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Initial magnetization, remanence, and coercivity of the random anisotropy amorphous ferromagnet

E. Callen*
American University, Washington, D.C. 20016
and Naval Surface Weapons Center, Silver Spring, Maryland 20910

Y. J. Liu*
American University, Washington, D. C. 20016

J. R. Cullen†
Naval Surface Weapons Center, Silver Spring, Maryland 20910

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We analyze the coercivity, remanence, and initial magnetization of a single-domain, single-component amorphous material. We assume a uniform ferromagnetic exchange and concentrate on the effect of single-ion anisotropy. We find a threshold anisotropy below which the coercivity is zero. At large anisotropy, the coercivity is proportional to the anisotropy energy. Exchange pulling increases the remanence of amorphous materials. We suggest that exchange pulling between crystallites may also be operative in small-grain metallic polycrystals.

I. INTRODUCTION

The rare-earth–iron amorphous materials show unusual magnetic behavior. YFe_2 is a spin glass.[1] Alloys of the other rare earths, those with ionic magnetic moments, are ferrimagnetic, although due to nonalignment both the iron and the rare-earth "sublattices" can display average moments below those of the isolated ions.[2] The Curie temperatures are well below those of the crystalline phase.[3] The hysteresis curves show sharp discontinuities at the coercive field.[3-5] The low-temperature initial magnetization curves show what Rhyne *et al.*[3] call an "anomalous `coercive'-force-type spin transition." That is, at low temperatures the magnetization versus applied field curve rises with large positive curvature at fields far beyond the demagnetization field. Not only do experiments show the RFe alloys to be generally ferrimagnetic, but there is also some supportive theoretical evidence.[6] Chi and Alben in a paper which should be viewed as a companion to this one, and one in which the authors consider numerically the same questions we attack analytically, show that for the same Hamiltonian we assume, states of significantly reversed moment and spin glass states are of higher energy than the more aligned states.

The peculiarities we investigate derive from anisotropy not exchange. Therefore for simplicity we assume a single exchange field, the same at all rare earth sites, both in magnitude and direction. The anisotropy, on the other hand, could hardly be so uniform. The material is amorphous, so all directions are equally likely for the uniaxial crystal field easy axis of any particular spin. (The easy plane case will be considered in a separate section later in this paper.) We assume a uniform distribution of easy axes over the unit sphere, but the same magnitude of anisotropy energy, the same coefficient, for all sites. This is consistent with experimental Mössbaur evidence,[7] which shows the magnitude of local, single-ion anisotropy field to be as sharply defined in the amorphous materials as in the crystalline phase, and of about the same size in the two phases.

This model, the random anisotropy model, was first proposed by Harris, Plischke, and Zuckermann[8] and employed by them to explain (albeit not uniquely—the spread-in-exchange model can give the same results) the reduction in Curie temperature and saturation moment of amorphous materials. Harris and Zobin[9] have now used the random anisotropy model to explain the same properties—coercivity and remanence—that we analyze herein. Thus there are now three papers explaining the same observations on the same basic model.[6,9] We shall compare as we go along.

II. MODEL

We treat the problem classically at $T = 0$ in the molecular-field approximation. There is an exchange energy, a Zeeman energy, and a single-ion anisotropy energy. Suppose the magnetization and the external field to be along the z axis. An individual spin, of dipole moment $\vec{\mu}$, makes angles θ with the z axis and ϕ around the axis, measured from the xz plane, and has an easy axis with polar and azimuthal angles Θ and Φ. The magnetization

per unit volume is $\vec{M}$. Because the single-ion anisotropy is assumed to be purely uniaxial, for each spin $\phi = \Phi$. Letting D be the uniaxial crystal-field parameter, the anisotropy energy of a spin of easy axis Θ is $-D\cos^2(\Theta - \theta)$. The Zeeman energy in external field H is $-\mu H\cos\theta$ and the exchange energy of the spin is $-\mu\lambda M\cos\theta$. The energy of this spin is then

$$E = -\mu(H + \lambda M)\cos\theta - D\cos^2(\Theta - \theta)\,. \tag{1}$$

The magnetization density is $M = \mu n m$, with n the number of spins per unit volume and $m = \langle\cos\theta\rangle$. In terms of the reduced parameters

$$h \equiv \mu H/\lambda n\mu^2;\quad d \equiv D/\lambda n\mu^2\,, \tag{2}$$

the energy is

$$E/\lambda n\mu^2 \equiv e = -(h + m)\cos\theta - d\cos^2(\Theta - \theta)\,. \tag{3}$$

