Heavy-ion stopping powers and the low-velocity-projectile z^3 effect

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(Received 6 May 1977)

Recent heavy-ion stopping-power measurements with elemental solid targets have been analyzed in order to ascertain the influence on effective ion charge of incorporating the low-velocity-projectile z^3 effect in Bethe-Bloch calculations. Shell corrections and the mean excitation energy of a given target were held fixed while searching for the best-fit value of a single charge-state parameter. In general, excellent fits to the stopping powers at projectile energies above 0.3 MeV/amu were achieved. Results of the present study compare very favorably with those from other extant methods of analysis.

I. INTRODUCTION

In the course of conducting experiments in numerous areas of physics, it is frequently necessary to know with great precision the energy lost by massive charged particles traversing matter. A means of calculating energy losses of swift projectiles exists in the form of Bethe-Bloch theory,[1] from which the electronic stopping power of a given target can be found for a broad interval of incident ion energies. Unfortunately the Bethe-Bloch expression for stopping power contains quantities known as the mean excitation energy and shell corrections, neither of which can in general be reliably calculated. (At very high projectile velocities, a density-effect correction also becomes necessary, but the present discussion pertains to energies considerably below that relativistic region.) Hence the mean excitation energy and shell corrections are often obtained through fits of Bethe-Bloch theory to range and stopping-power measurements. Further complications arise from the fact that the validity of Bethe-Bloch theory rests on applicability of the first Born approximation, from which it follows that stopping power depends on the square of the projectile charge, ze. Deviations from the projectile z^2 dependence were observed more than two decades ago when results of experimental pion-mass determinations suggested that oppositely-charged projectiles of the same mass and velocity lost energy at different rates while traversing matter.[2] The basis of these departures from Bethe-Bloch theory, sometimes called the Barkas effect,[3] was conjectured by Barkas to be the failure of the first Born approximation, so that a projectile z^3 correction term was required in the stopping-power formula. A brief history of the investigation of this effect has been outlined recently.[4,5] Two quantitative formalisms were developed for treatment of the projectile z^3 effect—one for very low velocities[6] and the other for both very low and very high velocities.[7] Ramifications for data analyses including the low-velocity-projectile z^3 correction[6] have been explored for stopping-power measurements with composite targets[4,5,8] and with elemental targets.[9,10] All of these studies[4,5,8-10] dealt with projectiles no more massive than α particles, and with energies at which the projectile could be considered completely ionized.[1] At very low projectile velocities, however, the effective projectile charge, z^*e, depends[1] on the velocity as well as on the basic projectile charge, ze. Hence a Bethe-Bloch stopping-power calculation at low projectile velocity requires knowledge of three effects, all of whose magnitudes vary differently with projectile velocity: the shell corrections, the projectile z^3 correction, and the charge-state correction.

One effective method[11] of analyzing range and stopping-power measurements has utilized the Bethe-Bloch formula, with shell corrections obtained from the theoretical calculations of Walske[12,13] supplemented by empirically determined scaling parameters and with charge-state information based on numerous previous studies. The method was later systematized by interpolating and extrapolating original shell correction scaling parameters[11] to obtain suggested values for all elements.[14] With the advent of the low-velocity-projectile z^3 theory,[6] the original computer code[11] was modified so as to include this correction and thus to investigate the effects on data analysis of such inclusion.[4] That is, omission of the projectile z^3 effect conceivably would distort both the mean excitation energy and the shell-correction scaling parameters extracted from stopping-power measurements. However, an independent study by Ashley[9] of measurements made with elemental targets of atomic numbers between 20 and 30 and with hydrogen-isotope projectiles indicated that previously obtained mean excitation energies were essentially free of distortion. Ashley developed a technique for evaluating the shift in shell corrections induced by inclusion of the projectile z^3 effect[9]; Shepard and Porter extended the method in order to deal with composite targets and to utilize the revised shell corrections in analyzing stopping-power measurements made with α particles.[5] Further measurements with α particles traversing elemental targets[15,16] have recently been analyzed in the same way[10]; no intrinsic limitation exists to prohibit application of the formalism to heavy-ion stopping-power measurements. However, extant charge-state information arose from investigations which did not include a projectile z^3 effect in data analysis. Whereas deviations from the projectile z^2 dependence of the Bethe-Bloch formula were acknowledged in a recent review, these effects were then implicitly included in the charge-state parametrization.[1] Such a procedure merely combines two corrections to the Bethe-Bloch formula, of course. However, the two corrections possess very different dependences on projectile velocity. Reduction of the effective projectile charge with decreasing velocity produces a decrease in stopping power, whereas the low-velocity projectile z^3 effect causes stopping power to increase with decreasing velocity. Thus it appeared desirable to sort out the effects of the two corrections to Bethe-Bloch calculations. Fortunately, available shell-correction scaling parameters[11,14] had been extracted from measurements at projectile velocities sufficiently high to ensure that the basic projectile charge prevailed.[11] Consequently, the existing shell-correction modification technique[5,9] could be used to establish shell-correction scaling parameters free of projectile-z^3-effect distortions and, subsequently, to apply the modified Bethe-Bloch formalism to basic projectile charge and velocity combinations for which the effective charge required evaluation.

To this end some recent heavy-ion stopping-power measurements[16,17] were analyzed with a single parameter characterizing the effective charge.[18] The generally excellent fits achieved, especially in comparison with predictions of the popular semi-empirical tabulation of heavy-ion stopping powers,[19] encouraged analysis of further data. Thus measurements of stopping powers of six elemental targets for five heavy ions[20] were similarly investigated, with notable success.

