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Quantum-limit magnetoresistance for acoustic-phonon scattering

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The behavior of strong-field magnetoresistance is discussed under conditions where acoustic-phonon scattering in the high-temperature limit is considered to be the dominant mechanism of scattering. The magnetoresistance, both in the transverse and the longitudinal configuration, is found to increase linearly with magnetic field in the quantum limit.

Transverse and longitudinal magnetoresistance are the two most investigated properties of semiconductors in which the effect of the magnetic field on electronic transport properties is exhibited. While the Boltzmann transport equation has been adequate for description of longitudinal magnetoresistance, the theoretical analysis of transverse magnetoresistance, especially at high magnetic fields, has continued to puzzle solid-state physicists for some time.[1] The electronic motion is well known to be quantized at high magnetic fields. The effects of this quantization on the galvanomagnetic properties should be most pronounced at low temperatures and high fields, such that $\omega_c \tau \gg 1$ and $\hbar\omega_c \gg k_B T$, where $\omega_c = eB/m^*c$ is the cyclotron frequency of an electron with effective mass m^* in a magnetic field B, τ is the relaxation time, and T is the temperature of the sample. The condition under which $\hbar\omega_c \gg k_B T$ has been called[2] the "quantum limit."

A transport theory which was general enough to include quantum effects in crossed electric and magnetic fields was first developed by Adams and Holstein[2] and extended by other workers in the field. A review of these early works has been given by Kubo *et al.*[3] and by Roth and Argyres.[4] An exposition of these theories, especially those applicable to the magnetophonon resonance, has recently been given by Peterson.[1] The theoretical results predict the temperature and magnetic field dependence of the transverse magnetoresistance[2,4] and longitudinal magnetoresistance[4] for various scattering mechanisms. A divergence in the transverse magnetoresistance was encountered, which was removed by the assumed existence of several cutoff mechanisms such as collision broadening,[3] phonon drag,[5] inelasticity,[3,6] non-Born scattering,[4] and classical cutoff.[4,7]

For acoustic-phonon scattering, these theories predicted a quadratic dependence of the transverse magnetoresistance on the magnetic field.[2,4] In contrast, various experimental investigations of the transverse magnetoresistance in a variety of semiconducting materials[8-11] indicated an approximate linear dependence on magnetic field. Numerous attempts were made to resolve the contradiction between theory and experiment. Herring[12] postulated that the existence of inhomogeneities would give rise to a linear field dependence of the transverse magnetoresistance. On the other hand, experimental results obtained by various workers lead to the conclusion that there is very little correlation between the existence of inhomogeneities and the field dependence of the magnetoresistance.[13-15]

Recently, one of us (V. K. A.) has developed a theory[16,17] which, by extending the scattering dynamics beyond the strict Born approximation, predicts a linear magnetoresistance for the case of both acoustic- and optic-phonon scattering.[17-19] This theory also has the feature that it yields the classical results for the transverse magnetoresistance in the low-field limit. The above-mentioned results for the linear magnetic field dependence of the high-field magnetoresistance were obtained by numerical methods. Simple analytic expressions of these results, valid in the quantum limit, may be of interest to experimentalists, and thus are derived here.

For the case where acoustic-phonon scattering is taken as elastic, we have the following components of the conductivity tensor[18] in the quantum limit (only the $n = 0$ level being occupied under the condition $\hbar\omega_c \gg k_B T$):

$$\sigma_{xx}=\sigma_{yy}=C_1\int_0^\infty dy\,\frac{e^{-sy}}{y+A_{ac}^2/\hbar\omega_c^3}\,, \tag{1}$$

$$\sigma_{yx}=-\sigma_{xy}=C_2\int_0^\infty dy\,\frac{e^{-sy}y^{1/2}}{y+A_{ac}^2/\hbar\omega_c^3}\,, \tag{2}$$

