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Intraband magnetoabsorption in mixed semiconductors with composition fluctuations and in imperfect semiconductors *

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Theory of the intraband magnetoabsorption is derived in the quantum limit for imperfect semiconductors and mixed semiconductors with composition fluctuations. Both band edge and effective mass are regarded as position dependent. All harmonics of cyclotron resonance appear in Faraday configuration (for both cyclotron-resonance-active and -inactive polarizations). The effective-mass fluctuations give "steps" at the harmonics of cyclotron resonance on the magnetoabsorption curve for $\vec{E} \parallel \vec{H}$.

I. INTRODUCTION

Many semiconducting alloys can be prepared with composition varying over a wide range. Typical examples are alloys whose two components have similar crystal structure and lattice constants, e.g., $Hg_{1-v}Cd_vTe$ or $Cd_{1-v}Zn_vS$. When the composition of such a mixed crystal is not uniform [i.e., $v = v(\vec{r})$], band parameters (energy gap, effective mass, etc.) are position dependent,[1-6] which leads to a number of interesting phenomena. Variation of the energy gap produces a difference between fields acting on electrons and holes (so-called quasielectric fields[1]) and strongly modifies several physical effects. The effective-mass gradient leads to nonlinear free-carrier effects both at the optical[5,7-9] and at lower[10] frequencies.

The effective-mass gradient may also influence the cyclotron motion of free carriers.[5] This phenomenon was studied in detail for uniform effective-mass gradients.[11] A uniform band-edge gradient (i.e., the electric and/or quasielectric field) was also taken into account. A nondegenerate parabolic and spherical band was assumed, and spin effects were neglected. Carrier concentration was assumed to be low enough to neglect plasma effects. The modified Landau levels and the intraband magnetoabsorption was calculated. The effective-mass gradient (but not the band-edge gradient!) was shown to change the selection rules of cyclotron resonance, allowing transitions with $\Delta n = \pm 2$.

In the present paper we are interested mainly in mixed semiconductors which are macroscopically uniform but have microscopic fluctuations of composition. An unavoidable reason for the existence of microscopic fluctuations in an alloy is the fact that the atoms of one component substitute the atoms of the second component more or less at random (this gives rise to the so-called "residual resistance" of alloys). However, large microscopic fluctuations of composition may be due also to, e.g., fluctuations of temperature during the crystal growth, if the segregation coefficient differs from 1. As we will show, the intraband magnetoabsorption can be used as a tool to study these microscopic fluctuations of composition in detail.

To this end, we adopt all the assumptions of the Ref. 11, but we allow for the most general form of the position dependence of the effective mass and band edge. In Sec. II the radiation-induced intraband transition rates are calculated and it is shown that in the presence of composition fluctuations there are no selection rules for magneto-optical transitions. In Sec. III we restrict our considerations to the quantum-limit case and calculate the magnetoabsorption for $\Delta n \geq 2$ and for the radiation electric field perpendicular or parallel to the external magnetic field. It is then shown that there exist some effects due only to the effective-mass fluctuations. In Sec. IV we discuss the results and we present some numerical examples.

It should be mentioned that our results apply not only to mixed semiconductors with fluctuating composition but also to, e.g., doped semiconductors which are not alloys. Of course, in the latter case only the band edge fluctuates (the effective mass is constant). In Sec. IV we present results of the calculation of the intraband magnetoabsorption for a semiconductor with ionized impurities.

Our results may be applied also to crystals with internal strains. In this case, both band edge and effective mass may fluctuate.

II. MAGNETO-OPTICAL TRANSITION RATES

The effective-mass Hamiltonian for an electron of the charge $-e$ will be written in the form

$$\mathcal{H} = \tfrac{1}{4}\{[-i\hbar\vec{\nabla} + (e/c)\vec{A}(\vec{r},t)]^2, 1/m^*(\vec{r})\}_+ + U(\vec{r}) - e\phi(\vec{r},t) \ , \tag{1}$$

where $\{\cdots\}_+$ denotes the anticommutator. (Without symmetrization $\mathcal{H}$ is not Hermitian![3-5]) $m^*(\vec{r})$ and $U(\vec{r})$ are the position-dependent effective mass and conduction-band edge, respectively, $\vec{A}(\vec{r},t)$ is the vector potential, and $\phi(\vec{r},t)$ is the scalar potential of an external electromagnetic field.

The form (1) of $\mathcal{H}$ is a natural generalization of an effective-mass Hamiltonian for mixed semiconductors derived in the "virtual-crystal" approximation in the absence of magnetic field.[4,6] In the latter case relativistic and spin effects were neglected, and a nondegenerate parabolic and spherical band at Γ point was assumed for the high-symmetry crystal. An important assumption was that the effective mass and conduction-band edge change slowly on the dimensions of the unit cell. The over-all change of the effective mass and conduction-band edge on the electron wavelength was also assumed to be small. Recently, the effective-mass Hamiltonian of the form (1) was derived under the same assumptions for the case when an external magnetic field is present.[12,13]

In this paper we are interested in the effective-mass and band-edge fluctuations due to the fluctuations of composition and to the crystal imperfections. Therefore, it is natural to expand the position-dependent inverse effective mass and band edge into Fourier series:

$$m^{*-1}(\vec{r}) = m_0^{*-1} + m_0^{*-1} \sum_{\vec{q}}{}' \mu(\vec{q}) \exp(i\vec{q}\cdot\vec{r}) \ , \tag{2}$$

$$U(\vec{r}) = U_0 + \sum_{\vec{q}}{}' \nu(\vec{q}) \exp(i\vec{q}\cdot\vec{r}) \ . \tag{3}$$

m_0^{*-1} and U_0 are the average values of the inverse effective mass and band edge, respectively. The summation runs over all $\vec{q} \neq 0$ allowed by a periodicity box:

$$q_\alpha = (2\pi/L_\alpha)Q_\alpha \quad (\alpha = x, y, z) \ , \tag{4}$$

where Q_α are integers. To keep m^* and U real we have to assume

$$\mu(-\vec{q}) = \mu^*(\vec{q}) \ , \tag{5}$$

$$\nu(-\vec{q}) = \nu^*(\vec{q}) \ . \tag{6}$$

To justify the Hamiltonian (1) it is necessary to assume that μ and ν are small. We assume also μ and ν small enough for nonexistence of electron bound states. Moreover, the contribution to the calculated physical effect given by $q > q_{\text{np}}$ should be negligible. By q_{np} we mean the electron wave number at which the nonparabolicity of the conduction band becomes important.

