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High-field magnetoresistance of hopping transport in the disordered impurity system of transmutation-doped Ge

M. L. Knotek
Sandia Laboratories, Albuquerque, New Mexico 87115
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The magnetoresistance behavior for impurity conduction of a series of transmutation-doped Ge samples (majority impurity Ga, compensation $K = 0.4$) with Ga impurity density N_A ranging from 3×10^{15} cm^{-3} to 5×10^{17} cm^{-3} up to fields of 110 kOe is presented. The resistivity approximates the form $\rho = \rho_0 \exp(\epsilon_3/kT)$. At low densities ($N_A \lesssim 2 \times 10^{15}$ cm^{-3}) the pre-exponent obeys $\ln\rho_0(H) = \ln\rho_0(H=0) + CH^2 N_A^{-1}$ as Mikoshiba's theory predicts, and ϵ_3 is constant. At moderate densities, both ϵ_3 and p_0 increase with field, with the effect on ρ_0 being larger than expected from the simple theory, but with a weaker functional dependence on H and N_A. The ratios of transverse to longitudinal effects are consistent with percolation theory. Contrary to the results of Gadzhiev and Shlimak, we see a smooth decrease of $\rho_0(H)$ with N_A throughout the moderate-density region. At high densities, a metal-to-semiconductor transition is induced which appears to be an Anderson transition. The data are discussed in terms of the impurity polarization model of Mott and Davis, the Anderson delocalization model of Shklovskii and Shlimak, and the many-electron hopping model of Knotek and Pollak. The data indicate that while the former two models may contribute at the higher densities, the latter model more reasonably accounts for the lower-density behavior.

I. INTRODUCTION

In doped semiconductors the transport at low temperatures is through impurity bands which are formed by the hydrogen-like impurity orbitals centered on each impurity site. This has been called impurity conduction. The impurities are situated at random and there is a random potential due to the presence of charged compensating minority ions. Transmutation-doped Ge has fixed compensation and a truly random distribution of impurities and hence is an ideal model for studying transport in a disordered system. At low impurity densities, impurity conduction is well described in terms of classical hopping theory. At moderate to high impurity densities, there are marked departures from the classical behavior. Three physically distinct models involving, respectively, dielectric enhancement due to impurity polarization, Anderson delocalization, and many-electron correlated hopping have been advanced to describe the transport in the moderate- to high-density regime. It is the purpose of this paper to test these models against the results of magnetoresistance measurements in the moderate- to high-density regimes.

In this paper we first review in some detail the development of the theory and the supporting empirical evidence. We then present results of both transverse and longitudinal magnetoresistance measurements on a series of well-characterized transmutation-doped Ge samples in the moderate to high impurity density regimes as a function of temperature and field (to 110 kOe). We stress in particular the variation of the activation energy and the pre-exponential of the resistivity with density and field. The data are discussed qualitatively in terms of the applicable models with the conclusion that correlated many-electron hopping is most likely the dominant contributor in the transition region of densities, but that at the higher densities all of the proposed mechanisms could contribute.

II. REVIEW OF PREVIOUS WORK

Impurity conduction is the transport of charge between the impurity levels in a crystalline semiconductor. At low to moderate impurity densities the random impurity array comprises a Mott-Hubbard insulator,[1] i.e., the upper and lower Hubbard bands are separated in energy and the transport in the upper Hubbard band exhibits the activation energy, ϵ_2. At $T = 0$ the lower band is full and the upper band empty and the system is described in terms of singly occupied impurity orbitals. In order to achieve transport in the lower Hubbard band without first having to promote a carrier to the upper Hubbard band, compensating minority impurities are added to move the Fermi level into the lower band. The system is then describable in terms of the charged majority ions moving by the hopping of electrons in the random potential of the charged minority ions as first proposed by Mott[1] and Conwell.[2] In the most simplistic picture the dc charge transport ought to exhibit an activation energy which is roughly equal to the Coulomb interaction between a majority and a minority ion, $E_{\text{Coul.}} = e^2/K_0 r_A$ where K_0 is the dielectric constant of the host lattice and r_A is the average intermajority distance (interacceptor distance in p-type materials such as we are concerned with here) and $r_A = (\frac{4}{3}\pi N_A)^{1/3}$

where N_A is the acceptor density. Anderson's[3] concept that the wave functions in the majority impurity band can become localized due to the random potential set up by the minority impurity ions is crucial to the understanding of this problem. In this paper, we will assume unless otherwise noted that the entire lower Hubbard band is localized and that the eigenfunctions of the system are adequately described by a single electron residing on a single impurity center. This assumption is implicit in the model of Miller and Abrahams[4] who first treated the problem in its entirety using a phonon-assisted tunneling model. Miller and Abrahams (MA) reduce the problem to the solution of an impedance network where each site, i, in the system is connected to every other site, j, by an impedance, Z_{ij}, which can be written

$$Z_{ij} \propto e^{2\alpha R_{ij} + \epsilon_{ij}/kT},$$

where $\alpha = 1/a$, a being the Bohr radius of the hydrogenic impurity oribtal, R_{ij} the spatial separation of sites i and j, and ϵ_{ij} an energy term related to the random potential of the system, arising from the occupation statistics of both the electrons and the phonons involved in the phonon-assisted tunneling (hopping) process. There are three separate calculations which must be carried out in proceeding from MA's microscopic model to the determination of the bulk properties. First the Fermi level, E_F, must be determined, second, the distribution of energies for excitations which lead to conduction is determined, and finally, the impedance network is solved. MA treated the first problem by assuming an energy distribution for majority impurities given by the Coulomb interaction with nearest-neighbor minority ions and then solving for E_F with this distribution. This is seen to be nearly correct in that a given majority ion at $T = 0$ will see only its nearest-neighbor minority plus an additional background of neutral majority-minority ion pairs, the potential of which averages to zero. Miller and Abrahams solve for an average rather than a true Fermi level, however, since in the nearest-neighbor distribution there are of necessity configurations with arbitrarily large first nearest-neighbor separation. We must have $E_F = 0$ for the true value of E_F in such a distribution. This problem is correctly solved by Shklovskii and Efros[5] (SE) who consider the possibility that some minorities may have two majority ions in their vicinity (2 complexes) and an equal number will have none (0 complexes). They show that there is a net gain of energy for the system due to the presence of these complexes and solve for E_F, relative to the energy of an isolated impurity which is defined as $E = 0$, finding

$$E_F = 1.6(e^2/K_0 r_A) \quad \text{when } K \ll 1. \tag{1}$$

Miller and Abraham's calculation has 2.6 in place of SE's 1.6.

In determining the excitation spectrum MA once again use the nearest-neighbor minority-ion Coulomb distribution which peaks well away from $E = 0$. This is clearly incorrect. As discussed by Pollak[6] and Knotek and Pollak,[7] if a majority ion is excited away from its minority center, it will move in an atmosphere of neutral majority-minority ion pairs which have a potential distribution which must peak at $E = 0$. This produces a so-called Coulomb gap in the one-electron excitation spectrum and at very low K, this gap is equal to E_F as discussed in Refs. 5–7. As K is increased, the distribution of states at $E = 0$ is broadened and percolation at lower energies is possible. Hence increasing K at low K decreases ϵ_3, as is commonly observed.[8] In addition, the presence of the 0 and 2 complexes and their resultant potential fluctuations has some effect on ϵ_3, as discussed in Ref. 5.

