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Model of photoconduction in an amorphous medium

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A model for trap-controlled charge transport in an amorphous material is considered. The model is defined in terms of a set of coupled differential rate equations, in which each type of trap is characterized by a capture probability ω_i^* and a release probability r_i^*. This model is significant because it has recently been shown that the continuum limit of the Scher-Montroll master equation for anomalous dispersion is completely equivalent to the multiple-trapping equations. In this paper we develop the multiple-trapping model, and use asymptotic methods to treat traps whose release rates and capture rates are both different from one. We obtain an estimate of the increased transit time due to trapping, and also discuss the determination of both the number and types of different traps necessary to produce a disperse photocurrent transient.

I. INTRODUCTION

There has been considerable interest recently in the physics of charge transport in amorphous photoconductors. This problem is an interesting one conceptually and has many significant technological applications. Both the experimental and theoretical work has now developed to the point where it is possible to obtain new information about the electronic structure of these materials.

Traditionally, charge transport in amorphous photoconductors (Fig. 1) has been studied by the time-of-flight technique. A comprehensive program using this technique has recently been carried out by Pfister,[1] who has studied a-As_2Se_3, a-Se, and other materials in detail. In these experiments, a light flash of short duration is used to illuminate a sample of photoconducting material through a semitransparent electrode. Electron-hole pairs are created close to the surface of the material, charge carriers of the appropriate polarity are swept through the bulk of the sample by the applied electric field, and are collected by another electrode. Experimentally, either the voltage decay rate or the current decay (for fixed voltage across the sample) is monitored. For definiteness, we consider the decay of photocurrent transients for fixed voltage.

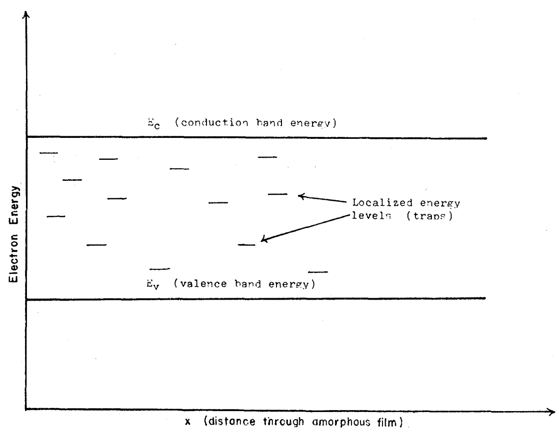

FIG. 1. Schematic diagram for energy levels of charges in an amorphous material with a large number of defects or traps. Transport of charge through the conduction band, under the influence of an electric field, is frequently interrupted by the capture of charge by traps. A distribution of release times from these traps leads to the highly dispersive type of current pulse shown in Fig. 2. This diagram is drawn for the case of no electric field. A term linear in x is superimposed on this diagram when the electric field is present.

The observed current transients $I(t)$ are found to be monotonically decreasing and relatively featureless. Sharfe,[2] however, has demonstrated that if $\log_{10} I(t)$ is plotted against $\log_{10} t$, the data lie along two straight lines of negative slope which intersect at a time τ_m. A typical dispersive current transient for As_2Se_3, say, is shown in Fig. 2. In particular, the measured current behaves like $t^{-(1-\alpha)}$ for $t < \tau_m$ and $t^{-(1+\alpha)}$ for $t > \tau_m$, with $0 < \alpha < 1$. This feature has been explained by Scher and Montroll[3] using a stochastic model involving a continuous-time random walk of charge carriers with a non-Gaussian waiting-time distribution $t^{-(1+\alpha)}$. Although the continuous-time random-walk formulation is very general, it is still necessary to assume a form for the waiting-time distribution function before quantities of physical interest can be calculated. It is easy to obtain this distribution function for extreme dispersion (power law) or no dispersion (exponential), but for cases of intermediate dispersion, the functional form is more difficult to determine. Recent experiments on a-

