and is equal to the density of conduction electrons in the metal. In general, $\vec{\xi}$ is obtained from a self-consistent solution of the acoustic-wave equation [Eq. (13)], Maxwell's equations, and the Boltzmann-transport equation. However, self-consistency is not important in the generation of acoustic waves of ultrasonic frequency because the ionic current is small compared to the electronic current and the electromechanical coupling is weak. In contrast, self-consistency becomes important in considering the attenuation of generated acoustic waves, because for this situation the electronic current is comparable to the ionic current. This is not of importance here since the reported experimental data are the actual generated amplitudes after making corrections for attenuation.

With the neglect of self-consistency, it can be shown that for a stress-free surface, [i.e., $(\partial\vec{\xi}/\partial z)_{z=0} = 0$], the amplitude of an acoustic wave propagating towards the opposite end of the surface is given by

$$\vec{\xi}(z, t) = \vec{A} \sin(q_0 z - \omega t),$$

with

$$\vec{A} \equiv -\frac{1}{\rho\omega s} \int_0^\infty \cos q_0 z' \vec{f}(z')dz'. \tag{15}$$

The observed acoustic-wave amplitude at infinity, $\xi(\infty)$, is therefore equal to A and we write it as

$$\vec{\xi}(\infty) = \vec{A}. \tag{16}$$

The force density $\vec{f}(z)$ depends on the mechanism for electron scattering at the boundary of the metal. In the following we shall assume that electrons are reflected specularly at the boundary. Influences of diffuse scattering will be considered in a separate paper. The assumption of specular scattering allows one to adopt a simplified, yet mathematically equivalent model to describe the problem of acoustic generation. That is, we may replace the semi-infinite metal by an infinite medium that is symmetrical about the plane $z = 0$. In order to include the effects of an incident electromagnetic wave at the boundary of the semi-infinite metal, we place a source at $z = 0$ in the infinite-medium model. In addition, the plane $z = 0$ remains stress-free, as in the semi-infinite case. $\vec{E}(z)$ is continuous across the boundary. However, because of the presence of the source at $z = 0$, components of $d\vec{E}/dz$ parallel to the surface is discontinuous at $z = 0$. The electronic current density $\vec{j}_e(z)$ is then a functional of $\vec{E}(z)$. From Eqs. (12) and (14), we see that the force density can also be written as a functional of $\vec{E}(z)$. Thus $\vec{f}(z)$ itself is an even function of z and we can write

$$\vec{\xi}(\infty) = -\frac{1}{2\rho\omega s} \int_{-\infty}^\infty e^{iq_0 z'} \vec{f}(z')dz' = -(\pi/\rho\omega s)\vec{f}(q_0), \tag{17}$$

with

$$\vec{f}(q_0) = \frac{1}{2\pi} \int_{-\infty}^\infty e^{iq_0 z'} \vec{f}(z')dz',$$

the Fourier transform of $\vec{f}(z)$. We now evaluate $\vec{f}(q_0)$ separately for the local and nonlocal limits.

A. Local limit

In this case the electric field $\vec{E}(z)$ and the electronic current density $\vec{j}_e(z)$ are related through a local conductivity tensor. We use circular coordinates to write this relation as follows:

$$j_e^\pm \equiv j_{ex} \pm i j_{ey} = \sigma^\pm E^\pm, \tag{18a}$$

with

$$\sigma^{\pm}=\sigma_0/(1\mp i\omega_c\tau)\,. \tag{18b}$$

Here, σ_0 is the zero-field conductivity as defined in Eq. (12), τ is a constant relaxation time, and $\omega_c=eH_0/m^*c$. Assuming a time dependence $\sim e^{-i\omega t}$ for the fields, the relevant Maxwell's equations, neglecting the displacement current, can be written

$$\frac{\partial^2E^{\pm}}{\partial z^2}=-\frac{4\pi i\omega}{c^2}j_e^{\pm}\pm\frac{2\omega}{c}H^{\pm}(0)\delta(z)\,, \tag{19}$$

where $H^{\pm}(0)=\pm iH_y(0)$, the magnitude of the ac magnetic field at the surface and $\delta(z)$ is the Dirac δ function. The second term on the right-hand side represents the source at $z=0$. As discussed earlier, we have neglected the ionic current. From Eqs. (18) and (19), we obtain for the Fourier component of the field,

$$E^{\pm}(q_0)=\frac{\mp(\omega/\pi c)H^{\pm}(0)}{q_0^2-4\pi i\sigma^{\pm}/c^2}\,. \tag{20}$$

