PHYSICAL REVIEW B VOLUME 16, NUMBER 7 1 OCTOBER 1977

Observability of Josephson pair-quasiparticle interference in measurements of flux entry into a superconducting interferometer

R. I. Gayley and T. C. Wang
Department of Physics, State University of New York at Buffalo, Amherst, New York 14260
(Received 19 August 1976)

We show that the pair-quasiparticle interference current in Josephson junctions will probably have a large effect on the amount of flux that will break into a superconducting interferometer. This shows promise as a new more-accurate way to measure this interference current. For this and other reasons, we argue that the calculation of this flux entry should be done more accurately than heretofore.

INTRODUCTION

In this paper, we propose a new approach to measuring the "$\cos\varphi$" term in the tunnel current in a Josephson junction. It is based on measuring the entry of flux into a superconducting interferometer.

For constant voltage, the tunnel current in a Josephson junction was given by Josephson as[1]

$$i = i_c(V,T)\sin\varphi + V\sigma_0(V,T) + V\sigma_1(V,T)\cos\varphi . \quad (1)$$

The first term on the right-hand side represents pair tunneling, $i_c(0,T)$ is the junction critical current, and φ is the superconducting phase difference across the junction. The second term gives the quasiparticle tunnel current, with conductivity $\sigma_0(V,T)$. The third term is referred to as the pair-quasiparticle interference term, or the $\cos\varphi$ term. It reflects the fact that the paired electrons and the quasiparticles are not really independent. Much of the discussion of the behavior of Josephson junctions has assumed that i_c and σ_0 were not functions of V and that σ_1 was negligible. This is reasonable when V is very small compared to the energy-gap voltage $V_g = 2\Delta/e$, where Δ is the half-gap. For larger voltages, these assumptions are invalid.

The function $\sigma_1(V,T)$ has proved very difficult to measure, even though it is not small when $V \cong V_g$. One reason for this is that at such voltages the resulting current oscillates at a very high frequency and tends to average to zero. So far, only a rather rough measurement has been possible in tunnel junctions[2] (we will not consider other types of weak links), and it disagrees with theory.[3] Therefore, a new approach to measuring σ_1 could be of great value.

ANALYSIS

The experiment we have in mind uses a superconducting loop that is interrupted at one point by a Josephson junction. Starting in zero field, a magnetic field, perpendicular to the plane of the loop, is increased slowly until flux abruptly breaks into the loop. The amount of flux now in the loop ϕ_{enter} is the measured quantity.

This experiment was analyzed by Smith and Blackburn[4] by computer simulation. In addition, they reported some measurements on one loop. However, they did not know the capacitance of their junction accurately, so they were not able to make a precise test of their calculation. They assumed the familiar equivalent circuit model for this system, consisting of the loop inductance L, the junction capacitance C, and the quasiparticle tunneling resistance R, all in parallel with an element obeying the relation $i = i_c \sin\varphi$. They took L, C, R, and i_c to be constants.

This system can be thought of as a damped LCR oscillator in parallel with an added element that exhibits Josephson oscillations. Smith and Blackburn were interested in the case in which the LCR oscillator was overdamped. They showed that the simple model predicted that ϕ_{enter} would change in a smooth, predictable way as the damping parameter $\beta \equiv \sqrt{LC}/RC$ was varied. They erred in supposing that their results could be extrapolated to small β, where the LCR oscillator is underdamped. Wang and Gayley[5] showed that in that case the behavior of ϕ_{enter} vs β becomes erratic and very sensitive to noise. In the present paper, we restrict ourselves to larger β, where the behavior is expected to be regular. The dividing point is at $\beta = 2.0$ when the parameter $\gamma \equiv Li_c/\phi_0$ is zero, where ϕ_0 is the flux quantum, and roughly 1.6 for the interesting range of values of γ.

In the larger β regime, we believe that, while Smith and Blackburn's general conclusions are probably valid, their results are not accurate enough for quantitative comparison with experiment. The main reason is that the voltage across the junction will not be small. As Smith and Blackburn point out, one can estimate the maximum voltage across the junction in this experiment to be roughly $[\phi_0/(LC)^{1/2}]\gamma/\beta$. This is just i_cR, which is of the order of V_g. Therefore one cannot assume $V \ll V_g$. One cannot treat i_c and σ_0 as independent of V, and neglecting σ_1 is also invalid. Another problem is that V is not a constant, which means that Eq. (1) does not really apply. During most of the period of flux entry, V will be roughly constant, so this error may not be serious. However, the other errors probably are. In particular, we will show that the $\cos\varphi$ term can have a very large influence, and, for that reason, this experiment may prove very interesting.

To examine this question we have modified Smith and Blackburn's program, which they very kindly gave us a copy of, by adding the term $\sigma_1 V\cos\varphi$ to the current. We treated σ_1 as a constant and used the values $\sigma_1 = -\sigma_0$, 0, and $+\sigma_0$. These values were chosen because $-\sigma_0$ is roughly the measured value[2] and $+\sigma_0$ is about the theoretical value.[3] The results of the simulation for $\gamma = 100$ are shown in Fig. 1. These graphs are still not suitable for quantitative comparison with experiment or for measurement of σ_1 because i_c, σ_0, and σ_1 were taken to be constants. However, they do show that including σ_1 changes the results considerably.