We consider the case of static uniform magnetic field $\vec{H}$, parallel to the z axis. We use the dipole approximation, i.e., we assume a uniform oscillating electric field of radiation of the form

$$\mathrm{Re}[\vec{E}\exp(-i\omega t)] \ , \tag{7}$$

where $\vec{E}$ is a complex vector. $\hbar\omega$ should be smaller than the energy gap. The vector potential is

$$\vec{A}(\vec{r},t) = (0, Hx, 0) + (c/\omega)\,\mathrm{Im}(\vec{E}e^{-i\omega t}) \ , \tag{8}$$

and the scalar potential vanishes:

$$\phi(\vec{r},t) = 0 \ . \tag{9}$$

In the weak-radiation approximation, using Eqs. (2), (3), (8), and (9), the Hamiltonian (1) can be written in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_f + \mathcal{H}_r + \mathcal{H}_{rf} \ , \tag{10}$$

where

$$\mathcal{H}_0 = -\frac{\hbar^2}{2m_0^*}\left[\frac{\partial^2}{\partial x^2} - \left(\frac{x}{\lambda^2} - i\frac{\partial}{\partial y}\right)^2 + \frac{\partial^2}{\partial z^2}\right] + U_0 \ , \tag{11}$$

$$\mathcal{H}_f = -\frac{\hbar^2}{2m_0^*}\sum_{\vec{q}}{}' \mu(\vec{q})\exp(i\vec{q}\cdot\vec{r})\left(\Delta + i\vec{q}\vec{\nabla} - \tfrac{1}{2}q^2 + 2i\frac{x}{\lambda^2}\frac{\partial}{\partial y} - q_y\frac{x}{\lambda^2} - \frac{x^2}{\lambda^4}\right) + \sum_{\vec{q}}{}' \nu(\vec{q})\exp(i\vec{q}\cdot\vec{r}) \ , \tag{12}$$

$$\mathcal{H}_r = -i\frac{\hbar e}{m_0^*\omega}\left[\mathrm{Im}(E_x e^{-i\omega t})\frac{\partial}{\partial x} + \mathrm{Im}(E_y e^{-i\omega t})\left(i\frac{x}{\lambda^2} + \frac{\partial}{\partial y}\right) + \mathrm{Im}(E_z e^{-i\omega t})\frac{\partial}{\partial z}\right] \ , \tag{13}$$

$$\begin{aligned}\mathcal{H}_{rf} = -i\frac{\hbar e}{2m_0^*\omega}\sum_{\vec{q}}{}' \mu(\vec{q})\exp(i\vec{q}\cdot\vec{r})\Big[&2\,\mathrm{Im}(E_x e^{-i\omega t})\frac{\partial}{\partial x} \\ &+ 2\,\mathrm{Im}(E_y e^{-i\omega t})\left(i\frac{x}{\lambda^2} + \frac{\partial}{\partial y}\right) + 2\,\mathrm{Im}(E_z e^{-i\omega t})\frac{\partial}{\partial z} + i\,\mathrm{Im}[(\vec{E}\cdot\vec{q})e^{-i\omega t}]\Big] \ ,\end{aligned} \tag{14}$$

and $\lambda = (c\hbar/eH)^{1/2}$ is the usual magnetic length.

In the present paper we are not calculating transitions which are allowed in the absence of fluctuations, i.e., the cyclotron resonance transitions. In fact, we will assume $\omega > \omega_{c0}$, where $\omega_{c0} = eH/cm_0^*$ is the cyclotron frequency for the average value of m^{*-1}. We are interested in the contributions to transition rate proportional to $|\vec{E}|^2$ and of not higher than the second order in μ and ν. To calculate the intraband magnetoabsorption we can use the second-order perturbation calculus, as it is done in the theory of free-carrier absorption in the absence of effective-mass fluctuations and magnetic field. (See, e.g., Ref. 14.) However, there is one important difference. Namely, the first-order matrix element of the time-dependent perturbation does not vanish in our case because of $\mathcal{H}_{rf}$. Moreover, it is of the same order in $|\vec{E}|$ and μ as the second-order terms given by products of matrix elements of $\mathcal{H}_f$ and $\mathcal{H}_r$. Therefore, some interference of the first- and second-order perturbation contributions will appear due only to the effective-mass fluctuations. To take this into account we use the general formula for the transition rate calculated up to the second order of perturbation.

In our problem this formula reads (see Appendix)

$$W_{|k_y k_z n\rangle \to |k_y' k_z' n'\rangle} = \frac{\pi}{2\hbar^2}\,\delta\left(\omega - \frac{1}{\hbar}(\mathcal{E}_{k_z' n'} - \mathcal{E}_{k_z n})\right)$$
$$\times \Bigg| \langle k_y' k_z' n' | \mathcal{H}_+ | k_y k_z n\rangle + \sum_{\substack{k_y'' k_z'' n'' \\ \mathcal{E}_{k_z'' n''} \neq \mathcal{E}_{k_z' n'}}} \langle k_y' k_z' n' | \mathcal{H}_f | k_y'' k_z'' n''\rangle \langle k_y'' k_z'' n'' | \mathcal{H}_+ | k_y k_z n\rangle (\mathcal{E}_{k_z' n'} - \mathcal{E}_{k_z'' n''})^{-1}$$
$$- \sum_{\substack{k_y'' k_z'' n'' \\ \mathcal{E}_{k_z'' n''} \neq \mathcal{E}_{k_z n}}} \langle k_y' k_z' n' | \mathcal{H}_+ | k_y'' k_z'' n''\rangle \langle k_y'' k_z'' n'' | \mathcal{H}_f | k_y k_z n\rangle (\mathcal{E}_{k_z'' n''} - \mathcal{E}_{k_z n})^{-1} \Bigg|^2 \tag{15}$$

for $\mathcal{E}_{k_z' n'} > \mathcal{E}_{k_z n}$. The transition rate for transitions in the opposite direction is the same. $|k_y k_z n\rangle$ is a Landau function, i.e., an eigenstate of $\mathcal{H}_0$, of the form

$$|k_y k_z n\rangle = (L_y L_z)^{-1/2} \exp[i(k_y y + k_z z)] f_n(x - x_0) \;, \tag{16}$$

where f_n is the nth harmonic-oscillator function of the characteristic length $\lambda, n = 0, 1, \ldots,$ and $x_0 = -\lambda^2 k_y$. $\mathcal{E}_{k_z n}$ is the energy corresponding to $|k_y k_z n\rangle$

$$\mathcal{E}_{k_z n} = \hbar\omega_{c0}(n + \tfrac{1}{2}) + (\hbar^2 k_z^2 / 2m_0^*) \;. \tag{17}$$

k_y and k_z have the form

$$k_\alpha = (2\pi/L_\alpha) K_\alpha \quad (\alpha = y, z) \;, \tag{18}$$

where K_α are integers. If the electron is in the periodicity box, $-\frac{1}{2}L_x \leq x_0 < \frac{1}{2}L_x$. This yields a condition

$$-L_x/2\lambda^2 < k_y \leq L_x/2\lambda^2 \;. \tag{19}$$

The operator $\mathcal{H}_+$ is

$$\mathcal{H}_+ = -\frac{\hbar e}{m_0^* \omega} \Bigg\{ E_x \frac{\partial}{\partial x} + E_y \left(i\frac{x}{\lambda^2} + \frac{\partial}{\partial y} \right) + E_z \frac{\partial}{\partial z}$$
$$+ {\sum_{\vec{q}}}' \mu(\vec{q}) \exp(i\vec{q}\cdot\vec{r}) \left[E_x \frac{\partial}{\partial x} + E_y \left(i\frac{x}{\lambda^2} + \frac{\partial}{\partial y} \right) + E_z \frac{\partial}{\partial z} + \tfrac{1}{2} i \vec{E}\cdot\vec{q} \right] \Bigg\} \;. \tag{20}$$