For ultrasonic frequencies ($\omega\sim10^7$ Hz) and low magnetic fields ($H_0\lesssim 20$ kG), $q_0^2\ll|4\pi\omega\sigma^{\pm}/c^2|$, and we obtain

$$E^{\pm}\simeq-cH_y(0)/4\pi^2\sigma^{\pm} \tag{21}$$

and

$$j_e^{\pm}\simeq-cH_y(0)/4\pi^2\,. \tag{22}$$

These results could be obtained from Eq. (19) by neglecting the left-hand side altogether. This is a consequence of weak electromechanical coupling.

After neglecting the ionic current, we obtain from Eq. (12) and the definition of $\vec{f}(z)$ as given in Eq. (14),

$$f^{\pm}(z)=\pm iH_0j_e^{\pm}(z)/c\,. \tag{23}$$

Using Eq. (22), the Fourier component $f^{\pm}(q_0)$ is given by

$$f^{\pm}(q_0)=\mp iH_0H_y(0)/4\pi^2\,. \tag{24}$$

Thus from Eq. (17), the amplitude of the generated acoustic wave can be written

$$\xi^{\pm}(\infty)=\pm iH_0H_y(0)/4\pi\rho\omega s\,, \tag{25}$$

and transforming back to Cartesian coordinates, final expressions for the x and y components of the generated acoustic-wave amplitude in the local limit are

$$\xi_x=0 \quad\text{and}\quad \xi_y=H_0H_y(0)/4\pi\rho\omega\rho\,. \tag{26}$$

The above expressions are exactly what one would have obtained from the free-electron theory of Quinn[1] in the local limit. Thus the Bragg-reflection force does not have any effect on the generated sound-wave amplitude in the local limit.

B. Nonlocal limit

When nonlocal conditions are included, the electrons experience spatially varying electric fields between collisions. Thus the conductivity tensor becomes wave-vector dependent and must be obtained from the solution of the Boltzmann-transport equation. For the boundary conditions appropriate for specular scattering, we shall use the results of Kjeldaas[11]:

$$j_e^{\pm}(q)=\sigma_0G_{\pm}(q)E^{\pm}(q)\,, \tag{27}$$

where $G_{\pm}(q)$ are even functions of q and $G_-(H_0)=G_+(-H_0)$. Exact expressions for $G_{\pm}(q)$ can be obtained from Ref. 11 replacing m by m^*. σ_0 in Eq. (27) is the zero-field conductivity and has been defined in Eq. (12). With the above constitutive relation, we proceed as in the local limit and obtain

$$f_{\pm}(q_0)=\mp\frac{CH_{\pm}(0)}{4\pi^2}\left[\frac{im}{e\tau}\left(\frac{1}{G_{\pm}}-1\right)-\frac{H_0}{c}\left(\frac{m}{m^*}-1\right)\right] \tag{28}$$

and

$$\xi^{\pm}(\infty)=\frac{H_{\pm}(0)}{4\pi\rho\omega s}\left[\pm\frac{imc}{e\tau}\left(\frac{1}{G_{\pm}}-1\right)-H_0\left(\frac{m}{m^*}-1\right)\right]\,. \tag{29}$$

