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Ginzburg-Landau model for a random magnet*

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A self-consistent Hartree approximation for the random Ginzburg-Landau model is used to calculate the susceptibility, the magnetic specific heat, and the critical dynamics of a random magnet. The effect of the quartic interaction on the above properties is studied in relation to the Gaussian approximation. The main features of the present calculation are in qualitative agreement with the experimental data on random magnets.

I. INTRODUCTION

The problem of random magnets has been actively investigated in the recent years from both theoretical and experimental points of view.[1-4]

The category of random magnets includes systems with quenched disorder where atomic moments are randomly arranged in space. In amorphous magnets the moments are assumed to have nearest-neighbor interactions and the disorder is usually described by an Ising system with randomly removed bonds. In a spin glass the randomly distributed spins are interacting via a long-range oscillatory Rudermann-Kittel-Kasuya-Yosida (RKKY) interaction. The theoretical approaches for spin glasses were mainly concerned with the existence and definition of an order parameter which describes an onset of spin ordering.[3] In spite of being successful in explaining the observed cusplike susceptibility, the specific-heat behavior predicted by these theories (cusplike or divergent) seems to be in contradiction with the experiments.

A different theoretical approach which is mainly concerned with the fluctuations of the order parameter is the one based on the Ginzburg-Landau (GL) free-energy functional. The latter formalism has been thoroughly studied via the renormalization-group techniques to explain the nature of the phase transition in random Ising model with quenched disorder.[4]

In the present work we use the so-called random Ginzburg-Landau model, originally suggested by Larkin and Ovchinnikov[5] for inhomogeneous superconductors, and later adapted to amorphous magnets by Shapero *et al.*[2] to treat random magnets above the ordering temperature. The latter authors have found a cusp in the susceptibility, using a Gaussian approximation which completely neglects the effect of the quartic term in the free-energy (GL) functional:

$$F[m]=\int d^3x\{A(\vec{x})m(\vec{x})^2+Bm(\vec{x})^4+C[\vec{\nabla}m(\vec{x})]^2\}, \tag{1.1}$$

where the order parameter $m(\vec{x})$ represents the magnetization averaged over a region of radius L surrounding the point $\vec{x}$, where $a \ll L < \xi(T)$, a and $\xi(T)$ being the lattice spacing and the temperature-dependent (GL) correlation length, respectively. B and C are the usual temperature-independent (GL) parameters with $C=\xi_0^2$, ξ_0 being the T-independent (GL) correlation length. In Eq. (1.1) $A(\vec{x})=A_0+\delta A(\vec{x})$, where $\delta A(\vec{x})$ is a Gaussian variable which phenomenologically represents the local fluctuations of the spin-spin interaction. This quantity $\delta A(\vec{x})$ is proportional to the fluctuation of the local field $\mathcal{H}(\vec{x})$ acting at the spin site $\vec{x}$ and averaged in a similar way as $m(\vec{x})$. In the case of amorphous magnets, the $A(\vec{x})$ fluctuations are due to the effect of the structural disorder on the short-range spin-spin interactions, whereas in the spin glasses the magnetic-impurities disorder combined with the long-range RKKY interaction is the origin of $\delta A(\vec{x})$.[6]

In our notation $A_0=T/T_{c0}-1\equiv\epsilon$, where T_{c0} is the transition temperature of the homogeneous model ($A=A_0$) which corresponds to a uniform distribution of the magnetic atoms of a given concentration in the Gaussian approximation ($B=0$).

The results of our calculation, in particular the persisting cusplike susceptibility associated with a possible broad peak of the magnetic specific heat point to an *a posteriori* justification for the applicability of the suggested model also to spin glasses.

Although our main interest is in the static properties of a random magnet we present here a dynamic generalization of the model introduced in Ref. 2 taking into account the quartic term. Within Hartree approximation which properly includes the screening effect[7] on the random field $\delta A(\vec{x})$ we calculate self-consistently the configurationally averaged propagator of the order parameter. Using this propagator we derive the T dependence of the relaxation rate of the critical slowing down Γ, and the magnetic specific heat C_m. Moreover, we calculate the $\vec{q}=0$ dynamic form factor which exhibits a central peak and finally, the $\vec{q}$-integrated form factor related to the NMR experiment. The main new feature of our work is to investigate the effect of the quartic term on the above quantities relative to the Gaussian approximation.

The outline of the paper is as follows: In Sec. II we define the general Hartree-random (HR) model; Sec. III is devoted to the static ($\omega = 0$) properties of the HR model and the ω dependence is discussed in Sec. IV. The numerical results are presented together with the analytical calculations in Secs. III and IV.