The matrix element of $\mathcal{H}_f$ between two arbitrary Landau states is

$$\langle k_y + q_y \; k_z + q_z \; n' | \mathcal{H}_f | k_y k_z n\rangle = {\sum_{q_x}}'' \exp(-i\lambda^2 k_y q_x)$$
$$\times \Big(\nu(\vec{q}) F(n', n, q_y, q_x) + \tfrac{1}{2}\hbar\omega_{c0}\mu(\vec{q}) \{ [2n + 1 + \tfrac{1}{2}\lambda^2 q^2 + \lambda^2 k_z (k_z + q_z)] F(n', n, q_y, q_x)$$
$$+ (\tfrac{1}{2}n)^{1/2} \lambda (q_y - i q_x) F(n', n-1, q_y, q_x)$$
$$+ [\tfrac{1}{2}(n+1)]^{1/2} \lambda (q_y + i q_x) F(n', n+1, q_y, q_x) \} \Big) \;, \tag{21}$$

where summing over q_x we omit $q_x = 0$ if $q_y = q_z = 0$. F is defined as

$$F(n', n, q_y, q_x) = \int f_{n'}(x + \lambda^2 q_y) e^{i q_x x} f_n(x)\, dx \;. \tag{22}$$

This integral is equal to[15]

$$\begin{aligned} F(n',n,q_y,q_x) &= 2^{-|n'-n|/2}(n!/n'!)^{\operatorname{sgn}(n'-n)/2} \\ &\quad\times \exp(-\tfrac{1}{2}i\lambda^2 q_x q_y)[i\lambda q_x + \operatorname{sgn}(n'-n)\lambda q_y]^{|n'-n|}\,\exp(-\tfrac{1}{4}\lambda^2 q_\perp^2)L_{\min(n',n)}^{|n'-n|}(\tfrac{1}{2}\lambda^2 q_\perp^2)\ , \end{aligned} \tag{23}$$

where $\operatorname{sgn}(n'-n)=(n'-n)/|n'-n|$ (and equals 0 for $n'=n$). We have also denoted

$$q_\perp^2 = q_x^2 + q_y^2\ , \tag{24}$$

and $L_l^{l'}(x)$ is the Laguerre polynomial

$$L_l^{l'}(x) = \sum_{m=0}^{l} (-1)^m \binom{l'+l}{l-m}\frac{x^m}{m!}\ . \tag{25}$$

As we are not interested in the contributions to the transition rate of higher than the second order in μ or ν, in the second-order perturbation terms of Eq. (15) we keep only μ-independent part of the matrix element of $\mathcal{H}_+$ which equals

$$\langle k_y+q_y\, k_z+q_z\, n'|\mathcal{H}_+|k_y k_z n\rangle_{\mu=0} = -(\hbar e/m_0^*\omega)\delta_{0q_y}\delta_{0q_z}\{iE_z k_z \delta_{nn'} + (1/\lambda)[n^{1/2}E_+\delta_{n',n-1} - (n+1)^{1/2}E_-\delta_{n',n+1}]\}\ , \tag{26}$$

where we have used the notation

$$E_\pm = 2^{-1/2}(E_x \pm iE_y)\ . \tag{27}$$

In the following we restrict our considerations to the case when for the initial and final states we have

$$n' - n > 1 \tag{28}$$

and $\mathcal{E}_{k_z' n'} > \mathcal{E}_{k_z n}$. For such n and n' the matrix element of the full operator $\mathcal{H}_+$ is

$$\begin{aligned} &\langle k_y+q_y\ k_z+q_z\ n'|\mathcal{H}_+|k_y k_z n\rangle_{|n'-n|>1} \\ &\quad= -\frac{\hbar e}{m_0^*\omega}\sum_{q_x}^{''} \exp(-i\lambda^2 k_y q_x)\mu(\vec{q})\{\tfrac{1}{2}i(\vec{E}\cdot\vec{q}+2E_z k_z)F(n',n,q_y,q_x) + (1/\lambda)[n^{1/2}E_+F(n',n-1,q_y,q_x) \\ &\qquad - (n+1)^{1/2}E_-F(n',n+1,q_y,q_x)]\}\ . \end{aligned} \tag{29}$$

The second-order perturbation contributions to Eq. (15) are visualized on Fig. 1. There also exists, of course, a first-order contribution which is not depicted on this figure.

As we want to calculate the absorption, we need only the value of $W_{|k_y k_z n\rangle \to |k_y+q_y\, k_z+q_z\, n'\rangle}$ averaged over all k_y. We will denote it by $\overline{W}_{(k_z,n;q_y,q_z,n')}$. To obtain $\overline{W}$ we use Fig. 1 and we insert Eqs. (17), (21), (23), (26), and (29), into Eq. (15). Making use of the functional relations between Laguerre polynomials, and of the formula

$$\sum_{k_y} \exp[i\lambda^2(q_x'-q_x)k_y] = \frac{L_x L_y}{2\pi\lambda^2}\delta_{q_x' q_x} \tag{30}$$

(strictly valid for $L_\alpha/2\pi^{1/2}\lambda$, $\alpha=x,y$, being integers), we find after some algebra

$$\begin{aligned} \overline{W}(k_z,n;q_y,q_z,n') &= \delta\Big(\omega - \frac{1}{\hbar}(\mathcal{E}_{k_z+q_z\,n'} - \mathcal{E}_{k_z n})\Big)\frac{\pi e^2 n!}{n'!\,2m_0^{*2}\omega^2} \\ &\quad\times\sum_{q_x}^{''}(\tfrac{1}{2}\lambda^2 q_\perp^2)^{n'-n}\exp(-\tfrac{1}{2}\lambda^2 q_\perp^2)\Big|\frac{1}{\hbar}\nu(\vec{q})L_n^{n'-n}(\tfrac{1}{2}\lambda^2 q_\perp^2)\Big(i\frac{E_x q_x + E_y q_y}{\omega^2-\omega_{c0}^2}\omega + \frac{E_y q_x - E_x q_y}{\omega^2-\omega_{c0}^2}\omega_{c0} + i\frac{E_z q_z}{\omega}\Big) \\ &\quad+\tfrac{1}{2}\mu(\vec{q})\Big\{i\frac{E_x q_x + E_y q_y}{\omega^2-\omega_{c0}^2}\Big[L_n^{n'-n}(\tfrac{1}{2}\lambda^2 q_\perp^2)\Big(\omega\omega_{c0}(\tfrac{1}{4}\lambda^2 q_z^2 + n' + n + 2) + \frac{m_0^*\omega}{\hbar q_z^2}[\omega - \omega_{c0}(n'-n)]^2 \\ &\qquad - \frac{2}{\lambda^2 q_\perp^2}\omega[(n'-n)\omega + (n'+n)\omega_{c0}]\Big) + L_{n-1}^{n'-n}(\tfrac{1}{2}\lambda^2 q_\perp^2)\frac{4}{\lambda^2 q_\perp^2}n'\omega\omega_{c0}\Big] \\ &\quad+\frac{E_y q_x - E_x q_y}{\omega^2-\omega_{c0}^2}\Big[L_n^{n'-n}(\tfrac{1}{2}\lambda^2 q_\perp^2)\Big(\omega_{c0}^2(\tfrac{1}{4}\lambda^2 q_z^2 + n' + n + 1) + \omega^2 + \frac{m_0^*\omega_{c0}}{\hbar q_z^2}[\omega - \omega_{c0}(n'-n)]^2 \\ &\qquad - \frac{2}{\lambda^2 q_\perp^2}\omega[(n'+n)\omega + (n'-n)\omega_{c0}]\Big) + L_{n-1}^{n'-n}(\tfrac{1}{2}\lambda^2 q_\perp^2)\frac{4}{\lambda^2 q_\perp^2}n'\omega^2\Big] \\ &\quad+i\frac{E_z q_z}{\omega}L_n^{n'-n}(\tfrac{1}{2}\lambda^2 q_\perp^2)\{\omega_{c0}(\tfrac{1}{4}\lambda^2 q_z^2 + n' + n + 1) - \frac{m_0^*}{\hbar q_z^2}[\omega^2 - \omega_{c0}^2(n'-n)^2]\}\Big\}\Big|^2\ . \end{aligned} \tag{31}$$

