PHYSICAL REVIEW B VOLUME 16, NUMBER 5 1 SEPTEMBER 1977

High-frequency conductivity of electrons on a helium surface

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(Received 1 March 1977)

We have calculated, within the mean-field theory, the density and frequency dependence of the electron-ripplon conductivity for electrons on helium. The numerical results suggest that such measurements will probe the short-range properties of this strongly interacting system.

Electrons trapped at the surface of liquid helium form an almost perfect two-dimensional electron gas whose concentration may be varied over some three orders of magnitude ($10^6 < n < 10^9$).[1] At low temperature, the electrons motion in the plane are unrestricted except insofar as the electrons collide occasionally with ripplons, thermally activated surface waves, and except insofar as they have Coulomb coupling to other electrons. From a theoretical point of view, this is a particularly fascinating system since the dimensionless strength of the Coulomb coupling parameter [$\Gamma_0 = e^2(2\pi n)^{1/2}/k_B T$ in the classical case and $r_s = me^2/(\pi n)^{1/2}$ in the quantum case] may be varied over many orders of magnitude. It has been suggested that condensation into a two-dimensional Coulomb solid may occur in an experimentally accessible range.[2]

The problem has been, and still is, to find ways of probing the short-range properties of this strongly correlated system. A recent experiment of Grimes and Adams[3] has graphically demonstrated the existence of long-wavelength two-dimensional plasmons propagating in this gas. In the regime below 0.6 K, they were able to show that the plasmon lifetime was dominated by ripplon scattering and that the lifetime $1/\tau$ appearing in a Drude fit to the high-frequency conductivity,

$$\sigma = \frac{ine^2}{m(\omega + i/\tau)} \simeq \frac{ine^2}{m\omega}\left(1 - \frac{i}{\omega\tau_{ac}}\right) \tag{1}$$

was empirically fitted by a function of the form

$$\tau_{ac}^{-1} = (eE_0 + eE_\perp)^2/C\,, \tag{2}$$

with $C \simeq 4\sigma_0\hbar$. Here $\sigma_0 = 0.36$ ergs/cm^2 is the surface tension, and $E_0 \simeq 230 \pm 12$ V/cm at $T = 0.5$ K. In Eq. (2), $E_\perp$ is the perpendicular electric field at the surface of the helium, which, along with the attractive image potentials and the 1-eV barrier preventing electrons from entering the helium, keeps a finite concentration n of the electrons at the surface.

In a recent publication,[4] hereafter called I, it was shown that a one-electron theory gave a good quantitative fit to the data. In this paper, we would like to show that there are very significant density- and frequency-dependent effects in a calculation of σ. These effects arise because of the dynamic screening of the ripplons by the electrons. In fact, the explicit size and nature of these effects does, as we will see, depend in a crucial way on the short-range properties of the interacting liquid. We will present numerical results assuming that a version of random-phase approximation (RPA) or mean-field theory is valid for this classical strongly interacting system, although we know from the start that such a description breaks down at high enough densities.[5] The experimental observation of this breakdown will ultimately lead to a better understanding of correlation effects.

For the system under consideration, the interaction potential between the ith electron and the ripplons is given by

$$U_i = S^{-1/2} \sum_{\vec{q}} (a^\dagger_{\vec{q}} + a_{-\vec{q}}) e^{i\vec{q}\cdot\vec{r}_i} \overline{V}_q\,, \tag{3}$$

where S denotes the area of the helium surface, $a^\dagger_{\vec{q}}$ is the creation operation for a ripplon, and $\vec{r}_i$ is the electron coordinate in the plane. In the limit of large perpendicular electric field ($E_\perp \gg E_0$), the interaction potential $\overline{V}_q$ is given by $\overline{V}_q = Q_q e E_\perp$ where $Q_q = (\hbar q/2\omega_q\rho_0)^{1/2}$ is the displacement of the surface due to the qth mode, and ρ_0 is the density of the helium. Here, ω_q is the frequency of the ripplons given approximately by

$$\omega_q^2 = q^3\sigma_0/\rho_0\,. \tag{4}$$

When the ripplon frequencies are low compared, for example, to typical electron plasma frequencies, we can treat the electron ripplon scattering as the scattering from a static random potential. In this case, it is possible to show[6] that the high-frequency Drude-like conductivity of such a system to lowest order in $|\overline{V}_q|^2$ can be written exactly in terms of the correct dielectric response function of the system $\epsilon(q,\omega)$, i.e.,

$$\sigma=(ine^2/m\omega)[1-I(\omega)/2m\omega^2], \tag{5}$$

where

$$I(\omega)=\frac{1}{2\pi}\int\frac{|\overline{V}_q|^2}{n}\left(\frac{2k_BT}{\hbar\omega_q}\right)\left(\frac{q}{2\pi e^2}\right)q^3\,dq \times\left(\frac{1}{\epsilon(q,0)}-\frac{1}{\epsilon(q,\omega)}\right). \tag{6}$$

Equations (5) and (6) are valid whenever the second term in the parentheses is small compared to one, i.e., at high enough frequencies. For electrons on liquid helium at temperatures of about 0.5 °K, this typically means frequencies above 100 MHz.

