PHYSICAL REVIEW B VOLUME 16, NUMBER 5 1 SEPTEMBER 1977

Quasiparticle scattering and current-voltage characteristics of superconductor-normal-superconductor film structures*

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(Received 4 June 1976)

The quasiparticle spectrum of a film with superconducting surface sheaths is calculated for states with energies less than the maximum value Δ of the pair potential. The influence of a magnetic field parallel to the film surface is discussed semiqualitatively. The probability of Andreev scattering (AS) in a surface sheath is found to be limited by ordinary reflection processes to a narrow range of values of momentum perpendicular to the film surface. The "normal" region behaves like a gapless superconductor. When a ground-state current flows in it, in thermal equilibrium quasiparticle states with momenta opposite to the ground-state flow are preferentially populated. Those with high probability of AS continuously transfer momentum to the condensate so that a voltage must appear in order to maintain a stationary current. Current-voltage characteristics are calculated and discussed for various temperatures and film thicknesses. They show step structures due to the spatial quantization of the quasiparticle states. The theory is compared to the experiments on quantized resistances.

I. INTRODUCTION

Current-voltage characteristics (CVC) of films with superconducting surface sheaths measured recently[1-3] exhibit two striking features: (a) finite voltages are sustained by the superconducting surface layers parallel to which stationary currents from a constant-current source are flowing; (b) there are steps in the CVC, and linear current branches with quantized resistances can be traced out. While steps and finite voltage drops in the CVC of superconducting films and microbridges have been observed and interpreted by various groups and authors,[4-7] their experimental situations and the magnitude of the observed currents and voltages differ significantly from the ones reported in Refs. 1–3. Also, special effects like the decrease of the quantized resistances with increasing magnetic field parallel to the phase boundaries and low-field resistances higher than the normal film resistance (see Fig. 9 of Ref. 2) are difficult to understand on the basis of vortex-channel[4] or phase-slip-center[6] mechanisms.

Hayler, Geppert, Chen, and Kim (HGCK) elaborate a phenomenological model[2] in order to describe their linear current branches in terms of quasiparticle currents through quantized states, bound between the pair potential walls.[1] Making a number of assumptions concerning the quasiparticle momentum $k_{\perp n}$ normal to the phase boundary, the quasiparticle relaxation time τ, and the quasiparticle energy E_n in the presence of current flow, they obtain quantized resistances after fitting the quasiparticle density n_q, and reproduce rather well cutoff voltages and current intercepts.[2]

Despite its success the simple phenomenological model of HGCK does not resolve the theoretical problems posed by the observed CVC. By its very nature it cannot explain the ability of the superconducting surface sheaths to sustain a finite voltage difference between their ends nor does it give the reason why the current should flow through a certain bound quasiparticle state. Furthermore, the assumption that the relaxation time τ, responsible for energy dissipation and the appearance of a resistance (or voltage), be equal to the time a quasiparticle needs to cross the normal region between the phase boundaries[1,2] is not self-explanatory, since reflections at the normal (N)–superconducting (S) interfaces *produce* the bound quasiparticle state; they do not limit its quantum-mechanical lifetime. Finally, the energy relevant for the definition of a bound state, i.e., a state with exponentially damped wave amplitudes in the S regions, is independent of the drift velocity of an applied current (see Sec. II and Ref. 8); this aspect differs from the point of view adopted in Ref. 2.

Therefore, a different approach towards a theory of superconducting-normal-superconducting films with the outer surface bordering on vacuum and with current flow parallel to the phase boundaries appears to be justified and necessary. Based on prior considerations of the processes involved in particle-hole scattering at N-S phase boundaries[9,10] and the spectrum of *ISNSI* films[8] (I, insulator), we will try to show how current and voltage steps can originate in suitable layer structures because of quasiparticle-induced spatial and temporal variations of the pair potential's (order parameter's) phase. Like HGCK we build our theory on their fundamental idea, that quantized bound quasiparticle states are responsible for the structure of the CVC,[1,2] but contrary to HGCK we do not assume current conduction through individual quasiparticle states. We rather follow Bardeen and Johnson[11] and look into quasiparticle countercurrents established in the low-lying bound states by relaxation with the lattice.

According to experiments reported in Ref. 3 and mentioned in Ref. 2, *ISNSI* sandwiches of Pb and Ag layers sustain finite voltages and show steps in the CVC even without an applied magnetic field $\vec{H}$. Therefore, and for the sake of simplicity, we will at first disregard $\vec{H}$ in Secs. II–IV. Section II presents the eigenvalue equations for all bound states, i.e., states with energies E^0 (in a frame moving with the ground-state flow) which are less than the maximum value Δ of the pair potential. They are solved for the lowest-lying states $E^0 \ll \Delta$ with any value of the Fermi momentum component k_{zF} normal to the phase boundaries. The higher bound states with $E^0 \leq \Delta$ are calculated for $k_{zF}^2 \gg 2m\Delta$, m being the electron mass. In Sec. III we show how "current-excited" quasiparticles in these states can induce electrical potential differences between the film ends, and discuss in Sec. IV the energy dissipation due to quasiparticle relaxation with the lattice and quasiparticle recombination into Cooper pairs. The treatment of the magnetic field in Ref. 8 (and in a number of papers quoted therein) is inadequate. We will look into its influence on our results in Sec. V. Section VI presents numerical calculations of: (i) the very-low-temperature CVC of *ISNSI* systems whose normal regions are so wide that several states with high particle-hole scattering probability can satisfy the condition $E^0 \ll \Delta$; (ii) the voltage as a function of the ground-state current when all states with $E^0 < \Delta$ are involved. Qualitative and quantitative features of the calculated CVC are compared to the experimental results of[1-3] HGCK in Sec. VII. Appendix A presents the derivation of the quasiparticle eigenvalue equations and in Appendix B the probability of particle-hole scattering from *NSI* boundaries is calculated.

II. EIGENVALUE EQUATIONS AND BOUND-STATES SPECTRUM

Let us consider a pure metal film of thickness $2D$ with a central normal layer of thickness $2a$ and two superconducting surface sheaths, each of thickness $D-a$. As in Ref. 8 we choose the usual idealized model of a steplike pair potential which has the constant value Δ for $a < |z| < D$ and is zero for $|z| < a$.[8-16] Since the work function of a metal against the insulating vacuum is much larger than Δ, we approximate the height of the surface potential barrier at $|z| = D$ by infinity. The situation is depicted by Fig. 1 of Ref. 8. We assume that no magnetic field is present, i.e., that the *SNS* structure is produced by sandwiching normal and superconducting layers[3] and that the proximity effect may be neglected.[11]

Investigating a normal layer imbedded between two practically infinite superconducting regions Ishii[12] concludes that the spectrum of the bound states with large Fermi momentum components k_{zF} normal to the phase boundaries is more like that of a gapless superconducting state than of the usual normal state, so that the N region is capable of carrying a supercurrent. As pointed out in Ref. 8, the N layer between two thin superconducting surface sheaths has the same property, due to the spectrum of the energetically low-lying quasiparticle states with $k_{zF} \lesssim (2m\Delta)^{1/2}$. Therefore, it is justified to consider an *ISNSI* system that carries a homogeneous ground-state flow in y direction parallel to the interface with an average momentum per electron $\vec{e}_y q$. [The spatial constancy of q simplifies the process of solving Eqs. (2.1). It does not mean that the total current, which includes the quasiparticle countercurrent due to relaxation with the lattice, is spatially homogeneous, see Eqs. (3.18)–(3.21). It will be largest at the film edges. Consequences of an increase of q towards the film edges will be discussed qualitatively in Sec. VII.] Thus, in the stationary state we assign the phase $2qy$ to the pair potential in both surface sheaths and continue it into the N region. The magnetic field of the current will be neglected.[11]

In this system the Bogoliubov equations for the electron and hole components of a quasiparticle in the state $\boldsymbol{k}$ are

$$E_k u_k(r) = -(1/2m)(\nabla^2 + k_F^2) u_k(r) + e^{i2qy}\Delta(z) v_k(r), \qquad (2.1a)$$

$$E_k v_k(r) = (1/2m)(\nabla^2 + k_F^2) v_k(r) + e^{-i2qy}\Delta(z) u_k(r), \qquad (2.1b)$$

$$\Delta(z) = |\Delta(z)| = \Delta\Theta(|z| - a), \quad \hbar = 1.$$

The quasiparticle energy E_k is measured relative to the Fermi energy $k_F^2/2m$. The solutions are

$$\begin{pmatrix} u_k \\ v_k \end{pmatrix} = e^{i(k_x x + k_y y)} \begin{pmatrix} e^{iqy} \\ e^{-iqy} \end{pmatrix} \Big\{ \Theta(a-|z|)\Big[a_1 \begin{pmatrix}1\\0\end{pmatrix}\alpha(z) + a_2\begin{pmatrix}1\\0\end{pmatrix}\alpha^{-1}(z) + b_1\begin{pmatrix}0\\1\end{pmatrix}\beta(z) + b_2\begin{pmatrix}0\\1\end{pmatrix}\beta^{-1}(z)\Big]$$

$$+\Theta(z-a)\Big[A_1\begin{pmatrix}\nu\\ \nu^{-1}\end{pmatrix}\gamma(z) + A_2\begin{pmatrix}\nu\\ \nu^{-1}\end{pmatrix}\gamma^{-1}(z) + A_3\begin{pmatrix}\nu^{-1}\\ \nu\end{pmatrix}\delta(z) + A_4\begin{pmatrix}\nu^{-1}\\ \nu\end{pmatrix}\delta^{-1}(z)\Big]$$

$$+\Theta(-z-a)\Big[B_1\begin{pmatrix}\nu^{-1}\\ \nu\end{pmatrix}\delta(z) + B_2\begin{pmatrix}\nu^{-1}\\ \nu\end{pmatrix}\delta^{-1}(z) + B_3\begin{pmatrix}\nu\\ \nu^{-1}\end{pmatrix}\gamma(z) + B_4\begin{pmatrix}\nu\\ \nu^{-1}\end{pmatrix}\gamma^{-1}(z)\Big]\Big\}, \qquad (2.2)$$

where

$$\alpha(z) \equiv \exp[ik_{ZF}(1+\epsilon)^{1/2}z], \qquad \beta(z) \equiv \exp[ik_{ZF}(1-\epsilon)^{1/2}z], \qquad (2.3a)$$

$$\gamma(z) \equiv \exp[ik_{ZF}(1+i\delta_E)^{1/2}z], \qquad \delta(z) \equiv \exp[ik_{ZF}(1-i\delta_E)^{1/2}z], \qquad (2.3b)$$

and

$$k_{ZF} \equiv (k_F^2 - k_x^2 - k_y^2)^{1/2};$$

k_x, k_y are wave numbers,

$$\epsilon \equiv 2mE^0/k_{ZF}^2, \quad E^0 \equiv E_k - k_y q/m, \qquad (2.4)$$

$$\delta_E \equiv 2m(\Delta^2 - E^{02})^{1/2}/k_{ZF}^2,$$

$$\nu \equiv \exp[i\tfrac{1}{2}\arccos(E^0/\Delta)].$$

k symbolizes the complete set of quantum numbers, including the spin, that characterize the quasiparticle state.