II. ANALYSIS AND RESULTS

The effective-charge theory of the electronic stopping power of matter for heavy ions originated by Knipp and Teller[21] has recently been reviewed and revised by Sauter and Bloom.[22,23] Basic assumptions of the original theory[21] are two in number[22]: (i) The electronic stopping power S_c of a given material for a heavy ion of atomic number z and velocity v can be written as the product of two factors:

$$S_c = (\gamma^2 z^2) S_0 \,, \tag{1}$$

where S_0 is the stopping power of a proton with the same velocity. The factor γ, called the effective-charge parameter, should be independent of the traversed medium. (ii) The factor γ^2 should possess a functional dependence only on the quantity $\epsilon/z^{4/3}$, where ϵ is the ion energy (in MeV/amu) in the laboratory reference frame. Of course, assumption (i) rests on applicability of first-order perturbation theory.

A satisfactory fit of considerable experimental data was achieved in a study by Pierce and Blann[24] by means of the parametrization

$$\gamma = 1 - \exp(-0.95 v_r) \,. \tag{2}$$

Here the reduced velocity parameter v_r is the ratio of ion velocity in the laboratory frame v to the Thomas-Fermi electron velocity $(e^2/\hbar) z^{2/3}$; hence $v_r = \beta/\alpha z^{2/3}$, where β is the relativistic velocity parameter v/c, and α is the fine-structure constant. This expansion, whose consistency with basic assumption (ii) above is difficult to establish,[23,24] yielded electronic stopping powers accurate to within 8% for ion energies above 0.3 MeV/amu. At lower energies, a more complicated expression for γ was necessary.[24]

Development of the low-velocity-projectile-z^3-effect theory[6,7] and reports of considerable heavy-ion stopping-power data[16,17,20] subsequent to the Marshall and Blann study[24] suggested the present investigation of effective charge in the presence of the projectile z^3 correction.[6] In analyzing accurate measurements of the stopping power of Si for ^{12}C, ^{14}N, and ^{16}O projectiles at energies from 2 to 10 MeV/amu, inclusion of the projectile z^3 effect was found to be necessary and the consequence of such inclusion on effective charge was discussed.[17] However, in contrast to the present investigation, Bethe-Bloch stopping power theory was not directly utilized in the earlier study.[17] Measurements by the Chalk River group[16,20] were orignally analyzed with neither Bethe-Bloch theory nor the projectile z^3 modification. All of these data, for projectile energies above 0.3 MeV/amu, have been subjected to fits with Bethe-Bloch calculations[1,11] as modified by the low-velocity-projectile z^3 correction.[6] The effective projectile charge has been characterized by the form of Pierce and Blann,[24] with the constant 0.95 appearing in Eq. (2) replaced by a search parameter λ. Every other parameter of the formulation was fixed at a value based on previous fits to experimental data. The form and notation of the shell corrections have recently been described in detail.[10] (In brief, the shell corrections of Walske[12,13] for the K and L shells are utilized directly. The M- and N-shell corrections are given the same form as the L-shell correction, with provision for modification by scaling parameters.) Shell-correction scaling parameters obtained from extensions[14] of numerous analyses[11] of experiments with $z=1$ projectiles were used at the outset. In the course of calculations, the shell corrections were revised as a sum to reflect inclusion of the projectile z^3 effect.[5] Mean excitation energies I_0 were selected from values cited in several compilations,[1,25-29] as described previously.[10] The various recommended and selected values are displayed in Table I, along with utilized shell-correction scaling parameters in the notation of Ref. 11. The single available parameter of the low-velocity-projectile z^3 correction was set at the value recommended[6] on the basis of fits to crucial Barkas effect data.[30] Only the effective-charge parameter λ remained to be established in the present investigation. The possibility of determining all parameters simultaneously clearly exists, but generally in such attempts the solution set lacks uniqueness,[1] especially if the measurements are not of great accuracy. A manifestation of this difficulty arose in an earlier attempt to determine merely two parameters in fits of stopping-power data for α particles traversing various composite targets of low atomic number.[31] Thus the variation of a single-effective-charge parameter with different projectile-target pairs was studied, using preselected values of all other parameters.

The most accurate experimental data[17] analyzed were the 2% measurements of stopping powers of Si for ^{12}C, ^{14}N, and ^{16}O projectiles with energies from 1 to 10 MeV/amu. In this case, the basic measurements had been smoothed by Kelley *et al.* through a three-parameter fit.[17] Measurements of stopping powers of Ni, Ge, Y, Ag, and Au for ^{16}O and ^{35}Cl projectiles with energies from 1 to 3 MeV/nucleon featured an accuracy of 4%; these data had been smoothed through an interpolation procedure by Ward *et al.*[16] Finally, measurements encompassing stopping powers of Ti, Fe, Ni, Cu, Ag, and Au for ^{19}F, ^{24}Mg, ^{27}Al, ^{32}S, and ^{35}Cl projectiles with energies from 0.15 to 4.00 MeV/nucleon were considered accurate to 4% or, in cases of short extrapolations, to 6%; these data had been smoothed by means of an interpolation technique by Forster *et al.*[20] In the present analysis, only energies above 0.3 MeV/nucleon were included, so that the few cases of extrapolation studied were those at the highest energies. Results of the best-fit calculations with modified Bethe-Bloch theory are summarized in Table II, were the value of λ is displayed along with the corresponding figure of merit σ for each combination of projectile-target atomic numbers (z, Z). (σ represents the root-mean-square relative deviation of calculated from measured stopping-power values.) The Si target data[17] manifest worsening fits with increasing z, but the average value of σ is 1.53, indicating an overall fitting capability of about 3%. The data of Ward *et al.*[16] for ^{16}O projectiles yield an average σ of 0.82, and for ^{35}Cl projectiles a corresponding average of 1.11; hence these measurements have been fitted within 4%.