Equation (6) contains all the essential physics in the problem.[7] Unlike the usual Boltzmann equation approach in I, it includes both frequency dependence and self-consistent screening in the collision term $I(\omega)$. The real part of $I(\omega)$ can be simply related to a frequency-dependent mass or reactive effect, while the imaginary part of $I(\omega)$ is related to an inverse collision time. In this paper, we will focus on the collisional or absorptive term. Substituting the value of $|\overline{V}_q|^2$ into Eq. (6) and using our Drude formula, Eqs. (1) and (5), we can define a frequency- and density-dependent relaxation time $\tau_{ac}(\omega)$, i.e.,

$$\frac{1}{\tau_{ac}(\omega)}=\frac{1}{\tau_0}\left(\frac{k_BT}{2\pi ne^2}\right)\frac{\hbar}{m\omega}\int q^2\,dq\,\mathrm{Im}\left(\frac{1}{\epsilon(q,\omega)}\right), \tag{7}$$

where $1/\tau_0=(eE_\perp)^2/4\sigma_0\hbar$ is the frequency- and density-independent inverse collision time found in I. The

$$\mathrm{Im}\,\frac{1}{\epsilon(q,\omega)}\equiv\frac{4\pi e^2}{q^2}\,\frac{\mathrm{Im}Q(q,\omega)}{|\epsilon(q,\omega|^2}$$

arises from the imaginary part of the polarizability Q, i.e., the single-particle continuum and the zeros of $\epsilon(q,\omega)$, i.e., the collective mode contributions.

In order to explicitly evaluate Eq. (7), we must use some approximate form for $\epsilon(q,\omega)$. Since there are no known analytic results for arbitrary Coulomb coupling ($\Gamma_0=e^2 2^{1/2}\pi^{1/2}n^{1/2}/k_BT$), we choose to evaluate our expression utilizing a classical RPA dielectric function,

$$\epsilon(q,\omega)=1+(2\pi e^2/q)Q^0(q,\omega) \tag{8}$$

with

$$Q^0(q,\omega)=\frac{2}{\hbar^2}\int\frac{d^2p}{(2\pi)^2}\left(\frac{f_{p+\hbar q/2}-f_{p-\hbar q/2}}{\epsilon_{p+\hbar q/2}-\epsilon_{p-\hbar q/2}-\hbar\omega-i\delta}\right). \tag{9}$$

In Eq. (9), f_p is the Fermi-Dirac distribution function for momentum p at a finite temperature T.

The classical RPA dielectric function has much of the correct physics in it. While it neglects explicit correlation effects, it is known to be exact at long wavelengths ($q\to 0$), and to be quite accurate at short wavelength ($q\to\infty$). It has a collective mode, the plasmon which is exact for a small plasma parameter ($\Gamma_0\to 0$), and is a pretty good approximation to the phonon mode in the solid as ($\Gamma_0\to\infty$). While it is clear that the details of this calculation will be incorrect at intermediate coupling, the qualitative features will indeed be reproduced correctly by such an approximation. The difference between our calculations and an experiment, when it gets done, will enable us to empirically get at the role of correlations in such plasmas.

Since our system is classical, we would like to take the classical limit of Eq. (9). However, such a strict classical limit leads to a divergent integral in Eq. (7). This problem and the connection with the conventional handling, via the Boltzmann equation, of the transport problem can best be analyzed by examining the integral in Eq. (7) which is proportional to,

$$\Delta(\omega)\equiv\int d^2p\int\frac{q\,dq}{|\epsilon(q,\omega)|^2}(f_{p+\hbar q/2}-f_{p-\hbar q/2}) \times\delta\left(\frac{\hbar q^2}{2m}-\frac{\vec{p}\cdot\vec{q}}{m}-\omega\right). \tag{10}$$

For the moment, let us neglect the screening factor $|\epsilon(q,\omega)|^2$ in the denominator, since for any finite ω in the limit of infinite q, $\epsilon\to 1$. The main difference between the more conventional formula [see Eq. (17) of I] and Eq. (10) is the dynamic screening in the effective interaction and the explicit inclusion of ω in the energy conserving delta function.