The solutions are subject to the boundary conditions

$$u_k(z=\pm D) = 0 = v_k(z=\pm D). \qquad (2.5)$$

They are normalized:

$$\int (u_k^2 + v_k^2)\, d^3r = 1.$$

In Ref. 8 it is explained how three different types of quasiparticle surface scattering mix together all degenerate solutions with the same k_{ZF}.

As a result of the matching conditions and Eq. (2.5) there are 12 equations for the 12 integration constants in (2.2). The energy eigenvalues as usual result from the requirement that the determinant of the coefficients of the integration constants vanish. Expanding this determinant in Appendix A we obtain the following results:

(a) The eigenvalues of the energies $E^0 \ll \Delta$ for any value $k_{ZF} < k_F$ are

$$E_n^0(k_{ZF}) = (n - \alpha')\pi k_{ZF}/2ma, \quad n = 1, 2, 3\ldots, \qquad (2.6a)$$

where

$$\begin{aligned}\alpha' &= \alpha'(k_{ZF})\\ &\equiv (1/2\pi)P\arccos\Big\{\Big[\big\{[\tfrac{1}{4}\delta^2 - (1+\delta^2)^{1/2}+1]\cosh 2\kappa + [\tfrac{1}{4}\delta^2 + (1+\delta^2)^{1/2}+1]\cos 2K\\ &\qquad - 2\delta\sinh\kappa\sin K + \delta^2(\cosh\kappa\cos K + \tfrac{1}{2})\big\}\,4\cos(4k_{ZF}a)\\ &\qquad - \big([(1+\delta^2)^{1/2}-1]\sinh 2\kappa + [(1+\delta^2)^{1/2}+1]\sin 2K\\ &\qquad + \sqrt{2}\,\delta\{[(1+\delta^2)^{1/2}+1]^{1/2}\sin\kappa\cos K + [(1+\delta^2)^{1/2}-1]^{1/2}\cosh\kappa\sin K\}\big)\,4\sin(4k_{ZF}a)\\ &\qquad - 2\{[(1+\delta^2)^{1/2}+1]\cosh 2\kappa - [(1+\delta^2)^{1/2}-1]\cos 2K - 2 - 4\delta\sinh\kappa\sin K\}\Big]\\ &\qquad \times\{[(1+\delta^2)^{1/2}+1]^2\cosh 2\kappa + [(1+\delta^2)^{1/2}-1]^2\cos 2K + 4 + 4\delta^2(\cosh\kappa\cos K + \tfrac{1}{2})\}^{-1}\Big\}. \qquad (2.6b)\end{aligned}$$

Thus, $0 < \alpha' < \frac{1}{2}$. ($P$ indicates that the principal value is to be taken.) The meanings of the abbreviations are

$$\delta \equiv 2m\Delta/k_{ZF}^2, \quad \kappa \equiv \sqrt{2}\,[(1+\delta^2)^{1/2}-1]^{1/2}k_{ZF}(D-a), \quad K \equiv \sqrt{2}\,[(1+\delta^2)^{1/2}+1]^{1/2}k_{ZF}(D-a). \qquad (2.7)$$

The more specialized cases of Ref. 8 are included in Eq. (2.6).

(b) The eigenvalue equation for the higher bound states with $E^0 \lesssim \Delta$ and k_{ZF} so large that $(1 \pm i\delta_E)^{1/2} \approx 1$ is

$$\begin{aligned}0 = &-\cos(4mE^0a/k_{ZF}) + 2[1-(E^0/\Delta)^2]\cos(4k_{ZF}D) + \{1 - \cosh[4m(\Delta^2 - E^{02})^{1/2}(D-a)/k_{ZF}]\}\\ &- [2(E^0/\Delta)^2 - 1]\cos(4mE^0a/k_{ZF})\cosh[4m(\Delta^2 - E^{02})^{1/2}(D-a)/k_{ZF}]\\ &+ 2(E^0/\Delta)[1-(E^0/\Delta)^2]^{1/2}\sin(4mE^0a/k_{ZF})\sinh[4m(\Delta^2 - E^{02})^{1/2}(D-a)/k_{ZF}]. \qquad (2.8)\end{aligned}$$

For $E^0 > \Delta$ this equation also yields the eigenvalues of the "continuum states" whose wave functions do not decay in the S regions.

It is not possible to give analytical solutions of Eq. (2.8) in the complete range of its validity. However, it simplifies for k_{ZF} values such that $(1 \pm i\delta_E)^{1/2} \approx 1$ and yet $4m(\Delta^2 - E^{02})^{1/2}(D-a)/k_{ZF} \geq 3$, because then the dominant terms in Eq. (2.8) are the ones involving the exponential functions and we obtain

$$0 = -1 + (2x^2 - 1)\cos(px) + 2x(1-x^2)^{1/2}\sin(px), \tag{2.9}$$

with $x \equiv E^0/\Delta$ and $p \equiv 4m\Delta a/k_{ZF}$.

When p is large, one might expect that, if x_G is a solution of (2.9), then

$$x_n \approx x_G + n2\pi/p, \tag{2.10}$$

with n an integer, is an approximate solution, too. In fact, the numerical calculation of the zeros of the right-hand side of Eq. (2.9) for $p = 45$ yields eight practically equidistant solutions obeying the relation

$$x_n = 0.07 + n\,0.13, \quad 0 \leq n \leq 7 \quad (\tfrac{2}{45}\pi = 0.14).$$

Therefore, in the k_{ZF} range where Eq. (2.9) is valid we have equidistant bound energy levels

$$E_n^0 = E_G(k_{ZF}) + n\pi k_{ZF}/2ma, \quad n = 0, 1, 2, \ldots \tag{2.11}$$

according to Eq. (2.10). This spectrum has the same structure as that of Eq. (2.6). (Note the different ways of counting n.) It agrees with the density of states curve calculated by Ishii[12] for thick S layers. As is shown in Appendix B the values of k_{ZF} and $D-a$, for which Eq. (2.11) holds, are just the ones that allow a nonvanishing probability of particle-hole scattering[9] in the thin surface sheaths. We will see in Sec. III that these states are of fundamental importance to voltage induction.

If

$$g \equiv 4m(\Delta^2 - E^{02})^{1/2}(D-a)/k_{ZF}$$

and

$$i \equiv 4mE^0(D-a)/k_{ZF}$$

are so small that the approximations $\cosh g \approx 1$, $\sinh g \approx g$; $1 \approx \cos i$, $i \approx \sin i$ are justified, Eq. (2.8) becomes

$$\cos(4k_{ZF}D) = \cos(4mE^0D/k_{ZF}). \tag{2.12}$$

The solutions of this equation are the energies of a normal film of thickness $2D$.[8] [Equation (2.12) also results from Eq. (2.8) in the limit $\Delta \to 0$.] This is not surprising, because small values of g and i mean that the quasiparticles pass the surface sheaths without hardly ever noticing them.

III. SCATTERING AND VOLTAGE INDUCTION

Three different quasiparticle scattering mechanisms in joint action are responsible for the appearance of the voltage jumps we are about to calculate. (i) Particle-hole scattering, also called Andreev scattering[9] (AS), at the N-S phase boundaries is a process possible only in (inhomogeneous) superconductors; it is of fundamental importance to the effect. It has been investigated and described by many authors.[9-16] (ii) Thin superconducting surface sheaths cannot completely damp out quasiparticle waves with large k_{ZF} despite $E^0 < \Delta$. Therefore, these waves can be reflected at the outer film surfaces. This type of scattering, together with ordinary reflection from the pair potential walls[8] for very small k_{ZF}, limits finite probabilities of AS to a narrow range of k_{ZF}. (iii) Quasiparticle scattering from phonons establishes thermal equilibrium with the lattice and causes decaying quasiparticles in the S regions to recombine into Cooper pairs. Let us look into these processes one after another.

A. Andreev scattering (AS)

When a quasiparticle in a state above the Fermi surface passes across an inhomogeneity of the superconducting order parameter $\Delta(\vec{r})$, it will be scattered into a state below the Fermi surface with a certain probability, and vice versa. This mixing of states is responsible for the Tomasch effect[17] and for the combination of wave functions in the mixed state of type-II superconductors,[18] and in superconductors with magnetic impurities, etc. Its nature was first described and analyzed by Andreev[9] for the intermediate state. Consider an electron of excitation energy E^0 with a momentum $\vec{k}^+$ above the Fermi sea which moves in z direction from a normal into a superconducting region where the pair potential is Δ. If $E^0 < \Delta$, the electron encounters a zero density of states in the S region. Around itself it creates an unstable normal region by pushing a Cooper pair out of the volume element it occupies. The charge deficit in this volume element pulls in a second electron, of the same energy and with momentum $\vec{k}^-$ below the Fermi surface, out of the normal region. A hole is left there close to the phase boundary which is filled up by an electron in an adjacent volume element deeper in the N region and so on, so that effectively the hole moves away from the phase boundary with a group velocity $-\vec{k}^-/m$.[13] Via the phonon-mediated attractive electron-electron interaction the two new electrons in the energetically forbidden S region form a Cooper pair, transferring their total momentum $\vec{k}^+ + \vec{k}^- = 2\vec{k}_F$ to the condensate into which they merge. This momentum and the charge $2e$, transported from the N into the S region in a single-particle → hole-scattering process, are carried away by a supercurrent, which results in a phase shift $2S_k$ of the order parameter. In Ref. 13 the equation for this phase shift was derived:

$$n_s \operatorname{div} \vec{\nabla} S_k = 4m \operatorname{Im}[\Delta(z) v_k^* u_k]; \qquad (3.1)$$

n_s is the electron density in the ground state. [The factor $1-f_l$ in Ref. 13 is due to the non-equilibrium situation assumed in the derivation of Eq. (3.1). It can be replaced by 1 here. The value of the right-hand side of Eq. (3.1) for semi-infinite N and S regions is given in Ref. 13.]