Finally, MA solve the impedance network problem by essentially assuming that at each site in the path followed through the solid the carrier takes the first smallest impedance in proceeding to its succeeding site. If this formula is followed rigorously, one must encounter some arbitrarily large first smallest impedances for any infinite sample. Thus any bulk resistivity so calculated will be infinite. The correct solution of this problem lies in the use of percolation theory which considers the total impedance of the system by finding the critical impedance Z_c such that an arbitrarily long path may be found through the sample in which no intersite impedance greater than Z_c is encountered. Due to the exponential distribution of Z's, the bulk impedance is then merely proportional to Z_c. When a Coulomb gap is present, the percolation problem involves only the intersite separations R_{ij} (the so-called R-percolation problem). This has been treated by Pollak,[9] Shklovskii and Efros,[10] and Seager and Pike[11] who find

$$\rho \propto e^{c(2\alpha r_A)}, \tag{2}$$

with $c = 1.2$, $c = 1.2$, and $c = 1.4$, respectively, for the three calculations. Thus we expect to find a resistivity of the form

$$\rho = \gamma \exp[1.4(2\alpha r_A) + \beta\theta e^2/K_0 r_A], \tag{3}$$

$$= \rho_0 \exp(\beta\epsilon_3), \tag{3a}$$

where $\beta = 1/kT$ and $\theta = 1.6$ at $K \approx 0$ and decreases as K increases. This form is seen to be widely obeyed for low-density samples, especially for small K. An orbital size can be deduced from the $\ln\rho$ vs r_A curves, and for the Ga-doped Ge, we deduce from the data of Fritzsche and Cuevas[12] $a = 75$ Å using

Seager and Pike's value of c. This value of a is in good agreement with the value deduced by Golin[13] from ac conductivity measurements on p Ge.

The functional dependence of ϵ_3 on r_A, which by Eq. (3) is given by $\epsilon_3 = \theta e^2/K_0 r_A$, is also found to be correct, and at small K, the agreement with this form is adequate, e.g., the data of Fritzche[14] on n Ge follows this form to densities of $\sim 10^{16}$ cm^{-3}. In the samples studied in this paper an r_A^{-1} behavior of ϵ_3 is followed to $\sim 2\times 10^{15}$ cm^{-3}.[12,15]

At densities above the low-density regime, which we define as that range of densities where Eq. (3) is followed, a systematic departure from Eq. (3) occurs. As is widely observed,[8,12,14,16] ϵ_3 falls below the r_A^{-1} dependence and eventually *decreases* with increasing density (see our Fig. 7). Simultaneously, as pointed out by Pollak and Knotek[17] there is a break in ρ_0 to values larger than that obtained by extrapolation from low densities. We denote the density at which the peak in ϵ_3 occurs as N_p. There have been three suggestions put forward to explain this behavior. Mott and Davis[18] suggest that intrasite polarization of impurity levels leads to a dielectric constant κ significantly larger than that of the host lattice, K_0, which reduces ϵ_3. However, this larger value of κ should also be observable in the ϵ_1 (excitation to valence-band conduction) and ϵ_2 mechanisms. In general, breaks in ϵ_2 and ϵ_1 do not accompany the break in ϵ_3. Also the break in ρ_0 does not follow from this model. A second suggestion by Shklovskii and Shlimak[19] is that at concentrations above N_p, Anderson delocalization starts to occur in the peak of states at $E = 0$. As the concentration increases, the delocalization region broadens and the activation energy from the Fermi level to the edge of the delocalized states lessens. However, if the Anderson delocalization is operative, ρ_0 should *drop* to a value no greater than the maximum metallic resistivity[20] since (by postulate) transport occurs in delocalized states. The break to higher values of ρ_0 above N_p is difficult to explain with such a model as Pollak and Knotek[17] point out.

The third suggestion made by Pollak[21] and Knotek and Pollak[22] is that the decrease in ϵ_3 is caused by correlated multi-electron hopping. The salient feature of this mechanism is the possibility of reducing the activation energy by correlated transitions. As Knotek and Pollak[23] explain, there are two distinct regimes in density of Eq. (3). In the low-density regime, the process is matrix-element limited, i.e., the first exponential term is the larger of the two and Eq. (3) should be followed. In the high-density regime, the one-electron process is energy limited, i.e., the second term in Eq. (3) dominates. The crossover between the two regimes occurs at $r_A = r_c$, where r_c is given by

$$r_c = (\beta e^2\theta/2\alpha K_0)^{1/2} = (ae^2\theta/2kTK_0)^{1/2}. \tag{4}$$

In the high-density regime, multi-electron processes which lower ϵ_3 at the expense of increasing the first term in Eq. (3) (a smaller quantity) will dominate the transport. In such a process, the pre-exponential of the transition rate includes the factor $\exp(-2\sum \alpha r)$ where the sum is over the hopping distances, r, of the electrons which hop simultaneously. Thus the pre-exponential of the resistivity must increase when in the correlated hopping regime, in comparison with the one-electron hopping regime.

Mikoshiba and Gonda[24] and Mikoshiba[25] treated the magnetoresistance in impurity conduction in the context of Miller and Abrahams' theory by considering the effect of a magnetic field on the microscopic transition rates. Primarily the transition rates transverse to the magnetic field become exponentially smaller than longitudinal transitions. There are two contributions to this effect arising from (i) orbital shrinkage transverse to the field, and (ii) magnetic-field-induced phase shifts between neighboring orbitals. The latter effect was first pointed out by Holstein[26] and Miller.[27] Mikoshiba's microscopic picture was treated in terms of percolation theory using scaling arguments by Shklovskii.[28] He finds

$$\rho_0(H)/\rho_0(0) = \exp[t(a/\lambda)(r_A/\lambda)^3], \tag{5}$$

where $t = 0.17$, and λ is the magnetic length

$$\lambda = (c\hbar/eH)^{1/2}. \tag{5a}$$

In the case where unaltered percolation paths were treated, $t = 0.255$, and in Mikoshiba's microscopic model, $t = 0.35$. In n-type Ge, this form is followed quite well although the effect is highly anisotropic due to the orbital anisotropy. The value of a deduced from magnetoresistance is very close to that deduced from the density dependence and Eq. (3).[29] However, in p Ge, although the field dependence [i.e., $\ln\rho_0(H)/\rho_0(0) \propto H^2$] is obeyed at low densities, the value of a deduced from Eq. (5) is considerably smaller than that deduced from Eq. (3). Gadzhiev and Shlimak[15] find $t = 0.07$ for their lowest-density p Ge sample. This discrepancy is not understood.

Shklovskii[10] also points out that although the transition rates transverse to the field are exponentially smaller than the longitudinal transitions, the transverse to longitudinal ratio of the magnetoresistance should differ only in the pre-exponential. This is because both processes make transitions predominantly at low angles to the field, but in the transverse case a meandering path is taken which involves more steps and hence a pre-exponential increase over the longitudinal case as

has been observed by Sladek.[30]

The simple theories like MA's do not predict an effect on ϵ_3 due to a magnetic field since in the simple theories ϵ_3 is a function only of the impurity distribution which is not affected by a magnetic field. Chroboczek *et al.*[31] observed that in some samples large increases in ϵ_3 could be induced at high fields in n Ge and subsequently Gadzhiev and Shlimak[15] observed the same effect on samples of p Ge similar to those studied here. Gadzhiev and Shlimak found that ϵ_3 increases with field in the moderate- and high-density regimes ($N_A > N_p$) where ϵ_3 does not follow the form of Eq. (3). For samples with $N_A < N_p$, ϵ_3 is not a function of field. A similar effect was observed by Shklovskii and Shlimak[19] in the piezoresistance in n Ge. For $N_A > N_p$, ϵ_3 increased drastically under uniaxial stress while for $N_A < N_p$, no increase was seen. Pollak[32] saw the opposite effect in p Ge, i.e., ϵ_3 decreased above N_p. This is because the acceptor levels in Ge *expand* with uniaxial stress while donor levels *contract*. This points out that the effect on ϵ_3 is due to orbital-size variation and not due to, say, the electron spin interacting with the magnetic field.