Classically the equilibrium position of the spin is at that θ at which its energy is an extremum

$$c\sin\theta = \sin2(\Theta - \theta)\,, \tag{4}$$

with

$$c \equiv (h + m)/d\,. \tag{5}$$

One must check that the energy is not merely an extremum but a minimum

$$e'' > 0\,. \tag{6}$$

The reduced magnetization is of course the spherical average

$$m = \left(\int \cos\theta\,\sin\Theta\,d\Theta\right)\Big/\int \sin\Theta\,d\Theta\,. \tag{7}$$

In this equation we must substitute the function $\theta(\Theta)$ from Eq. (4).

III. REMANENCE AND SUSCEPTIBILITY

It is conceptually nicest to start from the unique state of complete magnetic saturation at infinite field, and then relax h back to zero. At $T = 0$, and ignoring domain effects, what is the remanent magnetization? Since all the spins were pulled to $\theta = 0$ by the external field, when the field is turned off they will all remain in the upper hemisphere. First consider the two limits. At one end, $d = 0$, all spins will lie along the exchange field, and $m = 1$. At the other limit, $d = +\infty$, each spin will lie along its easy axis and $m = 0.5$. At intermediate d the spins will on the average be pulled from their easy axes toward the z axis by the exchange, increasing m. The reader should contrast this with the calculation of the remanence of a polycrystal.[10] In the usual polycrystal average (we shall have more to say about this later) the exchange does not pull the magnetization of each crystallite from its easy axis nor, mutatis mutandis, does the anisotropy reduce the crystallite moment. The exchange plays no part. The polycrystal average is simply the crystallite z components of moment averaged over assumed directions of easy axes.[11]

In the amorphous magnet, when d is small each θ is small, and we find m by iteration from $m = 1$. From Eq. (4),

$$\sin\theta \simeq [d/(h + 1)]\sin2\Theta$$

and

$$\cos\theta \simeq 1 - \tfrac{1}{2}[d/(h + 1)]^2\sin^2 2\Theta\,.$$

Integrating in Eq. (7), we find

$$m \simeq 1 - \tfrac{4}{15}[d/(h + 1)]^2\,. \tag{8}$$

This is good when $d \ll \sqrt{2}\,(h + 1)$. Thus, when $d \ll \sqrt{2}$, the remanence is

$$m(0) \simeq 1 - \tfrac{4}{15}d^2\,, \tag{9}$$

and the reduced susceptibility at remanence is

$$\left.\frac{dm}{dh}\right|_{h=0} \simeq \tfrac{8}{15}d^2 \quad (d \ll \sqrt{2})\,. \tag{10}$$

The same calculation allows us to find the high-field susceptibility, for when h is large θ is also small. From Eq. (8) one finds that at large fields

$$\frac{dm}{dh} \simeq \frac{8}{15}\,\frac{d^2}{h^3} \quad \left(h \gg \frac{d}{\sqrt{2}} \text{ and } h \gg 1\right)\,. \tag{11}$$

Near the other limit, when d is large, each spin is pulled to a θ slightly less than its easy axis Θ in the upper half space

$$\theta \simeq \Theta - [(h + 0.5)/2d]\sin\Theta\,,$$

$$\cos\theta \simeq \cos\Theta + [(h + 0.5)/2d]\sin^2\Theta\,.$$

Therefore,

$$m \simeq 0.5 + (h + 0.5)/3d\,. \tag{12}$$

This is good when $d \gg \frac{1}{3}(2h + 1)$. The remanence is then

$$m(0) \simeq 0.5 + 1/6d \quad (d \gg \tfrac{1}{3})\,, \tag{13}$$

and the susceptibility at remanence is

$$\left.\frac{dm}{dh}\right|_{h=0} \simeq \frac{1}{3d} \quad (d \gg \tfrac{1}{3})\,. \tag{14}$$

Figure 1 shows the remanence as a function of d, for positive d.