The solutions of Eq. (3.1) are subject to boundary conditions determined by the positions of the external current leads. For instance, if these are connected to two points on the z axis of the N-S system which are on either side of and far away from the phase boundary at $z=0$, we have the solution of Ref. 13 with a supercurrent in z direction in the stationary state. On the other hand, if both semi-infinite regions were completely surrounded by particle reservoirs so that there were no constraints on the directions the quasiparticle-induced currents may take, then the solution of Eq. (3.1) in the stationary state would yield the supercurrent density

$$\vec{j}_{Sk} = n_s(e/m)\vec{\nabla} S_k = 2\vec{k}_F(e/m)(1-e^{-\kappa z})/V_N, \qquad (3.2)$$

V_N being the volume of the normal region and $\kappa = k_{ZF}\delta_B$. Note that although the momentum of a quasiparticle excitation is nearly completely conserved in a single process of AS,[13] in a stationary situation charge and momentum flow is associated with a continuous flow of electrons towards the interface and holes away from it. Close to the interface curl $\vec{j}_{Sk} \neq 0$. The two penetrating electrons with forbidden energy and a surrounding unstable normal region form decaying vortices as they scatter into Cooper pairs. In the situation of interest to us, a net current can flow parallel to the y axis only. Before discussing this in detail it is necessary to find out how probable AS is in a thin superconducting surface sheath.

B. Ordinary surface scattering

In Appendix B we calculate the probability of Andreev scattering in a superconducting layer

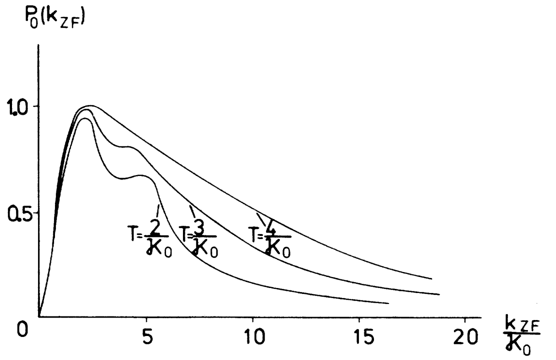

FIG. 1. Probability $P_0(k_{ZF})$ of Andreev scattering in a single surface sheath of thickness T for various T values. The slight energy dependence of the scattering probability is not shown. It smears out $P_0(k_{ZF})$ by about 0.05.

of thickness T backed by an infinite potential wall at $z=T$. A semi-infinite normal region extends to the left of $z=0$. Figure 1 shows this probability $P_0(k_{ZF})$. We see that for the quasiparticle states under consideration with energies $E^0 < \Delta$ the particle-hole scattering probabilities have their maximum values equal or close to 1 in the range

$$2\kappa_0 < k_{ZF} \lesssim 2.5\kappa_0, \quad \kappa_0 \equiv (m\Delta)^{1/2}. \qquad (3.3)$$

With decreasing layer thickness T the range of finite $P_0(k_{ZF})$ shrinks, because less and less waves will be damped out completely in the S region. Rather the probability increases that the electron (hole) component of the quasiparticle is reflected into itself at the outer film surface and returns into the N region without having been scattered into the hole (electron) component. Excitations with very small k_{ZF} cannot penetrate at all into the S region but suffer ordinary reflection from the pair potential wall at $z=0$. Therefore, $P_0(k_{ZF})$ drops to zero for $k_{ZF} < \kappa_0$.

Multiple-scattering processes from opposite surface sheaths in *ISNSI* systems should peak the AS probability—designated by $P(k_{ZF})$ then—even more sharply than in Fig. 1 around its maximum.

C. Relaxation with the lattice

Let us consider an *ISNSI* system of length L, width W, and thickness $2D$, with an electric current from an external current source flowing in y direction parallel to the phase boundaries. Following Bardeen and Johnson[11] we may divide this current into two components: a ground-state flow with a net momentum $\vec{q} = \vec{e}_y q$ per electron and a countercurrent in $-y$ direction carried by the quasiparticles which are "excited" (in a reference frame moving with velocity q/m), in order that the electron system be in thermal equilibrium with the lattice. In thermal equilibrium the quasiparticle distribution is given by the Fermi function

$$f(E_k)=[\exp(E_k/kT)+1]^{-1} \tag{3.4}$$

where according to Eqs. (2.4)

$$E_k=E^0+k_y q/m \tag{3.5}$$

is the quasiparticle energy in the laboratory system (i.e., in a reference frame fixed to the lattice). Equations (3.4) and (3.5) show that quasiparticle states with a momentum k_y opposed to the ground-state momentum per electron q will be preferentially populated. Especially in the zero-temperature limit, $T\to 0$, only those quasiparticle states are occupied for which

$$E_k<0,\quad \text{i.e.,}\quad k_y q<0,\quad |k_y q|/m>E^0. \tag{3.6}$$

In a normal metal, relations (3.6) lead to a quasiparticle countercurrent which exactly cancels the ground state flow, i.e., a current without an applied voltage will die out because of quasiparticle relaxation with the lattice. In our system, however, the structure of the excitation spectrum, especially of its lowest part where the energies are given by Eq. (2.6), only allows for excitation of relatively few quasiparticles so that the countercurrent is not sufficient to cancel the ground-state flow (see Figs. 3 and 4). Therefore, in films with superconducting surface sheaths at low temperatures currents can flow without an applied voltage, as long as q remains below a critical value q_0 to be determined later. For this reason *SNS* and *ISNSI* systems behave as gapless superconductors.[8,12]

The states with finite probability $P(k_{ZF})$ of Andreev scattering have k_{ZF} values close to k_0 defined by

$$P(k_{ZF}=k_0)=\max, \tag{3.7}$$

where

$$k_0\approx 2.5(m\Delta)^{1/2} \tag{3.8}$$

according to Fig. 1 and Eq. (3.3) for a single process of AS. We do not expect that k_0 is shifted appreciably, when multiple AS occurs in surface sheaths facing each other. The energies of these states are given by Eqs. (2.6) and (2.11) which can be unified into

$$E_n^0(k_{ZF})=(n-\alpha)\pi k_{ZF}/(2ma),$$

$$n=1,2,3,\ldots,\quad k_{ZF}\approx k_0, \tag{3.9}$$

$$\alpha=\alpha(k_{ZF})=\begin{cases}\alpha'(k_{ZF}) & \text{if } E^0\ll\Delta,\\ 1-2maE_G(k_{ZF})/(\pi k_{ZF}) & \text{if } E^0\leq\Delta.\end{cases}$$

If bias current and ground state momentum q are such that Eq. (3.6) will be satisfied for a number of states given by Eq. (3.9), then even at very low temperatures quasiparticles will be present which carry rather large momenta $-|k_y|$ into the surface sheaths when suffering Andreev scattering. Their momenta in z direction normal to the surface sheaths follow from Eqs. (2.3), (2.4), (B1), and (3.9). In the N region their magnitude is

$$k_z^+=k_{ZF}+(n-\alpha)\pi/2a \tag{3.10a}$$

for the electron component of the quasiparticle and

$$k_z^-=k_{ZF}-(n-\alpha)\pi/2a \tag{3.10b}$$

for the hole component. Here we have expanded

$$(1+\epsilon)^{1/2}\approx 1+\tfrac{1}{2}\epsilon.$$

Since $P(k_0)\approx 1$, the wave functions (2.2), with $k_{ZF}=k_0$ in good approximation, divide into two degenerate, independent solutions: one with $+k_z^\pm$ $(a_2=0=b_2)$ and one with $-k_z^\pm$ $(a_1=0=b_1)$, because perfect Andreev scattering only mixes electron and hole states of nearly equal momentum. Figure 2 shows the corresponding excitations.

As discussed in Sec. III A, each particle-hole scattering leads to a transfer of charge $2e$ and momentum $2\vec{k}_F$ to the ground state. On the other hand, hole-particle scattering pulls two electrons of momentum $2\vec{k}_F'$ out of the S region (and the ground state) into the N region in a process time reversed to the one described above. Therefore, the net momentum and charge which are effectively transferred to the ground state in one surface sheath depend upon the difference in the rate at which electrons and holes hit upon the phase boundary.

In the stationary state the wave functions (2.2), with $k_{ZF}=k_0$ and properly chosen coefficients

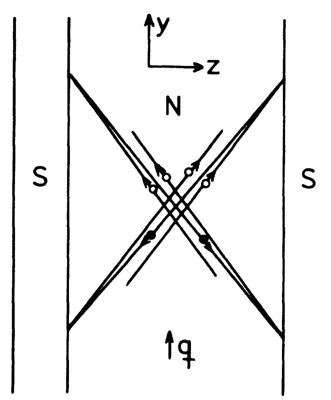

FIG. 2. Current-excited quasiparticles in states with perfect Andreev scattering.

a_i and b_i, describe a continuous flow of electrons and holes towards either phase boundary. Let us consider the surface sheath on the right-hand side of Fig. 2. The number N_k^+ of current-excited electrons of momentum $\vec{k}^+ = \vec{e}_x k_x + \vec{e}_y k_y + \vec{e}_z k_z^+$, group velocity $\vec{v}_g^+ = \vec{k}^+/m$ and density ρ_0, and the number N_k^- of holes of momentum $-\vec{k}^- = -(\vec{e}_x k_x + \vec{e}_y k_y + \vec{e}_z k_z^-)$, group velocity $\vec{v}_g^- = +\vec{k}^-/m$ and density ρ_0 which per unit time hit the N-S interface is

$$\dot{N}_k^{\pm} = \rho_0 LW(\vec{e}_z \vec{v}_g^{\pm}) = \rho_0 LWk_z^{\pm}/m. \tag{3.11}$$

The BCS model[19] of noninteracting quasiparticles is assumed. The rate at which charge $2e$ and momentum $2\vec{k}_F$ are transferred to the ground state by particle-hole scattering anywhere in one surface sheath is, with Eqs. (3.10) and (3.11),

$$\begin{aligned}\tau_k^{-1} &= \dot{N}_k^+ - \dot{N}_k^- = \rho_0 LW(k_z^+ - k_z^-)/m \\ &= \rho_0 LW\pi(n-\alpha)(ma)^{-1}.\end{aligned} \tag{3.12}$$