Gadzhiev and Shlimak interpret their data in terms of Shklovskii and Shlimak's model for the dropoff in ϵ_3. It is the purpose of this paper to extend the measurements of Gadzhiev and Shlimak to higher densities and higher fields, as well as to the transverse case, both to query their interpretation and to include interpretation in terms of the other two models for ϵ_3[7,18] in that density range.

III. EXPERIMENTAL DETAILS

The experiments were carried out on a series of transmutation-doped Ge samples, the preparation of which is described by Fritzsche and Cuevas.[12] Transmutation doping with low-energy neutrons produces Ga acceptor impurities of density N_A with As and Se compensating donor impurities of effective density N_D with a fixed compensation $K = N_D/N_A = 0.4$. Thus it is possible to vary impurity density while knowing with a certainty that the compensation, K, is fixed and that the impurities are truly randomly distributed. The impurity density is determined by Hall effect and conductivity measurements. These experiments were carried out on a series of 12 samples, the acceptor densities of which are listed in Table I.

TABLE I. Transmutation-doped Ge, $K = 0.4$.

Sample	Acceptor density N_A
1	3.56×10^{15}
2	5.11×10^{15}
3	6.9×10^{15}
4	1.8×10^{16}
5	2.42×10^{16}
6	2.94×10^{16}
7	3.54×10^{16}
8	4.9×10^{16}
9	7.2×10^{16}
10	1.05×10^{17}
11	1.5×10^{17}
12	5.0×10^{17}

Contact to the samples was made using Cerroseal solder and zinc chloride flux. Samples were bar shaped with dimensions 1–2 mm×1–2 mm×1–2 cm. Current contacts covered the ends, while the voltage probe contacts for four-probe measurements were points 3–5 mm apart centered on the sample. Standard four-probe techniques were used with applied electric fields ranging from 10 to 100 mV/cm. The samples were immersed in liquid He and the temperature was varied by pumping on the He. The temperature was determined from the He vapor pressure. Measurements were made from 4.0 to 1.6 K with data taken only on cooling to avoid stratification in the He above the λ point.

Two different superconducting magnets were used, one capable of 110 kOe and the other 120 kOe. The fields quoted herein are accurate to within ±1%. Bore diameter (2 cm) allowed both transverse and longitudinal measurements to be made on samples up to ~1.9 cm long.

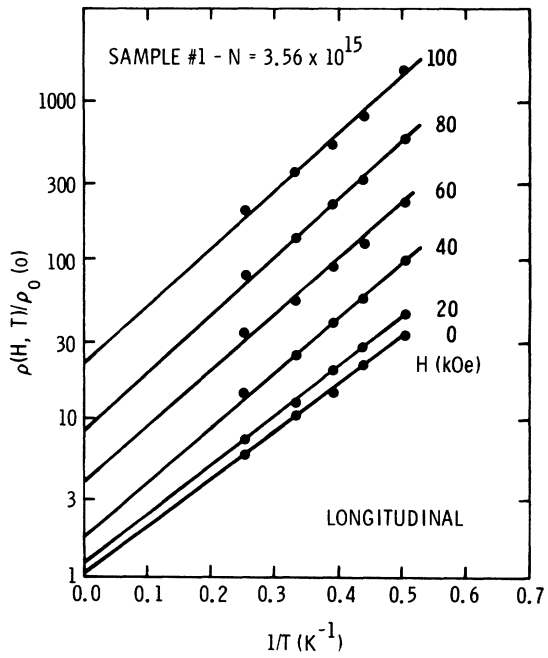

FIG. 1. Normalized resistivity $\rho(H,T)/\rho_0(0)$ vs $1/T$ as a function of field H (kOe) in the longitudinal case, $\vec{H} \parallel \vec{I}$, for Sample 1, $N = 3.56\times10^{15}$. On a longitudinal scale $\rho(H,T) = \rho_0(H)\exp(\epsilon_3/kT)$.

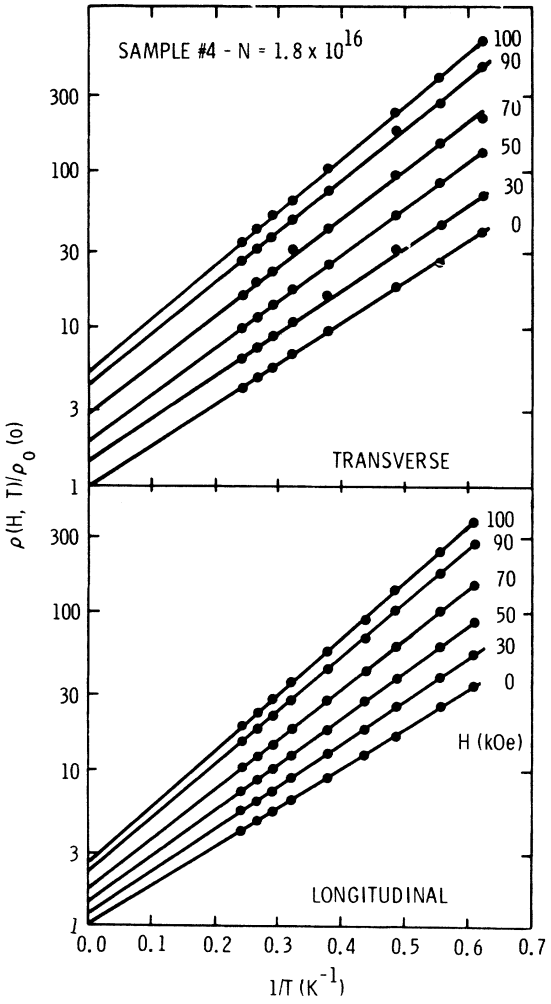

FIG. 2. Same as Fig. 1 for both transverse ($\vec{H}\perp\vec{I}$) and longitudinal ($\vec{H}\parallel\vec{I}$) cases for Sample 4, $N=1.8\times10^{16}$. Note that there are magnetically induced shifts of both the pre-exponent and the activation energy with the activation energy shifts being orientation independent.