TABLE I. Mean excitation energies of the target elements extracted from various sources, along with the values selected for the present study, I_0, and shell correction scaling parameter values used in the notation of Ref. 11.

Target	Mean excitation energies (eV)							Shell correction scaling parameters[a]			
	Ref. 1	Ref. 25	Ref. 26	Ref. 27	Ref. 28	Ref. 29	I_0	HM	HN	VM	VN
Si	174	170	172	158	···	···	172	1.4	0.0	0.70	0.0
Ti	230	227	246	222	229 ±2	···	230	4.6	0.0	1.00	0.0
Fe	285	273	285	290	280 ±2	···	284	5.5	0.0	1.00	0.0
Ni	305	312	305	320	303 ±4	···	304	5.5	0.0	1.00	0.0
Cu	320	320	315	350	321 ±3	320 ±3	320	5.0	0.0	1.00	0.0
Ge	360	350	345	370	···	···	358	5.5	0.0	0.45	0.0
Y	···	···	400	395	···	···	400	2.7	0.0	1.10	0.0
Ag	475	465	480	505	···	469 ±8	476	3.0	2.5	0.40	1.0
Au	780	780	770	830	···	771 ±20	790	2.3	9.8	1.60	2.7

[a] Shell corrections are obtained from the sum, $\sum_i C_i = B1 \times C_k(\beta^2) + B2 \times C_L(\beta^2) + VM \times C_L(HM \times \beta^2) + VN \times C_L(HN \times \beta^2)$, where C_K and C_L are the K- and L-shell corrections, respectively, calculated by Walske (Refs. 12 and 13). In the present study both $B1$ and $B2$ were set at 1.0, indicating no modification of the C_K and C_L values of Walske (Refs. 12 and 13). Corrections for the M and N shells are taken to be of the same form as the L-shell correction, with the adjustable scaling parameters HM, HN, VM, and VN.

TABLE II. Values of the effective-charge parameter λ which provided the best fit to the data for each projectile-target pair along with the corresponding root-mean-square relative deviation of calculated from measured values σ.

λ and corresponding σ for each projectile-target (z, Z) combination												
Ref. 16 data[a]			Ref. 20 data[b]						Ref. 17 data[c]			
Z	z: 8	17	Z	z: 9	12	13	16	17	Z	z: 6	7	8
28	0.780	0.724	22	0.836	0.849	0.845	0.820	0.802	14	1.282	0.923	1.090
	0.38	1.22		1.27	1.07	0.99	0.73	0.82		0.94	1.32	2.32
32	0.780	0.700	26	0.793	0.792	0.792	0.789	0.779				
	0.33	1.76		0.53	0.70	0.95	0.80	0.77				
39	0.810	0.729	28	0.755	0.740	0.743	0.747	0.738				
	0.63	0.48		0.60	1.30	1.35	1.70	1.75				
47	0.724	0.711	29	0.780	0.765	0.766	0.768	0.756				
	1.14	0.73		0.47	0.83	0.94	1.26	1.15				
79	0.694	0.687	47	0.754	0.760	0.758	0.755	0.742				
	1.62	1.35		1.30	0.93	0.88	0.27	0.18				
			79	0.770	0.754	0.752	0.730	0.723				
				1.92	1.67	1.81	1.43	1.53				

[a] Ward *et al.* [b] Forster *et al.* [c] Kelley *et al.*

Similarly, the measurements of Forster *et al.*[20] accept fits with an overall average σ of 1.06, with the average for each projectile lying in the interval from 1.02 (^{19}F) to 1.15 (^{27}Al). These data have therefore been fitted within 5%. Variations in λ as a function of target and projectile atomic numbers will be discussed below.

Graphs of calculated and measured stopping powers are shown in Figs. 1–8. Predictions from the Northcliffe and Schilling tables[19] appear for many of the target-projectile combinations. Figure 1 shows that the calculated stopping powers of Si for ^{12}C and ^{14}N agree well with measurements; a poorer fit in the case of ^{16}O projectiles can likely be attributed to the rather large amount of scatter in the measurements. In Fig. 2 an excellent correlation between calculated and measured stopping powers for ^{16}O projectiles on Ni, Ge, and Y is obvious; fits to Ag and Au are less satisfactory in the sense that calculations exceed measurements at all but the lowest energies, where calculated curves turn over so sharply as to fall considerably short of the measured values. Northcliffe and Schilling predictions[19] for the cases of Ni and Ge are clearly inferior to the present calculation, but those for Ag and Au evince essentially the same disagreements as those based on modified Bethe-Bloch theory. Each of the latter calculations could be improved by choosing a larger mean excitation energy—perhaps a value closer to that characteristic[10] of the α particle data[16] for the same target—but this sort of departure from the fitting procedure could be defended only if the suspiciously high mean excitation energies from the α-particle study[10] were to be proved correct by future corroboration. Data for ^{35}Cl projectiles, shown in Fig. 3 along with calculated curves, do not agree so well with theory as the ^{16}O data. Only the fits of measurements with Y and Ag targets qualify as excellent. In the case of the Au target, the low-energy data are fitted very well whereas calculated values exceed measurements elsewhere. The low-velocity-projectile z^3 correction[6] is large for combinations of large projectile and target atomic numbers. For example, at a ^{35}Cl projectile energy of 12 MeV the projectile z^3 contribution to the total calculated stopping power of Au is slightly more than a third, so that the agreement of calculation with measurement at low energies is quite remarkable. Figure 4 contains stopping powers of six elemental targets for the ^{19}F projectiles. Whereas the fit to the Ti data is good, agreement of theory and experiment is excellent for the Fe, Ni, and Cu data. A fit of the Au data again proved difficult; only in this case did the Northcliffe and Schilling prediction[19] prove superior. The stories of ^{24}Mg and ^{27}Al projectiles, told in Figs. 5 and 6, respectively, are essentially the same as that of ^{19}F, except that the fits for Fe, Ni, and Cu have deteriorated slightly. The cases of ^{32}S and ^{35}Cl projectiles, summarized in Figs. 7 and 8, respectively, indicate an improvement in quality of fit to Ti and Au, and especially to Ag, measurements, but a minor worsening in quality of fit to Ni and Cu data. Whereas the present calculation furnishes a fit markedly superior to that of Northcliffe and Schilling[19] for the Ti and Ag targets, the two types are roughly equal in quality for Ni and Au targets. Calculated stopping powers consistently larger than measurements at the higher energies for all projectiles on Au targets suggested again an upward revision of the mean excitation energy for that elemental target. A revision was considered unacceptable for the reasons stated above, but some