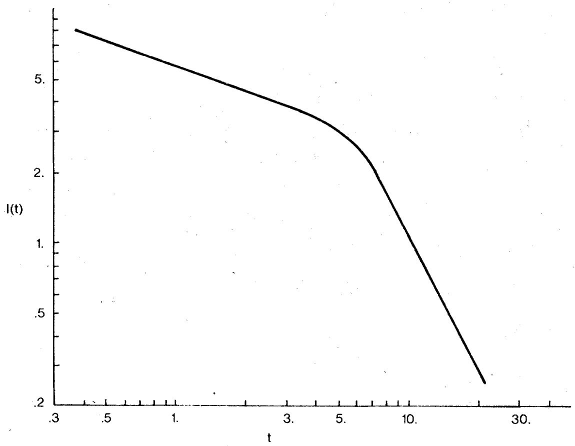

FIG. 2. Typical $\log_{10}I$-$\log_{10}t$ plot showing the current $I(t)$ associated with charges moving, in the presence of an electric field, through an amorphous photoconductor film. The dispersion in the current trace is due to the presence of traps in the material, which capture charges and retain them for varying lengths of time. The scale factors for both axes have been chosen for convenience.

Se,[1] however, show that the extreme dispersion observed at low temperatures ($T = 123$ K) becomes less with increasing temperature, and disappears altogether at room temperature. In view of these results it is clearly important to formulate a theory of charge transport which has the flexibility to deal with a broad range of dispersion.

Recently,[4] a model for trap-controlled transport in which the charge capture and release events are treated explicitly has been examined in detail, and found to be capable of predicting the anomalous dispersion which had previously been analyzed only by the Scher-Montroll theory. Of greater interest, it was found that the multiple-trapping model could be used to analyze photocurrent transients which were very disperse, or showed little dispersion. Some of the results of the theoretical analysis of the photocurrent transients in a-Se were reported earlier.[4] In the present paper we restrict ourselves mainly to the mathematical analysis of the multiple-trapping equations. One of the most interesting results arising out of the a-Se analysis, the basic equivalence of the Scher-Montroll and multiple-trapping theories, will be dealt with in a separate publication.[5]

We now consider a continuum model, corresponding to a homogeneous spatial distribution of a finite number (n, say) of distinct types of traps. Each type of trap is characterized by a lifetime τ_i and a release time $\tau_{r,i}$ ($i = 1, \ldots, n$). Hence, release probabilities per unit time are $r_i^\dagger = \tau_{r,i}^{-1}$, capture probabilities per unit time are $\omega_i^\dagger = \tau_i^{-1}$, and $\omega_i^\dagger T_0$ is the expected number of times a carrier will be captured by the ith kind of trap, where T_0 is the transit time of free carriers.

Let the (dimensional) variables t^* and x^* denote time and position through the amorphous film with $x^* = 0$ corresponding to the film's surface, and $x^* = L$ to the substrate. General differential equations for the volume concentrations of free carriers $p^*(x^*, t^*)$ and carriers trapped in the ith type of trap $p_i^\dagger(x^*, t^*)$ have been given by Rudenko and others.[6] We will restrict ourselves here to a small signal flash with constant electric field E. The photogeneration rate in this case is closely approximated by a δ function. Consequently, the solution for the flash is a Green's function for the case of continuous illumination which occurs in xerographic applications. The trap-dominated photoconduction equations are now

$$\frac{\partial p^*}{\partial t^*} + \mu E \frac{\partial p^*}{\partial x^*} + \sum_{i=1}^{n} (p^* \omega_i^\dagger - p_i^\dagger r_i^\dagger) = \eta N_0 \delta(x^*) \delta(t^*) \tag{1.1}$$

and

$$\frac{\partial p_i^\dagger}{\partial t^*} + p_i^\dagger r_i^\dagger = p^* \omega_i^\dagger \quad (i = 1, \ldots, n) , \tag{1.2}$$

where N_0 is exposure in photons per unit area and η is the efficiency of conversion into free carriers. Associated initial conditions are

$$p^*(x^*, 0) = p_i^\dagger(x^*, 0) = 0 . \tag{1.3}$$

The transient photocurrent I^* per unit area is obtained from p^* through the relation

$$I^*(t^*) = \frac{q}{L} \int_0^L \mu E p^*(x^*, t^*)\, dx^* , \tag{1.4}$$

where q is the magnitude of the moving charge.