$$\sigma_{zz}=C_3\int_0^\infty dy\,e^{-sy}y\,, \tag{3}$$

with

$$A_{ac}=E_1^2k_BT(2m^*)^{1/2}/2\pi\rho u^2\hbar^2\lambda^2\,, \tag{4}$$

$$\lambda=(\hbar/m^*\omega_c)^{1/2}\,, \tag{5}$$

$$s=\hbar\omega_c/k_BT\,, \tag{6}$$

$$C_1=n_ee^2\hbar^3A_{ac}/\pi^{1/2}m^{*3}(k_BT)^{3/2}s\omega_c^3\lambda^4\,, \tag{7}$$

$$C_2=n_ee^2\hbar^3/\pi^{1/2}m^{*5/2}(k_BT)^{3/2}s\omega_c\lambda^3\,, \tag{8}$$

$$C_3=2n_ee^2\hbar^2\omega_c^2/\pi^{1/2}m^*(k_BT)^{3/2}A_{ac}\,. \tag{9}$$

Here E_1 is the deformation potential constant, ρ is the density of the semiconducting material, u is the velocity of sound, and n_e is the carrier density. The expressions (1)–(3) can be further simplified, with the result

$$\sigma_{xx}=\sigma_{yy}=\frac{2n_ee^2\hbar}{3\pi m^*k_BT\omega_c\tau_0}\times\exp\left(\frac{2\hbar/\tau_0}{3\pi^{1/2}k_BT}\right)^2E_1\left(\left(\frac{2\hbar/\tau_0}{3\pi^{1/2}k_BT}\right)^2\right), \tag{10}$$

$$\sigma_{yx}=-\sigma_{xy}=\frac{n_ee^2}{m^*\omega_c}\left[1-\frac{2\hbar/\tau_0}{3k_BT}\exp\left(\frac{2\hbar/\tau_0}{3\pi^{1/2}k_BT}\right)^2\times\operatorname{erfc}\left(\frac{2\hbar/\tau_0}{3\pi^{1/2}k_BT}\right)\right], \tag{11}$$

$$\sigma_{zz}=3n_ee^2\tau_0/m^*s\,, \tag{12}$$

where

$$\tau_0^{-1}=3(2m^*k_BT)^{3/2}E_1^2/8\pi^{1/2}\rho u^2\hbar^4 \tag{13}$$

is the zero-field average relaxation time for electron–acoustic-phonon scattering, and $E_1(x)$ and $\operatorname{erfc}(x)$ are the exponential integral[20] and complementary error function,[21] respectively:

$$E_1(x)=\int_x^\infty dt\,e^{-t}/t\,, \tag{14}$$

$$\operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty dt\,e^{-t^2}\,. \tag{15}$$

Under most conditions $2\hbar/3\pi^{1/2}k_BT\tau_0$ is much less than unity. For example at 77 °K with parameters for InSb,[22] $2\hbar/3\pi^{1/2}k_BT\tau_0=1.466\times10^{-4}$ if the deformation-potential constant E_1 is set equal to 7.2 eV, a value quoted by Rode.[23] In this case small-argument forms of the exponential integral and the complementary error function can be used:

$$E_1(x)\approx-\ln x-\gamma\,, \tag{16}$$

$$\operatorname{erfc}(x)\approx1\,, \tag{17}$$

where $\gamma=0.577$ is Euler constant. In this limit σ_{xx} and σ_{yx} are given by

$$\sigma_{xx}=\frac{2n_ee^2\hbar}{3\pi m^*k_BT\omega_c\tau_0}\left[\ln\left(\frac{3\pi^{1/2}k_BT}{2\hbar/\tau_0}\right)^2-\gamma\right]\,, \tag{18}$$

$$\sigma_{yx}=n_ee^2/m^*\omega_c\,. \tag{19}$$

The transverse magnetoresistance is given by

$$\frac{\rho_{xx}}{\rho_0}=\frac{\sigma_{yy}}{\rho_0(\sigma_{xx}\sigma_{yy}+\sigma_{xy}^2)}\approx\frac{\sigma_{yy}}{\rho_0\sigma_{xy}^2}=\frac{2}{3\pi}\frac{\hbar\omega_c}{k_BT}\left[\ln\left(\frac{3\pi^{1/2}k_BT}{2\hbar/\tau_0}\right)^2-\gamma\right]\,, \tag{20}$$