II. HARTREE-RANDOM MODEL

As a generalization of the random magnets theory of Shapero *et al.*[2] the local time-dependent susceptibility $g(\vec{x},\vec{x}',t)$ obeys, in Hartree approximation, the following equation

$$\{\partial/\partial t + \gamma[A(\vec{x}) + 2B\langle m(\vec{x},t)^2\rangle + C\vec{\nabla}^2]\}g(\vec{x},\vec{x}',t) = T\gamma\delta(\vec{x}-\vec{x}')\,\delta(t)\,, \quad (2.1)$$

where $\gamma = 8T_{c0}/\pi$. Treating $\delta A(\vec{x})$ perturbationally, and performing a configurational average on $g(\vec{x},\vec{x}',t)$ we define an averaged translational invariant propagator $G(\vec{x}-\vec{x}',t)$

$$G(\vec{x}-\vec{x}',t) = \langle g(\vec{x},\vec{x}',t)\rangle_c \quad (2.2)$$

which obeys the diagrammatic equation given by Fig. 1(a). The full thin lines represent the dynamical propagator of the Gaussian-homogeneous (GH) model ($B=0, A=A_0$) which in $(\vec{q},\omega)$ space has the expression

$$G_0(\vec{q},\omega) = T(\vec{q}^{\,2}\xi_0^2 + \epsilon - i\tilde{\omega})^{-1}\,, \quad (2.3)$$

where $\tilde{\omega} = \omega/\gamma$.

The full dynamic propagator is written, as usual in the $(\vec{q},\omega)$ space, as

$$G^{-1}(\vec{q},\omega) = G_0^{-1}(\vec{q},\omega) + \Sigma(\vec{q},\omega)\,, \quad (2.4)$$

where the self-energy $\Sigma(\vec{q},\omega)$ is according to Fig. 1(a)

$$\Sigma(\vec{q},\omega) = (B/T)\,G(\vec{x}=\vec{x}',t=0) - T^{-2}(2\pi)^{-3}\int d^3q'\,S(\vec{q}-\vec{q}')\times\nu^2(\vec{q}-\vec{q}')\,G(\vec{q}',\omega)\,. \quad (2.5)$$

In Eq. (2.5) $S(q)$ is the Fourier transform of the correlation function of $\delta A(\vec{x})$,

$$\langle\delta A(\vec{x})\,\delta A(\vec{x}')\rangle_c = (\langle\delta T_c^2\rangle_c/T_{c0}^2)\times\exp(-|\vec{x}-\vec{x}'|^2/\lambda^2)\,, \quad (2.6)$$

where $\delta T_c(\vec{x})$ is proportional to $\delta\mathcal{H}(\vec{x})$, and λ is the correlation length of the frozen disorder of impurities sites. The screening of $\delta A(\vec{x})$ fluctuations due to the interactions of $m(\vec{x},t)$ (B term) is taken into account via the quantity $\nu(q)$. The three-dimensional expression of $\nu(q)$ was previously calculated by Ferrell and Scalapino[7] and its dimensionless form appropriate to our framework is given below by Eq. (2.13).

In the process of configurational average we discarded crossing diagrams [Fig. 1(b_2)] relative to noncrossing diagrams [Fig. 1(b_1)] since even in Gaussian case ($B=0$) $\Sigma_{b_2}/\Sigma_{b_1} = (\kappa/2\pi)\ln(2e/\kappa) \ll 1$ in the critical region. The dimensionless quantity κ is related to the temperature-dependent inverse correlation length $\kappa = \xi(T)^{-1}\xi_0$, and is defined as

$$\kappa^2 = \epsilon + T\Sigma(0,0)\,. \quad (2.7)$$

We calculate the quantity $G(\vec{x}=\vec{x}',t=0)$ as follows:

$$G(\vec{x}=\vec{x}',t=0) = \frac{T}{(2\pi)^3}\int d^3q\,(\vec{q}^{\,2}\xi_0^2+\kappa^2)^{-1} = (T/2\pi^2\xi_0^3)(1-\pi\kappa/2)\,, \quad (2.8)$$

where in the integration process we used a momentum cutoff ξ_0^{-1}. Using the above result and Eq. (2.5), the zero-momentum self-energy $\Sigma(0,\omega)$ obeys the equation

$$T\Sigma(0,\omega) = (T/T_{c0})w(-\kappa+2/\pi) - \frac{\lambda^3\langle\delta T_c^2\rangle_c}{2\pi^{1/2}T_{c0}^2}\int_0^\infty \frac{q^2\,dq\,e^{-q^2\lambda^2/4}\nu(q)^2}{q^2\xi_0^2+\epsilon+T\Sigma(\vec{q},\omega)-i\tilde{\omega}}\,. \quad (2.9)$$

In Eq. (2.9) the parameter w, proportional to the strength of the quartic interaction, is defined by

$$w = BT_{c0}/2\pi\xi_0^3\,. \quad (2.10)$$

This quantity is related to the so-called Ginzburg critical width $\Delta\epsilon_G$, $\Delta\epsilon_G = 8w^2$.[8]

With the usual approximation $\Sigma(\vec{q},\omega)\to\Sigma(0,\omega)$ in Eq. (2.9) we define a frequency- (ω) dependent

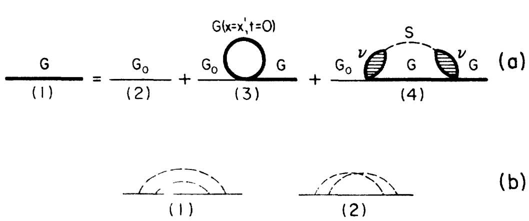

FIG. 1. (a) Diagrammatic Dyson equation for the configurationally averaged dynamic Green's function G. The thin full lines represent the GH propagator; the bold lines are for the full propagators; the interrupted lines represent the frozen disorder correlation function; ν contains the screening correction. (b) Two typical self-energy diagrams appearing in the averaging process for Σ.