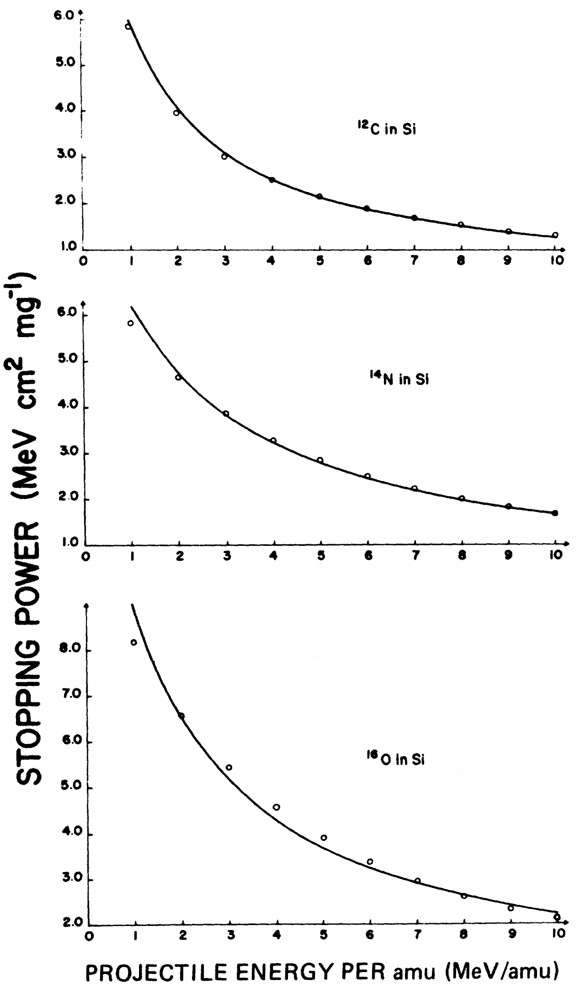

FIG. 1. Stopping powers of Si for ^{12}C, ^{14}N, and ^{16}O projectiles as measured by Kelley *et al*. (data circles—Ref. 17), and as calculated in the present study (solid curve). Uncertainties in these 2% measurements are indicated roughly by the diameter of the data circles.

variations of σ with λ and I values were studied; a higher I value induces a compensatory upward shift in λ, of course, and the resulting quality of fit does not always improve. An obvious explanation for these results with Au would be the presence of errors in the projectile z^3 correction[6] for high-Z targets; yet in a previous investigation of the stopping powers of Ag and Au for α particles, the best-fit mean excitation energies remained anomalously large even when the projectile z^3 correction was excluded from the Bethe-Bloch formalism.[10] Hence the mean excitation energies of Ag and Au were left fixed at the values based on several other compilations,[1,25-29] and the projectile z^3 effect was retained intact.

Variations of λ as a function of z and Z show general trends which are apparent in Table II. For a