In this formula $L_l^{l'}(x)$ should be replaced by zero if l is negative. It follows from Eq. (31) that in the presence of effective-mass and band-edge fluctuations there are no selection rules for magneto-optical transitions: any change of k_y, k_z, or n is allowed.

III. INTRABAND MAGNETOABSORPTION IN THE QUANTUM LIMIT

We restrict the following considerations to the quantum-limit case at 0 K. In other words, we assume that the electrons occupy only all the states with $n=0$ and $|k_z| \leqslant k_{zF}$, where

$$k_{zF} = \pi^2\lambda^2 N_e \tag{32}$$

and N_e is the electron concentration. The condition of the quantum limit reads

$$\hbar\omega_{c0} > \hbar^2 k_{zF}^2 / 2m_0^* \tag{33}$$

or

$$\tfrac{1}{2}\lambda^2 k_{zF}^2 < 1 \ . \tag{34}$$

Power absorbed per unit volume because of transitions from all occupied states (of $n=0$) to all empty states of quantum number n' $(n' \geqslant 2)$ will be denoted by $P_{n'}$. Using Eq. (31) (for $n=0$) and replacing the summation over k_z by integration we obtain

$$\begin{aligned} P_{n'} = {} & \epsilon\left(\omega \geqslant \omega_{c0}n' - \frac{\hbar k_{zF}^2}{2m_0^*}\right)\frac{e^2\omega_{c0}}{n'!\,4\pi\hbar\omega} \\ & \times \sum_{\vec{q}}{}' \epsilon\left\{\left|k_{zF} - \left[k_{zF}^2 + 2\left(\frac{m_0^*}{\hbar}\right)(\omega-\omega_{c0}n')\right]^{1/2}\right| \leqslant |q_z| \leqslant k_{zF} + \left[k_{zF}^2 + 2\left(\frac{m_0^*}{\hbar}\right)(\omega-\omega_{c0}n')\right]^{1/2}\right\}\frac{1}{|q_z|}\left(\tfrac{1}{2}\lambda^2 q_\perp^2\right)^{n'} \\ & \times \exp\left(-\tfrac{1}{2}\lambda^2 q_\perp^2\right)\left|\frac{1}{\hbar}\,\nu(\vec{q})\left(i\,\frac{E_x q_x + E_y q_y}{\omega^2-\omega_{c0}^2}\,\omega + \frac{E_y q_x - E_x q_y}{\omega^2-\omega_{c0}^2}\,\omega_{c0} + i\,\frac{E_z q_z}{\omega}\right)\right. \\ & + \tfrac{1}{2}\mu(\vec{q})\left[i\,\frac{E_x q_x + E_y q_y}{\omega^2-\omega_{c0}^2}\left(\omega\omega_{c0}\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 2\right) + \frac{m_0^*\omega}{\hbar q_z^2}(\omega-\omega_{c0}n')^2 - \frac{2}{\lambda^2 q_\perp^2}\,n'\omega(\omega+\omega_{c0})\right)\right. \\ & + \frac{E_y q_x - E_x q_y}{\omega^2-\omega_{c0}^2}\left(\omega_{c0}^2\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 1\right) + \omega^2 + \frac{m_0^*\omega_{c0}}{\hbar q_z^2}(\omega-\omega_{c0}n')^2 - \frac{2}{\lambda^2 q_\perp^2}\,n'\omega(\omega+\omega_{c0})\right) \\ & \left.\left. + i\,\frac{E_z q_z}{\omega}\left(\omega_{c0}\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 1\right) - \frac{m_0^*}{\hbar q_z^2}\left(\omega^2 - \omega_{c0}^2 n'^2\right)\right)\right]\right|^2 , \end{aligned} \tag{35}$$

where by definition ϵ is equal to 1 if the indicated inequality is fulfilled, and equals zero otherwise.

It should be noted that the wave vectors $|\vec{q}| \gg n'^{1/2}/\lambda$ give practically no contributions to $P_{n'}$. Therefore, our assumption $q < q_{\text{np}}$ is fulfilled if only $n'\hbar\omega_{c0} < \mathcal{E}_{\text{np}}$, where $\mathcal{E}_{\text{np}}$ is the electron energy at which the nonparabolicity of the conduction band becomes important.

If $\mu(\vec{q})$ and $\nu(\vec{q})$ are random variables, we can replace in Eq. (35) $|\mu(\vec{q})|^2$, $|\nu(\vec{q})|^2$, and $\mu(\vec{q})\nu^*(\vec{q})$, by their average values $\langle|\mu(\vec{q})|^2\rangle_{\text{av}}$, $\langle|\nu(\vec{q})|^2\rangle_{\text{av}}$, and $\langle\mu(\vec{q})\nu^*(\vec{q})\rangle_{\text{av}}$. We use the notation

$$\langle|\mu(\vec{q})|^2\rangle = L_x L_y L_z \langle|\mu(\vec{q})|^2\rangle_{\text{av}} \ , \tag{36}$$

$$\langle|\nu(\vec{q})|^2\rangle = L_x L_y L_z \langle|\nu(\vec{q})|^2\rangle_{\text{av}} \ , \tag{37}$$

$$\langle\mu(\vec{q})\nu^*(\vec{q})\rangle = L_x L_y L_z \langle\mu(\vec{q})\nu^*(\vec{q})\rangle_{\text{av}} \ . \tag{38}$$

We can then replace the summation over $\vec{q}$ in Eq. (35) by integration.