quantity $z(\omega)$,

$$z^2=\epsilon+T\Sigma(0,\omega)-i\tilde{\omega}\,. \qquad (2.11)$$

In the $\omega=0$ limit z is obviously identified with the dimensionless temperature-dependent inverse correlation length κ defined by Eq. (2.7). Using Eqs. (2.7), (2.9), and (2.11) we obtain the following self-consistent equation for $z(\omega)$

$$z^2=w(T/T_{c0})(2/\pi-\kappa)+(\epsilon-i\tilde{\omega}) - \frac{2\alpha}{\pi^{1/2}}\int_0^\infty \frac{t^2e^{-t^2}\nu(t;\kappa)^2}{t^2+z^2/r^2}dt\,, \qquad (2.12)$$

where $r=2\xi_0/\lambda$ and $\alpha=2(\langle\delta T_c^2\rangle_c/T_{c0}^2)/r^2$. The explicit expression of $\nu(t;\kappa)$ is given by

$$\nu(t;\kappa)=[1+(wT/rT_{c0})\tan^{-1}(rt/2\kappa)/t]^{-1}\,. \qquad (2.13)$$

In order to solve Eq. (2.12) for $\mathrm{Re}\,z(\omega)$ and $\mathrm{Im}\,z(\omega)$ for a given set of parameters w, r, α, and ϵ, we need the value of $\kappa=\mathrm{Re}\,z(\omega=0)$. This quantity κ and another static ($\omega=0$) property of the Hartree-random model will be discussed in Sec. III.

III. STATIC PROPERTIES OF THE HR MODEL

Putting $\tilde{\omega}=0$ and replacing z by κ in Eq. (2.12) we obtain the self-consistent equation for κ which should be solved numerically. Before this we would like to discuss two simple limiting cases which can be solved analytically.

A. Gaussian-random model ($w=0$)

In the absence of the interaction between the fluctuations ($B=0$) the Ginzburg critical width is zero and consequently $\nu(w=0)=1$. In this case the self-consistent integral equation (2.12) becomes simply algebraic

$$\kappa_G^2=\epsilon-\alpha\,[1-\pi^{1/2}(\kappa_G/r)\exp(\kappa_G^2/r^2)\,\mathrm{erfc}(\kappa_G/r)]\,, \qquad (3.1)$$

where erfc(x) is the usual error function.[9] In order to investigate the behavior of the κ_G solutions in the vicinity of the ordering temperature, where we expect $\kappa_G\ll 1$, we approximate (Ref. 9)

$$xe^{x^2}\mathrm{erfc}(x)\simeq x(1-2x/\pi^{1/2})+O(x^3)\,.$$

Then Eq. (3.1) becomes quadratic with real positive solutions $\kappa_G\geqslant\kappa_c^G=\alpha r(\pi^{1/2}/2)/(r^2+2\alpha)$ for any $\epsilon\geqslant\epsilon_c^G$, where

$$\epsilon_c^G=\alpha\,[1-(\pi/4)\,\alpha/(r^2+2\alpha)]\,. \qquad (3.2)$$

In any practical cases $\alpha/r^2\ll 1$ and $\epsilon_c^G\simeq\alpha$.[10] In the vicinity of ϵ_c^G, κ_G behaves like

$$\kappa_G/\kappa_c^G=1+[(\epsilon-\epsilon_c^G)/(\alpha-\epsilon_c^G)]^{1/2}\,. \qquad (3.3)$$

The exact numerical calculation of κ_G via Eq. (3.1) shows that the value of κ_G at $\epsilon=\epsilon_c^G$ is finite and very close to κ_c^G calculated above. The apparent divergence of $d\kappa_G/dT$ [see Eq. (3.3)] seems to be an artifact of the approximation used for $e^{x^2}\mathrm{erfc}(x)$. Since κ_G^{-1} is not divergent as in a real phase transition, neglecting higher terms in the $e^{x^2}\mathrm{erfc}(x)$ expansion is *asymptotically* incorrect. For any practical range of temperatures, the behavior of κ_G will be assumed to be given by Eq. (3.3), and the "divergence" seen in the magnetic specific heat, and the nuclear relaxation rate in the Gaussian-random (GR) model should be taken within the above assumption.