IV. RESULTS

The results for the resistivity vs temperature for most of these samples at $H=0$ have been reported in the paper by Fritzsche and Cuevas.[12] We assume for all samples that the resistivity can be parameterized in the form

$$\rho(H,T)=\rho_0(H)\,e^{\epsilon_3(H)/kT}. \tag{7}$$

Shown in Figs. 1–6 are the resistivity $\rho(H,T)$ normalized to $\rho_0(0)$ vs T^{-1} as a function of the magnetic field H for Samples 1, 4, 7, 9, 11, and 12, respectively. For the remainder of the paper we set $N\equiv N_A$ for simplicity and all values are in cm^{-3}. In Sample 1, ϵ_3 increases with H up to 60 kOe and then saturates at ~0.75 meV which agrees well with

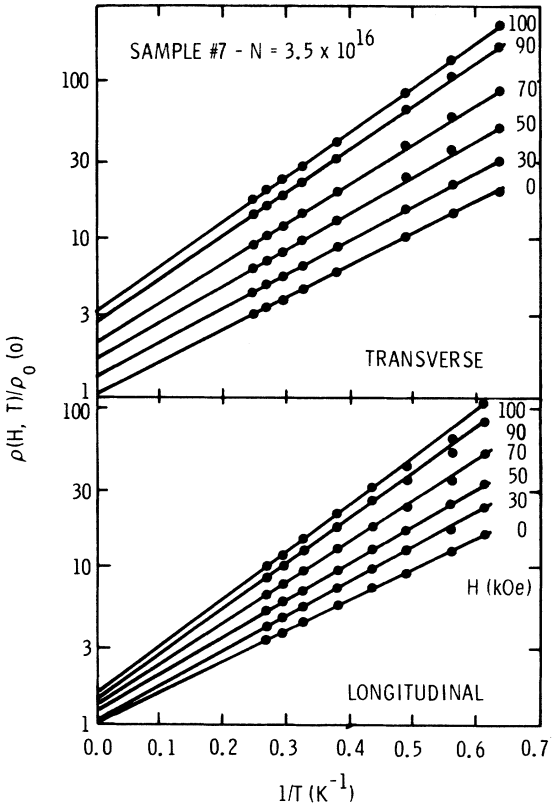

FIG. 3. Same as Fig. 2 for Sample 7, $N=3.5\times10^{16}$.

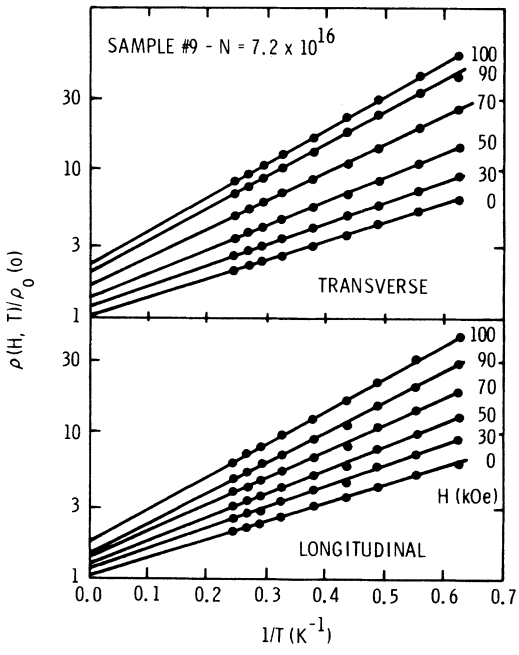

FIG. 4. Same as Fig. 2, for Sample 9, $N=7.2\times10^{16}$.

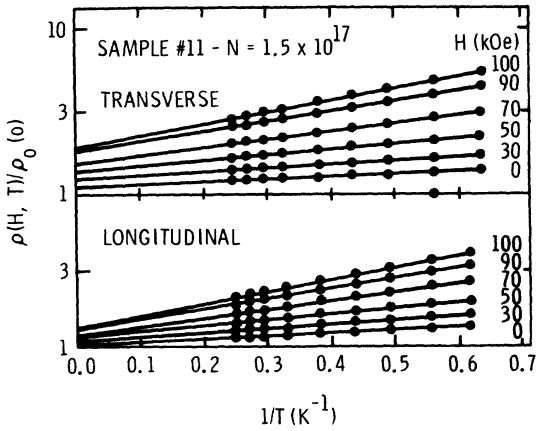

FIG. 5. Same as Fig. 2, for Sample 11, $N = 1.5\times 10^{17}$.

GS's value at 55 kOe for a similar sample. Figure 7, which plots ϵ_3 vs $N^{1/3}$ ($N^{1/3}$ is proportional to $1/r_A$, where r_A is the average interimpurity separation) shows that the ϵ_3 has increased up to the value predicted by the simple theory but does not increase beyond that point. Gadzhiev and Shlimak's value of ϵ_3 for $N = 1.2\times 10^{15}$, which is H independent, is plotted in Fig. 7 for comparison.

Our values of $\rho_0(0)$ and $\epsilon_3(0)$ are very close to those of Fritzsche and Cuevas (FC) except for the lowest-density samples where our values of $\epsilon_3(0)$ (see Fig. 7) are somewhat smaller than theirs (FC's ϵ_3 for Sample 1 is plotted for reference). This may be due to the fact that our temperature range did not extend as low as theirs, and the plateaus evident in their data at the higher temperatures have a stronger influence on our data.

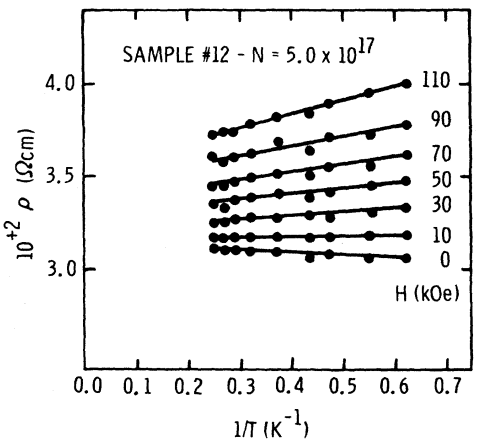

FIG. 6. Resistivity, ρ, and conductivity, σ, vs $1/T(K^{-1})$ as a function of H (kOe) in the longitudinal case $H \| I$ for Sample 12. $N = 5.0\times 10^{17}$. The resistivity is metallic at $H = 0$ and becomes semiconducting at $H = 30$ kOe. The minimum metallic conductivity is $3.16\times 10^{1}(\Omega\text{-cm})^{-1}$ (at 10 kOe).

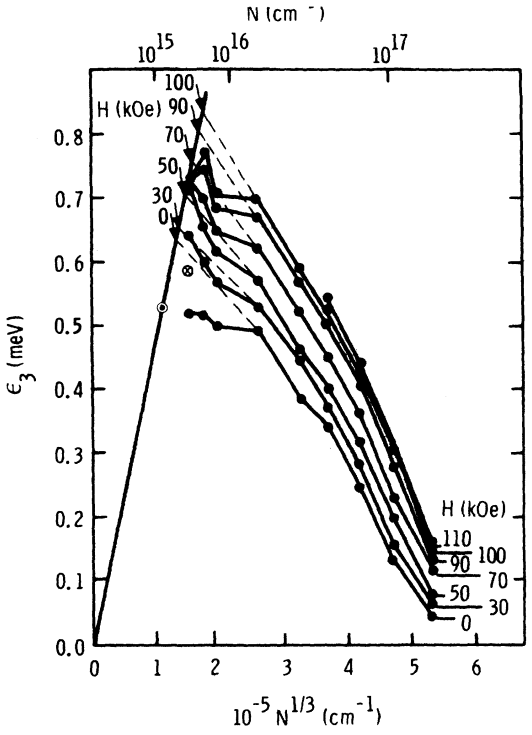

FIG. 7. Activation energy, ϵ_3 (meV) vs $N^{1/3}$ (where $r_A^{-1} = 1.6\,N^{1/3}$) as a function of field H (kOe). The straight line shows the dependence expected from Eq. (3). The arrows are the calculated transition points for the onset of correlated hopping from Eq. (8). The smoothly varying curves at higher density have been extrapolated by dashed lines to denote the upper limit on ϵ_3 expected for correlated hopping. ⊗ From Fritzsche and Cuevas (Ref. 12) for $N = 3.5\times 10^{15}$. ⊙ from Gadzhiev and Shlimak (Ref. 15) for $N = 1.2\times 10^{15}$.