For circularly polarized radiation in the Faraday configuration we obtain $P_{n'+}$ if $E_+ \neq 0, E_- = E_z = 0$, i.e., for CRI polarization (cyclotron resonance inactive), and $P_{n'-}$ if $E_- \neq 0, E_+ = E_z = 0$, i.e., for CRA polarization (cyclotron resonance active), where

$$
\begin{aligned}
P_{n'\pm} = {} & |E_\pm|^2 \epsilon\left(\omega \geq \omega_{c0} n' - \frac{\hbar k_{zF}^2}{2m_0^*}\right) \frac{e^2 m_0^* \omega_{c0}^2}{n'!\,8\pi^4 \hbar^2 \omega(\omega \pm \omega_{c0})^2} \\
& \times \int d^2 q_\perp \left(\tfrac{1}{2}\lambda^2 q_\perp^2\right)^{n'+1} \exp\left(-\tfrac{1}{2}\lambda^2 q_\perp^2\right) \int_{q_{z1}}^{q_{z2}} dq_z \frac{1}{q_z} \\
& \times \left[\frac{1}{\hbar^2}\langle|\nu(\vec{q})|^2\rangle + \frac{1}{\hbar}\left(\omega_{c0}\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 1\right) + \frac{m_0^*}{\hbar q_z^2}(\omega - \omega_{c0} n')^2 \mp \omega - \frac{4}{\lambda^2 q_\perp^2} n' \omega \epsilon(\mp 1 > 0)\right)\right. \\
& \times \operatorname{Re}\langle \mu(\vec{q}) \nu^*(\vec{q})\rangle + \tfrac{1}{4}\left(\omega_{c0}\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 1\right)\right. \\
& \left.\left. + \frac{m_0^*}{\hbar q_z^2}(\omega - \omega_{c0} n')^2 \mp \omega - \frac{4}{\lambda^2 q_\perp^2} n' \omega \epsilon(\mp 1 > 0)\right)^2 \langle|\mu(\vec{q})|^2\rangle\right] \ . \qquad (39)
\end{aligned}
$$

Deriving Eq. (39) we have used relations (5) and (6), and we have denoted

$$q_{z1} = \left|k_{zF} - \left[k_{zF}^2 + 2(m_0^*/\hbar)(\omega - \omega_{c0} n')\right]^{1/2}\right| \ , \qquad (40)$$

$$q_{z2} = k_{zF} + \left[k_{zF}^2 + 2(m_0^*/\hbar)(\omega - \omega_{c0} n')\right]^{1/2} \ . \qquad (41)$$

For radiation polarized linearly along the magnetic field ($E_+ = E_- = 0, E_z \neq 0$) we obtain in a similar way

$$
\begin{aligned}
P_{n'\parallel} = {} & |E_z|^2 \epsilon\left(\omega \geq \omega_{c0} n' - \frac{\hbar k_{zF}^2}{2m_0^*}\right) \frac{e^2 \omega_{c0}}{n'!\,16\pi^4 \hbar \omega^3} \\
& \times \int d^2 q_\perp \left(\tfrac{1}{2}\lambda^2 q_\perp^2\right)^{n'} \exp\left(-\tfrac{1}{2}\lambda^2 q_\perp^2\right) \\
& \times \int_{q_{z1}}^{q_{z2}} dq_z\, q_z \left[\frac{1}{\hbar^2}\langle|\nu(\vec{q})|^2\rangle + \frac{1}{\hbar}\left(\omega_{c0}\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 1\right) - \frac{m_0^*}{\hbar q_z^2}(\omega^2 - \omega_{c0}^2 n'^2)\right) \operatorname{Re}\langle \mu(\vec{q}) \nu^*(\vec{q})\rangle\right. \\
& \left. + \tfrac{1}{4}\left(\omega_{c0}\left(\tfrac{1}{4}\lambda^2 q_z^2 + n' + 1\right) - \frac{m_0^*}{\hbar q_z^2}(\omega^2 - \omega_{c0}^2 n'^2)\right)^2 \langle|\mu(\vec{q})|^2\rangle\right] \ . \qquad (42)
\end{aligned}
$$

It is interesting to observe that the expression $P_{n'-} - [(\omega + \omega_{c0})/(\omega - \omega_{c0})]^2 P_{n'+}$ does not depend on $\langle|\nu(\vec{q})|^2\rangle$. For $\omega = n'\omega_{c0}$ the expression (39) has a divergency. Close to $\omega = n'\omega_{c0}$, i.e., for

$$|\omega - \omega_{c0} n'|/\omega_{c0} \ll \tfrac{1}{2}\lambda^2 k_{zF}^2 \ , \qquad (43)$$

there is

$$
\begin{aligned}
P_{n'\pm} \approx {} & |E_\pm|^2 \frac{e^2 m_0^*}{n'!\,16\pi^4 n'(n' \pm 1)^2 \hbar^2 \omega_{c0}} \ln\left(\frac{\frac{1}{2}\lambda^2 k_{zF}^2 \omega_{c0}}{|\omega - \omega_{c0} n'|}\right) \\
& \times \int d^2 q_\perp \left(\tfrac{1}{2}\lambda^2 q_\perp^2\right)^{n'+1} \exp\left(-\tfrac{1}{2}\lambda^2 q_\perp^2\right) \left\{\frac{1}{\hbar^2}\langle|\nu(\vec{q}_\perp)|^2\rangle + \frac{\omega_{c0}}{\hbar}\left[1 + 2n'\left(1 - \frac{2n'}{\lambda^2 q_\perp^2}\right)\epsilon(\mp 1 > 0)\right] \operatorname{Re}\langle \mu(\vec{q}_\perp) \nu^*(\vec{q}_\perp)\rangle\right. \\
& \left. + \tfrac{1}{4}\omega_{c0}^2 \left[1 + 2n'\left(1 - \frac{2n'}{\lambda^2 q_\perp^2}\right)\epsilon(\mp 1 > 0)\right]^2 \langle|\mu(\vec{q}_\perp)|^2\rangle\right\} \ , \qquad (44)
\end{aligned}
$$

where

$$\vec{q}_\perp = (q_x, q_y, 0) \ . \qquad (45)$$

$P_{n'\parallel}$ has no divergency [see Eq. (42)]. However, its derivative with respect to ω has divergencies at $\omega = n'\omega_{c0} - (\hbar k_{zF}^2/2m_0^*)$ and at $\omega = n'\omega_{c0}$. The former corresponds to the square-root shape of the absorption edge for the considered indirect transitions. Close to the latter point, i.e., for ω fulfilling the condition (43), there is

$$
\begin{aligned}
\frac{d}{d\omega} P_{n'\parallel} \approx {} & -|E_z|^2 \frac{e^2 m_0^*}{n'!\,8\pi^4 n'^2 \hbar^3 \omega_{c0}} \ln\left(\frac{\frac{1}{2}\lambda^2 k_{zF}^2 \omega_{c0}}{|\omega - \omega_{c0} n'|}\right) \\
& \times \int d^2 q_\perp \left(\tfrac{1}{2}\lambda^2 q_\perp^2\right)^{n'} \exp\left(-\tfrac{1}{2}\lambda^2 q_\perp^2\right) \left[\operatorname{Re}\langle \mu(\vec{q}_\perp) \nu^*(\vec{q}_\perp)\rangle + \tfrac{1}{2}\hbar\omega_{c0}(n' + 1)\langle|\mu(\vec{q}_\perp)|^2\rangle\right] \ . \qquad (46)
\end{aligned}
$$