Chi and Alben[6] examine the ground state of a cluster of 100 classical spins with uniform ferromagnetic exchange, uniaxial anisotropy, and external field. Their remanence drops below 0.5 when the anisotropy is more than 40 times the exchange energy, and is 0.45 at 500 times. Chi and Alben argue that at very large d (but not infinite)

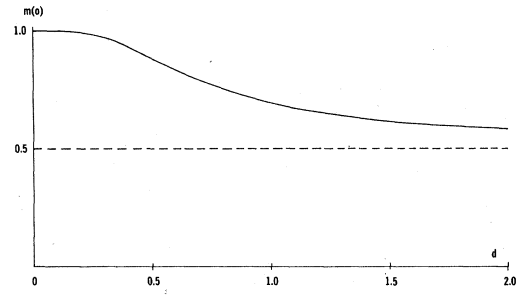


FIG. 1. Reduced remanence as a function of d (d = the ratio of anisotropy energy to exchange energy).

local fluctuations in the exchange will cause an occasional spin to be flopped down into the lower hemisphere as the external field is relaxed back to zero. It is our understanding, however, that this could only happen to spins with easy axes very close to the xy plane. These spins although relatively heavily populated contribute very little to the remanent magnetization. In any case at physically attainable ratios of anisotropy to exchange, we believe that the generalized pulling by the exchange force on all of the spins more than compensates for the occassional flopping of a few spins to the lower hemisphere, and the remanence rise above 0.5, as in Eq. (13).

In real materials, inhomogeneities in the exchange interaction—occassional antiferromagnetic bonds—reduce the moment from alignment, and this cannot be neglected in estimating the remanence. In the RFe_2's, for example, nonsaturation because of antiferromagnetic rare-earth–rare-earth coupling is large.[2]

The analysis of Harris and Zobin[9] is more difficult to comprehend. Those authors improve on molecular-field theory by introducing explicitly another local molecular field which varies from site to site. They claim that for $d > 0.1$ a spin-glass state lies lowest. We do not understand how this can be correct at large d. Yet Harris and Zobin also cite Harris, Plischke, and Zuckermann[8] to show the remanence at $d = 1$ to be 0.9 (we find 0.7). There seem to be some differences in definitions here.

IV. HYSTERESIS LOOP AND COERCIVE FORCE

The state of magnetization depends not only on domains, which we ignore, and on the crystal field and the external field, but upon the past history; except for $m = \pm 1$ there are different ways of arranging individual spins consistent with a particular magnetization.

Suppose one relaxes the spins back to remanence from infinite field. If the anisotropy is zero and one puts on an infinitesimal negative field, the magnetization will switch from $+1$ to -1. [Ultimately for all d, $m(-h)$ joins the line $m = -1 + \frac{4}{15}[d/(h+1)]^2$ of Eq. (8), as h becomes more negative.]

The valid argument that for $d = 0$, m switches to $-m$ at $h = 0$ appears to be in tension with the following: When there is no effective field on the spins they lie along their easy axes in the upper half space (so prepared). In this state their moment is 0.5. There will then be no effective field when $h = -0.5$. Therefore it would seem that all hysteresis curves should pass through the point $h = -0.5, m = 0.5$, irrespective of d. This is clearly not right at $d = 0$, but we shall see that for $d > 1$ all hysteresis curves do indeed go through this point.

Molecular field theory is not a very satisfactory way to calculate the coercive force. If the magnetization were truly uniform there would be no coercive force at any value of d, since the random anisotropy, evenly spread over all directions, would be averaged out to zero, and the only angularly dependent term in the total energy would be the Zeeman term. It is the local irregularities in the magnetization induced by the local anisotropy which allow for coercivity. But molecular-field theory should be a good approximation at large external field and/or large anisotropy field, the single-ion terms in the energy. In the large anisotropy regime we expect the coercive force to be proportional to the anisotropy energy ($H_c \propto D$), and here we shall find that molecular-field theory and geometry gives

$$\mu H_c = 0.964D \quad (d > 5.3), \tag{15}$$

which we expect to be not far off the mark. Chi and Alben, in their Fig. 3, show H_c having a constant value $h_c \simeq 2.2$ for all $d > 5$, at least up to $d = 500$ (at which d we expect $h_c = 482$).