Gadzhiev and Shlimak[14] (GS) also report similar data on samples with N ranging from 1.2 to 35×10^{15} and the same general agreement at $H = 0$.

Samples 4, 7, 9, and 11 (Figs. 2–5) all have several common features. Figures 2–5 show both transverse and longitudinal values of $\rho(H,T)/\rho_0(0)$ vs $1/T$. The ϵ_3 vs H behavior is not a function of sample orientation to within our experimental accuracy. Hence the mechanism for the reduction in ϵ_3 in the moderate-density regime must also be independent of the direction of H. The magnitude of the increase in ϵ_3 with field gradually decreases as the density of impurities is increased. This is shown clearly in Fig. 8 which plots ϵ_3 vs H for most of the samples studied.

The pre-exponential shifts are larger in the transverse than in the longitudinal directions. The ratio of transverse to longitudinal effects increases as the magnetoresistance effect increases. Figures 9 and 10 plot $\rho_0(H)/\rho_0(0)$ vs H for the longitudinal and transverse measurements, respec-

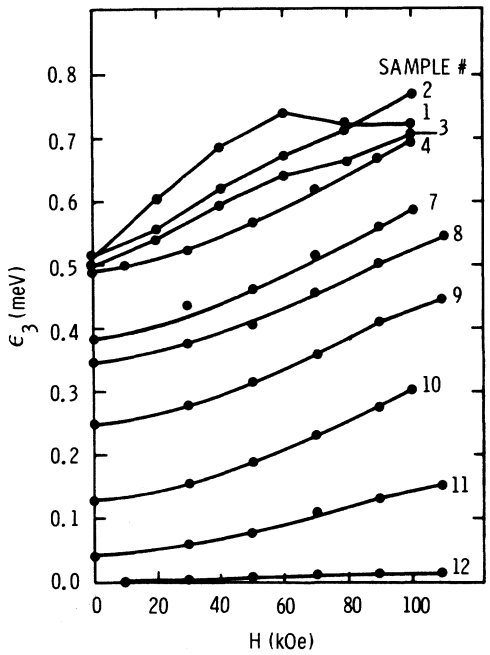

FIG. 8. Activation energy, ϵ_3 vs H for data of Fig. 7 with Sample 12 added. Note that ϵ_3 saturates at $H=60$ kOe for the lowest concentration (Sample 1). The effect gradually decreases as density is increased.

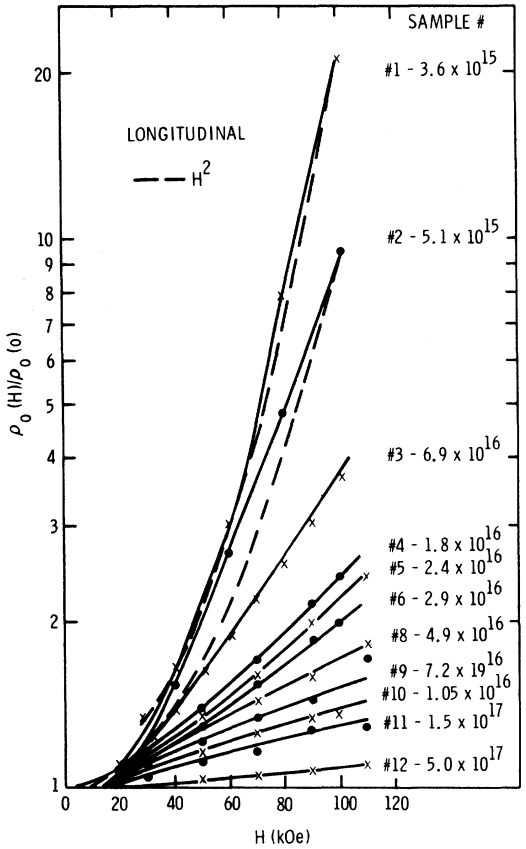

FIG. 9. $\rho_0(H)/\rho_0(0)$ vs H for the longitudinal case, $\vec{H}\|\vec{I}$, as a function of density. The H^2 dependence expected from Mikoshiba's theory are shown as dashed curves for comparison.

tively. The transverse to longitudinal ratio for the two lowest densities is quite a bit larger than expected compared to our high-density data, an effect which we cannot explain. Only the lowest-density samples approximate an H^2 dependence of the exponent. Two H^2 curves, normalized to the highest field point for the two lowest densities (dashed lines), have been plotted for reference. For all densities the magnitude of $\rho_0(H)/\rho_0(0)$ decreases monotonically as the density is increased for both longitudinal and transverse results. This differs markedly from the results of GS, who report that for densities greater than $\sim 9\times10^{15}$ the effect *increases* with density. This is clearly not the case for our data and we note that our ranges of density and field are considerably larger than GS.

Figures 11 and 12 plot $\rho_0(H)/\rho_0(0)$ vs $N^{-1/3}$. Mikoshiba's theory predicts an R^3 (or $1/N$) dependence to the curves, and at the highest fields this is obeyed for the lowest-density samples in the longitudinal case as the $1/N$ curves demonstrate. In the transverse case, we could not make measurements to the highest fields for the lowest-density samples due to the high impedance. We note that in all cases the magnetoresistance effect is *larger* at high densities than extrapolation from low density would predict, i.e., at high densities the data lie *above* both the H^2 and $1/N$ curves which nominally fit the data at low densities and high fields.

The longitudinal curves (Fig. 11) show a distinct inflection point at $N^{-1/3}\approx(2-3)\times10^6$ cm to lower values of $\rho_0(H)/\rho_0(0)$ as density increases which is not present in the transverse case of Fig. 12. Note, however, that when one looks only at the *magnitude* of $\rho_0(H)/\rho_0(0)$ the transverse and longitudinal curves exhibit roughly the same functional form, e.g., for $H\geq 50$ kOe and $\rho_0(H)/\rho_0(0)>1.3$, both transverse and longitudinal effects are superlinear in $N^{-1/3}$. The difference at lower fields may indicate either that there is more than one contributing mechanism and the transverse field essentially quenches one of them out or that the range of densities does not extend high enough to see the inflection in the transverse case. This difference in transverse and longitudinal effects is not attributable to percolation effects as discussed by Shklovskii due to the differing functional dependences on H. Note, however, that the effects on ϵ_3 are essentially identical for the transverse and longitudinal cases throughout this density range.

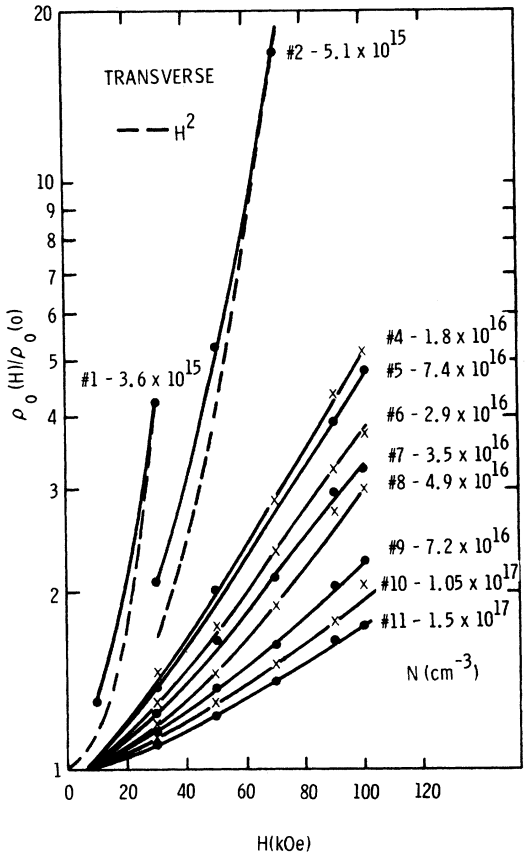

FIG. 10. Same as Fig. 9 for the transverse case, $\vec{H} \perp \vec{I}$.