The momentum transfers in $\pm x$ direction from degenerate states of opposite k_x cancel. The supercurrents in positive and negative z direction induced in both surface sheaths are reflected at the outer film surfaces, flow back and eliminate each other. They spread the net transferred momentum $2k_y<0$ as a uniform ground-state flow opposite to q across the total cross section of the gapless superconducting *ISNSI* film. Consequently, the phase shift of the ground-state wave function, i.e., half of the phase shift of the order parameter in the S regions, has the gradient

$$\vec{\nabla} S_k = \vec{e}_y \nabla_y S_k = -\vec{e}_y 2|k_y| P(k_{ZF})/n_s V. \tag{3.13}$$

Here V is the volume of the film, so that $n_s V$ is the total number of electrons in the ground state of the film which receive the y momentum of a quasiparticle suffering Andreev scattering with a probability $P(k_{ZF})$ close to unity. The time change of this phase shift is

$$\frac{d}{dt}\nabla_y S_k = \frac{\nabla_y S_k}{\tau_k} = \frac{-\pi(n-\alpha)|k_y| P(k_{ZF})}{ma^2 n_s V}, \tag{3.14}$$

where we have put

$$\rho_0 = (LW\,2a)^{-1}. \tag{3.15}$$

The total phase-shift gradient $\nabla_y S$, due to all excited quasiparticles, is the sum of Eq. (3.14) over all states k occupied according to the Fermi distribution function:

$$\frac{d}{dt}\nabla_y S = -\frac{\pi}{ma^2 n_s V}\sum_{k,k_y>0}(n-\alpha)P(k_{ZF})k_y\left[f\left(E^0 - \frac{k_y q}{m}\right) - f\left(E^0 + \frac{k_y q}{m}\right)\right]. \tag{3.16}$$

Therefore, Andreev scattering of current excited quasiparticles would continuously decrease the ground state flow until no current at all would be flowing unless a potential difference eU that accelerates the ground-state electrons in the $+y$ direction is established between the film ends. If the ground-state momentum gain from eU just balances the loss given by Eq. (3.16), we have the stationary situation of constant ground-state flow and continuous AS we are investigating.[20] Thus, eU is just the integral of Eq. (3.16) along a straight path of length L from one film end to the other. (We could also say that Andreev scattering produces an internal electric field which just cancels the electric field from the potential difference eU.) Alternatively, we can also calculate eU using the gauge transformation properties of the pair potential and the vector potential in the Bogoliubov equations[21]: A superconductor with pair potential $\Delta e^{i2S(t)}$, vector potential $\vec{A}'$ and scalar potential Φ' is identical to one with Δ, $\vec{A} = \vec{A}' - (\hbar c/e)\vec{\nabla} S$ and $\Phi = \Phi' + (\hbar/e)\partial S/\partial t$. (It is convenient to reintroduce $\hbar$ here.) Therefore, in the stationary state considered by us, with pair potential Δ and zero internal electric and magnetic fields, the potential difference between the film ends at $y=-L$ and $y=0$ is

$$\begin{aligned} eU(q) &= e[\Phi(-L) - \Phi(0)] \\ &= \hbar\int_0^{-L}\frac{d}{dt}\nabla_y S\,dy = \frac{\pi\hbar^2 L}{ma^2 n_s V}\sum_{k,k_y>0}(n-\alpha)P(k_{ZF})k_y\left[f\left(E^0 - \frac{\hbar^2 k_y q}{m}\right) - f\left(E^0 + \frac{\hbar^2 k_y q}{m}\right)\right]. \end{aligned} \tag{3.17}$$

The density of the total current flowing in this situation is the sum of the ground-state flow

$$\vec{j}_G = \vec{e}_y n_s e\hbar q/m \tag{3.18}$$

and the quasiparticle countercurrent

$$\vec{j}_{qp} = -\frac{\vec{e}_y 2e\hbar}{mWL2a^*}\sum_{k,k_y>0}k_y(f_1 - f_2). \tag{3.19}$$

$2a^*$ is the average effective part of the film thickness over which quasiparticles spread and

$$f_1 - f_2 \equiv f(E^0 - \hbar^2 k_y q/m) - f(E^0 + \hbar^2 k_y q/m). \tag{3.20}$$

The total current in y direction is

$$I(q) = j_G 2DW + j_{qp} 2a^* W. \tag{3.21}$$

IV. ENERGY DISSIPATION AND EFFECTIVE RESISTANCE

Let us discuss Eqs. (3.17)–(3.21) from the point of view of energy dissipation. A fundamental condition for the validity of these equations is that of thermodynamic equilibrium of the electron system with the lattice. The quasiparticle countercurrent (3.19) is a result of this equilibrium. A number of quasiparticles in it would leak away into the ground state, inducing a ground-state countercurrent, because of Andreev scattering. Therefore the quasiparticle countercurrent would decrease by the amount

$$I_R = \frac{e\hbar}{mL} \sum_{k,k_y>0} 2k_y P(k_{ZF})(f_1 - f_2) = \frac{e}{m} \hbar n_s W 2D \sum_{k,k_y>0} |\nabla_y S_k| (f_1 - f_2) \tag{4.1}$$

during the average time τ, defined by

$$\sum \frac{\nabla_y S_k}{\tau_k}(f_1 - f_2) \equiv \tau^{-1} \sum \nabla_y S_k (f_1 - f_2): \quad \tau^{-1} = \sum_{k,k_y>0} \tau_k^{-1} k_y P(k_{ZF})(f_1 - f_2) \Big/ \sum_{k,k_y>0} k_y P(k_{ZF})(f_1 - f_2), \tag{4.2}$$

if not, in order to maintain thermal equilibrium via relaxation with the lattice, quasiparticles would be created from the ground-state flow at the same rate at which they return to it by Cooper pair formation in the S layers. In these relaxation processes energy is being dissipated from the electrons to the lattice which has to be supplied by the external source that maintains the potential difference eU between the film ends. [If no current is flowing, there are nevertheless quasiparticle excitations with $E_k > 0$ present at finite temperatures. Since the electrons with $k_{ZF} = k_0$ disappear more rapidly into one S layer than they are being pulled out of the other one by the slightly slower reflected holes, new excitations have to be created from the ground state at the rate τ_k^{-1} given by Eq. (3.12) in order to have the quasiparticle state "incident electron-reflected hole" permanently occupied in both components, as is demanded by thermal equilibrium. The lattice receives the energy necessary to create these excitations from the electron pairs which recombine into Cooper pairs in the S regions, lowering their energy to the ground-state energy. The induced ground-state currents cancel.]

We may calculate the dissipated power from a "classical" point of view, if we put it equal to the work per unit time done by the voltage source on the electron system. In order to do that we use the classical relaxation time model[22] where one assumes that during a time τ_R the electrons may be accelerated freely by the voltage. Then collisions inhibit a further increase of the current. The stationarity condition that all power transferred from the voltage source to the electrons be dissipated to the lattice means, in the model, that the current decreases to zero within the relaxation time τ_R after the voltage has been switched off.

If a voltage U is applied to a conductor of length L, cross section A, and electron density n_s, the current increase according to Newton's law: $m\,dv = e(U/L)\,dt$, during a time interval dt, is

$$dI = (e^2/m) n_s A (U/L)\,dt.$$

When the current increases from

$$I(t) = (e^2/m) n_s A (U/L) t \tag{4.3}$$

to $I(t) + dI$, the increase in kinetic energy of the electrons is equal to the work dW done by the voltage source:

$$dW = n_s A L \left(\frac{e}{m}\frac{U}{L} t\right) \frac{eU}{L}\,dt = \frac{mL}{e^2 n_s A} I\,dI. \tag{4.4}$$

In the stationary state the current in the relaxation-time model is obtained from Eq. (4.3) replacing t by τ_R:

$$I_s = (e^2 n_s A \tau_R / mL) U, \tag{4.5}$$

and all the work done on the electrons is transferred to the lattice, i.e., the energy Pdt dissipated during dt is given by Eq. (4.4) with $t = \tau_R, I = I_s$, so that $dI = I_s\,dt/\tau_R$:

$$Pdt = dW = I_s^2 \frac{mL}{e^2 n_s A} \frac{dt}{\tau_R} = I_s^2 R \tau_N \frac{dt}{\tau_R}, \tag{4.6}$$

where we have defined

$$R\tau_N \equiv mL/e^2 n_s A. \tag{4.7}$$

In a truly normal conductor the relaxation time is

$$\tau_R = \tau_N \equiv mL/e^2 n_s A R, \tag{4.8}$$

R being the normal-state resistance. The dissipated power has the familiar form

$$P = \frac{dW}{dt} = I_s^2 R. \tag{4.9}$$

In our *ISNSI* system we have

$$\tau_R = l\tau, \tag{4.10}$$

where τ is given by Eq. (4.2) and

$$l^{-1} \equiv I_R/I_s \tag{4.11}$$

is the fraction the quasiparticle current of Eq. (4.1) constitutes of the total current I_s. Equation (4.11) expresses the fact that, if after each time interval τ the lth part of the total current changes into the countercurrent $-I_R$, then in the relaxation-time model the total current is zero, when the time $\tau_R = l\tau$ will have passed after switching off the voltage. We recall that only the component $-I_R$ of the quasiparticle countercurrent participates repeatedly in the dissipative processes. For the consideration of the stationary state we neglect the initial dissipation due to the establishment of the remaining part of the countercurrent.

Inserting Eq. (4.10) into Eq. (4.6), we obtain the dissipated power

$$P = \frac{dW}{dt} = I_s^2 R\,\frac{\tau_N}{l\tau} = I_s^2 R_{\text{eff}}, \tag{4.12}$$

where the effective resistance of the *ISNSI* system is being defined as

$$R_{\text{eff}} \equiv R\tau_N/l\tau. \tag{4.13}$$

With Eqs. (4.13), (4.7), (4.11), (4.1), (4.2), and (3.12) the voltage U related to the stationary current I_s by R_{eff} is

$$U = I_s R_{\text{eff}} = \frac{mL}{e^2 n_s A}\,\frac{I_R}{\tau} = \frac{\hbar^2\pi}{ema^2 n_s A}\sum_{k,k_y>0}(n-\alpha)k_y P(k_{zF})(f_1 - f_2). \tag{4.14}$$

This is U as given by Eq. (3.17) with $V = LA$. U and I_s are the values of voltage and current shown by the instruments in a stationary experiment. The effective resistance R_{eff}, defined by Eq. (4.13) or as U/I_s according to Eq. (4.14), consists of components proportional to $n - \alpha$ whose weight increases with increasing ground-state momentum q according to the difference of Fermi functions $f_1 - f_2$.