We can understand why the $\cos\varphi$ term does not average out in this experiment by a simple argument. Just prior to the entry of flux into the loop, there is a large energy stored in the loop inductance by virtue of the circulating current that has been induced. Then, entry of flux corresponds to the decay of this circulating current. A large voltage appears across the junction as the flux begins to move rapidly in; the junction becomes lossy; and energy is extracted.

The circulating current need not decay to zero, however. There is a set of quasistationary quantum states, each with a different circulating current, that the system might settle into. As the current decays and flux enters, we can regard the loop as passing continuously from one state to the next. Meanwhile, there is an oscillating interchange of some of the energy between the loop inductance and the junction capacitance, due to Josephson oscillations in the junction. (The LCR oscillator is overdamped but the Josephson oscillations are not.) The capacitive energy, and therefore the voltage, passes through a minimum each time another quantum state is reached. As more and more energy is dissipated, a point eventually will be reached such that this minimum voltage is zero. The system becomes trapped in that quantum state. For very large damping, this occurs while the circulating current is large, and ϕ_{enter} will be small. For smaller damping, the circulating current decays further before trapping occurs, and ϕ_{enter} is larger.

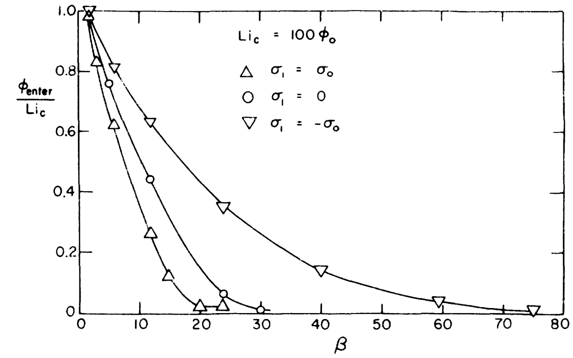

FIG. 1. Results of computer simulation of the flux ϕ_{enter} that will enter a single-junction interferometer, showing the effect of the $\cos\varphi$ term, using the simplified model described in the text. ϕ_{enter} is normalized by dividing by Li_c, which is the maximum flux that can be generated by a supercurrent in the loop. The calculation assumed an initial applied field of magnitude such that it would produce a flux of $100.3\phi_0$ in the loop, where ϕ_0 is the flux quantum. The flux in the loop, just before the entry of the external flux, was taken to be $0.25\phi_0$. These are the appropriate initial conditions for the experiment described in the text, except that $0.05\phi_0$ was added to the applied field to ensure that the external flux would indeed break in. For further discussion of this type of calculation, see Refs. 4 and 5. σ_0 is the coefficient of the pair tunnel current, σ_1 is the coefficient of the $\cos\varphi$ term, and β is the damping parameter $(LC)^{1/2}/RC$.

The above allows us to understand the shape of the $\sigma_1 = 0$ curve in Fig. 1. Now, when we include the pair-quasiparticle interference current, the damping will change and so will ϕ_{enter}. If we use P_i for the power dissipated because of this interference term, then

$$P_i = \sigma_1 V^2 \cos\varphi \,. \tag{2}$$

As we said, V will not be constant because of Josephson oscillations in the junction. It turns out that V will be lowest when $\cos\varphi$ is -1 and largest when $\cos\varphi$ is $+1$. This means that P_i does not average to zero even though $\cos\varphi$ oscillates between plus and minus one. Further, it says that the period near $\cos\varphi = +1$ is more important in determining the average of P_i, $\langle P_i\rangle$, because that is when V is largest. If σ_1 is positive, then P_i is positive in that period, and we expect $\langle P_i\rangle$ to be positive. Then the net effect of the $\cos\varphi$ term will be to increase the damping and reduce ϕ_{enter}. If σ_1 is negative, the reverse is true. This agrees with the results shown in Fig. 1.

CONCLUSION

We believe that the calculation of the flux entering a superconducting interferometer should be done again, using a more complete treatment of the Josephson tunnel current. The arguments presented here indicate that, when that is done, the results will be quite sensitive to the sign and magnitude of the coefficient σ_1 of the pair-quasiparticle interference term. This should form the basis of an accurate method of measuring this quantity.

[1]B. D. Josephson, Phys. Lett. 1, 251 (1962).
[2]N. F. Pederson, T. F. Finnegan, and D. N. Langenberg, Phys. Rev. B 6, 4151 (1972).
[3]For a review of this question and for a discussion of the details of the current in a Josephson tunnel junction, see D. N. Langenberg, Rev. Phys. Appl. 9, 36 (1974); and R. E. Harris, Phys. Rev. B 10, 84 (1974).
[4]H. J. T. Smith and J. A. Blackburn, Phys. Rev. B 12, 940 (1975).
[5]T. C. Wang and R. I. Gayley, Phys. Rev. B (to be published).