The behavior of the coercivity in the small d regime, where the exchange plays a more important role in averaging out the anisotropy, is more subtle. We defer discussion of these results until after describing the method of analysis.

For each value of c one is to solve Eqs. (4) and (7) for m, starting from the assumed high-field, fully aligned state. A particularly simple case is $c = 1$. Then we have, from Eq. (4), $\theta = \frac{2}{3}\Theta$, and Eq. (7) gives $m = 0.76$. Another simple case is the one discussed previously, $c = 0$. Then $\theta = \Theta$ and $m = 0.5$. A third case is $c = -1$, for which $\theta = 2\Theta$ for $0 \leq \Theta \leq \frac{1}{4}\pi$ and $\theta = \frac{1}{3}(\pi + 2\Theta)$ for $\frac{1}{4}\pi \leq \Theta \leq \frac{1}{2}\pi$, and $m = -0.053$.

At other values of c one must solve Eq. (4) numerically, being careful to consider the stability criterion. For $c > -1$ all spins remain stably in the upper hemisphere, but at $c = -1$ the spins whose easy axes are on the $\Theta = 45°$ cone become unstable. As c is made more negative an annulus symmetrical around 45° widens. The spins with easy

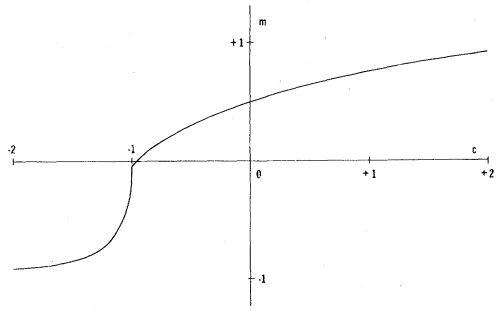


FIG. 2. Reduced magnetization as a function of c, starting from the saturated state (c is the ratio of reduced effective field to d).

axes in this annulus overcome the local anisotropy energy barriers and find lower minima in the lower hemisphere. At each c one calculates the proper θ for each spin easy axis Θ and sums to find the magnetization, $m(c)$. Only when $c=-2$ are all spins flopped down; memory of past history is lost, and reversibility restored. Figure 2 shows $m(c)$.

It is worth noting that this procedure of obtaining $m(c)$ using Eqs. (4) and (7) is identical with that used by Stoner and Wohlfarth in getting the hysteresis curve for their isolated particle model of polycrystals (we will comment on this matter in Sec. VII), the correct loop being given by Chikazumi.[11]

From Fig. 2 we can now, by a geometrical construct, find the self-consistent magnetization. From Eq. (5) we have

$$m = dc - h\,. \tag{16}$$

For each assumed d and applied field h one plots on Fig. 2 a straight line whose m intercept is $-h$ and whose slope is d. The intersections of the straight line with the curve $m(c)$ are possible self-consistent solutions, but not all are thermodynamically stable against small fluctuations in the magnetization. Only those solutions with m a positively increasing function of h can be realized.

Suppose d is small and h positive. The straight line intersects the curve at large $\pm c$ and near $m=\pm 1$, of which the physical solution is of course $m \simeq 1$. As h is decreased the physical solution moves toward smaller m, and toward the remanence. At h equal to zero the interesting solution switches abruptly to the negative m (and negative c) intersection. There is thus no coercive force and no hysteresis for small d.

Now consider another limit—large d. Even for large positive fields the straight lines of very large slope intersect the $m(c)$ curve close to $c=0$ and near $m=0.5$. Clearly the remanence, as d becomes larger, approaches 0.5 (see discussion at end of Sec. III).

At what anisotropy energy does coercivity first appear? This is the d at which a straight line with slope d, going through the origin, tangents the $m(c)$ curve in the third quadrant. It occurs at $d=0.6$. For $d<0.6$ there is no coercivity. A sudden drop in magnetization occurs at $h=0$. At negative fields the proper solution of the three intersections is that of the most negative m. But for $d>0.6$ there is a finite coercivity.