V. MAGNETIC FIELDS AND SURFACE CURRENTS

When solving the Bogoliubov equations (2.1) we neither considered a magnetic field parallel to the film surface nor the associated screening currents in the surface sheaths. Therefore, the results of Sec. I–IV are valid strictly speaking only for the *ISNSI* systems where current and voltage have been observed in zero magnetic field.[2,3] However, the majority of the experiments reported in Refs. 1–3 were done in the presence of parallel magnetic fields and the authors interpret their effect as consisting mainly of an increase of the width $2a$ of the N region and a suppression of the proximity effect.[1-3] The theoretical justification of this interpretation given in Ref. 8 is questionable, because it did not care about constant components of the vector potential outside the N region,[23] which only seldom bear any physical relevance otherwise.[24]

The electron velocity in a supercurrent

$$\vec{v}_s = [\hbar\vec{\nabla}\chi - 2(e/c)\vec{A}\,]/2m \tag{5.1}$$

relates the phase χ of the order parameter and the vector potential $\vec{A}$ to each other. Therefore, in two parallel S layers of extreme type I, where a parallel magnetic field cannot penetrate, the condition $\vec{v}_s = 0$ results in phases χ of the pair potential which have equal magnitudes but opposite signs in the two S layers according to the change of sign of the piecewise constant vector potential $\vec{A}$.[15,16] Thus, the usual simple gauge transformation[21] to a real pair potential is not possible[15,16] and the treatment of the vector potential in Ref. 8 is inadmissible in *SNS* systems with S layers of extreme type-I behavior.

Gogadze and Kulik[16] find that no spatially quantized (Andreev) states show up in the density-of-states function of an *SNS* system with zero penetration depth λ_p of the magnetic field and no screening currents in the S layers. Rather they obtain states with the energy separation of Landau levels. On the other hand they demonstrate that Andreev levels appear in quantum effects where local interaction is significant.

Let us consider an *ISNSI* system of extreme type I ($\lambda_p \to 0$). There, a vector potential

$$\vec{A} = \vec{e}_z yH_0\{\Theta(a - |z|) + \Theta(|z| - a)\exp[-[(|z| - a)/\lambda_p]]\} \to \vec{e}_z yH_0\Theta(a - |z|) \quad \text{as } \lambda_p \to 0, \tag{5.2}$$

giving rise to a magnetic field

$$\vec{H} = \vec{e}_x H_0\Theta(a - |z|), \tag{5.3}$$

results in $\chi = 0$. The physics is the same as with[15]

$$\vec{A} = \vec{e}_y H_0[-z\Theta(a - |z|) - (\text{sgn}z)\,a\Theta(|z| - a)]$$

and $\chi \neq 0$. In the Bogoliubov equations the terms in A^2 must not be neglected, because y varies along the total length of the film. Thus, we have the term $(i\partial/\partial z \mp eH_0 y/c\hbar)^2$ in the momentum operators of the electron (−) and hole (+) wave functions. If we use Gogadze and Kulik's concept of a local, i.e., spatially dependent, energy[16] the following changes result in Appendix A and Sec. II.

We define a phase factor

$$\lambda \equiv \exp[i\tfrac{1}{2}\pi\Phi(y)/\Phi_0], \tag{5.4}$$

where

$$\Phi(y) \equiv H_0 2ay\,;\ \ \Phi_0 = hc/2e$$

is the magnetic flux quantum. Let D_{kl} symbolize the element in row k and column l of a determinant. Then, in Eq. (A1) the elements $D_{5,9}$, $D_{9,9}$, $D_{5,10}$, $D_{9,10}$, $D_{8,11}$, $D_{12,11}$, $D_{8,12}$, and $D_{12,12}$ are multiplied by λ, and the rest of the elements in columns 9–12 are multiplied by λ^{-1}. In Eq. (A4) λ multiplies the elements $D'_{3,8}$, $D'_{6,9}$, $D'_{6,10}$, $D'_{7,7}$, and $D'_{10,9}$ and λ^{-1} the remaining ones in columns 7–10. The first of the large square brackets [ ] in Eq. (A5) is multiplied by $(\lambda^4+\lambda^{-4})$ and the last term in the numerator of Eq. (2.6b), which is

$$-2\{[(1+\delta^2)^{1/2}+1]\cosh 2\kappa - [(1+\delta^2)^{1/2}-1]\cos 2K - 2 - 4\delta\sinh\kappa\sin K\},$$

is multiplied by $\cos[2\pi\Phi(y)/\Phi_0]$. In the limit $\delta \ll 1$ and $D-a$ large compared to the coherence length, this term is the leading one in the numerator of Eq. (2.6b). In kilogauss fields with 2a being of the order of some 10^3 Å the concept of local energies then only makes sense within regions of a length of some 10^2 Å where $\Phi(y)/\Phi_0$ varies little. On a macroscopic scale the energy levels of Eq. (2.6) appear as being smeared out. This is the situation analyzed by Gogadze and Kulik.[16] For arbitrary δ and thin superconducting layers the relative weight of the y-dependent term decreases in Eq. (2.6) and the validity of the local energy concept improves. If, furthermore, a constant N-layer thickness exists in limited sections of the film only, either because the evaporation technique[2] results in a spatially varying film thickness or/and because of the very mechanism of N-region formation in parallel fields,[25] only local energy spectra will exist. Therefore, it is not impossible that the states with maximum Andreev scattering probability induce the voltage locally in a number of uncorrelated regions and show up experimentally as being spatially quantized despite the magnetic field.

In the first report on quantized resistances the films were specified as type-I superconductors.[1] In the second report[2] the classification is changed to type II. In any case, the supposition that the magnetic field does not penetrate the thin surface layers is unrealistic, and screening currents in the S layers should be taken into account in view of the fact that the phase of the order parameter so decisively may determine the character of the solutions of the Bogoliubov equations. The corresponding quantitative, electrodynamically self-consistent calculations are difficult and have yet to be performed. Qualitatively for type-II films one may argue that in thin surface sheaths about half a coherence length thick[25] the magnetic field will be minimum in the center and increase towards the edges. The screening current at the outer edge of a surface sheath will flow in a direction opposite to that of the current at the inner edge. In a gauge where $\vec{A}$ has the direction of y and depends on z, sufficiently strong screening currents may change the sign of the phase χ which the quasiparticles "see" when moving through the S layer. Therefore, the net influence of surface currents and vector potential on the quasiparticle wave functions may approximately cancel in the S layers. Then $\vec{A}$ has to be considered in the N region only, as it was done in Ref. 8, so that indeed the effect of the magnetic field only would consist of an increase of the width of the N region and suppression of the proximity effect.[2,3]

It should be emphasized that measurements on *ISN* and *SNI* systems show little qualitative difference from the ones on *ISNSI* systems.[2,3] In these systems vector potential and phase of the order parameter are not fixed, because no magnetic flux is enclosed between S layers. Therefore, the problems with the magnetic field discussed above do not exist there, while voltage inducing quantized states with high Andreev scattering probability can be present in these samples, too.

The preceding discussion gives some possible reasons for the relatively little qualitative differences observed in the CVC of *ISNSI* systems with and without applied magnetic fields.[3] More detailed and more exact analyses, however, are necessary in order to fully clarify the role of magnetic fields and surface currents.

VI. VOLTAGE AND CURRENT (REF. 26)

Equations (3.17)–(3.21) give the voltage U and the current I as a function of the ground-state momentum q.

When computing U we replace the sharply peaked probability of Andreev scattering $P(k_{zF})$ by a weighted δ function,

$$P(k_{zF}) = r\kappa_0\delta(k_{zF}-k_0), \tag{6.1}$$

where the positive number r has to be chosen so that the integral over k_{zF} of both sides of Eq. (6.1) yields the same result. For $P_0(k_{zF})$ this is the area under the respective curves in Fig. 1 for which r would vary between 4 and 10.

Inserting Eq. (6.1) into Eq. (3.17) with k labeling the set of quantum numbers $(k_x, k_y, n, \text{spin})$, performing the spin sum and the integrals over k_x and k_y and observing that two quasiparticle states with opposite z momentum belong to each energy value (3.9) with nearly perfect Andreev scattering we obtain

$$eU = eU_0 \sum_{n\geq 1}^{E_n^0<\Delta} (n-\alpha_0)\,\mathrm{Re}\left[\left(1-\frac{q_0^2(n-\alpha_0)^2}{q^2(1-\alpha_0)^2}\right)^{1/2}\right]$$

$$\equiv e\sum_{n\geq 1}^{E_n^0<\Delta} U_n \qquad (6.2)$$

at $T=0$ K, where

$$f_1 - f_2 = \Theta(-E_n^0 + \hbar^2 |k_y| q/m)\,. \qquad (6.3)$$

We have defined

$$eU_0 = \hbar^2 L r \kappa_0 k_0 (k_F^2 - k_0^2)^{1/2}/\pi m n_s D a^2\,, \qquad (6.4)$$

$$\alpha_0 \equiv \alpha(k_0)\,,$$

$$q_0 = \pi(1-\alpha_0)/2a(k_F^2/k_0^2 - 1)^{1/2}\,. \qquad (6.5)$$

Equation (6.3) with Eq. (3.9) has determined the range of integration of k_y. As a consequence nonzero contributions to the voltage occur only if $q > q_0$. At $T=0$ K there is a lower critical current $I_0 \equiv I(q_0)$ below which no voltage appears between the film ends, because no bound quasiparticle states with finite AS probability are occupied. When

$$q = q_{n-1} \equiv q_0(n-\alpha_0)/(1-\alpha_0)\,, \qquad (6.6)$$

the quasiparticle states with the quantum number n start contributing to the voltage.