Equations (1.1)–(1.3) may be solved directly using Laplace transforms, and Schmidlin has shown[6] that an exact solution for the current involves an n-fold convolution of modified Bessel functions of the first kind of order one. However, because of its complicated form, information can be easily extracted from this exact solution only in special cases such as $\omega_i^\dagger T_0$ large. The present work examines this model using asymptotic techniques. The equations are first nondimensionalized with respect to appropriate reference scales. Simplifications of the equation for the concentration of free carriers are then obtained, which remove all types of traps except those with $\omega_i^\dagger T_0$ and $r_i^\dagger T_0$ both order one. The asymptotic results also allow estimation of the increased mean transit time due to trapping. A final portion of this work considers the inverse problem of determining the types of traps responsible for a given current trace.

II. SCALING AND EXACT SOLUTION

To nondimensionalize the model equations, define new independent variables x and t by

$$x = x^*/L \quad \text{and} \quad t = t^*/T_0 , \tag{2.1}$$

where the free transit time T_0 is now simply $L(\mu E)^{-1}$. If σ_0 is a charge density, appropriate scaled-dependent variables are now

$$p(x,t) = qp^*(x^*,t^*)/\sigma_0$$

and (2.2)

$$p_i(x,t) = qp_i^*(x^*,t^*)/\sigma_0 .$$

With these scalings, Eqs. (1.1)–(1.3) become

$$\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} + \sum_{i=1}^{n} (p\omega_i - p_i r_i) = \lambda \delta(x)\delta(t) , \tag{2.3}$$

$$\frac{\partial p_i}{\partial t} + r_i p_i = \omega_i p , \tag{2.4}$$

and

$$p(x,0) = p_i(x,0) = 0 , \tag{2.5}$$

where

$$r_i = r_i^* T_0, \quad \omega_i = \omega_i^* T_0 ,$$

and (2.6)

$$\lambda = \eta N_0 q T_0/\sigma_0 .$$

Also, if $I(t) = I^*(t^*)T_0/\sigma_0 L$, then

$$I(t) = \int_0^1 p(x,t)\,dx . \tag{2.7}$$

An exact but somewhat involved solution of (2.3)–(2.5) may be obtained by using Laplace transforms without further analysis of these equations. If $\tilde{p}(x,s) = \mathfrak{L}[p(x,t)]$, then

$$\tilde{p}(x,s) = \lambda e^{-a(s)x} \tag{2.8}$$

for $x > 0$, where

$$a(s) = s\left(1 + \sum_{i=1}^{n} \frac{\omega_i}{s + r_i}\right) . \tag{2.9}$$

Denoting $\tilde{I}(s) = \mathfrak{L}[I(t)]$, this gives

$$\tilde{I}(s) = \lambda\left(\frac{1}{a(s)} - \frac{e^{-a(s)}}{a(s)}\right) . \tag{2.10}$$

The first term in (2.10) may be easily treated using a partial fraction expansion for $1/a(s)$. In particular, if

$$\frac{1}{a(s)} = \sum_{j=0}^{n} \frac{A_j}{s + s_j} , \tag{2.11}$$

then $s_0 = 0, s_1, \ldots, s_n$ are real and distinct, and

$$\phi(t) = \mathfrak{L}^{-1}\left(\frac{\lambda}{a(s)}\right) = \lambda \sum_{i=0}^{n} A_i e^{-s_i t} . \tag{2.12}$$

The principal difficulty with immediately applying the Laplace transform approach involves the complicated nature of

$$\psi(t) = \mathfrak{L}^{-1}\left(\frac{\lambda e^{a(s)}}{a(s)}\right) . \tag{2.13}$$