For a linearly polarized incident electromagnetic wave, $H_{\pm}(0)=\pm iH_y(0)$ and using the symmetry of $G_{\pm}$, namely, $\text{Re}G_+=\text{Re}G_-$ and $\text{Im}G_+=-\text{Im}G_-$, we finally obtain the following expressions for the components of the acoustic-wave amplitude at infinity:

$$|\xi_x(\infty)|=\frac{CH_y(0)}{4\pi\omega\rho s}\frac{m}{m^*}\frac{(3\pi^2n)^{1/3}\hbar\omega}{seq_0l}\left|\frac{\text{Re}G_+}{|G^+|^2}-1\right| \tag{30}$$

and

$$|\xi_y(\infty)|=\frac{CH_y(0)}{4\pi\omega\rho s}\left|\frac{m}{m^*}\frac{(3\pi^2n)^{1/3}_{\hbar\omega}}{seq_0l}\frac{\text{Im}G_+}{|G_+|^2}-\frac{H_0}{c}\left(\frac{m}{m^*}-1\right)\right|\,, \tag{31}$$

with $l=v_0\tau$ and v_0 the Fermi velocity. The quantity measured by Chimenti *et al.*[7] is $|\xi(\infty)|$ and is given by

$$|\xi(\infty)|=(|\xi_x(\infty)|^2+|\xi_y(\infty)|^2)^{1/2}\,. \tag{32}$$

On comparing the above expressions with those obtained from the free-electron theory, it is observed that the amplitude $|\xi_x(\infty)|$ is modified by the factor m/m^*. The corresponding term in $|\xi_y(\infty)|$ also contains this factor. In addition $|\xi_y(\infty)|$ contains an extra term proportional to m/m^*-1. This term can enhance or suppress the contribution from the first term depending on whether m/m^*-1 is less than or greater than zero. These features are possible only at low magnetic fields for which $\omega_c\tau/q_0l<1$. If $\omega_c\tau/q_0l \gg 1$, then $\mathrm{Im}G_+/|G_+|^2 \approx \omega_c\tau$, and $\mathrm{Re}G_+/|G_+|^2 \simeq 1$, so that one obtains the results as found in the local limit. Thus the Bragg-reflection force may modify the generated amplitude only at small magnetic fields. The large field amplitudes, however, should behave linearly as in the local case.

Using the parameters appropriate for potassium, we have calculated the amplitudes as given in Eqs. (30)–(32) at a frequency of 10 MHz. The results are shown in Figs. 4–8 for various values of m/m^* and q_0l. The results corresponding to $m/m^*=1$ are the same as those one would have obtained from free-electron theory. The results for $m/m^*>1$ and $q_0l>1$ are encouraging. Here the Bragg-reflection force plays a significant role and produces the same features as found in the experimental data. (See Figs. 1 and 2.) When $m/m^* \sim 2$, our theory fits surprisingly well with the experiment. However, the band-structure value for m/m^* in potassium as calculated by Ham[13] is only 0.83. Consequently, if the Fermi surface of potassium is simply connected, this theory does not explain the data. In a subsequent paper we shall show that the anomalous behavior of aluminum is explained by this theory even though aluminum is nearly-free-electron-like. The Bragg-reflection force becomes very important when the Fermi surface is not simply connected.

IV. ULTRASONIC ATTENUATION AND THE HELICON-PHONON INTERACTION

In the nonlocal case, the self-consistent force on lattice ions is considerably modified due to the Bragg-reflection force. The dispersion relations for coupled electromagnetic and sound waves will be accordingly modified. One would, therefore, suspect that the phenomena such as the attenuation of sound and the helicon-phonon interaction, which are directly linked with the dispersion relations, might exhibit some new features when the Bragg-reflection force is taken into account. We have investigated this matter and present our findings briefly in Secs. IV A and IV B.

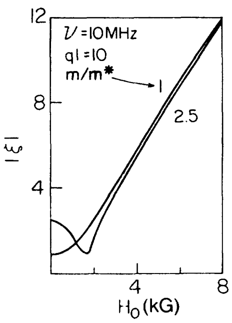

FIG. 5. Disappearance of the effects of the Bragg-reflection force in the region of higher field. The two curves are for two values of m/m^*, but with $ql=10$ for both.

A. Ultrasonic attenuation

Here we assume that a plane-polarized transverse ultrasonic wave is impressed upon a metal surface at $z=0$, propagating into the metal along the z direction perpendicular to the surface. As in Secs. II and III, we consider the presence of a static magnetic field normal to the surface of the metal. As a result of the impressed ultrasonic wave, the lattice ions will vibrate about their mean positions producing an electric field that causes the conduction electrons in the metal to accelerate. The electrons in turn, transfer the excess energy to thermal phonons through collisions with impurities in the lattice. Thus the energy of the ultrasonic phonon is attenuated as it propagates through the metal. In the presence of the static magnetic field there is also a rotation of the plane of polarization of the ultrasonic wave. The plane-polarized ultrasonic wave can be considered to consist of right- and left-circularly-polarized waves of equal amplitudes. These two waves suffer different velocity changes as they propagate through the metal in the presence of the magnetic field. This causes the plane of polarization of the ultrasonic wave to rotate by one-half the phase difference between the right- and left-circularly-polarized waves.