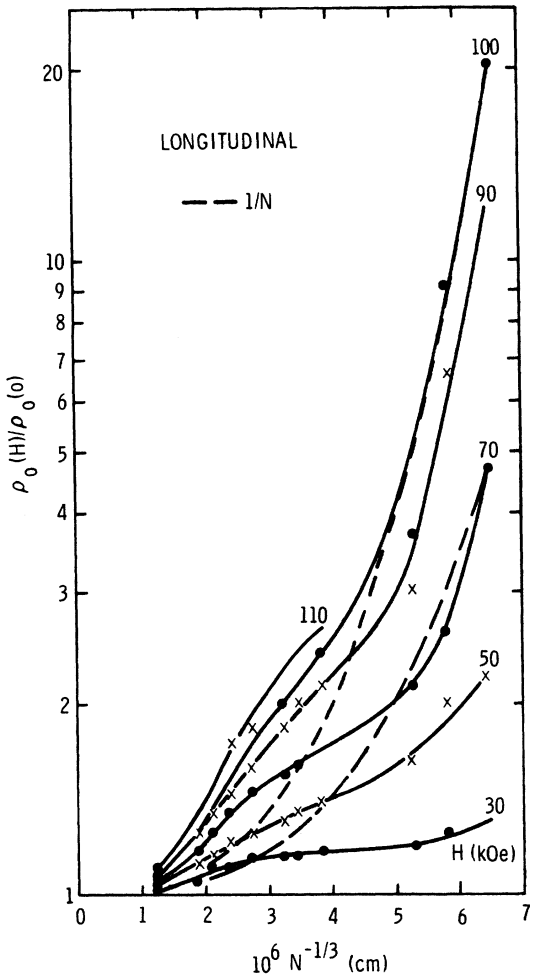

FIG. 11. $\rho_0(H)/\rho_0(0)$ vs $N^{-1/3}(r_A = 0.62N^{-1/3})$ for the longitudinal case, $\vec{H} \| \vec{I}$ as a function of field H (kOe). The $1/N$ dependence expected from Mikoshiba's theory is roughly followed for the smallest densities and highest fields. The effect is stronger at higher densities, i.e., the data lie above the $1/N$ curves.

Finally, we consider Fig. 6 for Sample 12 with $N = 5.0\times10^{17}$, which is distinctly metallic at $H = 0$. At an applied field of 10 kOe, ρ is essentially temperature independent and further increase of H causes the temperature dependence to become semiconductorlike (an Anderson transition). The value of the conductivity just before the transition to the semiconductor state occurs corresponds to Mott's[20] minimum metallic conductivity and the value we obtain from the 10-kOe curve is $\sigma_{\text{min met}} = 3.16\times10^{1}\ (\Omega\ \text{cm})^{-1}$. We emphasize the extreme sharpness of this transition and its implications regarding the so-called mobility edge which we discuss below. These results are an analog of the experiments of Allen *et al.*[33] who see the same type of transition in n-type Ge at $K = 0.3$ by varying the *density* of impurities. They find $\sigma_{\text{min met}} = 1.5\times10^{1}\ (\Omega\ \text{cm})^{-1}$ and a critical density of $N = 4.5\times10^{17}$, both of which are in reasonable agreement with our values. Note that at $N = 5\times10^{17}$ the Fermi level appears to be extremely close to the mobility edge and yet the data in Fig. 7 suggest that ϵ_3 goes to zero at a density of only $\sim1.7\times10^{17}$ ($N^{1/3} = 5.6\times10^{5}$).

V. DISCUSSION

Any model or combination of models which is envisioned to explain impurity conduction in the moderate- to high-density case must possess several qualitative features which are well established experimentally. First of all, at $H = 0$, for $N > N_p$, ϵ_3 decreases with density but much more slowly than the dropoff in ϵ_2 seen by, say, Fritzsche[14] or Davis and Compton.[18] In our samples, $N_p \approx 2\times10^{15}$ and ϵ_3 decreases almost linearly with $N^{1/3}$ until at $N \sim 2\times10^{17}$, $\epsilon_3 \approx 0$, and some form of semiconductor-to-metal transition is observed. Simultaneously, in the same density range, ρ_0 breaks to values higher than the R-percolation value.[17] In addition, it has generally been observed that at

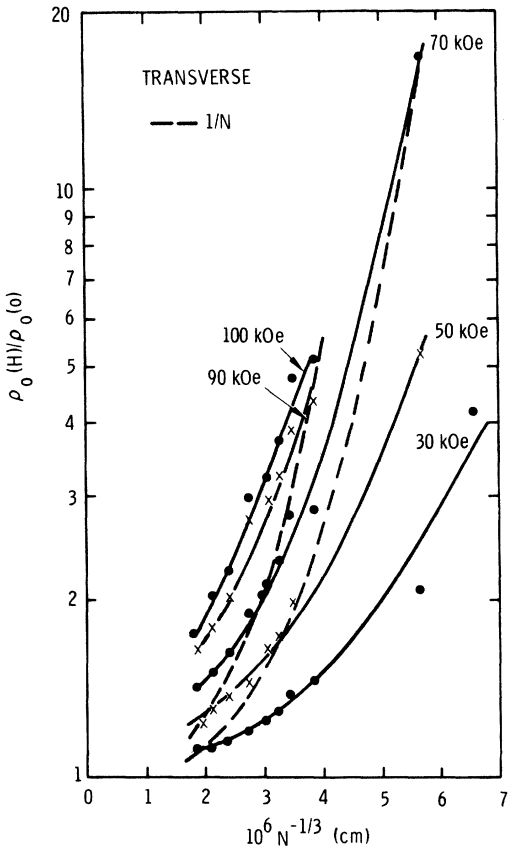

FIG. 12. Same as Fig. 11 for the transverse case. As in Fig. 11 the effect at the higher densities is larger than the simple theory predicts.

ultralow temperatures the process is not truly activiated but exhibits a $\ln\rho \propto T^{-x}$ behavior with $x<1$.[34-37] It must also be noted that for $N>N_p$, parameterizing our data in terms of ρ_0 and ϵ_3 is actually an approximation, although a fairly good one, in that the data goes smoothly over to the nonactivated behavior for temperatures somewhat below those used here and in Refs. 12 and 15.[34,35]

Fritzsche and Cuevas saw some signs of a measurable Hall mobility for samples with $N>2\times10^{16}$. They were limited to the measurement of Hall mobilities $\mu_H>3$ cm^2/V sec and their data do not extend over a wide temperature range. Our magnetoresistance results do not suggest the emergence of a mechanism with a higher mobility in that density regime and it seems possible that the measured Hall mobility (~3 cm^2/V sec) may actually be due to some other mechanism with considerably higher mobility, but which makes only a minor contribution to the total conductivity, such as a weak contribution from the ϵ_2 mechanism or excitation above a mobility edge. Fritzsche[13] observed similar effects in n Ge when ϵ_2 was only of minor importance for $N\gtrsim2.4\times10^{16}$. In that case, a clear ϵ_2 mechanism does not dominate until $N\sim7.4\times10^{16}$ and $\mu=55$ cm^2/V sec at 2.5 K. Certainly more extensive measurements with techniques capable of measuring smaller values of μ_H would seem in order.