B. Hartree-homogeneous model ($\alpha=0$)

Assuming a homogeneous distribution of the magnetic impurities, the local field $\mathcal{H}(\vec{x})$ does not fluctuate and then $\delta T_c=0$. In this case $\alpha=0$ and the self-consistent equation for κ in the Hartree-homogeneous (HH) model is simply quadratic

$$\kappa_H^2+w(\epsilon+1)\,\kappa_H-[\epsilon+(2/\pi)w(\epsilon+1)]=0\,. \qquad (3.4)$$

This equation allows real non-negative solutions $\kappa_H\geqslant 0$ only for $\epsilon\geqslant\epsilon_c^H$, where $\epsilon_c^H=-2w/(\pi+2w)$. Therefore, for any finite w (critical width $8w^2$) there is a different (negative) shift in the ordering temperature. In order to see the critical behavior of κ_H and other different static quantities as a function of the reduced temperature relative to every $T_c(w)$ we define $\tau=T/T_c(w)-1$ and easily obtain

$$\kappa_H(\tau)=\begin{cases}\tau^{1/2} & \text{for } w=0\,,\\ \left(\dfrac{\pi+2w}{\pi w}\right)\tau & \text{for } \tau\ll 1 \text{ and } w\neq 0\,.\end{cases} \qquad (3.5)$$

Then in HH model the temperature-dependent inverse correlation length always goes to zero at $T_c(w)$ as $T-T_c(w)$ [except in the case $w=0$ (GH model) when $\kappa\sim(T-T_{c0})^{1/2}$]. This fact leads to a divergence of $\xi(T)$ at $T_c(w)$, which is associated, as usual, with a *real* phase transition. In Sec. III E we shall see how the randomness affects these results, i.e., the disappearance of the sharp phase transition existing in the homogeneous case(s).

C. Critical slowing down

The relaxation rate of the critical slowing down of the order parameter Γ is calculated via Suzuki-Igarashi[11] formula

$$\Gamma^{-1}=[G(0,0)/\gamma T]\,[1-\gamma T\partial\Sigma(0,\omega)/\partial(i\omega)\,|_{\omega=0}]\,. \qquad (3.6)$$

The first factor on the right-hand side of Eq. (3.6) represents the conventional slowing down $\Gamma_{\mathrm{conv}}^{-1}$, and is proportional via $G(0,0)$ to κ^{-2}. Performing the derivative of both sizes of Eq. (2.9) with $\Sigma(\vec{q},\omega)$ replaced by $\Sigma(0,\omega)$ in the integrand, and taking the limit $\omega \to 0$, we obtain the following result[12]:

$$\frac{\Gamma}{T_{c0}} = \frac{8}{\pi}\kappa^2\left(1 - \frac{2\alpha}{\pi^{1/2}r^2}\int_0^\infty \frac{t^2 e^{-t^2}\nu(t;\kappa)^2}{(t^2+\kappa^2/r^2)^2}dt\right). \quad (3.7)$$

Since κ remains finite for any $\alpha \neq 0$ (see below the numerical results for $w \neq 0$ case), the conventional relaxation rate of the order parameter $\Gamma_{\text{conv}} = (8/\pi) T_{c0}\kappa^2$ is always *finite* in the random case.

As before, the $w=0, \alpha\neq 0$ case (GR model) can be solved analytically and the Γ_G expression is given by

$$\Gamma_G/T_{c0} = (4/\pi)\kappa_G^2\{1 + [(\epsilon - \alpha)/\kappa_G^2] + 2[(\epsilon - \kappa_G^2)/r^2]\}. \quad (3.8)$$

In the derivation of the expression (3.8) we used the self-consistent equation for κ_G [Eq. (3.1)]. It can be verified that, if $\alpha = 0$, Eq. (3.1) gives $\kappa_G^2 = \epsilon$ and from Eq. (3.8) we have immediately $\Gamma_G/T_{c0} = (8/\pi)\epsilon$, the well-known result of the GH model.

The $\vec{q}=0, \omega=0$ configurationally averaged susceptibility $\langle\chi(0,0)\rangle_c$, which is proportional to $\Gamma_{\text{conv}}^{-1}$, remains finite even at $T=T_c$ since $\kappa > 0$ at this temperature. In Sec. III E we will see that for any $\alpha \neq 0$ we obtain a cusplike behavior for $\langle\chi(0,0)\rangle_c$, even for very strong interaction between the fluctuations of the order parameter.

On the other hand, in $\alpha = 0, w\neq 0$ case (HH model) the relaxation rate is given by the conventional one and equals $(8/\pi)\kappa_H^2$. Since $\kappa_H = 0$ at $T = T_c(w)$, the relaxation rate Γ_H, as a function of τ, approaches zero with a zero slope, except for the $w=0$ case, when the slope is finite and equal to $8/\pi$. [See Eq. (3.5)].

D. Specific heat

The configurationally averaged magnetic part of the entropy $\langle S_m\rangle_c$ is calculated via the order parameter with the formula

$$\langle S_m\rangle_c = -\frac{\partial A_0}{\partial T}\int d^3x \langle\langle m(\vec{x})^2\rangle\rangle_c, \quad (3.9)$$

where as usual we neglect a term proportional to $\int d^3x\langle\langle F\rangle\rangle_c$ which is not "critical" in the vicinity of T_c.[13] Using, as before, a momentum cutoff ξ_0^{-1} we have

$$\langle S_m\rangle_c = -\frac{V}{2\pi^2}\frac{T}{T_{c0}}\int_0^{\xi_0^{-1}} \frac{q^2\,dq}{q^2\xi_0^2+\kappa^2}. \quad (3.10)$$