Both the attenuation and the rotation of the plane

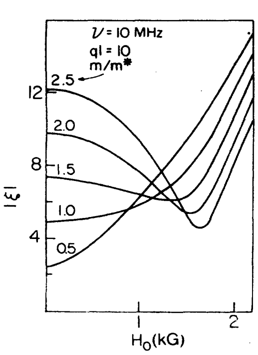

FIG. 4. Effects of the Bragg-reflection force on the amplitude $|\xi|$, plotted as a function of static magnetic field. Different curves are for different values of m/m^*, but with $ql=10$ for all. Vertical scale is in relative units.

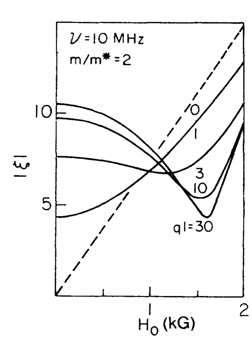

FIG. 6. Effects of the nonlocality parameter ql on the shape of the dip, appearing in the amplitude vs magnetic field curve when the Bragg-reflection force is included. $m/m^*=2$ for all curves. The broken line represents the local limit.

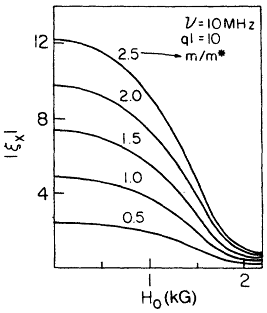

FIG. 7. Enhancement of the zero-field amplitude and the rapid fall of the NMDG amplitude $|\xi_x|$ as a function of magnetic field after including the Bragg-reflection force. Different curves are for different values of m/m^*, but with $ql=10$ for all.

of polarization can be described in terms of complex wave numbers $q^\pm$ of the right- and left-circularly-polarized components of the acoustic wave in the metal. The attenuation coefficient α, defined as the energy of the ultrasonic wave attenuated per unit length of the metal, is then given by

$$\alpha = \mathrm{Im}q^+ + \mathrm{Im}q^- . \tag{33}$$

The rotation angle Φ of the plane of polarization can be written

$$\Phi = \tfrac{1}{2}(\mathrm{Re}q^+ - \mathrm{Re}q^-) . \tag{34}$$

The quantities $q^\pm$ are obtained from the dispersion relation of the acoustic wave. The dispersion relation, in turn, is obtained self-consistently from the elastic wave equation, Maxwell's equations, and the Boltzmann-transport equation. We can, therefore, carry over the notation of Secs. II and III, provided we make the necessary changes in the definitions of variables. For example, $\xi^\pm$ will now represent the right- and left-circularly-polarized components of the amplitude of the impressed ultrasonic wave, not of the generated wave. Similarly $E^\pm$ will be the self-consistent electric fields instead of the electric fields associated with the incident electromagnetic wave. Since it is the local motion of ions that gives rise to these electric fields, we must not neglect the ionic current here as we have done in previous sections. With these modifications we can write the force on a lattice ion as

$$F_\pm = \frac{emE^\pm}{m^*} + \left(\frac{im\omega}{\tau} \mp \frac{eH_0}{c}\omega\right)\xi^\pm - \left[\frac{m}{ne\tau} \mp \frac{iH_0}{nc}\left(\frac{m}{m^*} - 1\right)\right] j_e^\pm . \tag{35}$$

The constitutive equation, Eq. (27) is now,

$$j_e^\pm(q) = \sigma_0 G_\pm[E^\pm(q) - J_i^\pm/\sigma_0] , \tag{36}$$

where, $J_i^\pm \equiv neu_\pm$ is the ionic current density. From Maxwell's equation with the neglect of displacement current, we obtain

$$\frac{d^2E^\pm}{dz^2} = -\frac{4\pi i\omega}{c^2}(j_e^\pm + J_i^\pm) . \tag{37}$$