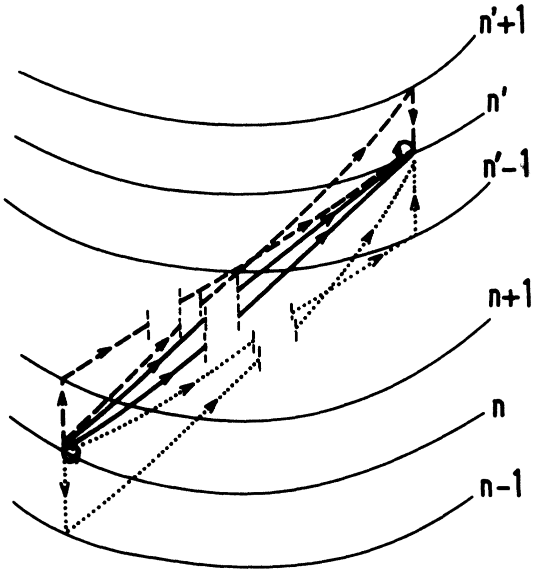

FIG. 1. Second-order perturbation contributions to the transition rate [Eq. (15)] for transitions $n \to n'$. Vertical transitions are due to radiation and the oblique ones to fluctuations.

The expression (46) is symmetric with respect to the point $\omega = n'\omega_{c0}$. Therefore, there is a "step" on the function $P_{n'\parallel}(\omega)$. This "step" is independent of $\langle|\nu(\vec{q})|^2\rangle$. It is due only to $\langle|\mu(\vec{q})|^2\rangle$ and $\langle\mu(\vec{q})\nu^*(\vec{q})\rangle$. If there are no strains in the crystal $\langle|\mu(\vec{q})|^2\rangle$ is given solely by fluctuations in composition. To $\langle\mu(\vec{q})\,\nu^*(\vec{q})\rangle$ only this part of band-edge fluctuations contributes which is correlated in phase with these composition fluctuations. Such band-edge fluctuations follow from the dependence of the band edge on composition, but may be due also to correlation of the concentration of impurities, defects, etc., with composition.

We are interested in mixed semiconductors of the type $A_{1-v(\vec{r})}B_{v(\vec{r})}$, $A_{1-v(\vec{r})}B_{v(\vec{r})}C$, etc., $[0 \leq v(\vec{r}) \leq 1]$. The average value of $v(\vec{r})$ will be denoted by v_0. For small $|v(\vec{r}) - v_0|$ and in the case of a nondegenerate spherical and parabolic band minimum in the Γ point of a high-symmetry crystal there is[6]

$$1/m^*(\vec{r}) = 1/m_0^* + (1/m_0^*)\alpha_0[v(\vec{r}) - v_0] \ , \tag{47}$$

$$U_c(\vec{r}) = U_0 + \beta_0[v(\vec{r}) - v_0] + \gamma_0\Delta[v(\vec{r}) - v_0] \ , \tag{48}$$

where α_0, β_0, and γ_0, are some constants given for the composition v_0. m_0^{*-1} and α_0 can be determined from the measurements on uniform mixed semiconductors. Of course, the expression (48) involves the position dependence of the band edge due only to the fluctuations of composition, and not to, e.g., potentials of ionized impurities or defects (we have indicated this by the subscript c).

The function $v(\vec{r})$ can be expanded into Fourier series

$$v(\vec{r}) = v_0 + {\sum_{\vec{q}}}' \rho(\vec{q}) \exp(i\vec{q}\cdot\vec{r}) \ , \tag{49}$$

where

$$\rho(-\vec{q}) = \rho^*(\vec{q}) \ . \tag{50}$$

From Eqs. (2), (3), and (47)–(49), we obtain the relations

$$\langle|\mu(\vec{q})|^2\rangle = \alpha_0^2\langle|\rho(\vec{q})|^2\rangle \ , \tag{51}$$

$$\mathrm{Re}\langle\mu(\vec{q})\nu_c^*(\vec{q})\rangle = \alpha_0(\beta_0 - \gamma_0 q^2)\langle|\rho(\vec{q})|^2\rangle \ , \tag{52}$$

$$\langle|\nu_c(\vec{q})|^2\rangle = (\beta_0 - \gamma_0 q^2)^2\langle|\rho(\vec{q})|^2\rangle \ , \tag{53}$$

where $\langle|\rho(\vec{q})|^2\rangle$ is defined in the same way as $\langle|\mu(\vec{q})|^2\rangle$ [see Eq. (36)].

Let us note that the terms of expression (49) corresponding to $|\vec{q}|$ of the order of reciprocal-lattice vector are meaningless, as the unit cell of the mixed semiconductor is always of the type A or B, AC or BC, etc. It was already mentioned, however, that such high q values do not contribute to $P_{n'}$.

To calculate $\rho(\vec{q})$ for $|\vec{q}|$ much smaller than the reciprocal-lattice vector we use Eq. (49) and assume that $v(\vec{r}) = 0$ in the unit cells of the type A, AC, etc., and $v(\vec{r}) = 1$ in the unit cells B, BC, etc. Strictly speaking, the exact form of $v(\vec{r})$ is unimportant if only the integral of $v(\vec{r})$ over a unit cell A, AC, etc., is equal to zero, and that over the unit cell B, BC, etc., is equal to 1. We obtain (for $\vec{q} \neq 0$)

$$\rho(\vec{q}) = (L_xL_yL_zN)^{-1}\sum_{\vec{R}} \exp(-i\vec{q}\cdot\vec{R})\theta_{\vec{R}} \ , \tag{54}$$

where the summation is over all Bravais lattice sites $\vec{R}$ in the volume $L_xL_yL_z$, N is the number of unit cells per unit volume, and $\theta_{\vec{R}}$ is a random variable of the values 0 or 1 for the unit cell corresponding to $\vec{R}$ being of the type A, AC, etc., or B, BC, etc., respectively. The random variables $\theta_{\vec{R}}$ fulfill the condition

$$v_0 = (L_xL_yL_zN)^{-1}\sum_{\vec{R}} \theta_{\vec{R}} \ . \tag{55}$$

Using Eq. (54) and (55) we find (for $\vec{q} \neq 0$)

$$\langle|\rho(\vec{q})|^2\rangle = v_0/N + (L_xL_yL_zN^2)^{-1} \times \sum_{\substack{\vec{R}'\vec{R} \\ \vec{R}' \neq \vec{R}}} \exp[i\vec{q}(\vec{R}' - \vec{R})]\langle\theta_{\vec{R}'}\theta_{\vec{R}}\rangle_{\mathrm{av}} \ . \tag{56}$$

In particular, for a perfectly random alloy (and sufficiently large $L_xL_yL_z$) we have for $\vec{R}' \neq \vec{R}$

$$\langle\theta_{\vec{R}'}\theta_{\vec{R}}\rangle_{\mathrm{av}} = \langle\theta_{\vec{R}'}\rangle_{\mathrm{av}}\langle\theta_{\vec{R}}\rangle_{\mathrm{av}} = v_0^2 \tag{57}$$

and

$$\langle |\rho(\vec{q})|^2\rangle = (1/N) v_0 (1 - v_0) \ . \tag{58}$$

As it should be, $\langle |\rho(\vec{q})|^2\rangle$ vanishes for both $v_0 = 0$ and $v_0 = 1$.