The formally exact classical limit of Eq. (10) with $|\epsilon|=1$ and f_0 the Boltzmann distribution is given by

$$\Delta(\omega)=\int d^2p\int q\,dq\,\vec{q}\cdot\frac{\vec{\delta}\vec{f}_0}{\partial p}\,\delta\left(\omega-\frac{\vec{q}\cdot\vec{p}}{m}\right) \tag{11}$$

which diverges. For any finite ω, q is allowed to go to infinity for every momentum p. The cutoff comes from the $\hbar^2q^2/2m$ in the distribution function and in the δ function. At the frequencies of interest ($\hbar\omega\ll k_BT$), both cutoffs occur at roughly the same place, i.e., $\hbar^2q^2/2mk_BT\sim 1$ and the cutoff is Gaussian. Thus, we choose to replace $\Delta(\omega)$ in our numerical calculations by

$$\Delta(\omega)=\int d^2p\int\frac{q\,dq}{|\epsilon(q,\omega)|^2}\,\vec{q}\cdot\frac{\vec{\delta}f_0}{\partial p} \times\delta\left(\omega-\frac{\vec{q}\cdot\vec{p}}{m}\right)e^{-\hbar^2q^2/2mk_BT}. \tag{12}$$

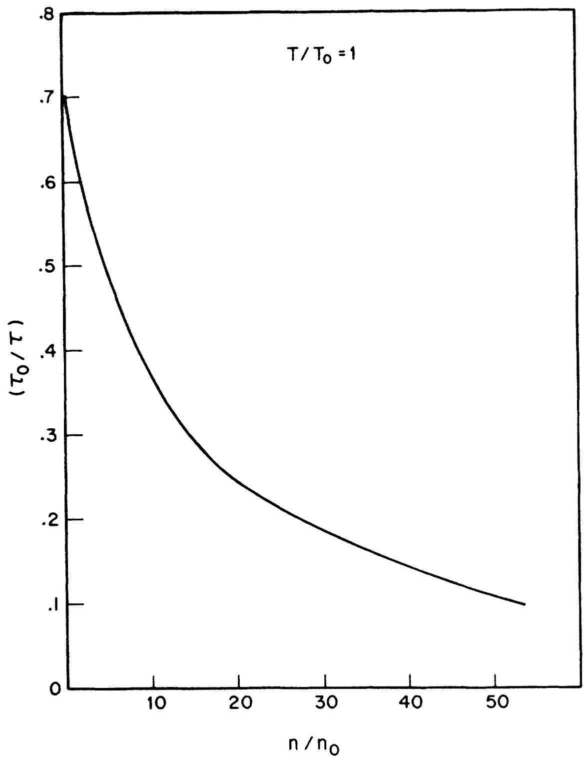

FIG. 1. Mean-field density dependence of the low-frequency electron collision time with $T = 1$ K and $n_0 = 10^6$ cm^{-2}.

The more standard Boltzmann-equation (see I) approach is equivalent to neglecting ω in the δ function, i.e.,

$$\Delta(\omega) = \int d^2p \int q\, dq\, \vec{q} \cdot \frac{\partial f_0}{\partial \vec{p}}\, \delta\left(\frac{\hbar q^2}{m} - \frac{\vec{q}\cdot\vec{p}}{m}\right) \qquad (13)$$

or $q = (2p/\hbar)\cos\theta$. We will see in our numerical results, that in the limit of zero screening and zero frequencies the two approaches give roughly (30%) the same answers. The differences arise from the detailed handling of the cutoff.

In Fig. (1), we plot the density dependence of Eq. (7). The quantity $n_0 = 10^6/\text{cm}^2$ and $T_0 = 1\ ^\circ$K. We have divided out $(1/\tau_0)$ so that the answer given in I for $1/\tau$ would be unity in these units. The resistivity decreases rapidly with increasing n. This is primarily due to the static screening of the ripplons. Assuming $\epsilon = 1 + q_{sc}/q$ has its static mean-field value with $q_{sc} = 2\pi n e^2/k_B T$, it is possible to show that to a good approximation the resistivity decreases linearly from its zero density value, i.e.,

$$\frac{1}{\tau} = \frac{1}{\tau_0}\left[1 - \left(\frac{2\pi^2 e^4 \hbar^2 n^2}{m(k_B T)^3}\right)^{1/2}\right]. \qquad (14)$$

This analytical result was obtained by integrating over q from a lower cutoff given by q_{sc} to an upper cutoff $k_{\max} = (2mk_BT)^{1/2}/\hbar$.