The Fourier component of the self-consistent electric fields can then be written

$$E^\pm(q) = \frac{(4\pi i\omega/c^2)J_i^\pm(1-G_\pm)}{q^2 - 4\pi i\omega\sigma_0 G_\pm/c^2} . \tag{38}$$

For ultrasonic frequencies, $q^2 \ll \omega\sigma_0/c^2$ one can neglect q^2 in the denominator of Eq. (38). This is Kjeldaas approximation. In this approximation, as can be seen by substituting Eq. (38) into Eq. (36), $j_e^\pm(q) = -J_i^\pm(q)$. In other words, the screening of the ionic current by the electronic current is complete. In this case, the fields may be written

$$E^\pm(q) = (ine\omega/\sigma_0)\xi^\pm(1/G_\pm - 1) , \tag{39}$$

and the Fourier component of the total force on the lattice ion is given by

$$F_\pm(q) = \frac{m}{m^*}e\left(E^\pm(q) \mp \frac{H_0\omega}{c}\xi^\pm(q)\right) . \tag{40}$$

If the mass of the lattice ion is M, then from Eqs. (13), (39), and (40), one obtains the dispersion relations for right- and left-circularly-polarized waves:

$$q_\pm^2 = \frac{\omega^2}{s^2}\left[1 + \frac{im}{M\omega\tau}\left(\frac{1}{G_\pm} - 1\right) \mp \frac{\Omega_c}{\omega}\frac{m}{m^*}\right] , \tag{41}$$

where $\Omega_c \equiv eH_0/Mc$ is the ion cyclotron frequency. Since unity is large compared to other terms in the brackets in Eq. (41), we may rewrite this relation as

$$q_\pm \simeq \frac{\omega}{s}\left[1 + \frac{im}{2M\omega\tau}\left(\frac{1}{G_\pm} - 1\right) \mp \frac{\Omega_c}{2\omega}\frac{m}{m^*}\right] . \tag{42}$$

From Eqs. (33) and (34), we then obtain the final expressions for the angle of rotation of the plane of polarization Φ and the attenuation coefficient α:

$$\Phi = \frac{m}{m^*}\frac{\Omega_c}{2s}\left(1 - \frac{1}{\omega_c\tau}\frac{\mathrm{Im}G^+}{|G^+|^2}\right) \tag{43}$$

and

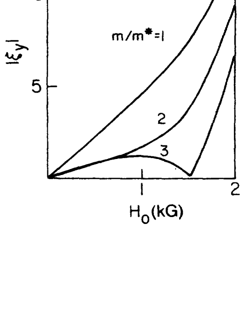

FIG. 8. Nonmonotonic dependence of the MDG amplitude $|\xi_y|$ on the static magnetic field when the Bragg-reflection force is included (curve with $m/m^*=3$). $ql=10$ for all curves.

$$\alpha = \frac{m}{m^*}\left[\frac{\hbar(3\pi^2 n)^{1/3}}{Msl}\left(\frac{\mathrm{Re}G_+}{|G_+|^2} - 1\right)\right] . \qquad (44)$$

Comparing Eqs. (43) and (44) with the standard results (e.g , Boyd and Gavenda[14]), we see that the only effect the Bragg-reflection force has on the attenuation of ultrasound in the Kjeldaas approximation is an overall multiplying factor m/m^* for both the rotation of the plane of polarization and the attenuation coefficient *after* replacing m by m^* in the usual free-electron results.