The magnetic part of the specific heat C_m is calculated easily from Eq. (3.10) with the result:

$$\xi_0^3 C_m = (T/4\pi T_{c0})[(\kappa - 2/\pi) + T(d\kappa/dT)]. \quad (3.11)$$

Replacing z by κ and putting $\tilde{\omega}=0$, and taking the derivative with respect to T of Eq. (2.12) we have

$$T_{c0}\frac{d\kappa}{dT} = \frac{1 + w[(2/\pi) - \kappa + (2\alpha/r)I_1(\kappa)]}{(2\kappa\Gamma/\Gamma_{\text{conv}}) + w(T/T_{c0})[(8\alpha/r^2)I_2(\kappa)+1]}, \quad (3.12)$$

with $\Gamma/\Gamma_{\text{conv}}$ given by the expression in the large parentheses of the right-hand side of Eq. (3.7). The integral expression for $I_1(\kappa)$ and $I_2(\kappa)$ are given by

$$I_1(\kappa) = \pi^{-1/2}\int_0^\infty \frac{2te^{-t^2}\tan^{-1}(rt/2\kappa)}{t^2+\kappa^2/r^2}[\nu(t;\kappa)]^3 dt, \quad (3.13a)$$

$$I_2(\kappa) = \pi^{-1/2}\int_0^\infty \frac{t^2 e^{-t^2}}{(t^2+\kappa^2/r^2)(t^2+4\kappa^2/r^2)}[\nu(t;\kappa)]^3 dt. \quad (3.13b)$$

As we pointed out before, in the $w=0$ case, $d\kappa/dT \sim (T-T_c)^{-1/2}$ as $T\to T_c$ and the magnetic specific heat C_m diverges with the critical exponent $\frac{1}{2}$.[14] The fact that, in general case $(w, \alpha \neq 0)$, neither κ nor Γ approach zero as $T\to T_c$, will prevent the divergence of C_m. Moreover we shall see in the next section that the behavior of C_m changes from a divergence in $w=0$ case (Gaussian) through a cusplike one for $w \ll 1$ to a broad peak behavior when a strong interaction between the order-parameter fluctuations is switched on $(w \lesssim 1)$.

E. Numerical results

The numerical results for the magnetic specific heat C_m and the relaxation rate of the critical slowing down as a function of ϵ for various values of w are exhibited in Fig. 2. The calculations were

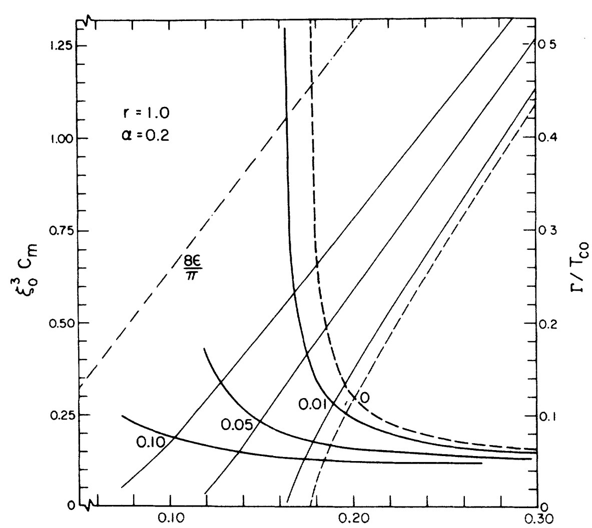

FIG. 2. Numerical results for the relaxation rate Γ (thin lines) and the magnetic specific heat C_m (bold lines) as a function of ϵ for different values of w. The interrupted lines represent the Gaussian $(w=0)$ results.

done for $r = 1$ and $\alpha = 0.2$, values which we believe are suitable for a random magnet. In particular, it is reasonable to assume that the frozen disorder correlation length λ is comparable to ξ_0 giving $r \sim 1$. The value $\alpha = 0.2$ implies $\langle \delta T_c^2 \rangle^{1/2} \simeq 0.31\, T_{c0}$ and corresponds to a partial smoothing out of the discrete lattice structure characteristic for a GL formalism. The Γ curves were obtained using Eq. (3.7), after solving the self-consistent equation for κ. Using, for a given ϵ, the κ and Γ values, and calculating numerically the integrals $I_1(\kappa)$ and $I_2(\kappa)$ given by Eqs. (3.13), we obtain the C_m value given by Eqs. (3.11) and (3.12). In each graph, the $w = 0$ curves represent the GR model results. As w progressively increases the behavior of C_m and Γ shows a drastic change relative to $w = 0$ case. The Γ curves show different nonlinear behavior and in particular the curves *never* reach the zero value in contrast with the homogeneous Hartree model ($\alpha = 0$) which shows a zero relaxation rate at T_c. The magnetic specific-heat curves change from a divergent behavior in GR case, through a cusplike one, to a possible broad peak as $w \simeq 0.1$. Relating the GL parameters A, B, and C to the $\sum_{ij} J_{ij} S_i S_j$ Hamiltonian, it is possible to show that $w \sim (d/\xi_0)^3 (J_0/T_{c0})^2$, where $J_0 = \sum_{ij} J_{ij}$, d being the mean distance between the magnetic atoms. Since in the first order (d, ξ_0) and (J_0, T_{c0}) have the *same* concentration dependence (c), we take w as c independent, which universally describes the strength of the interaction between the order-parameter fluctuations. Physical arguments show that due to the first-order effects, for small concentrations ($c < 10\%$), $T_{c0} \sim c$ and $\xi_0^3 \sim c^{-1}$,[15] a fact that gives a magnetic specific heat C_m proportional to the concentration c for all temperatures considered, which has been found experimentally.[16]