Is there an anisotropy energy above which there is no discontinuity in m? There is not. At $d=5.3$ the straight line tangent to $m(c)$ in the third quadrant intersects the $m(c)$ curve at $m=0$ and $c=-0.964$. For $0.6<d<5.3$ there is a sudden drop in magnetization at the coercive force, obtained from the m intercept of the tangent line. This is observed.[4,5] For more negative fields, the proper solution is again that of the most negative m. For $d>5.3$ the discontinuity in m starts at a small negative m. The discontinuity in m becomes smaller, and the m at which the discontinuity occurs becomes more negative with increasing d, approaching $m=-0.053$ in the limit. For $d>5.3$ the coercivity is that field for which the straight line passes through $m(c)$ at $m=0$, and is therefore given by

$$h_c = 0.964d \quad (d>5.3)\,, \tag{17}$$

which is Eq. (15).

Figure 3 shows the coercivity as a function of d. It should be contrasted with Chi and Alben's result. While we are confident of a proportionality of h_c on d at large d, we do not have confidence in our small-d behavior. The major question is whether a more sophisticated analytical treatment would

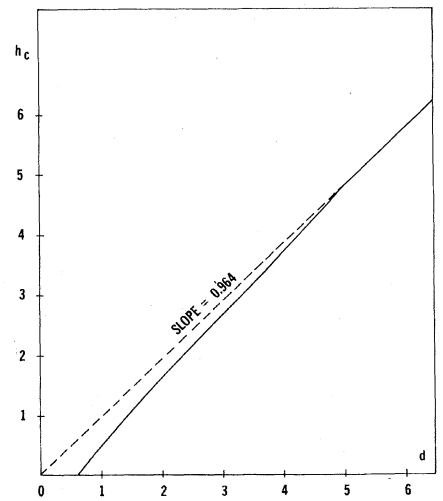
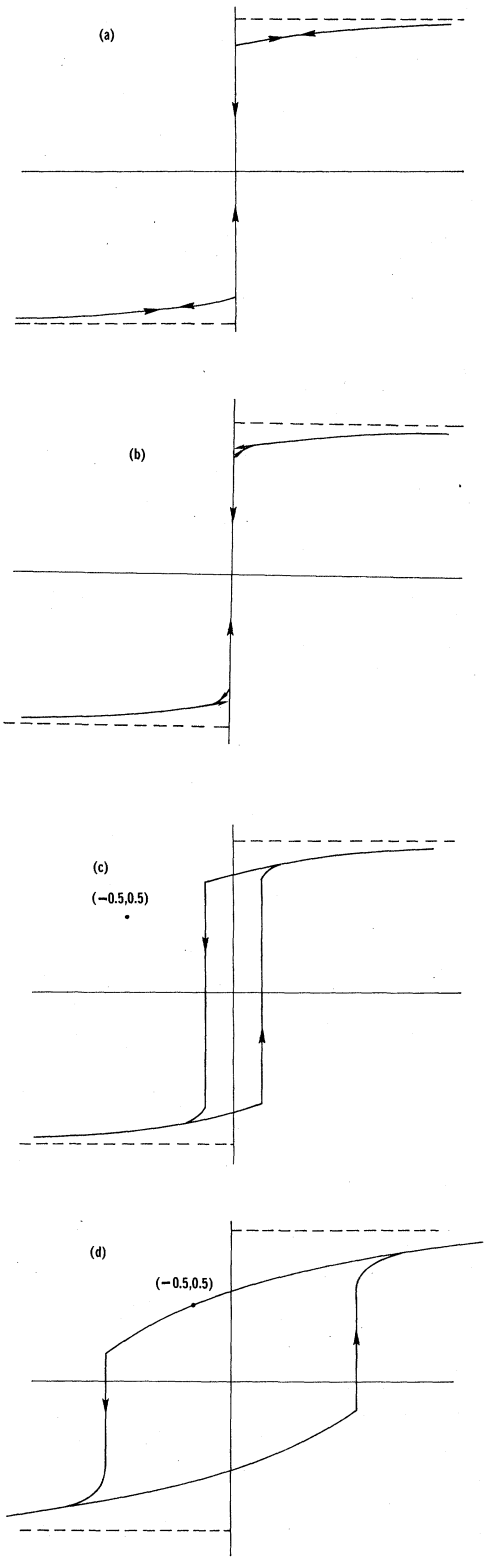


FIG. 3. Reduced coercive force as a function of d.