B. Helicon-phonon interaction

When a low-frequency electromagnetic (em) wave with wave vector $\vec{q} \parallel \hat{z}$, is incident on a metal surface, it cannot propagate into the metal. On the other hand, if a constant magnetic field $\vec{H}_0$ is applied parallel to $\vec{q}$ with sufficient magnitude that the electron cyclotron frequency is much larger than the frequency of the em wave, one of the two circularly polarized components of the incident plane-polarized em wave can propagate through the metal without appreciable damping. These propagating em waves are known as helicons. The phase velocity of helicons is very small and can be made to match the velocity of transverse sound waves in the metal. In such a situation, the degeneracy in the velocity of the helicon mode and the phonon mode is lifted due to a coupling between the two modes. This phenomenon is known as the helicon-phonon interaction. The extent of coupling between the two modes is best described by a coupling parameter η which is obtained from the dispersion relation of these coupled modes. The geometries used in Sections II–IV A also apply here, and when the Bragg-reflection force is included, we obtain the dispersion relation for the coupled modes:

$$\left(\omega - \frac{q^2c^2}{4\pi i\sigma_0 G_\pm}\right)\left[\omega^2 - q^2s^2 + \frac{im\omega}{M\tau}(1-G_-) + \Omega_c\omega + \left(\frac{m}{m^*} - 1\right)\Omega_c\omega G_-\right]$$
$$= i\omega^2\left(1 - \frac{1}{G_-}\right)\left[\frac{m}{M\tau}(1-G_-) - i\Omega_c G_-\left(\frac{m}{m^*} - 1\right)\right] . \qquad (45)$$

This dispersion relation takes a different form than the standard expression[15] due to the presence of terms containing the factor $m/m^* - 1$, which arises from the Bragg-reflection force. However this is not of importance in the region of interest. Helicons are propagating modes only at comparably large values of the magnetic field. The condition for appreciable coupling between the two modes is that their velocities be approximately equal. This happens, for instance, in potassium for $H_0 \simeq 10^2$ kG at a frequency of 20 MHz. At such high static magnetic fields, G_- is approximately equal to $1/(1+i\omega_c\tau)$ and the dispersion relation reduces to that given in Ref. (15):

$$\left(\frac{u}{s} - \frac{v_H}{s}\right)\left[\left(\frac{u}{s}\right)^2 - 1 + \frac{\Omega_c}{\omega}\left(\frac{u}{s}\right)^2\right] = \frac{\Omega_c}{\omega}\frac{v_H}{s} , \qquad (46)$$

where v_H ($\equiv c^2 q\omega_c/\omega_p^2$) is the helicon velocity in the absence of interaction with phonons, ω_p is the electron plasma frequency, and u is the phase velocity of the coupled modes. The coupling parameter η is thus given by

$$\eta = \Omega_c/\omega . \qquad (47)$$

The dispersion relation and the coupling parameter η as given in Eqs. (46),and (47) are not altered by the inclusion of the Bragg-reflection force. Therefore, this force has no effect on the helicon-phonon interaction.

V. SUMMARY AND CONCLUDING REMARKS

The Bragg-reflection force was shown to be a natural consequence of the concept of effective mass of the conduction electrons in metals. Though it cannot produce any noticeable effect on the helicon-phonon interaction, it modifies the rotation of the plane of polarization and the attenuation coefficient of the shear ultrasonic wave by a multiplying factor m/m^*. The phenomenon most significantly modified by this force is the electromagnetic generation of ultrasound in metals in the presence of small magnetic fields. Here it is capable of producing a nonmonotonic dependence of the generated amplitude as a function of the magnetic field. These effects will be most important for metals with m/m^* appreciably different from unity.

We have considered only a single isotropic effective mass for all conduction electrons. However one can easily extend the ideas of this paper to the consideration of two or more groups of electrons or holes with different effective masses. The Bragg-reflection force will be obtained by the condition that the total momentum of the ions balance that of the carriers. In this way, metals with complicated band structures can be studied. The Bragg-reflection force is expected to play a significant role in predicting the amplitude of electromagnetic generation of ultrasound in many metals.

Although this study was undertaken in order to understand the anomalous behavior of potassium, we must confess failure if the Fermi surface of potassium is simply connected. On the other hand, if the Fermi surface is multiply connected, as the torque anisotropy data of Holroyd and Datars[16] require, then inclusion of the Bragg-reflection force is necessary. At the present time the only physical mechanism that could lead to a multiply connected Fermi surface in potassium is a charge-density-wave instability[17] of the electronic ground state.