Solving Eq. (2.12) numerically for $z(\omega = 0) = \kappa$, we obtain for every $w \geq 0$ a positive solution for any $\epsilon \geq \epsilon_c(w) = T_c(w)/T_{c0} - 1$, a solution which remains *finite* even at $T = T_c(w)$. The fact implies the existence of a finite susceptibility at $\epsilon_c(w)$ even in the Gaussian-random model ($w = 0$ case). The finite value of $\xi(T) = \xi_0 \kappa^{-1}$ at $T_c(w)$ corresponds to an absence of a long-range ordering due to the random positions of the spins together with the (possible) oscillatory nature of their interaction. The curves of $\Gamma_{\text{conv}} = (8/\pi) T_{c0} \kappa^2$ and $T_{c0} \langle\chi(0,0)\rangle_c = (T/T_{c0}^2 \kappa^2)$ as a function of ϵ are exhibited in Fig. 3 for various values of w. This time we can see that the minimum value of κ [at $T = T_c(w)$] is almost independent of w and is very close to the Gaussian value κ_G, given by Eq. (3.3). The cusplike behavior of $\langle\chi(0,0)\rangle_c$ resembles the results obtained experimentally for spin glasses like Mn in Cu or Fe in Au.[16] The weak w dependence of $\langle\chi(0,0)\rangle_c$ contrasts with the behavior of C_m and Γ, exhibited in Fig. 2. This fact may explain the cusplike type curve for $\langle\chi(0,0)\rangle_c$ and the broad peak in C_m curve observed in the above-mentioned spin glasses. A value for $w \simeq 0.05$ seems appropriate for a glass regime as also corroborated by preliminary NMR results for the same glasses.[17]

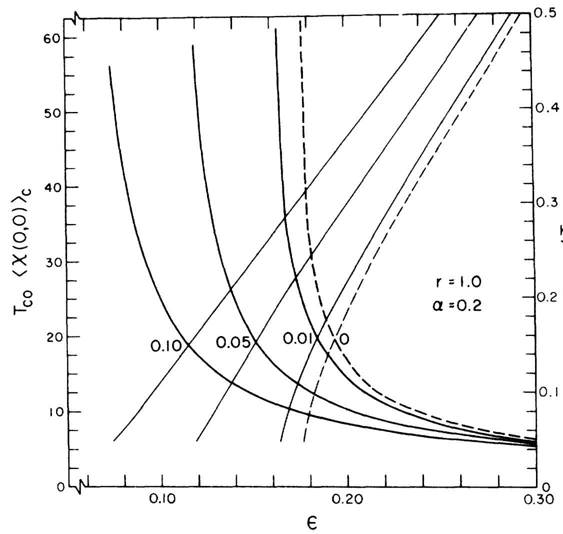

FIG. 3. Numerical results for the conventional relaxation rate Γ_{conv} (thin lines) and $\vec{q} = 0$, $\omega = 0$ susceptibility (bold lines) as a function of ϵ for different values of w. The interrupted lines represent the Gaussian ($w = 0$) results.

We note that Γ can be measured by studying the dynamic form factor obtained in the neutron diffraction measurements. For small values of w, $\Gamma_{\text{conv}}/\Gamma \gg 1$ implying the existence of two distinguished time constants leading to the presence of a central peak in the dynamic form-factor function. The central-peak problem and the relation of the $w = 0$ susceptibility to the NMR signal are the main subjects of the next section.

IV. DYNAMIC PROPERTIES OF HR MODEL

In order to calculate the $\vec{q}, \omega$ configurationally averaged susceptibility we have to solve first the integral equation (2.12) for the complex quantity $z = x + iy$. For the values of r, α, and w used above, we calculate the ω dependence of x and y using the already calculated value of $\kappa = x(\omega = 0)$. Defining $\langle\chi(\vec{q}, \omega)\rangle_c = G(\vec{q}, \omega)/T_{c0}^2$ and approximating again $\Sigma(\vec{q}, \omega)$ by $\Sigma(0, \omega)$ in the G expression we have

$$\langle\chi(\vec{q}, \omega)\rangle_c = (T/T_{c0}^2)(\vec{q}^{\,2} \xi_0^2 + z^2)^{-1}, \tag{4.1}$$

where z is defined by Eq. (2.11). We note that for the $w = 0$ case the integral equation for z becomes an analytic one

$$z_G^2-(\epsilon-i\tilde{\omega})$$
$$+\alpha[1-\pi^{1/2}(z_G/r)\exp(z_G^2/r^2)\operatorname{erfc}(z_G/r)]=0\,, \tag{4.2}$$

which can be solved, for $x(\omega)$ and $y(\omega)$, as a system of two coupled nonlinear equations. Using the Newton method, and starting for $\tilde{\omega}\ll 1$ with $x=\kappa$ and $y=0$ as initial guesses, the values for $x(\omega), y(\omega)$ are easily obtained. In the general case ($w\neq 0$) we have a much more difficult numerical task, to solve two coupled integral equations.