The Davis and Mott intrasite polarization model would seem to predict roughly the proper behavior of ϵ_3 with density. However, the dielectric enhancement may be too small below $N=5\times10^{16}$ to have an effect of the magnitude observed. Recent measurements by Castner *et al.*[38,39] on the dielectric enhancement due to impurity levels in both Si and Ge show that intrasite polarization of impurity levels leads to a doubling of the effective dielectric response at $N\sim2\times10^{18}$ in Si and a tripling at $\sim5\times10^{16}$ in Ge for $K\ll1$. In our case, where $K=0.4$, the dielectric enhancement should not be appreciable until $N\sim10^{17}$ cm^{-3}. Clearly there should be *some* effect above $N=5\times10^{16}$, but this effect does not seem to be contributing when $N<10^{16}$ cm^{-3}, where ϵ_3 has already dropped to half the value predicted by Eq. (3).

In addition, there should be, if anything, a break to decreased values of ρ_0 in this density regime by the following argument. Consider a system with a gap of width E_G separating the Fermi level and a distribution of states of width Δ centered at $E=0$. The transport is determined by a percolation solution in the distribution of states centered at $E=0$. This has some variable range hopping aspect to it since by hopping to sites with larger separations the carriers can find sites at lower energies through which to percolate, somewhat similar to the model of Grant and Davis.[40] When there is a dielectric enhancement p such that the effective dielectric constant becomes pK_0, $p>1$, both E_G and Δ are reduced by the factor $1/p$. In this case the carriers will prefer to hop to nearer-lying sites because of the smaller energies involved. Hence ρ_0 should break to smaller values in this regime.

Neither the appearance of a measurable Hall mobility nor the nonactivated behavior at lower temperatures follows from the polarization model, as applied to conduction in the lower Hubbard band, nor do these two factors preclude its applicability.

The model of Shklovskii and Shlimak[19] (SS), which they have presented only in a qualitative form, also predicts a dropoff in ϵ_3. The drop is coupled to the width of the peak (band) of states at $E=0$, ϵ_3 being given by the separation of the Fermi level and the mobility edge in the band. Thus ϵ_3 should vary linearly with the resonance energy, which varies exponentially with density, so we would expect a sharp drop in ϵ_3. A critical feature

of this model is the determination of the density at which there is Anderson delocalization in the band of states at $E = 0$. The SS model also predicts a break to lower values of ρ_0 (to the maximum metallic value) for $N > N_p$, which is the opposite of what is observed.[17] The observation of a measurable Hall mobility is critical to this model and if FC's measured mobility is actually due to the predominant conduction mechanism it would support this model for $N \gtrsim 2\times10^{16}$, but we note that the conductivity at this density is still over two orders of magnitude below the minimum metallic value. The nonactivated behavior observed at lower temperatures also does not follow from this model.

The correlated hopping model can result in the proper ϵ_3 vs density dependence as shown by Knotek and Pollak.[7] The break to higher values of ρ_0 is a natural consequence of the model[17] as is the nonactivated behavior at lower temperatures.[23] Mott[41] has recently proposed that a $T^{-1/4}$ temperature dependence can follow from this behavior. As presently conceived, there is no reason to expect a measurable Hall mobility with this mechanism.

It is conceivable that at high enough density a transition to transport by correlated hopping with no activation energy may be effected. In this case, the many-electron wave functions would become delocalized as opposed to the one-electron wave functions in the Anderson case. The disappearance of ϵ_3 in this data could be due in part to correlation effects, but surely the intrasite polarization as well as Anderson delocalization must play a role for $N > 5\times10^{16}$.

When a magnetic field is applied the orbitals are compressed and the mechanisms for the decrease in ϵ_3 are quenched. In the case where $N \sim N_p$, ϵ_3 can be increased to its one-electron Coulomb gap value as is observed for Sample 1. For samples which have been so "quenched" we expect that Mikoshiba's magnetoresistance theory should apply. For samples such that $N < N_p$, Mikoshiba's theory should apply throughout, save for complications at higher fields where it is not really applicable, and indeed Gadzhiev and Shlimak observe such a behavior for their sample with $N = 1.2\times10^{15}$. They observe an ϵ_3 invariant with field and a preexponential behavior following Eq. (5) with $t = 0.07$ which is over a factor of 2 smaller than expected [$t = 0.17$ in Eq. (5)]. The small value of t has been observed by others and is unexplained.[29]

In the case of the correlated hopping model, the point in density where a transition to many-electron processes should set in is calculable using the simple model recently discussed in Ref. 23. In this we must derive the magnetic field dependence of r_c of Eq. (4). In a magnetic field we must solve the equation

$$2r_c/a + 0.07(ar_c^3/\lambda^4) - \beta e^2/K_0 r_c = 0 \qquad (8)$$

as a function of H. At $H = 0$ and $T = 2$ K, $r_c = 447$ Å ($N_c = 2.7\times10^{15}$ cm^{-3}) using $a = 75$ Å and $K_0 = 16$. In Fig. 7 the calculated values of the upper limit on ϵ_3 and the critical density from Eq. (8) for the transition are denoted by arrows. The smoothly varying curves at higher densities have been extrapolated by the dashed lines to denote the upper limit of ϵ_3 we expect from this mechanism. We regard this as remarkable agreement, somewhat better than we could reasonably expect. If a somewhat larger value of t were used the calculated magnitude of the effect would be somewhat larger, but it varies rather weakly with t.

For the polarization model, the transition density (N_p) is determined by a critical value of Na^3 which is proportional to the polarizability per unit volume or the dielectric enhancement.[8] Thus we expect $r_c(H)$ to vary linearly with the effective $a(H)$ to the first approximation. This is a stronger function of field than the solutions of Eq. (8). The Anderson delocalization model should yield a similar dependence of the transition point with field because it involves a critical value of an *exponential* quantity with $(2R/a + 0.07aR^3/\lambda^4)$ in the exponent.

The data show that the effect on ϵ_3 is largest near the transition point and gradually gets smaller at the higher densities. Thus the curves in Fig. 7 are converging slightly at the higher densities. All of the models would predict that the density N_p should increase with H because the effective orbital size in Eq. (4) shrinks with H and thus ϵ_3 should follow the $N^{1/3}$ behavior to higher densities. Likewise, the point in density where $\epsilon_3 = 0$ should scale by the same factor as N_p, when the effective orbital is shrunk, to the first approximation. Thus the curves in Fig. 7 should uniformly shift to higher densities and higher energies with the field to a first approximation. For densities $N < 2.5\times10^{16}$, this is roughly what is seen, but the shift becomes progressively smaller above that density, which may indicate the presence of more than one of the mechanisms above that density. All three models would predict that ϵ_3 not be dependent on the orientation of the field since the energy ϵ_3 is determined microscopically while the resistivity is a percolation effect and should have some orientation dependence.