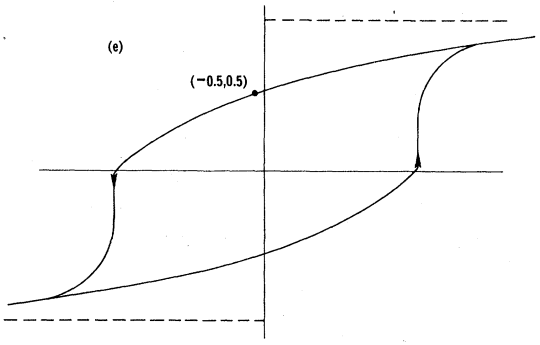


FIG. 4. Characteristic hysteresis loops in various ranges of d: (a) $d<0.46$; (b) $0.46<d<0.6$; (c) $0.6<d<1$; (d) $1<d<5.3$; (e) $d>5.3$.

eliminate a threshold d at which hysteresis first occurs, and recover Chi and Alben's numerical result.

Those authors, and Alben, Budnick, and Cargill,[12] also give an insightful dimensional argument. Following Imry and Ma's[13] analysis of the energetics of domain size, Chi and Alben conclude that at small d the coercivity should vary as d^4 with no small-d cutoff.

Harris and Zobin identify as the coercive force the field at which a discontinuity in m occurs when a field is applied to an initially unmagnetized material. We defer discussion of this phenomenon to our analysis of initial magnetization.

One comment on reversible magnetization. With $d<0.46$ and the material in the remanent state, an infinitesimal negative field reverses $m(0)$ to $-m(0)$ and an infinitesimal positive field switches the magnetization back to $m(0)$ again. But for $d>0.46$ this process is irreversible. Although d is below the coercivity threshold, so that the magnetization switches negative with an infinitesimal negative field, it is reduced in magnitude. Reverting to a small positive field does not restore the original remanence, but something less.

From the foregoing analysis, and using a symmetry argument for the return loop of the magnetization m^*,

$$m^*(-h) = -m(h)\,, \tag{18}$$

we are now in a position to construct hysteresis loops for any value of anisotropy. Figure 4 illustrates characteristic loops in various ranges of the anisotropy parameter.

V. INITIAL MAGNETIZATION

Even without domains there are many ways to achieve zero magnetization. We assume one of these, the uniform distribution of spin vectors in both hemispheres, over the unit sphere. It can be

seen that this state is self-consistent: Since m and h are zero, c is zero, and Eq. (4) has the solution $\theta = \Theta$ for each Θ. Easy axes are uniformly distributed, each spin lies along its easy axis, and the spins sum to zero moment.

Now an external field is turned on. What is the response? One can proceed as before. The distribution of spins is the sum of uniform populations of the upper and lower hemispheres. Therefore the appropriate $m_0(c)$ curve is one half the sum of $m_u(c)$ of Fig. 2 and its inverse m_u^*, with $m_u^*(-c) = -m_u(c)$:

$$m_0(c) = \tfrac{1}{2}[m_u(c) - m_u(-c)] . \tag{19}$$

By a load line analysis paralleling that explained in Sec. IV we conclude that for $d < 0.63$ any small field will flop the unstable spin system into a state of large remanence.

For $d > 0.63$ and for small fields one can again iterate from $m = 0$ and $\theta = \Theta$, more or less as precedes Eq. (12), and find

$$m \simeq h/(3d - 1) \quad (d > 0.63) . \tag{20}$$

But this linear response does not long continue. As a magnetization is induced by the external field an exchange field is built up. Positive feedback leads to a catastrophic collapse of the initial state at a certain field, as in the case of coercive force discontinuity. This field h_j at which a sudden jump in magnetization takes place can be found as a function of d by graphical analysis, using Eqs. (16) and (19). h_j is zero for $d < 0.63$ and rises linearly at large d, much as does the coercivity shown in Fig. 3. Something like this behavior has already been observed,[3] and has been explained by Harris and Zobin,[9] who, however, identify the critical field at which the jump in m occurs as the coercive force. We do not think it is the coercive force.

Ultimately at large fields all spins are at small θ, and the magnetization is again given by Eq. (8) irrespective of the initial state. Figure 5 illustrates the jump in initial magnetization.