Calculation of the total current according to Eqs. (3.18)–(3.21) is difficult. In order to compute the quasiparticle countercurrent (3.19) we must know the complete quasiparticle spectrum for all k_{ZF} at least in the energy range up to the last quasiparticle states with $k_{ZF} = k_0$ which enter the voltage sum (6.2).

If the width $2a$ of the N region is so large that $E_n^0(k_0) \ll \Delta$ for several integers $n \leq n_0$, then all "current excited" quasiparticle states have the spectrum (2.6) as long as $q \leq q_{n_0-1}$. In this case at low temperatures an approximate calculation of the current can be done, if we replace $\alpha'(k_{ZF})$ of Eq. (2.6) by an average value $\bar{\alpha}$. This approximation is unavoidable because of the complex structure of $\alpha'(k_{ZF})$. Figures 3–5 show the low-temperature CVC for $\bar{\alpha} = 0.2$. The quasiparticle countercurrent never exceeds 15% of the ground-state current.[27] Because of the structure of the spectrum (2.6) only relatively few quasiparticle states are available. Therefore the *ISNSI* system behaves like a gapless superconductor in the zero-voltage region where $q < q_0$.

If the N region is so narrow that only one or two states with k_0 have an energy $E_n^0(k_0) \ll \Delta$, while the other ones involved in the current fill in the range up to Δ, we only know about their spectrum that it changes from the form (3.9) for $k_{ZF} \approx k_0$ to the normal-state relation for $k_{ZF} \lesssim k_F$ and small $D-a$ which results from Eq. (2.12). We expect that the

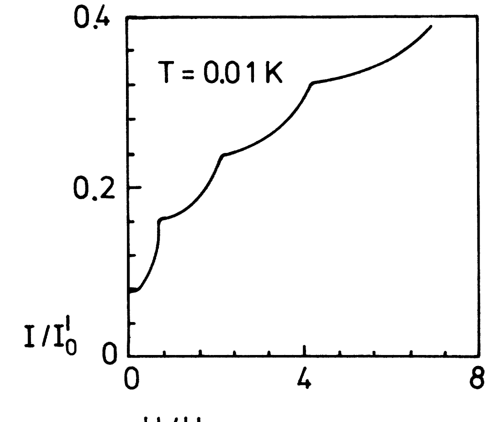

FIG. 4. CVC of films with wide N regions at $T=0.01$ K. $I_0' = e\hbar W k_F^3/m\pi^2$, $2a = 15\,000$ Å.

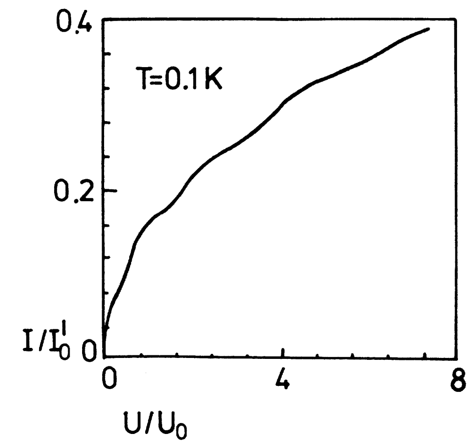

FIG. 5. CVC of films with wide N regions at $T=0.1$ K.

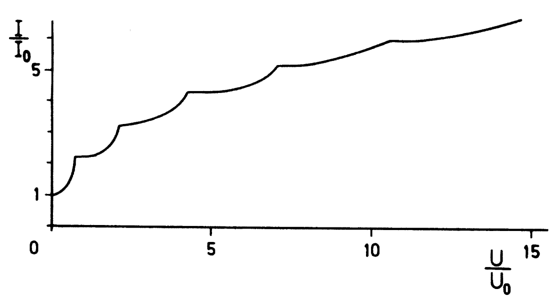

FIG. 3. Current-voltage characteristic (CVC) of films with wide N regions at temperature $T=0$ K. U_0 is given by Eq. (6.4), $I_0 = (e/m)\hbar n_s q_0 2DW$, $\alpha_0 = 0.2 = \bar{\alpha}$.

quasiparticle countercurrent becomes comparable to the ground-state current, because a good number of quasiparticles will have nearly normal-state energies. Preliminary computations indicate that, as in the simpler case of Bardeen and Johnson,[11] great care will have to be taken, when calculating the net current as the difference of two big quantities, once the full energy spectrum can be handled. At the present stage of the theory, for the case of narrow N regions, we can only calculate the voltage as a function of the ground-state momentum q. Figure 6 shows the contributions U_n from the different bound quasiparticle states to

$$U = \sum_{n}^{E_n^0<\Delta} U_n$$

at various temperatures T. $\sum_n U_n$ is obtained from Eq. (3.17) after performing all integrations and summations except for that over n.

VII. COMPARISON TO EXPERIMENT

A. Qualitative comparison

It should be emphasized that one of the basic presumptions of the theory is that of a ground-state current flowing in the N region. Therefore, the CVC of Figs. 3 and 4 must be compared to the returning sections of the experimental CVC,[1-3] since when the current is lowered after the film has become completely normal, it should flow in the whole cross section of the film, even after reappearance of the surface sheaths. A sudden withdrawal of current into the surface sheaths would lead to thermodynamical instabilities because of the formation of supercritical surface currents. Furthermore, being limited to the S layers the current would be associated with zero voltage, as is the current of the first sweep up to I_c.[1,2] This would be contrary to experimental observation. In the final part of the returning sections of the observed CVC, see Figs. 3 and 4 of Ref. 2, there are current and voltage steps that resemble the ones of our Figs. 3 and 4. The quantitative differences will be discussed later.

Tracing out of the linear current branches with the quantized resistances is performed by different techniques.[2] One way is to increase the current at the end point of a voltage jump.[1,2] It is reasonable to assume that the *additional* current from the increase preferably flows in the surface sheaths, in order to keep the energy dissipation, described in Sec. IV, as small as possible. This assumption is consistent with the observation of the resistanceless first current increase up to I_c. In the rising current the average ground-state momentum per electron in the N region q_N is less

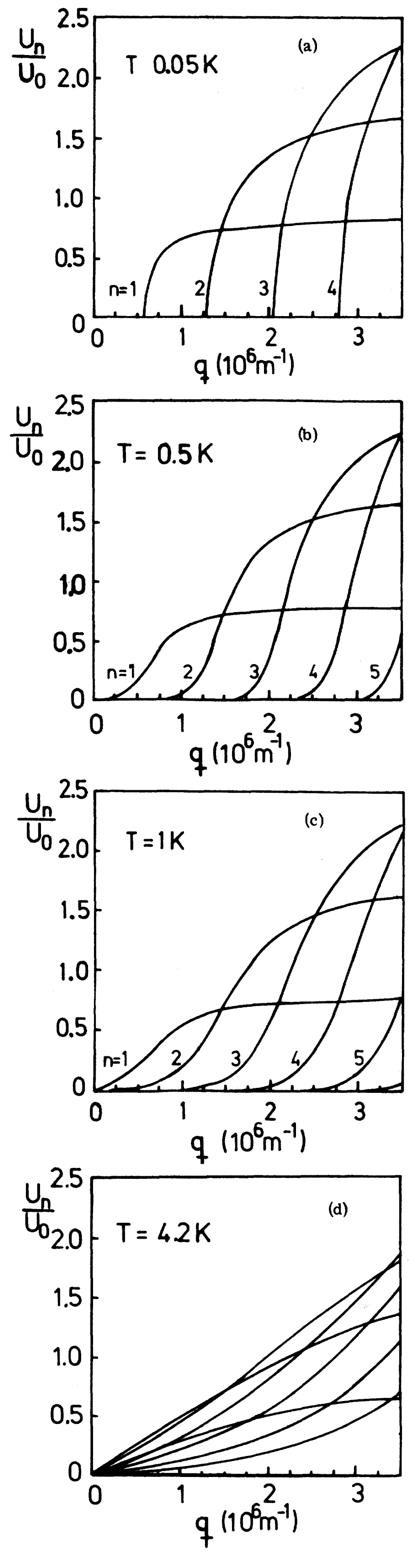

FIG. 6. Voltage contributions U_n from quasiparticle states with quantum number n vs ground-state momentum q at various temperatures T. It is assumed that the mean free path $l_N >> 2a = 3000$ Å.

than q_S in the S region. This results in a phase-matching problem of the wave functions at $z=\pm a$ which has not yet been resolved. Qualitatively we expect that q_N will replace q in Eq. (3.17), because the relevant quasiparticles are limited to the N region. With $q_N < q_S$ the total current (3.21) corresponding to a given voltage $U(q_N)$ will be greater than the current on the returning section of the CVC associated with a voltage of the same magnitude. Therefore, a suitable distribution of the increasing current over the N and S layers can lead to a nearly linear increase with current of the voltage contributions proportional to $n-\alpha_0$. In this mechanism we see the origin of the quantized resistances.

A linear current branch should terminate when in the S layers the critical current is reached and the whole film becomes normal. Alternatively, when the current has become near critical, part of it may shift into the N region and cause population of the next quantized level. This may account for the different ways of leaving a given current branch reported in connection with Figs. 3 and 4 of Ref. 2. When $q_N < q_0$ the whole cross section of the gapless superconductor can carry the current $I < I_0$ without appearance of a voltage. This explains the observed lower critical currents.[1-3]

The voltage of Eqs. (3.17) or (4.14), (6.2) and the corresponding resistances R_{eff} or dU_n/dI decreases as a^{-2} with the N-layer thickness $2a$. This agrees with the decrease of resistance with the magnetic field $\vec{H}$ which HGCK relate to a widening of the N layer by $\vec{H}$.[1-3]

Our Figs. 3–5 show low-temperature CVC for wide N regions with $2a = 15000$ Å $\approx 10\xi$, ξ is the coherence length. The qualitative similar returning sections of HGCK were taken from samples less than 10^4 Å thick at $T=4.2$ K. The theoretical limitation to large a is due to the necessity of having only states with $E_n^0 \ll \Delta$ contributing to the countercurrent. In a plot of voltage versus q for narrow N regions, i.e., $2a < 10^4$ Å, at $T \leq 0.05$ K the sum of the U_n from Fig 6(a) yields a dependence of $U=\sum_n U_n$ from q which is very similar to the CVC of Fig. 4. The expected large quasiparticle *counter*current in this case should result in current steps much smaller than the q steps.