A. $\vec{q}=0$ form-factor function

Defining the dimensionless $\vec{q}=0$ form-factor function as

$$F_{\vec{q}=0}(\omega)=T_{c0}^2[\operatorname{Im}\langle\chi(0,\omega)\rangle_c]/\omega\,, \tag{4.3}$$

with the definition (4.1) for $\langle\chi(0,\omega)\rangle_c$ we have

$$F_{\vec{q}=0}(\omega)=-\frac{T}{\omega}\frac{2xy}{(x^2+y^2)^2}\,. \tag{4.4}$$

The $\omega=0$ value of $F_{\vec{q}=0}$ is calculated with the same method as Γ using the fact that $y(\omega)/\omega$ goes to a finite value as $\omega\to 0$ and $x(\omega=0)=\kappa$. The result is

$$F_{\vec{q}=0}(\omega=0)=(T/\Gamma)/\kappa^2\,, \tag{4.5}$$

where Γ is the value of the relaxation rate of the critical slowing down calculated before [see Eq. (3.7)]. In order to compare our theoretical-numerical results with experimental data we chose for $w=0$ and $w=0.1$ such values for ϵ that the real temperature will be situated relative to $T_c(w=0)$ and $T_c(w=0.1)$, respectively, at the same distance. The curves for $F_{\vec{q}=0}(\omega)$ for $\tau=T/T_c(w)-1$ equal to 5×10^{-4}, 10^{-3}, and 5×10^{-3} are exhibited in Fig. 4. In the Gaussian case ($w=0$) the central peak rapidly becomes more and more pronounced as the temperature approaches (from above) the ordering temperature. When a strong interaction between order-parameter fluctuations is allowed ($w=0.1$) the change of the peak with the temperature is not so rapid and eventually the peak is finite height and broadened, even at $T_c(w=0.1)$, due to the finite values of Γ and κ. Moreover, away from the transition the form-factor curve for $w\neq 0$ is above the correspondent one for $w=0$ due to the fact that the Γ value in $w\neq 0$ becomes smaller than that in the Gaussian case at the same τ. We point out that the curves *are not* simply Lorentzian and look like a combination of two Lorentzians with different time constants.[18]

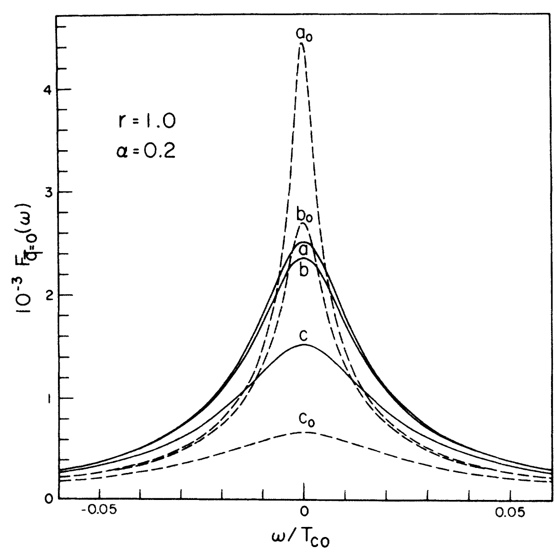

FIG. 4. ω dependence of the $\vec{q}=0$ form-factor function for the GR model ($w=0$, interrupted lines) and HR model ($w=0.1$, full lines). The curves (a_0,a), (b_0,b), and (c_0,c) are calculated for the τ values 5×10^{-4}, 10^{-3}, and 5×10^{-3}, respectively.

B. Local susceptibility

The nuclear spin-lattice relaxation rate due to the electron-spin fluctuations is related to the imaginary part of the local susceptibility

$$\chi(\omega)=\sum_{\vec{q}}\langle\chi(\vec{q},\omega)\rangle_c\,. \tag{4.6}$$

Taking the imaginary part of Eq. (4.6), and using Eq. (4.1), we transform the $\vec{q}$ summation to a three-dimensional $\vec{q}$ integration, and after a contour integration in the complex q plane we obtain the simple result

$$\xi_0^3\operatorname{Im}\chi(\omega)=-(T/4\pi T_{c0}^2)\,y\,. \tag{4.7}$$

The $\vec{q}$-integrated form-factor function is defined

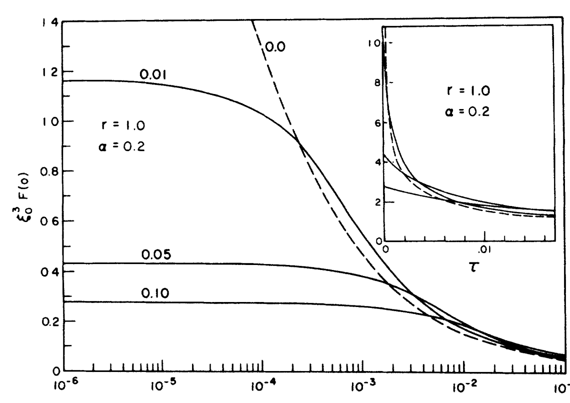

FIG. 5. Logarithmic plot of the τ dependnece of the $\vec{q}$-integrated form-factor function in the limit $\omega\to 0$. The curves are labeled by the values of w. Insert: the same, in a linear τ scale.