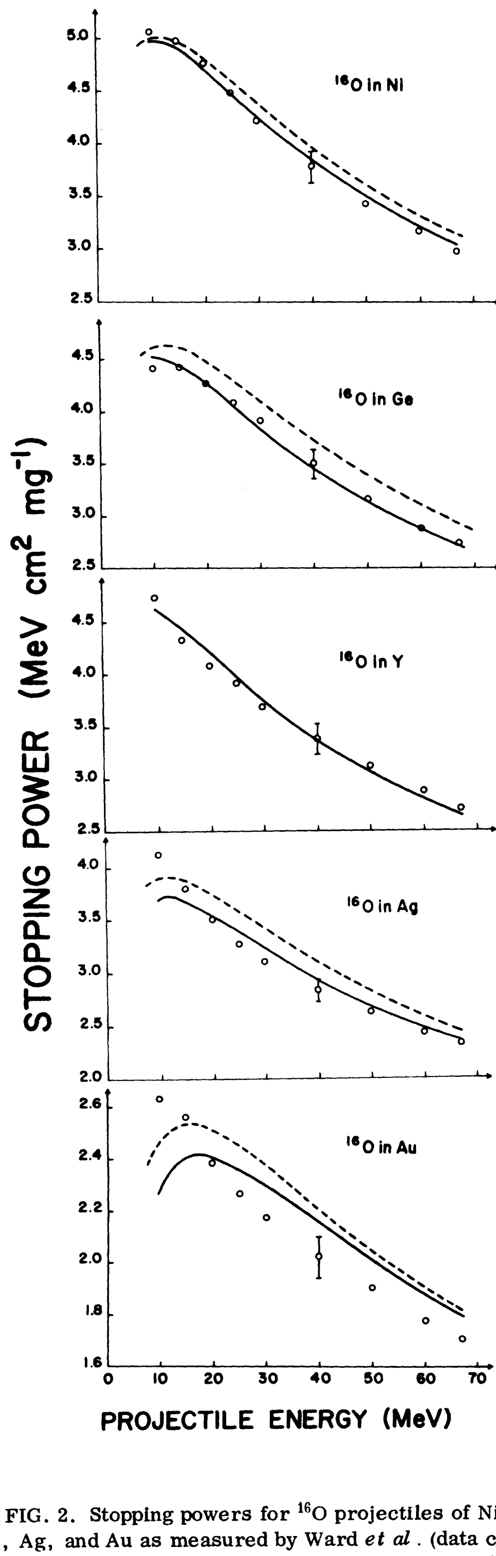

FIG. 2. Stopping powers for ^{16}O projectiles of Ni, Ge, Y, Ag, and Au as measured by Ward *et al*. (data circles—Ref. 16), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

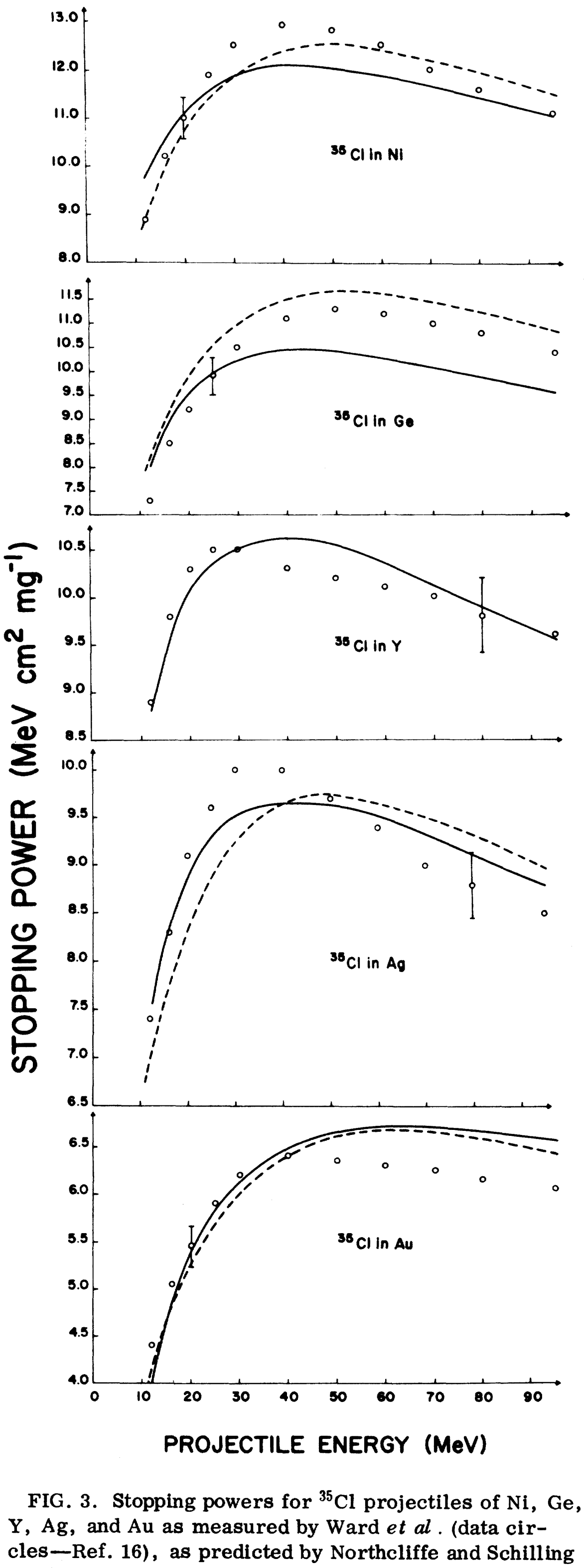

FIG. 3. Stopping powers for ^{35}Cl projectiles of Ni, Ge, Y, Ag, and Au as measured by Ward *et al*. (data circles—Ref. 16), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

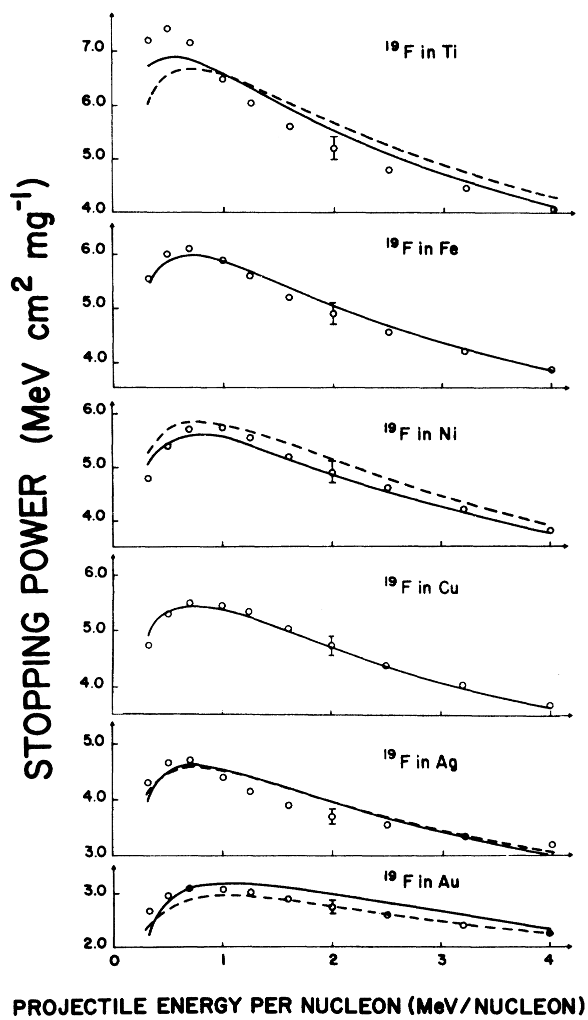

FIG. 4. Stopping powers for ^{19}F projectiles of Ti, Fe, Ni, Cu, Ag, and Au as measured by Forster *et al*. (data circles—Ref. 20), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