From Fig. 5 we note that the steps in the CVC become less pronounced when kT becomes comparable to the spacing between the relevant quasiparticle levels. Figure 6(d), which exhibits $U_n(q)$ for narrow N regions at $T=4.2$ K, results in a practically smooth function $U(q)=\sum_n U_n(q)$. Apparently all structure is gone. It will reappear for two reasons.

First, we have to expect a large quasiparticle countercurrent in samples with narrow N regions at $T=4.2$ K. The total current I, which will be much less than the ground-state current (3.18), should increase proportional to q^η where $\eta < 1$, because the number of populated quasiparticle states will increase drastically as q makes higher states with energies close to that of a normal metal ($k_{ZF} \lesssim k_F$) available for occupation. Therefore, the voltages U_n should rise more steeply with $I(q)$ than they do with q in Fig. 6(d). A second and more important reason is given in the following discussion of mean free path and Pauli principle.

B. Quantitative comparison

We have calculated voltage and current for pure films with ideal lattices where the quasiparticles have such large mean free paths that spatially quantized levels can form between the pair potential walls. Only inelastic scattering from phonons has been taken into account implicitly when assuming that it establishes thermal equilibrium with the lattice so that the Fermi function (3.4) describes the quasiparticle distribution.[11] HGCK report[2] that the samples which yielded good results, had mean free paths l_N of slightly less than 10^3 Å (in the completely normal film). Scattering from surface roughness will contribute to the limitation of l_N. It will not be felt in the presence of superconducting surface sheaths by the quasiparticles with maximum Andreev scattering probability, since, decaying in the S layers, they are not influenced by the film surface. Also, in the *ISNSI* system there are less states into which quasiparticles with $E_n^0 < \Delta$ can be scattered than in the completely normal film. Thus, its mean free path should be larger than l_N. Still it remains doubtful, if these restrictions on scattering can increase the mean free path by an order of magnitude. This will be necessary in order to have well quantized levels, because the quasiparticles should travel unperturbed at least once or twice across the N region of several 10^3 Å thickness. We see from Eqs. (3.4)–(3.6) that the Fermi function $f(E)$ approaches 1 for states with $E<0$, and inelastic scattering with energy loss and elastic scattering out of these states become very unprobable because Pauli's exclusion principle strongly inhibits transitions to states with high occupation probability. Therefore, the mean free path of quasiparticle states, which in the presence of ground-state flow have $E_k < 0$, will become large enough to allow for spatial quantization with well-defined wavelengths despite lattice defects and impurities. Probably it is unjustified to count the states with positive E_k in the voltage equation (3.17), if one considers real films. Only states pulled below a certain temperature-dependent energy value E_T

≤ 0 according to Eq. (3.6), will have sufficiently long mean free paths which allow them to enter Eq. (3.17). Consequently, the difference of Fermi functions $f_1 - f_2$ in Eqs. (3.17), (4.1), (4.2), (4.14) should be replaced by the step function $\Theta(-E_k + E_T)$. This results in voltage steps like that of Fig. 3 and Eq. (6.2) even at the "high" temperature of 4.2 K.

Next, let us calculate the number N of states with k_0 that satisfy Eq. (3.9) with

$$\Delta \geq E_n^0(k_0) = \pi(n - \alpha_0)\hbar^2 k_0/2ma \ .$$

We define

$$k_0 \equiv s\kappa_0 = s(m\Delta/\hbar^2)^{1/2} \ , \quad \Delta \equiv p \times 10^{-3}\ \text{eV}, \quad a \equiv y \times 10^3\ \text{Å} \ , \tag{7.1}$$

and obtain

$$E_n^0(k_0) \approx [s(p)^{1/2}/y](n - \alpha_0) \times 1.37 \times 10^{-4}\ \text{eV} \ , \tag{7.2}$$

so that

$$N = 7.3(y/s)(p)^{1/2} + \alpha_0 \ . \tag{7.3}$$

HGCK report that their data were taken from films whose thickness varied between 2000 and 6000 Å. The film thickness was estimated by use of a quartz oscillator during evaporation.[2] Errors up to 30% of these estimates apparently are not excluded by the authors (see their comment below Fig. 14). Such relatively thin films produced by evaporation do not have a constant thickness along a length of several millimeters. Plane film and phase boundaries may exist only in certain local regions and be bigger or thinner there than the estimated average. The film from which Fig. 1 of Ref. 1 and Fig. 4 of Ref. 2 were taken is reported to have an average thickness of 4000 Å. Each surface sheath should have a thickness of several hundred Å. Thus, the normal region may have a thickness $2a$ between 3000 and 4000 Å (allowing for errors of 30%) which locally may be even larger. Therefore, we may estimate that in Eqs. (7.1) and (7.3) $1.5 \leq y \leq 2$. According to Fig. 1, $2 \lesssim s \lesssim 2.5$ for reflection from a single surface sheath. $(p)^{1/2}$ will not differ much from 1. For the states with $E^0 \ll \Delta$, $0 \leq \alpha_0 \leq \frac{1}{2}$. For the higher states the range of α_0 could not be determined accurately but trivially it does not exceed 1. Consequently we have $5 \leq N \leq 8$. While 5 is too low, 8 comes closer to the number of 10 quantized resistance branches.[1,2] The remaining difference may be due to local thickness variations.

Finally, let us come to the magnitude of the quasiparticle induced voltage. As we can see from Fig. 6(a)—or Fig. 3 if there we replace I by q for the narrow N-region case—the length of an individual voltage step is of the order of U_0 defined by Eq. (6.4). We take the data of the lead film whose CVC is given in Ref. 1: length $L = 5$ mm average thickness $2D = 4000$ Å. We define

$$k_F \equiv x\ \text{Å}^{-1} \ , \tag{7.4}$$

use

$$n_s = k_F^3/3\pi^2$$

and the definitions (7.1). This yields

$$U_0 = 0.71 prs/x^2 y^2 \times 10^{-3}\ \text{V} \ . \tag{7.5}$$

As discussed below Eq. (6.1), for a single Andreev reflection $4 \lesssim r < 10$. We may assume that multiple reflections reduce the upper and lower limits of r by a factor of about 2. The value of x is approximately 1. (If one determines k_F of Pb from the free-electron density model one finds $k_F = 1.57$ Å^{-1}. Specific-heat data and the BCS relation between Fermi velocity and coherence length yield $k_F \approx 0.5$ Å^{-1}.) Several reasonable combinations of the parameters p, r, s, x, and y from their respective ranges of value are possible which yield $U_0 \approx 0.5 \times 10^{-3}$ V. This is the average length of the voltage steps one obtains from Fig. 1 of Ref. 1 or Fig. 4 of Ref. 2.

The quasiparticle induced voltage has been calculated microscopically without resorting to phenomenological models. The five parameters appearing in Eq. (7.5) have relatively narrow ranges of value. The experimental and theoretical order of magnitude of the voltage agree. This agreement and the common qualitative features of theory and experiment justify some confidence that the present theory is not irrelevant to the phenomenon of the quantized resistances. Nevertheless, the problems left open in this paper require further investigations.

ACKNOWLEDGMENTS

I wish to thank J. T. Chen, L. M. Geppert, L. G. Hayler, and Y. W. Kim for information on their work prior to its publication.

APPENDIX A: DERIVATION OF THE EIGENVALUE EQUATIONS

We demand that the electron components $u(\vec{r})$ and the hole components $v(\vec{r})$ of the quasiparticle wave functions as well as their gradients each join smoothly at the phase boundaries at $z = \pm a$. Observing furthermore Eq. (2.5) we obtain 12 homogeneous equations for the integration constants the determinant D of which is

$$\frac{D}{k_{ZF}^4}=\begin{vmatrix}
\nu\rho & \nu/\rho & \tau/\nu & 1/\nu\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\rho/\nu & 1/\nu\rho & \nu\tau & \nu/\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1/\nu\tau & \tau/\nu & \nu/\rho & \nu\rho & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \nu/\tau & \nu\tau & 1/\nu\rho & \rho/\nu & 0 & 0 & 0 & 0\\
\nu\gamma & \nu/\gamma & \delta/\nu & 1/\nu\delta & 0 & 0 & 0 & 0 & \alpha & 1/\alpha & 0 & 0\\
\gamma/\nu & 1/\nu\gamma & \nu\delta & \nu/\delta & 0 & 0 & 0 & 0 & 0 & 0 & \beta & 1/\beta\\
0 & 0 & 0 & 0 & 1/\nu\delta & \delta/\nu & \nu/\gamma & \nu\gamma & 1/\alpha & \alpha & 0 & 0\\
0 & 0 & 0 & 0 & \nu/\delta & \nu\delta & 1/\nu\gamma & \gamma/\nu & 0 & 0 & 1/\beta & \beta\\
\nu\gamma\theta & -(\nu/\gamma)\theta & (\delta/\nu)\omega & -\omega/\nu\delta & 0 & 0 & 0 & 0 & \alpha & -1/\alpha & 0 & 0\\
(\gamma/\nu)\theta & -\theta/\nu\gamma & \nu\delta\omega & -\nu\omega/\delta & 0 & 0 & 0 & 0 & 0 & 0 & \beta & -1/\beta\\
0 & 0 & 0 & 0 & \omega/\nu\delta & -\delta\omega/\nu & \nu\theta/\gamma & -\nu\gamma\theta & 1/\alpha & -\alpha & 0 & 0\\
0 & 0 & 0 & 0 & \nu\omega/\delta & -\nu\delta\omega & \theta/\nu\gamma & -(\gamma/\nu)\theta & 0 & 0 & 1/\beta & -\beta
\end{vmatrix}. \tag{A1}$$

Supposing that ϵ is small, see Eq. (2.4), we have approximated

$$k_{ZF}(1\pm\epsilon)^{1/2}\approx k_{ZF}, \tag{A2}$$

wherever it appeared as a factor multiplying an exponential. $\alpha,\beta,\gamma,\delta$ are $\alpha(a),\beta(a),\gamma(a),\delta(a)$ according to Eq. (2.3) and $\rho\equiv\gamma(D),\tau\equiv\delta(D)$, $\theta\equiv(1+i\delta_E)^{1/2}$, $\omega\equiv(1-i\delta_E)^{1/2}$.