This function can be shown to vanish for $t < 1$ (the scaled free transit time) and hence, $\psi(t)$ may be interpreted as an exit function. However, because of the exponential factor in (2.13), $\psi(t)$ is a sum of terms involving n convolutions of Bessel and δ functions. In particular, Schmidlin has shown[6]

$$I(t) = \phi(t) - \psi(t-1) , \tag{2.14}$$

where

$$\psi(t) = \phi(t) * (\delta_1 + g_1) * (\delta_2 + g_2) * \cdots * (\delta_n + g_n) , \tag{2.15}$$

$$\delta_j(t) = e^{-\omega_j}\delta(t) ,$$
$$g_j(t) = e^{-\omega_j - r_j t}(\omega_j r_j/t)^{1/2} I_1[2(\omega_j r_j t)^{1/2}] , \tag{2.16}$$

and I_1 is the modified Bessel function of order 1.

In the following sections, it is shown that, by applying asymptotic techniques to the differential equations *before* taking Laplace transforms, the number of terms in both the summation (2.12) and the convolutions (2.14) can be reduced. Results for the exit function will also yield an approximation to the mean transit time for photoconduction with trapping.

III. LIFETIME AND RELEASE TIME ASYMPTOTICS

Except in the case of variables p_k, for which ω_k and r_k are of order one, asymptotic methods may be used to simplify Eq. (2.3).

Case 1. $r_k \gg 1$ and $\omega_k \leq O(r_k)$

Exact solutions of Eq. (2.4) have the form

$$p_i(x,t) = \omega_i \int_0^t p(x,\tau)e^{-r_i(t-\tau)}\,d\tau . \tag{3.1}$$

When r_i is large, this integral may be analyzed by the method of Laplace. Assume that $r_k \gg 1$ for $k = m+1, \ldots, n$. Then, for these p_k, the major contribution to the integral in (3.1) comes from the small interval near the upper limit where $t - \tau = O(r_k)$. The usual Laplace first approximation is now[7]

$$p_k(x,t) \sim (\omega_i/r_i)p(x,t) . \tag{3.2}$$

The present application, however, requires second approximations to p_k. These may be obtained by expanding $p(x,\tau)$ in a power series about $\tau = t$, giving, for $t = O(1)$,

$$p_k(x,t) = \frac{\omega_k}{r_k}\left[p(x,t) - \frac{1}{r_k}\frac{\partial p}{\partial t}(x,t) + O\left(\left(\frac{1}{r_k}\right)^2\right)\right] . \tag{3.3}$$

Hence, for these $n-m$ variables,

$$\sum_{i=m+1}^{n}(p\omega_i - p_i r_i) = \frac{\partial p}{\partial t}\sum_{i=m+1}^{n}\frac{\omega_i}{r_i} + O\left(\frac{1}{r^2}\right), \quad (3.4)$$

and Eq. (2.3) becomes

$$\frac{\partial p}{\partial x} + \left(1 + \sum_{i=m+1}^{n}\frac{\omega_i}{r_i}\right)\frac{\partial p}{\partial t} + \sum_{i=1}^{m}(p\omega_i - p_i r_i) = \lambda\delta(x)\delta(t) + O\left(\frac{1}{r^2}\right), \quad (3.5)$$

where r is a scale for the magnitude of the large r_i.

Case 2. $\omega_k \gg 1$ and $r_k \leqslant O(\omega_k)$

Assume that ω_k is large for $k = l+1, \ldots, m$. From Eq. (2.4), an appropriate balance for $t = O(1)$ is then $p(x,t) = O(1)$ but $p_k(x,t) = O(\omega)$ where ω is a magnitude scale for the large ω_k. This suggests expanding p and these p_k in powers of ω, i.e.,

$$p(x,t) = p^{(0)}(x,t) + \omega^{-1}p^{(1)}(x,t) + \cdots,$$

$$p_k(x,t) = \omega p_k^{(0)}(x,t) + p^{(1)}(x,t) + \cdots \quad (k = l+1, \ldots, m). \quad (3.6)$$