IV. DISCUSSION OF RESULTS

Summarizing our results we should note, first of all, that for any polarization and frequency the absorption coefficient is nonvanishing, contrary to the intraband magnetoabsorption in an ideal crystal, but similar to the free-carrier absorption in a nonideal crystal in the absence of magnetic field. Second, all harmonics of cyclotron resonance, i.e., absorption peaks at $\omega = n'\omega_{c0}$, should appear in both cyclotron-resonance-active and inactive polarizations in the Faraday configuration. These peaks are due to the fluctuations of both the effective mass and the band edge.

The most striking effect is the appearance of the "steps" at $\omega = n'\omega_{c0}$ on the absorption curve for $\vec{E} \parallel \vec{H}$, due only to the effective-mass fluctuations (with an influence of the correlated band-edge fluctuations). This effect offers a method of determining the composition fluctuations in a mixed crystal from the strength of peaks on the derivative-of-absorption curve. To avoid superposition of peaks at $\omega = n'\omega_{c0}$ and at $\omega = n'\omega_{c0} - \hbar k_{zF}^2/2m_0^*$, one should have $\frac{1}{2}\lambda^2 k_{zF}^2$ not too close to either 0 or 1.

To estimate the order of magnitude of the expected effects, let us assume that the band-edge fluctuations are due only to the composition fluctuations and that $\gamma_0 = 0$. Moreover, we will assume a perfectly random alloy. From Eqs. (42), (51)–(53), and (58), and from the relation between $P_{n'\parallel}$ and the absorption coefficient $K_{n'\parallel}$, i.e.,

$$K_{n'\parallel} = (8\pi / c\kappa^{1/2} |E_z|^2) P_{n'\parallel} \tag{59}$$

(κ is the dielectric constant of the crystal), we obtain

$$K^c_{n'\parallel} = [e^2 v_0(1-v_0)/\pi^2 c\hbar N \kappa^{1/2}\lambda^4]\,\epsilon(X \geq 1-Y)(\alpha_0^2/X^3)\big((X-1+Y)^{3/2}Y^{1/2}(\tfrac{4}{3}Y+C) + (X-1+Y)^{1/2}Y^{1/2}(1+CY+C^2)$$
$$+ (X^2-1)C\ln\{|X-1|^{1/2}/[Y^{1/2}+(X-1+Y)^{1/2}]\}\big) \ . \tag{60}$$

We have used here the notations

$$X = \omega/n'\omega_{c0} \ , \tag{61}$$

$$Y = (1/2n')\lambda^2 k_{zF}^2 \ , \tag{62}$$

$$C = 1 + (1/n')(1 + 2\beta_0/\alpha_0\hbar\omega_{c0}) \ . \tag{63}$$

Let us now assume the following material parameters which correspond roughly to the n-type $Hg_{0.75}Cd_{0.25}Te$ at low temperatures: $v_0 = 0.25$, $N = 1.5\times 10^{22}$ cm^{-3}, $\kappa = 10$, $m_0^* = 0.02 m_0$, $\alpha_0 = -10$, and $\beta_0 = 0.5$ eV. We assume also $N_e = 6.8\times 10^{16}$ cm^{-3} and $H = 50$ kG, and we will discuss the case of $n' = 2$. Plasma effects should not be important for the normal incidence on a semiconductor plate, as $\hbar\omega_p = 21$ meV (ω_p is the plasma frequency) and $\hbar\omega_{c0} = 29$ meV. The assumption $n'\hbar\omega_{c0} < \mathcal{E}_{np}$ will also be fulfilled if the band gap is of the order of 200 meV (as in $Hg_{0.75}Cd_{0.25}Te$) or larger. The quantum-limit condition is fulfilled as $\frac{1}{2}\lambda^2 k_{zF}^2 = 0.5$. The magnetic length $\lambda = 115$ Å. We have $Y = 0.25$ and $C = -0.23$.

It can be seen from the formula (60) that positive C corresponds to "downwards steps," and negative C to "upwards steps." Note that for our parameters C is positive for $n' \geq 3$.

$K^c_{2\parallel}$ vs X is plotted on Fig. 2. Its derivative $dK^c_{2\parallel}/dX$ is shown on Fig. 3. The "step" on $K^c_{2\parallel}$ curve is weakly visible while the divergency of the derivative $dK^c_{2\parallel}/dX$ should be rather easily observable.

As already mentioned, the actual fluctuations of composition in a mixed semiconductor may be much larger than those in the perfectly random alloy. Consequently, $K^c_{n'\parallel}$ may be much higher than this given by Eq. (60).

Our general formulas, e.g., Eq. (42), can be also used to calculate intraband magnetoabsorption in imperfect semiconductors (which are not necessarily alloys). Let us study perhaps the most interesting case of a semiconductor doped highly enough to have all the impurities (donors) ionized. Of course, only $\langle |\nu(\vec{r})|^2\rangle$ contributes to magnetoabsorption in this case. One can observe from Eq. (29) that the first-order perturbation to the transition rate vanishes. For

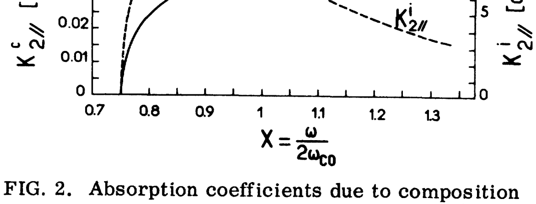

FIG. 2. Absorption coefficients due to composition fluctuations ($K_2^c{}_\parallel$) and to ionized impurities ($K_2^i{}_\parallel$) vs frequency for the numerical example discussed in the text.

simplicity we will neglect screening of impurity by free electrons. H_f is given by the charge density

$$d(\vec{r}) = eZ \sum_{\vec{R}_i} \delta(\vec{r} - \vec{R}_i) - eN_e \ , \tag{64}$$

where eZ is the impurity charge and the summation is over the random positions R_i of the impurities (N_e/Z impurities in a unit volume). Using the Poisson equation we obtain

$$\langle |\nu(\vec{q})|^2 \rangle = 16\pi^2 e^4 Z N_e / \kappa^2 q^4 \ . \tag{65}$$

From Eqs. (42) and (59) we have

$$\begin{aligned} K^{i}_{n'\parallel} = {} & \epsilon\left(\omega \geqslant \omega_{c0} n' - \frac{\hbar k_{zF}^2}{2m_0^*}\right) \frac{4e^6 Z N_e \omega_{c0}}{c\kappa^{5/2} n'!\hbar^3 \omega^3} \\ & \times (-1)^{n'} \Bigg((\tfrac{1}{2}\lambda^2 q_{z2}^2)^{n'} \exp(\tfrac{1}{2}\lambda^2 q_{z2}^2)\, \mathrm{Ei}(-\tfrac{1}{2}\lambda^2 q_{z2}^2) - (\tfrac{1}{2}\lambda^2 q_{z1}^2)^{n'} \exp(\tfrac{1}{2}\lambda^2 q_{z1}^2)\, \mathrm{Ei}(-\tfrac{1}{2}\lambda^2 q_{z1}^2) \\ & + \sum_{l=1}^{n'-1} (-1)^{l-1}(l-1)! \left[(\tfrac{1}{2}\lambda^2 q_{z2}^2)^{n'-1} - (\tfrac{1}{2}\lambda^2 q_{z1}^2)^{n'-1} \right] \Bigg) \ , \end{aligned} \tag{66}$$

where the exponential integral function is defined (for $x < 0$) as

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{1}{t} e^{-t} dt \ . \tag{67}$$

We have stated already that there is no divergency at $\omega = n'\omega_{c0}$ of the absorption coefficient for $\vec{E} \parallel \vec{H}$, and that if $\mu(\vec{q}) = 0$, the derivative of the absorption coefficient is also finite. In fact, the derivatives of $K^{i}_{n'\parallel}$ are finite up to the $(2n' - 1)$-fold one.