fixed z, the dependence of λ on Z is extremely complicated. However, for a fixed Z, the variation of λ with z follows fairly simple patterns, as shown in Fig. 9 for the Chalk River group data.[16,20] The value of λ for ^{32}S projectiles on a Ag target should be assigned a large uncertainty because of the paucity of measurements on which the value is based.[20] In cases where measurements of Ward *et al.*[16] were superseded by those of Forster *et al.*,[20] values of λ deduced from the latter experiment[20] are shown. Although the two values of λ for ^{35}Cl projectiles on Ni targets do not differ greatly, those for Ag and Au targets differ sufficiently to invoke concern over the lack of consistency in the results of the two experiments.[16,20] Similarly, the value of λ for ^{16}O projectiles on Ni targets[16] seems

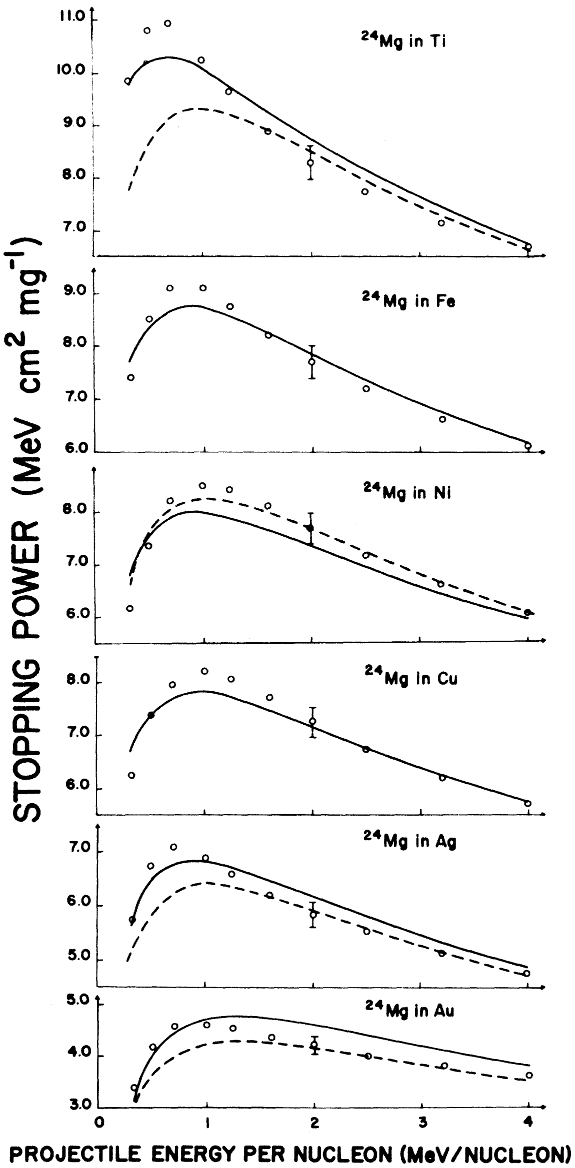

FIG. 5. Stopping powers for ^{24}Mg projectiles of Ti, Fe, Ni, Cu, Ag, and Au as measured by Forster *et al*. (data circles—Ref. 20), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

to follow quite smoothly the trend established by other projectiles on that material, but the values for Ag and Au targets[16] do not clearly follow the patterns established by analysis of the data from the later experiment.[20]

Interpolation of $\lambda(z, Z)$ should be practicable for a fixed Z in order to obtain λ values for missing z integers. However, an interpolation in Z for fixed z would be a far more tenuous procedure, even if one were to assume no more structure in