We expect that the mean-field results given here are inaccurate for large values of (n/n_0). Where it breaks down, one cannot say *a priori*, however, it is clear that for a strongly correlated system, the screening length in this mean-field theory becomes much smaller than the interparticle spacing, i.e., $q_{sc}(\pi n)^{-1/2} = 2(2\pi n e^2)^{1/2}/k_BT \gg 1$. Physically we know that the screening length for a strongly correlated system will be limited by the interparticle spacing. The deviations of the experiment from these RPA results will give important information about the effects of correlations.

It has been pointed out to us[8] that there may be some experimental evidence for such a strong density dependence. Three measurements of the low-temperature ($T \sim 0.5$ K) low-frequency ripplon-dominated mobility have been made.[3,9,10] For[9] $n \simeq 10^4$ cm^{-2}, $\mu \simeq 4 \times 10^5$; for[10] $n \simeq 3 \times 10^5$, $\mu \simeq 2 \times 10^6$; and for[3] $n \simeq 3 \times 10^7$, $\mu \simeq 1.3 \times 10^7$. The first two measurements are dc measurements, while the last is a plasmon linewidth measurement. The trend is in the right direction to be accounted for by a screening effect. However, it is safe to say that a detailed comparison must await a systematic study, on a single apparatus, of the density dependence of τ for constant perpendicular electric field.

The frequency dependence of the conductivity is even more interesting and informative. In order to clarify the physics somewhat, we show a plot (solid curve) of τ_0/τ for $n/n_0 = 300$, as a function of ω/ω_0, where $\omega_0 = [(2\pi)^{3/2} n^{3/2} e^2/m]^{1/2}$ is the zone-boundary plasmon. The striking increase in resistivity comes almost entirely from the contribution due to the pole [zero of $\epsilon(q,\omega)$] in the integrand of Eq. (7) (dotted curve).[11] In essence, the

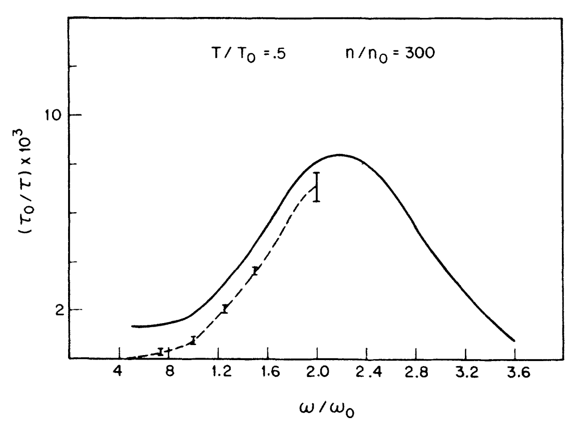

FIG. 2. Mean-field frequency dependence of the collision time (solid curve) for $T = 0.5$ K and an $n = 3 \times 10^8$ cm^{-2}. Dotted curve is the pole contribution.

ripplon potential breaks the momentum selection rule and permits excitation of arbitrary wave-vector plasmons by the long-wavelength electromagnetic field. The details of this contribution will depend on the detailed dispersion relation of these short-wavelength modes, plasmons in the gas, and phonons in the two-dimensional Coulomb solid if it exists.

While RPA, as we have stated, is quantitatively inaccurate at large values of n/n_0, it does give us a pretty good qualitative description of the collective mode as a function of q even for a two-dimensional Coulomb solid. We would expect on a qualitative basis that a correct description of the two-dimensional solid would give a more rapid frequency variation, i.e., the single-particle portion would be totally surpressed, and the pole contribution would remain. For a harmonic solid, one can rewrite Eq. (7) as

$$\frac{1}{\tau}=\frac{1}{\tau_0}\,\frac{k_BT}{m\omega}\sum_{\lambda}\int\frac{d^2q}{4\pi}\,\frac{(\vec{q}\cdot\vec{\epsilon}_{q\lambda})^2}{2m\omega_{\vec{q}\lambda}}\,\delta(\omega-\omega_{\vec{q}\lambda})\,, \qquad (15)$$

and the frequency dependence is qualitatively similar to the dotted curve in Fig. (2). It does not pay to belabor the detailed numerics since the results given here are only approximate quidelines. Again we must turn to the experiments for confirmation of these ideas. The essential point is simply that the frequency dependence of the $q=0$ resistivity is sensitive to the spectrum of collective modes at $q\neq 0$. We know of no current experimental work in this area.

ACKNOWLEDGMENTS

We would like to thank G. Beni and C. C. Grimes for many informative conversations, and C. K. N. Patel for a careful reading of the manuscript.

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[11]We have stopped our dotted curve at $\omega/\omega_0=2$, since it becomes difficult to say numerically, how much of the integral is the pole (error bar). However, the integrand in Eq. (7) is peaked around $\epsilon(q,\omega)=0$, and we could in fact say that the entire contribution for $\omega/\omega_0\geq 2$ is polelike.