Assuming $Z = 1$ we have calculated $K^{i}_{2\parallel}$ for the values of parameters used in the previous example. The result is plotted on Fig. 2. $K^{i}_{2\parallel}$ is two orders of magnitude higher than $K^{c}_{2\parallel}$ (for a perfect random alloy) but has no peculiarity near $\omega = 2\omega_{c0}$.

It was already stated that the magnetoabsorption coefficient for cyclotron-resonance-active and -inactive polarizations (in the Faraday configuration) has logarithmic divergences at $\omega = n'\omega_{c0}$ [see Eq. (44)]. These divergences do not vanish for $\mu(\vec{q}) = 0$, i.e., they will exist also in a semiconductor with ionized impurities, which is not an alloy, and will be proportional in amplitude to the impurity concentration [see Eq. (65)]. It is interesting to note that relatively strong absorption peaks at harmonics of cyclotron resonance have already been observed in n-type InSb,[16–19] and were interpreted as transitions involving plasmon emission.[19–23] One can speculate if these harmonics are not due, at least partially, to the mechanism described in the present paper, corresponding to electron-ionized impurity interaction.

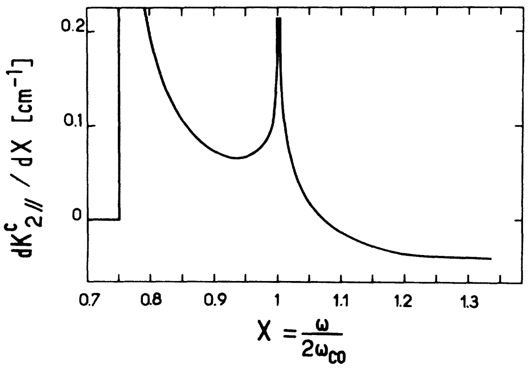

FIG. 3. Derivative of absorption coefficient due to composition fluctuations with respect to frequency vs frequency for the numerical example discussed in the text.

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APPENDIX

Suppose some quantum system is perturbed by the operator

$$\mathcal{H}_1 \cos\omega t + \mathcal{H}_2 \sin\omega t + \mathcal{H}_3 \ , \tag{A1}$$

where $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ are Hermitian and time-independent operators. We are interested in transitions between two eigenstates of the unperturbed system $|a\rangle$ and $|b\rangle$, of different energies ($\mathcal{E}_a \neq \mathcal{E}_b$). Let us define

$$\mathcal{H}_{\pm} = \mathcal{H}_1 \pm i\mathcal{H}_2 \tag{A2}$$

and assume that we have two small parameters α and β in our problem. Suppose that for all eigenstates $|c\rangle$ there is

$$\langle c|\mathcal{H}_{\pm}|d\rangle \propto \alpha \ \text{ if } \mathcal{E}_d = \mathcal{E}_a \ , \tag{A3}$$

$$\langle d|\mathcal{H}_{\pm}|c\rangle \propto \alpha \ \text{ if } \mathcal{E}_d = \mathcal{E}_b \ , \tag{A4}$$

$$\langle b|\mathcal{H}_{\pm}|c\rangle\langle c|\mathcal{H}_{\pm,\mp}|a\rangle \propto \alpha^2\beta \ , \tag{A5}$$

and that there is

$$\langle d|\mathcal{H}_\pm|c\rangle \propto \alpha\beta \quad \text{if } \mathcal{E}_c = \mathcal{E}_a \text{ and } \mathcal{E}_d = \mathcal{E}_b \tag{A6}$$

and

$$\mathcal{H}_3 \propto \beta \; . \tag{A7}$$

By the relations (A3)–(A7) we mean that the corresponding matrix elements are at least of the indicated order. We assume also that if

$$\mathcal{E}_c \neq \mathcal{E}_a, \quad \langle b|\mathcal{H}_\pm|c\rangle \propto \alpha \; , \tag{A8}$$

then

$$\hbar\omega \neq \pm(\mathcal{E}_b - \mathcal{E}_c) \; , \tag{A9}$$

respectively. If

$$\mathcal{E}_c \neq \mathcal{E}_b, \quad \langle c|\mathcal{H}_\pm|a\rangle \propto \alpha \; , \tag{A10}$$

then

$$\hbar\omega \neq \pm(\mathcal{E}_c - \mathcal{E}_a) \; , \tag{A11}$$

respectively.

In the transition rate $W_{a\to b}$ we neglect the terms of higher than the second order in β, and we keep only the terms proportional to α^2. Up to the second order of perturbation calculus we have for $\mathcal{E}_a < \mathcal{E}_b$

$$W_{a\to b} = \frac{\pi}{2\hbar^2}\,\delta\left(\omega - \frac{1}{\hbar}(\mathcal{E}_b - \mathcal{E}_a)\right)\Bigg|\langle b|\mathcal{H}_+|a\rangle + \sum_{\substack{c \\ \varepsilon_c \neq \varepsilon_b}} \langle b|\mathcal{H}_3|c\rangle\langle c|\mathcal{H}_+|a\rangle(\mathcal{E}_b - \mathcal{E}_c)^{-1} - \sum_{\substack{c \\ \varepsilon_c \neq \varepsilon_a}} \langle b|\mathcal{H}_+|c\rangle\langle c|\mathcal{H}_3|a\rangle(\mathcal{E}_c - \mathcal{E}_a)^{-1}\Bigg|^2 \; . \tag{A12}$$

The transition rate in the opposite direction is the same:

$$W_{a\to b} = W_{b\to a} \; . \tag{A13}$$

Let us now establish the correspondence:

$$\mathcal{H}_1\cos\omega t + \mathcal{H}_2\sin\omega t = \mathcal{H}_r + \mathcal{H}_{rf} \; , \tag{A14}$$

$$\mathcal{H}_3 = \mathcal{H}_f \; , \tag{A15}$$

$$\alpha = |\vec{\mathrm{E}}| \; , \tag{A16}$$

$$\beta = |\mu(\vec{q})|, \quad |\nu(\vec{q})| \; . \tag{A17}$$

The eigenstates of the unperturbed Hamiltonian are the Landau functions. The assumptions (A3)–(A11) are fulfilled and for the transition rate we obtain the formula (15).

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