and the longitudinal magnetoresistance by

$$\frac{\rho_{zz}}{\rho_0}=\frac{1}{\sigma_{zz}\rho_0}=\frac{1}{3}\frac{\hbar\omega_c}{k_BT}\,, \tag{21}$$

where ρ_0, the zero-field resistivity, is given by[19]

$$\rho_0=m^*/n_ee^2\tau_0\,. \tag{22}$$

These expressions predict a linear dependence of the magnetoresistance, transverse or longitudinal, on the magnetic field and an inverse dependence on temperature T. The experimental results[24,25] support this behavior in the quantum limit. For example, the ratio $\rho_{xx}k_BT/\rho_0\hbar\omega_c$ at 77 °K according to Eq. (20) is 3.6 for parameters appropriate to n-InSb[22] (including[23] $E_1=7.2$ eV). For relatively pure materials, when impurity scattering could be neglected, the experimental results[24] tend to confirm this prediction in the quantum limit.

For the longitudinal case, experimental data of Sladek give an almost constant value of $\rho_{zz}/\rho_0\approx\frac{1}{3}$ in the quantum limit. But the experimental results of Haslett and Love[25] do show a linear dependence. More experimental work is therefore needed. The transverse magnetoresistance is always larger than unity; the ratio ρ_{xx}/ρ_{zz} is given by

$$\frac{\rho_{xx}}{\rho_{zz}}=\frac{2}{\pi}\left[\ln\left(\frac{3\pi^{1/2}k_BT}{2\hbar/\tau_0}\right)^2-\gamma\right]\,, \tag{23}$$

which for InSb at 77 °K is 10.9.

It is also worth noting that the logarithmic cutoff comes in naturally in this theory without having to be brought in artificially. Here, the cutoff energy is independent of magnetic field and arises from the so-called classical cutoff,[7] but has a form similar to that obtained by considering collision broadening. Essentially because of the divergence of the reciprocal of the electron relaxation time for low-energy electrons, there is no contribution to σ_1 from electrons for which $\omega_e\tau(\epsilon)<1$, where ϵ is the energy of an electron. Since in the quantum limit the electron relaxation time varies inversely with the magnetic field, the above condition is satisfied for electrons of a particular energy which is independent of magnetic field as long as $\hbar\omega_c \gg k_B T$.

Different cutoff mechanisms would just alter the argument of the logarithmic term, and since a logarithmic dependence on the field would be dominated by the linear dependence, we would still obtain a linear magnetoresistance. For example, inelasticity of the acoustic-phonon scattering could be important at extremely low temperatures and may provide a suitable cutoff. Actually, a theory has been developed[19] in which the effect of inelastic acoustic-phonon scattering has been approximately taken into account by replacing the energy of the phonon occurring in the arguments of the energy-conserving δ functions by $\hbar u/\lambda$. With the use of the approximations $s \gg 1$, $su/\lambda\omega_c \ll 1$, and $\hbar u \omega_c / A_{ac}^2 \lambda \gg 1$ (this is approximately 10^{+5} at 10 kG), we obtain the result for the magnetoresistance in that case to be

$$\frac{\rho_{xx}}{\rho_0} = \frac{\hbar\omega_c}{3\pi k_B T}\left[\ln\left(\frac{k_B T}{\hbar u/2\lambda}\right) - \gamma\right] . \tag{24}$$

The expression for the transverse magnetoresistance using this approximation for inelastic acoustic-phonon scattering is almost identical to that for elastic acoustic-phonon scattering, Eq. (20), except that the cutoff energy in the logarithmic term is different. However, since the linear term will dominate the logarithmic term, the magnetoresistance will still be linear in its dependence on the magnetic field. The effect of inelasticities, however, will reduce the coefficient of the linear term when the inelastic cutoff dominates the so-called classical cutoff. This also corresponds to the results of the exact numerical calculation.[19]

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