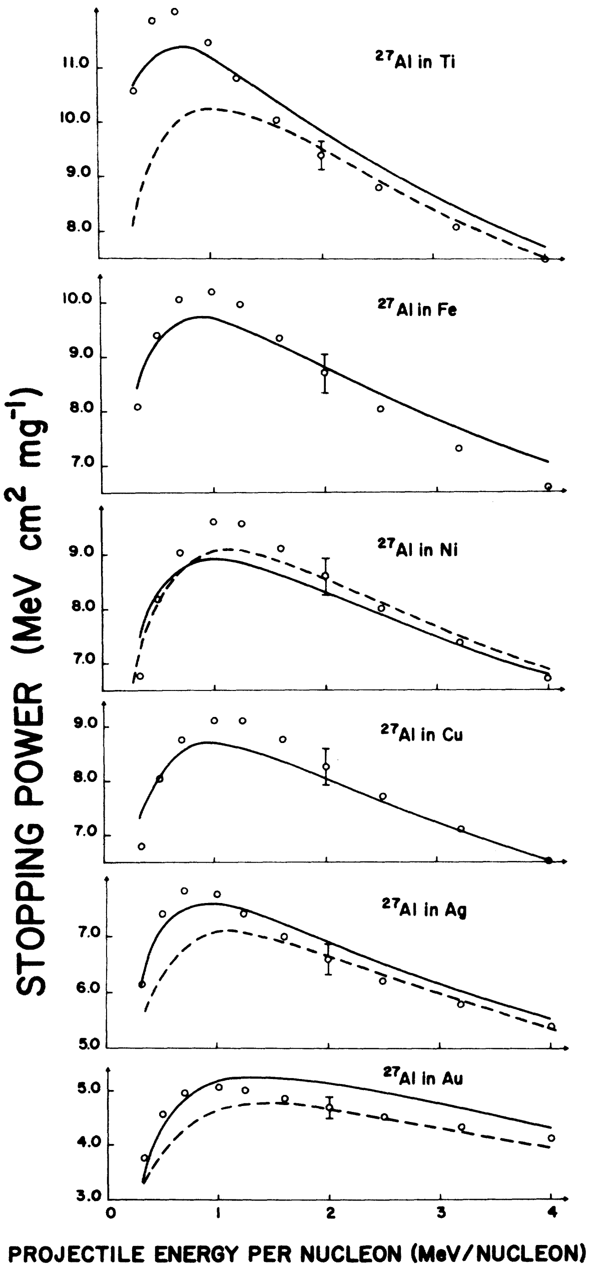

FIG. 6. Stopping powers for ^{27}Al projectiles of Ti, Fe, Ni, Cu, Ag, and Au as measured by Forster *et al*. (data circles—Ref. 20), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

the dependence of λ on Z than indicated by the few points tabulated thus far.

III. DISCUSSION AND CONCLUSIONS

The range of z values systematically investigated in the present study is rather limited, whereas the

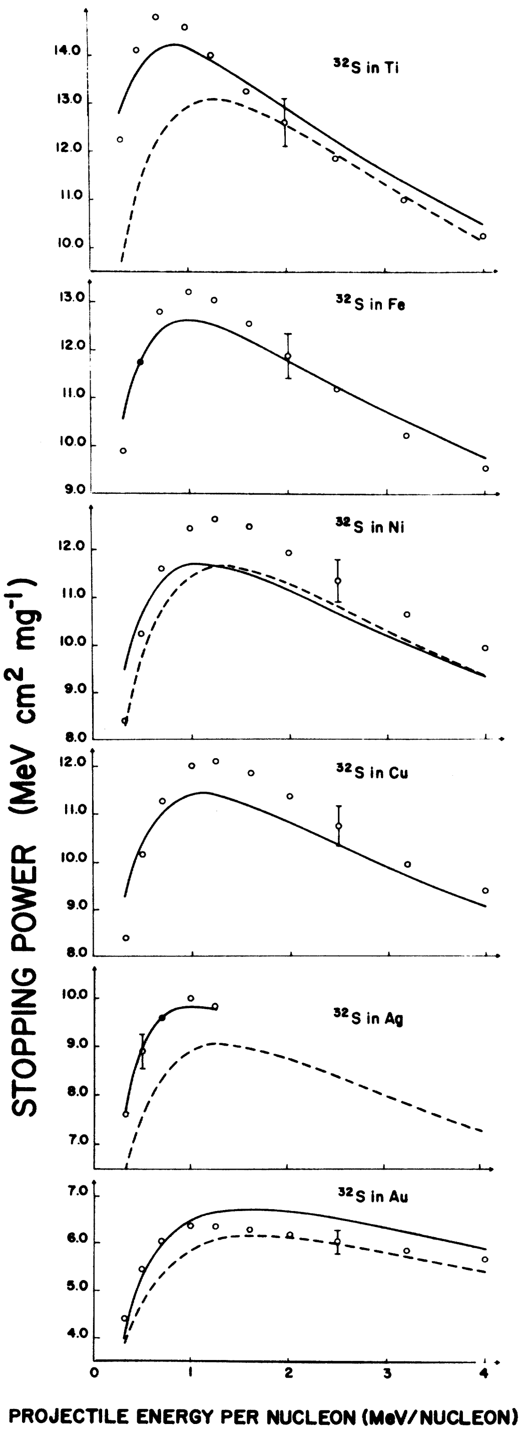

FIG. 7. Stopping powers for ^{32}S projectiles of Ti, Fe, Ni, Cu, Ag, and Au as measured by Forster *et al*. (data circles—Ref. 20), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