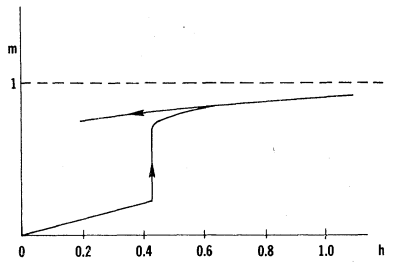


FIG. 5. Initial magnetization process for the case of $d = 1$.

VI. EASY PLANE ANISOTROPY

So far we have discussed only positive d. When d is negative the spins tend to lie not along their unique Θ axes but in the planes perpendicular to them. To discuss this case it is less confusing to take the anisotropy energy to be $+D\cos^2(\text{angle})$ and again consider positive d. Thus, if the hard axis makes angles Θ, Φ, the spin assumes angles $\theta, \Phi + \pi$, with θ given by the minimization of

$$E = -\mu\{H + \lambda M\}\cos\theta + D\cos^2(\theta + \Theta) .$$

We shall not detail the analysis, which parallels the foregoing, but we do calculate the remanence in limiting cases.

When d is small, c is large and θ is small. Then

$$\sin\theta \simeq \sin 2\Theta/c, \quad \cos\theta \simeq 1 - \sin^2 2\Theta/2c^2 ,$$

and

$$m \simeq 1 - \tfrac{4}{15}[d/(h+1)]^2 \quad [d \ll \sqrt{2}\,(h+1)] ; \tag{21}$$

the same as the easy axis case (m is even in d).

When d is large,

$$\theta \simeq \tfrac{1}{2}\pi - \Theta - \tfrac{1}{2}c\cos\Theta ,$$

$$\cos\theta \simeq \sin\Theta + \tfrac{1}{2}c\cos^2\Theta , \tag{22}$$

$$m \simeq \tfrac{1}{4}\pi + \tfrac{1}{6}[(h + \tfrac{1}{4}\pi)/d] \quad [d \gg \tfrac{1}{6}(4h/\pi + 1)] .$$

One might expect that there would be no coercive force when there is an easy plane—the spins would at $h = 0$ simply rotate in that plane to attain maximum projection along the magnetic field direction. We point out, however, that this is almost certainly not the case in an amorphous material. Rare earths with an "easy plane" are those whose charge density is prolate. Their orbital and spin angular momenta both lie along the elongated major axis. In a uniaxial crystal it may be possible to rotate this ellipsoid around in the basal plane. But in an amorphous material there are likely to be other uniaxial fields in the plane—the ellipsoid would "bump" into other ions.

VII. DISCUSSION

For simplicity and specificity we have presented our model as a single-site energy minimization—we have minimized the energy of each spin as a compromise between its individual anisotropy energy and the exchange and Zeeman energies. This will be most appropriate in the large anisotropy limit. If there were no exchange, each spin would lie along its easy axis. If there were no anisotropy, all spins would be parallel. At intermediate ratios of anisotropy to exchange there will be coherent regions. At large d these microdomains will be small but well defined (i.e., thin domain walls between them). The effective anisotropy en-

ergy of region i ($\langle D_i \rangle$) will be a little less than D itself (times the number of spins in the region), because of averaging, and the region magnetization will point close to its dominant easy axis. It is these clusters, rather than single ions, whose behavior we have been analyzing. At small d the exchange will dominate. Coherent regions will not be well defined. The magnetization will rotate smoothly and gradually through the material. Domain walls will be as large as domains. The effective D (per spin) of a "region" will be much less than the single ion D, because of averaging over the many spins in a large region. Alben[6,12] shows that in this limit $\langle D \rangle \sim d^4$. Thus $\langle D \rangle$ is a function of d, going as d^4 at small d and asymptoting to D at large d. In the small-d regime our "single ion" treatment is not very good.

There is good experimental evidence for the existence of microdomains in the amorphous RFe_2 compounds. Small-angle magnetic scattering of neutrons by[14] $TbFe_2$ suggests scattering by clusters of, very roughly, 100 Å diameter. The scattering is strong below the Curie point and diminishes as the temperature of the sample is raised into the paramagnetic state. The materials with microdomain structure are the ones that display large coercive forces and jumps in the initial magnetization. By our treatment they should have $d > 0.63$ and so perhaps there is at least internal self-consistency in applying our model at this reasonably large d.