The determinant can be simplified by adding and subtracting rows and columns multiplied by appropriate factors. With the abbreviations

$$A\equiv\tfrac{1}{2}[(\delta/\tau-\tau/\delta)+\omega(\delta/\tau+\tau/\delta)],\quad B\equiv-\tfrac{1}{2}[(\delta/\tau-\tau/\delta)-\omega(\delta/\tau+\tau/\delta)],$$
$$C\equiv\rho/\gamma-\gamma/\rho,\quad D\equiv\delta/\tau-\tau/\delta,\quad E\equiv\alpha^{-2}-\alpha^2,\quad F\equiv\beta^{-2}-\beta^2,\quad G\equiv\tfrac{1}{2}(1-\theta), \tag{A3}$$

this leads to

$$D/16k_{ZF}^4(\nu^2-\nu^{-2})^2\equiv D'=\begin{vmatrix}
C & \gamma/\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -C & \rho/\gamma & 0 & 0 & 0 & 0\\
0 & \nu & D/\nu & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 1/\nu & \nu D & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & -D/\nu & 0 & \nu & E & \alpha^2 & 0 & 0\\
0 & 0 & 0 & -\nu D & 0 & 1/\nu & 0 & 0 & F & \beta^2\\
\nu\theta & \nu G & A/\nu & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
\theta/\nu & G/\nu & \nu A & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & B/\nu & \nu\theta & \nu G & 1/\alpha^2 & 0 & 0 & 0\\
0 & 0 & 0 & \nu B & \theta/\nu & G/\nu & 0 & 0 & 1/\beta^2 & 0
\end{vmatrix}. \tag{A4}$$

Expansion of this determinant yields

$$D' = -\left[CD\theta\left(\frac{\rho}{\gamma}(A-DG)+\frac{\gamma}{\rho}(B+DG)\right)+C^2(A-DG)(B+DG)+D^2\theta^2\right]$$
$$+\left[BCDG\left(\frac{\rho}{\gamma}\theta-\frac{\gamma}{\rho}\theta+CG\right)+ABC^2G-ABC\theta\frac{\gamma}{\rho}-BD\theta^2\right]\left(\frac{\alpha^2}{\beta^2}(\nu^{-4}-1)+\frac{\beta^2}{\alpha^2}(\nu^4-1)\right)$$
$$+\left[ACDG\left(\frac{\gamma}{\rho}\theta-\frac{\rho}{\gamma}\theta-CG\right)+ABC^2G+ABC\theta\frac{\rho}{\gamma}+AD\theta^2\right]\left(\frac{\alpha^2}{\beta^2}(\nu^4-1)+\frac{\beta^2}{\alpha^2}(\nu^{-4}-1)\right)$$
$$-\left[ABCG\left(\frac{\rho}{\gamma}\theta-\frac{\gamma}{\rho}\theta+CG\right)-AB\theta^2\right](\nu^2-\nu^{-2})\left(\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}\right)+ACD\left(\frac{\rho}{\gamma}\theta+CG\right)\left(\frac{\alpha^2}{\beta^2}\nu^4+\frac{\beta^2}{\alpha^2}\nu^{-4}\right)$$
$$+BCD\left(\frac{\gamma}{\rho}\theta-CG\right)\left(\frac{\alpha^2}{\beta^2}\nu^{-4}+\frac{\beta^2}{\alpha^2}\nu^4\right)+\left[CD^2G\left(\frac{\gamma}{\rho}\theta-\frac{\rho}{\gamma}\theta-CG\right)+ABC^2+D^2\theta^2\right]\left(\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}\right)$$
$$+\alpha^2\beta^2(\nu^2-\nu^{-2})^2\left\{\left[BCDG\left(\frac{\gamma}{\rho}\theta-\frac{\rho}{\gamma}\theta-CG\right)-ABC^2G-ABC\theta\frac{\rho}{\gamma}+BD\theta^2\right]\right.$$
$$\left.+\left[ABCG\left(\frac{\rho}{\gamma}\theta-\frac{\gamma}{\rho}\theta+CG\right)+BC^2DG+BCD\theta\frac{\rho}{\gamma}-AB\theta^2\right]\right\}$$
$$+\frac{1}{\alpha^2\beta^2}(\nu^2-\nu^{-2})^2\left\{\left[ACDG\left(\frac{\rho}{\gamma}\theta-\frac{\gamma}{\rho}\theta+CG\right)-ABC^2G+ABC\theta\frac{\gamma}{\rho}-AD\theta^2\right]\right.$$
$$\left.+\left[ABCG\left(\frac{\rho}{\gamma}\theta-\frac{\gamma}{\rho}\theta+CG\right)-AC^2DG+ACD\theta\frac{\gamma}{\rho}-AB\theta^2\right]\right\}. \quad \text{(A5)}$$

In the limit $E^0 \ll \Delta$ we have $\nu = \exp(i\pi/4)$ and $\delta_E = 2m\Delta/k_{zF}^2$. After elementary but lengthy calculations and with the expansion $(1+\epsilon)^{1/2} \approx 1+\frac{1}{2}\epsilon$, Eq. (A5) and the condition $D'=0$ lead to Eq. (2.6). [As usual the set of negative energies $\sim(n+\alpha')$, $n<0$, is being discarded.]

If we allow for the full energy range $0<E^0<\Delta$ and regard k_{zF} values sufficiently large so that $(1\pm i\delta_E)^{1/2}\approx 1\approx\theta\approx\omega$, we have $A=\delta/\tau$, $B=\tau/\delta$, and $G=0$ in the set of definitions (A3). Then D' of Eq. (A5) simplifies considerably, and we find it to be equal to the right-hand side of Eq. (2.8).

APPENDIX B: PROBABILITY OF ANDREEV SCATTERING

Let us consider an electron above the Fermi surface of energy $E^0<\Delta$ coming from a semi-infinite normal region and incident on a superconducting layer with pair potential Δ and extension between $0\le z\le T$. At $z=T$ an infinitely high potential wall rises. The electron may either be scattered into a hole or reflected into itself at $z=0$ or $z=T$. The most general solution of the Bogoliubov equations for this situation can be obtained from Eq. (2.2) if there we replace $\Theta(a-|z|)$ by $\Theta(-z)$, $\Theta(z-a)$ by $\Theta(z)$, and put $b_2=0=B_1=B_2=B_3=B_4$. We may also choose $q=0$. Then, the probability of particle-hole scattering is given by $|b_1/a_1|^2\equiv P_0$.

The coefficients of the wave functions are determined by four matching conditions at $z=0$ and the boundary condition at $z=T$ in complete analogy to Sec. II and Appendix A. With the new set of abbreviations,

$$k_z^{\pm}\equiv k_{zF}(1\pm\epsilon)^{1/2},\quad k_\Delta^{\pm}\equiv k_{zF}(1\pm i\delta_E)^{1/2},$$
$$\alpha\equiv i(\Delta^2/E^{02}-1)^{1/2},\quad \theta_\pm\equiv\exp(ik_\Delta^\pm T), \quad \text{(B1)}$$

we obtain

$$b_1/a_1=-2(1-\alpha^2)^{1/2}\theta_+\theta_-k_z^+[(k_\Delta^+-k_\Delta^-)(1-\theta_+^{-2}\theta_-^{-2})+(k_\Delta^++k_\Delta^-)(\theta_+^{-2}-\theta_-^{-2})]$$
$$\times\big((\theta_+-\theta_+^{-1})\{(\theta_--\theta_-^{-1})2\alpha k_z^+k_z^-+(\theta_-+\theta_-^{-1})k_\Delta^-[(1-\alpha)k_z^-+(1+\alpha)k_z^+]\}$$
$$-(\theta_++\theta_+^{-1})\{(\theta_-+\theta_-^{-1})2\alpha k_\Delta^+k_\Delta^-+(\theta_--\theta_-^{-1})k_\Delta^+[(1-\alpha)k_z^++(1+\alpha)k_z^-]\}\big)^{-1}. \quad \text{(B2)}$$

Supposing that $\epsilon=2mE^0/k_{zF}^2$ is sufficiently small compared to 1 so that one may retain only the first order of it, we can further evaluate Eq. (B2).

With the dimensionless units

$$x\equiv k_{zF}/\kappa_E,\quad \tilde{T}\equiv\kappa_E T,\quad \text{where}\quad \kappa_E\equiv(m\Delta)^{1/2}(1-E^{02}/\Delta^2)^{1/4},$$

and the abbreviation $\xi_\pm\equiv(1+4/x^4)^{1/2}\pm1$, we finally obtain the probability of particle-hole scattering

$$P_0(k_{ZF}, E^0, T) \equiv |b_1/a_1|^2 = (\Delta/E^0)^2(1+2mE^0/k_{ZF}^2)\{(\xi_-)^{1/2}\sin[\sqrt{2}(\xi_+)^{1/2}x\tilde{T}] - (\xi_+)^{1/2}\sinh[\sqrt{2}(\xi_-)^{1/2}x\tilde{T}]\}$$
$$\times(\tfrac{1}{2}(\Delta^2/E^{02}-1)\{\xi_-\cos[\sqrt{2}(\xi_+)^{1/2}x\tilde{T}] + \xi_+\cosh[\sqrt{2}(\xi_-)^{1/2}x\tilde{T}]\}^2$$
$$+\{[x^{-2}(\xi_+)^{1/2} - (\xi_-)^{1/2}]\sin[\sqrt{2}(\xi_+)^{1/2}x\tilde{T}]$$
$$+[(\xi_+)^{1/2} + x^{-2}(\xi_-)^{1/2}]\sinh[\sqrt{2}(\xi_-)^{1/2}x\tilde{T}]\}^2)^{-1}. \quad \text{(B3)}$$

Figure 1 shows the probability $P_0(k_{ZF}, E^0, T) \to P_0(k_{ZF})$ as calculated numerically from Eq. (B3) for different thickness values T of the surface sheath. It depends only very little upon E^0.

The maximum of $P_0(k_{ZF})$ lies near $k_{ZF} \approx 2.5\kappa_0 \equiv k_0$, $\kappa_0 = \kappa_{E=0}$. Eq. (2.9) is valid, if $4m\Delta(D-a)/k_{ZF} > 3$. $4m\Delta(D-a)/k_0 = 4m\Delta(D-a)/2.5\sqrt{m\Delta}$ exceeds 3, if $T \equiv (D-a) > 2/\kappa_0$. As we see from Fig. 1, $P_0(k_{ZF})$ rises up to 1, when the surface layer thickness is $T > 2/\kappa_0$.

*Work supported in part by the Deutsche Forschungsgemeinschaft.

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