as

$$F(\omega) = T_{c0}^2[\mathrm{Im}\,\chi(\omega)]/\omega \tag{4.8}$$

which in the limit $\omega \to 0$ gives

$$\lim_{\omega\to 0} F(\omega) = \frac{1}{8\pi}\xi_0^{-3}\kappa(T/\Gamma). \tag{4.9}$$

The nuclear relaxation rate $(1/T_1)$ is proportional to the expression (4.9). Since $\Gamma_G \sim (T - T_c)^{1/2}$ the rate $1/T_1$ diverges with the same critical exponent as C_m, $\frac{1}{2}$.[14] The $\xi_0^3 F(0)$ curves as a function of τ in a logarithmic scale for various values of w are presented in Fig. 5. The fact that the $w \neq 0$ curves level off for such small values of τ indicates that we are not dealing with a real phase transition. Recalling that the Ginzburg critical width is in fact $8w^2$, we can see that the leveling off for each $w \neq 0$ starts around $\tau = 8w^2$. For $\tau > 10^{-2}$ the w dependence is very weak. Evidently, it is experimentally impossible to approach so close the transition temperature and we present in the insert of Fig. 5 the same data in a practical reasonable linear τ scale. Recent measurements on CuMn glasses[17] show that the $1/T_1$ enhances not more than twice when the relative temperature approaches the ordering temperature from above. This fact indicates that a value for w like 0.05 is appropriate for a spin-glass regime, which is also corroborated by the results on the magnetic specific-heat measurements.

V. CONCLUSIONS

We have discussed the static and the dynamic properties of a random magnet above its ordering temperature within a Ginzburg-Landau formalism which accounts for the quartic interaction between the order-parameter fluctuations. The possible long-range interaction between the impurity spins—via the conduction electrons—leads to a collectivelike "quasi-critical" behavior of the $\vec{q} = 0$, $\omega = 0$ susceptibility. The randomness, taken into account phenomenologically via the GL A term, combined with the fluctuations of the order parameter $m(\vec{x}, t)$, leads to short-range order effects like in the magnetic specific heat C_m, the relaxation rate of the critical slowing down Γ, and the central peak.[19]

In addition to the concentration-(c) dependence effects on the homogeneous- (Gaussian) transition temperature T_{c0}, the present model shows competitive second-order [in $\delta A(\vec{x})$ fluctuations] effects on the ordering temperature: the randomness shifts T_c to higher temperatures, whereas the interaction between the order-parameter fluctuations (the quartic term) leads to an opposite shift. A mean-field-approximation treatment shows that the structural disorder in a Heisenberg ferromagnet—via fluctuation of the exchange interaction—decreases T_c,[20] but an analysis of small random clusters of localized spins on the same model[21] shows an increase in T_c. Therefore, one would expect changes of T_c in both directions, the actual direction of change being governed by the details of the structural disorder included in the parameter r and α, and the corresponding changes in the exchange interactions represented by the quartic term B via the parameter w.[22] The proposed model accounts for the main experimental features that there is a persisting cusplike peak in the susceptibility associated with a broad peak in the magnetic specific heat. The different w dependence of C_m and $\langle\chi(0,0)\rangle_c$ is present already in the homogeneous models: The Hartree-approximation results which are equivalent to the so-called spherical model give a divergent susceptibility and a cusp in the specific heat, while in the Gaussian approximation both quantities are divergent at T_c.[23] We believe that the corrections beyond Hartree approximation are small, this fact being corroborated by the recent calculations based on the screening approximation by Bray[24] for inhomogeneous superconducting films.

In the context of the general central-peak problem at the phase transitions[25,26] we would like to emphasize the importance of the *self-consistency* in obtaining the correct dynamic form factor exhibiting a central peak. This fact may be of importance in the related problem of a central peak in the so-called model C of Halperin *et al.*[27] In the latter the order parameter is coupled to an entropy fluctuation which is a dynamic generalization of our static fluctuations of T_c. Using the renormalization-group technique the problem of central peak of the C model has been discussed in the "$4 - d = \epsilon$ expansion."[26] The diagrams of the present self-consistent scheme are obviously of any order in ϵ [see Fig. 1(b)], and hence beyond the ϵ-expansion results. This fact makes us believe that the central peaks experimentally observed in structural phase transition may be explained using the C model within a self-consistent scheme.

Finally, we point out that the present calculation can be also applied to an inhomogeneous superconductor. In view of the smallness of the Ginzburg critical widths in a three-dimensional superconductor the values of the parameter w to be used should be very small and the Gaussian results are expected to be valid in any practical range of temperatures.

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