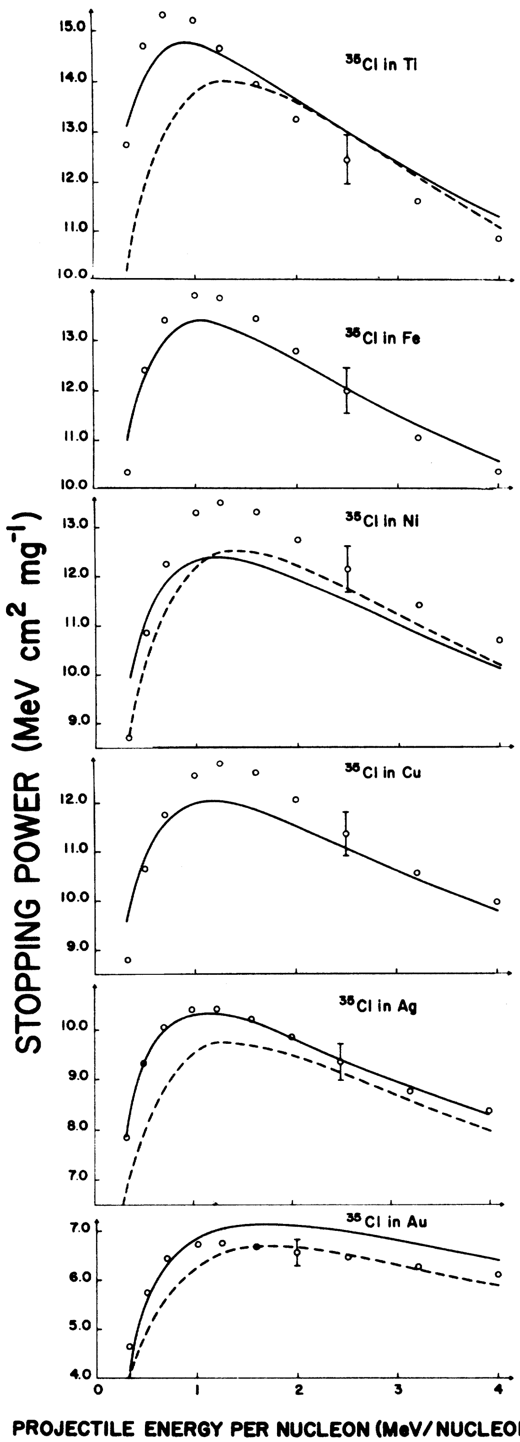

FIG. 8. Stopping powers for ^{35}Cl projectiles of Ti, Fe, Ni, Cu, Ag, and Au as measured by Forster *et al.* (data circles—Ref. 20), as predicted by Northcliffe and Schilling (dashed curve—Ref. 19), and as calculated in the present study (solid curve). A typical error bar indicates the uncertainty in these 4% measurements.

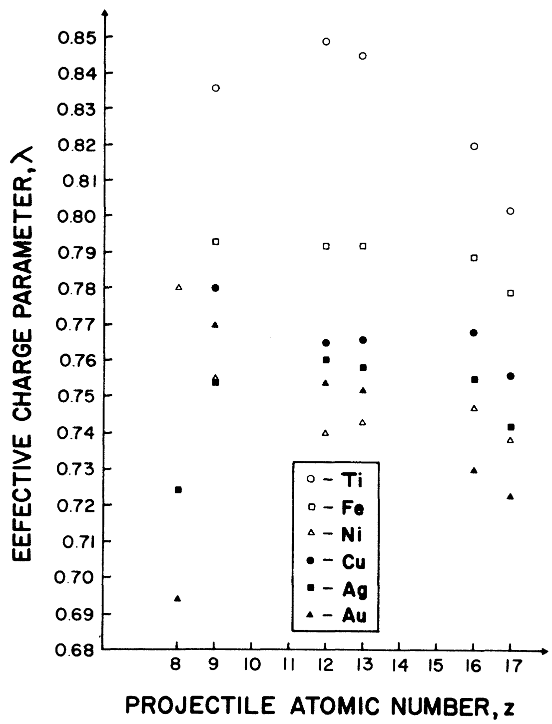

FIG. 9. Values of the single charge-state parameter λ as a function of projectile charge number (z) for various targets.

Z values encountered cover sparsely a broad interval. The resulting dependence of λ on Z with z fixed is so manifestly complex as to discourage any but the most obvious interpolations. Yet the generally good fits to experimental data with modified Bethe-Bloch theory were achieved by prior selection of all parameters of the formulation except the effective-charge parameter λ. These quantities comprise the mean excitation energy, the four shell-correction scaling parameters[11] associated with M and N shells (K- and L-shell corrections were utilized at full strength), one parameter for the low-velocity-projectile z^3 effect,[6] and the single effective-charge parameter chosen for this study. Four of the six parameters are associated with the shell corrections, whose contribution to the stopping number is usually small.[32] Of these four parameters, N-shell scaling parameters are zero except for Ag and Au targets, so that the actual total number of parameters used for $Z < 47$ is merely four. By way of comparison, construction of the Northcliffe and Schilling tables[19] required five parameters at energies above 0.5 MeV/amu and five other parameters at lower energies. If 0.5 MeV/amu is considered essentially equivalent to the lowest energy accepted in the present study, 0.3 MeV/amu, then the numbers of required parameters are roughly equal. It would be difficult to ascribe much physical significance to the shell-correction scaling parameters,[11] but the mean excitation energy falls in quite another category. Whereas the magnitude of the low-velocity-projectile z^3 parameter, b in the notation of Ref. 6, was known at the outset,[6] the specific value currently used, $b = 1.8$, was selected as the center of the interval established with fits of the Andersen *et al.* measurements of the stopping powers of Al and Ta using hydrogen and helium ions as projectiles.[30] Thus the parameters utilized possess varying degrees of physical significance. The fact that generally good fits could be attained by variation of the single effective-charge parameter lends credence to Bethe-Bloch theory and its modifications as a basis for analysis of stopping-power data. Continued analyses and investigations of effective-charge parametrization may lead to improvements in theory. However, revision of fundamental effective-charge theory[21] or of its more sophisticated descendants[23] so as to include a Z dependence in the effective-charge parameter(s) does not appear to be an immediate prospect. Ultimately, an adequate theory must reckon with the detailed description of projectile charge state as an average over many such ions of the same velocity, and with such very recent findings as those by Latta and Scanlon[33] that for 0.4–4.0-MeV He ions in solids the use of an effective charge without a corresponding modification of the basic stopping-power theory to take into account the departure from point-charge behavior leads to discrepancies from experimental data.

The present investigation of λ variations with z and Z will be continued by extending the scope of the study to other projectiles and targets. Since projectile z^3 corrections to stopping power are large for combinations of high z and Z, e.g., ranging from about a fifth to about a third of the total stopping power for ^{35}Cl projectiles on a Au target[20] in the energy interval considered, inclusion of higher-order corrections may be necessary. A recent extension of the theory to include projectile z^4 corrections[3] should be studied in order to test its efficacy in improvement of effective-charge analyses of stopping-power experiments.

ACKNOWLEDGMENT

The author appreciates the kindness of Dr. David Ward of the Chalk River Nuclear Laboratories in providing preliminary results of the most recent set of stopping-power measurements analyzed in the present study.

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