One wonders how the ratio of anisotropy to exchange energy can be so large—it is usually between 0.01 and 0.1. It may be that at intermediate d, when the regions are significantly larger than the range of the exchange interaction (especially in an amorphous material with short spin-spin longitudinal correlation length), the exchange interaction between regions is a surface effect, not a volume effect, decreasing the operative "J", and thus increasing the effective d.

That the causative agent of the large coercive force and initial magnetization jump is anisotropy and not inhomogeneities in the exchange interaction (i.e., a scattering of antiferromagnetic bonds, with regions pointing in different directions) is demonstrated by the behavior of amorphous $GdFe_2$, which should have much the same exchange interaction inhomogeneities as the other rare earths, but in which the Gd ion has no magnetic anisotropy. $GdFe_2$ displays no microdomain structure and a small coercive force of conventional origin.[15] It should be kept in mind that the important mechanism for coercive force, nucleation of reverse domains, has been neglected here.

Unlike the coercivity, the remanence will be affected in a major way by nonsaturation due to random antiferromagnetic exchange interactions. This should be taken into account[5] before results on the remanence are compared to experiment.

In comparing to the experimental evidence of the amorphous rare-earth–iron materials, we have gone even further than assuming a uniform ferromagnetic exchange field. In these materials there is not one magnetic component, but two. The rare earths tend to point in one direction, the irons opposite, but, depending on concentration, neither "sublattice" is saturated.[2] The first effect of an external field will then be to increase saturation of the magnetically dominant sublattice, at least. And at larger fields one can create angled structures.[16] Nevertheless, in taking magnetization data on RFe materials we do observe[3-5] curves reminiscent of Figs. 4 and 5, although the observed curves tend to be a little less sharply angular. This may be the result of temperature averaging. The theory needs to be repeated for nonzero temperatures. It will be useful to estimate how sensitive the technologically important energy product is to temperatures.

We feel that some form of the theory should be applicable not only to amorphous materials but to some polycrystals as well. In the usual analysis of polycrystal data one finds the direction of the magnetization in each crystallite as a minimum energy compromise between anisotropy energy and Zeeman energy—the crystallite magnetization is assumed constant in magnitude, and there is thought to be no exchange pulling of one crystallite magnetization on that of its neighbors. This is surely the case for nonconductors where the exchange interaction is of short range, and for large grain metallic samples when the range of exchange interaction is small compared to the crystallite size. But there should be circumstances, particularly in metals, and in samples of small grain size, when the magnetic interaction between grains cannot be ignored.

We think we see evidence of this. Permalloy ($Ni_{80}Fe_{20}$) films generally have an in-plane uniaxial anisotropy for one reason or another. Such films exhibit rather square hysteresis loops in the easy direction, although the coercive force is small (~2 Oe). Permalloy films have also been grown in the presence of a 50–400-Hz rotating magnetic field, which wipes out the preferred axis and produces films that are macroscopically magnetically isotropic. These films are determined to be polycrystalline with grain size estimated to be 400 Å. This is small enough for the grains to be of single domain, and thus the simplest analysis of polycrystals should be appropriate. The procedure of Stoner and Wohlfarth can be exactly followed in the case of two dimensions, with only the weighting

factors adjusted in the grain summation. One might then expect a hysteresis curve similar to that shown in Fig. 14.10 of Chikazumi, with a remanence of $2/\pi$ or 0.637. This is, however, not what is observed on a piece of isotropic Permalloy film[17]—the remanence is close to the saturation moment.

Taking into account magnetic interaction between grains of this size, we treat the polycrystal as a random anisotropy amorphous material, and the analysis in two dimensions similar to that of Secs. II–IV of this paper is applied. With a typical single-grain anisotropy field of 4 Oe and a strong and long-range exchange interactions by conduction electrons, d is expected to be very small, so that our simple analysis is not quite appropriate. But we can at least understand why the remanence is close to the high-field magnetization. Our theory, in two dimensions again, predicts zero coercivity at small d. The small coercivity observed (~0.5 Oe) could be due to film inhomogeneities and/or other local fluctuations, or may show that some more sophisticated analysis, along the lines being pursued by Alben or by Harris *et al.* is needed.

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