The density and magnetic field dependence of ρ_0 appears consistent with the following qualitative model for low to moderate densities. For densities $N < N_p$, the data should (and do) behave in the manner described by Mikoshiba, with the sole effect being confined to ρ_0 and no shift in ϵ_3 apparent. At moderate densities and fields, ϵ_3 lies considerably below the Coulomb gap value and is increasing with

field. When ϵ_3 saturates with field we expect that the H and R dependence of ρ_0 should revert to the Mikoshiba form once again and that is clearly seen in Fig. 11 where at low fields the curves are distinctly not R^3 but at higher field ($H > 50$ kOe) where roughly an R^3 (or $1/N$) dependence is assumed for the three lowest-density samples. This behavior is not inconsistent with that expected from the correlated hopping model.[17] It is not clear what magnitude effect to expect from the other two models in this density region. In the longitudinal case, the H^2 curve of Fig. 9 has $t = 0.035$ which is a factor of 2 smaller than Gadzhiev and Shlimak find at a similar density. Chroboczek *et al.*[43], however, find a value very similar to ours for p Ge and they also find rather strong orbital anisotropies. Since our samples were not crystallographically oriented, some of the discrepancies between our data and Gadzhiev and Shlimak could be due to the orbital anisotropy. In the transverse case, although our data are limited, the effect seems to be larger for the two lowest-density samples.

For $N \gg N_p$, the major features of the data are that there is a large disparity between longitudinal and transverse effects both in magnitude and functional dependence on H. This may suggest that there is more than one contributing mechanism and that the transverse fields are freezing out one mechanism much more efficiently than is the longitudinal field. Hall mobility measurements would be an extremely useful adjunct to this experiment in this density and field regime. The data in Figs. 9–12 show that the magnitude of $\rho_0(H)/\rho_0(0)$ at high density and low fields is larger than expected but that it is a weaker function of field and density. This is coincident with ρ_0 also being larger in this density regime.

Our data are not in agreement with that of Gadzhiev and Shlimak for $N > 10^{16}$ cm^{-3} where they see the effect on ρ_0 begin to *increase* with density. Although the effect is comparatively larger in that density regime in our data, it always monotonically decreases with density. Their conclusions concerning a high mobility for these densities would not seem to apply.

In Sample 12, we have interpreted the change in sign of the temperature coefficient of the conductivity with magnetic field in terms of an Anderson transition. Our value of $\sigma_{\text{min met}}$ is equal to $0.1\ e^2/\hbar r_A$, a value ~4 times larger than Mott's[20] prediction. There have been several observations of this same effect using somewhat different schemes. There are three separate parameters which can be varied to induce this transition. The relative position of the Fermi level E_F in the impurity band should be a function mainly of compensation. The random potential is a function of both density and compensation, and the intersite resonance energy or bandwidth is a function of the overlap between sites which is a function of density and effective orbital size. The relative size of the resonance energy and random potential in turn determines the position of the mobility edge E_c. In the case of transport in the upper Hubbard band, the bandwidth is also a function of compensation since that band is comprised of doubly occupied orbitals, and the density of these will be equal to the number of neutral majority impurities.

The experiments of Fritzsche[14] (n Ge), Fritzsche and Lark-Horowitz[42] (p Ge), and Davis and Compton[16] (n Ge) induce an Anderson transition by varying compensation at a fixed density, moving the Fermi level down in the band, and also increasing the random potential, and hence moving the mobility edge up in the band as compensation increases. When $E_F < E_c$, semiconducting behavior is assumed with excitation to the mobility edge dominating the transport. The value of minimum metallic conductivity observed by us is roughly a factor of 3 greater than seen by these three studies.

The experiment of Allen *et al.*[33] varies density at fixed compensation, thus varying both the resonance energy and the random potential. Since K is fixed, the Fermi level can be assumed to be relatively stationary in the band with the mobility edge moving slowly down in the band as density increases. In this case both the resonance energy and the random potential *increase* with density and so the transition is induced because one increases faster than the other. Our minimum metallic conductivity is a factor of 1.5 or 2 greater than theirs.

In our case we vary only the resonance energy because we vary only the effective orbital parameter, not intersite separations. The crucial question is whether the data are truly activated, i.e., is the activation energy given by $E_c - E_F$? If the data are not characterized by a single activation energy, then that would imply that the mobility edge is somewhat soft. A graded mobility edge will soften the temperature dependence since as temperature is increased the carriers will transport in lower mobility states which lie at lower energies. If the edge is truly a discontinuity in mobility then a well-defined activated behavior should be observed over quite a range in temperature, going over to a $T^{-1/4}$-like behavior at very low temperatures where transport at the Fermi level is dominant. This requires going to ultralow temperatures. The advantages of using a magnetic field to move the mobility edge is that it allows arbitrarily small increments in $E_c - E_F$ and if the range in $1/T$ can be increased (and typically the range in $1/T$ can be increased by a factor of at least 100 by use

of a ^{3}He dilution refrigerator) the shape of the mobility edge can be very accurately deduced. Such a series of experiments is presently being considered for Sample 12.

There is something slightly paradoxical in the data in that it appears that ϵ_3 goes to zero at $N \sim 1.7\times10^{17}$ and yet our results at $N \sim 5\times10^{17}$ suggest that $E_F \approx E_c$. We would expect that given the rather sharp decrease of ϵ_3 below $N = 1.5\times10^{17}$, $E_F - E_c$ would continue to increase rather strongly as density was further increased. One possibility is that for $N < 1.5\times10^{17}$ the transport is quite different than for $N = 5\times10^{17}$ when a magnetic field is applied. In the latter case it appears that transport is by excitation to a mobility edge, but in the former case it may still be hopping conduction at the Fermi level or across the Coulomb gap as we discussed above. A study of samples where $1.5\times10^{17} < N < 5\times10^{17}$ is needed to resolve this question.

VI. CONCLUSIONS

In the low-density regime ($N < N_p$), the basic concepts of Miller and Abrahams for the dc conductivity, given some refinements, give a good overall understanding of the data at $H = 0$. Mikoshiba's theory for the magnetoresistance, also suitably refined, adequately explains the magnetoresistance effects for $N < N_p$. The orbital parameters derived from the magnitude of the effect are anomalously small and there is a rather wide variation among the various studies. Crystallographic anisotropies shown by Chroboczek *et al.* could be responsible for some of this discrepancy.

Our discussion of the possible mechanism for the drop in ϵ_3 at higher densities has been necessarily qualitative in nature. The correlated hopping model possesses all of the qualitative features seen in both density and temperature dependence as well as magnetoresistance, especially the effect of field on ϵ_3. The rough calculation of critical density for onset of many-electron transitions gives better agreement with the data than we could reasonably expect but definitely shows that the data are not inconsistent with that model. This would seem to be the dominant effect below $N \sim (2-5)\times10^{16}$. Above 2×10^{16}, effects due to intrasite impurity polarization enhancement of the dielectric constant as well as Anderson delocalization in the impurity band may begin to contribute.

While ϵ_3 appears to go to zero at $N \sim 1.7\times10^{17}$, the ease with which the temperature coefficient is reversed with field at $N = 5\times10^{17}$ suggests that the system is extremely close to the classical Anderson transition (i.e., $E_F = E_c$) at that density. This suggests that ϵ_3 going to zero at $N = 1.7\times10^{17}$ could be due to other effects, possibly many-electron delocalization or a dielectric catastrophe or a combination of both effects.

The value of minimum metallic conductivity derived from our magnetically induced Anderson transition is somewhat larger ($\sigma_{\min\,\mathrm{met}} = 0.1\,e^2/\hbar r_A$) than that derived from experiments which vary compensation and density. These experiments carried out to ultralow temperatures promise a sensitive and accurate method for determining the structure of the mobility edge.

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