APPENDIX

For simplicity, we consider the one-dimensional electron gas of a monoatomic linear chain of atoms whose reciprocal lattice vector is $\vec{Q}$. We assume that the average potential energy due to the lattice ions is $2U_1 \cos Qx$ and the unperturbed electronic states are free-particle states. The action of the crystal potential is to cause mixing of the unperturbed state $\vec{k}$ with, for example, $\vec{k}-\vec{Q}$, producing a gap at $k=\frac{1}{2}Q$. Treating this mixing by degenerate perturbation theory, one obtains the energy eigenvalues for states above and below the gap.

$$\mathcal{E}_{\pm}(k)=\tfrac{1}{2}(\epsilon_k+\epsilon_{k-Q})\pm\tfrac{1}{2}[(\epsilon_k-\epsilon_{k-Q})^2+4U_1^2], \qquad (A1)$$

where, $\epsilon_k=\hbar^2k^2/2m$. The corresponding eigenfunctions are

$$\Psi_+=\cos\theta\, e^{ikx}-\sin\theta\, e^{i(k-Q)x}, \qquad (A2)$$

$$\Psi_-=\sin\theta\, e^{ikx}+\cos\theta\, e^{i(k-Q)x}, \qquad (A3)$$

for states above and below the gap, respectively. The coefficients are given by

$$\sin 2\theta=2U_1/(\mathcal{E}_+-\mathcal{E}_-)\equiv 2U_1/W. \qquad (A4)$$

The corresponding effective masses $m_\pm^*$, defined as

$$\frac{1}{m_\pm^*}\equiv\frac{1}{\hbar^2}\frac{\partial^2\mathcal{E}_\pm}{\partial k^2},$$

can be obtained as

$$m/m_\pm^*=1\pm 2\hbar^2Q^2U_1^2W^{-3}/m. \qquad (A5)$$

Under the action of an external electric field $\vec{E}$, the interaction Hamiltonian is $\mathcal{H}_{\rm int}=eE\hat{x}$, and $\Psi_\pm$ in Eqs. (A2) and (A3) are the new unperturbed states. Neglecting interband transitions, the action of the electric field has two distinct effects. Firstly it produces a uniform translation of the probability densities in k space. Secondly and most importantly for our purpose, it polarizes the individual electronic wave functions. As an example, we consider the change in the state $\Psi_-(k)$, below the gap, in the presence of the electric field. Treating $\mathcal{H}_{\rm int}$ by first-order perturbation theory, the polarized state Ψ_-' becomes

$$\Psi_-'=\Psi_-+\frac{\langle\Psi_-|eEi\nabla_k|\Psi_+\rangle\Psi_+}{\mathcal{E}_--\mathcal{E}_+}. \qquad (A6)$$

Thus there is an extra charge density $\rho_k(x)$ associated with the matrix element of $eEi\nabla_k$ between the lower and the upper bands:

$$\rho_k(x)=-e[|\Psi_-'|^2-|\Psi_-|^2]=e^2E\left(\frac{2\hbar^2QU_1}{mW^3}\right)\sin Qx. \qquad (A7)$$

This charge density interacts with the crystal potential to produce an additional force F_A on the electron in the state $\vec{k}$, given by

$$F_A=-\frac{1}{e}\frac{\partial U}{\partial x}\rho_k(x), \qquad (A8)$$

with $U\equiv 2U_1\cos Qx$. Averaging this force over one unit cell, we obtain,

$$\langle F_A\rangle=2eE\hbar^2Q^2U_1^2/mW^3 \qquad (A9)$$

and from Eq. (A5),

$$\langle F_A\rangle=-eE(m/m^*-1). \qquad (A10)$$

The condition of electrical neutrality then leads to the Bragg-reflection force F_B on the lattice ion, which balances the force $\langle F_A\rangle$, and is given by

$$F_B=eE(m/m^*-1). \qquad (A11)$$

From above it is clear that *virtual* interband transitions cause polarization of electronic charge densities which, in turn, produce a Bragg-reflection force on the lattice ion through the crystal potential.

The derivation of the Bragg-reflection force in the presence of a magnetic field is similar to the discussion of this Appendix, and the result is given by Eq. (11) of Sec. II of the text. The expression obtained here in Eq. (A11) is the limiting form of Eq. (11) for zero magnetic field.

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