Substitution of these expansions into Eq. (3.5) shows that for $k = l+1, \ldots, m$, $p\omega_k - p_k r_k$ must vanish to lowest order. Equation (2.4) now requires that $\partial p_k/\partial t$ should be of order one rather than of order ω, and thus,

$$p_k^{(0)} = (\omega_k/\omega r_k)p^{(0)}, \quad (3.7)$$

$$p^{(1)}\frac{\omega_k}{\omega} - p_k^{(1)}r_k = \omega\frac{\partial p_k^{(0)}}{\partial t} = \frac{\omega_k}{r_k}\frac{\partial p^{(0)}}{\partial t}, \quad (3.8)$$

etc. The equation for $p^{(0)}(x,t)$ now becomes (dropping the superscript zero)

$$\frac{\partial p}{\partial x} + \left(1 + \sum_{i=l+1}^{n}\frac{\omega_i}{r_i}\right)\frac{\partial p}{\partial t} + \sum_{i=1}^{l}(p\omega_i - p_i r_i) = \lambda\delta(x)\delta(t) + O\left(\frac{1}{r^2}, \frac{1}{\omega}\right) \quad (3.9a)$$

Hence, traps which capture carriers a large number of times act in a similar manner to traps with short release times.

The large ω_k results obtained above, directly from the governing equations, may also be obtained from the exact solution (2.14)–(2.16). As noted previously,[6] for this special case the Bessel functions in (2.16) may be replaced by their asymptotic expansions, giving Gaussians with means ω_k/r_k and variances $\sigma_k^2 = 2\omega_k/r_k^2$ for $k = l+1, \ldots, m$. These Gaussians may now be convoluted directly to produce another Gaussian with mean $\sum_{l+1}^{m}\omega_i/r_i$ and variance $\sigma^2 = \sum_{l+1}^{m}\sigma_i^2$.

Case 3. $r_k \ll 1$ and $\omega_k = O(1)$

Let r_k be small for $k = \nu+1, \ldots, \alpha$. For these p_k, the exponential in the exact solution [Eq. (3.1)] may be expanded in a uniformly convergent series in powers of r_k. Use of this series then gives

$$p_k(x,t) = \omega_k\left(\int_0^t p(x,\tau)\,d\tau - r_k\int_0^t (t-\tau)p(x,\tau)\,d\tau + O(r_k^2)\right). \quad (3.9b)$$

Hence, as $p\omega_k - p_k r_k = \partial p_k/\partial t$,

$$\sum_{i=\nu+1}^{\alpha}(p\omega_i - p_i r_i) = \gamma_1 p - \gamma_2\rho\int_0^t p(x,\tau)\,d\tau + O(\rho^2), \quad (3.10)$$

where ρ is a reference scale for the small r_k, and

$$\gamma_1 = \sum_{\nu+1}^{\alpha}\omega_i \quad\text{and}\quad \gamma_2 = \sum_{\nu+1}^{\alpha}\omega_i\frac{r_i}{\rho}. \quad (3.11)$$

In effect, Eq. (3.10) consolidates all traps with r_i small and ω_i of, at most, order 1.

Case 4. $\omega_k \ll 1$ and $r_k \leqslant O(1)$

Let ω_k be small and r_k of, at most, order 1 for $k = \alpha+1, \ldots, l$. By Eq. (3.1), these p_k are order ω_k. Hence, if θ is a reference scale for the small ω_k, expanding p in powers of θ shows that the first approximation to p will satisfy Eq. (3.9) with the terms $\sum_{\alpha+1}^{l}(p\omega_i - p_i r_i)$ removed. Traps with very long lifetimes thus will not contribute to the current at lowest order for $t = O(1)$.

Only variables p_k ($k = 1, \ldots, \nu$) with associated values of ω_k and r_k *both* of order one now remain in the equation for $p(x,t)$,

$$\frac{\partial p}{\partial x} + \gamma\frac{\partial p}{\partial t} + \omega(\alpha)p - \sum_{i=1}^{\nu}p_i r_i = \lambda\delta(x)\delta(t) + O(r^{-2}, \omega^{-1}, \rho, \theta), \quad (3.12)$$

$$\gamma = 1 + \sum_{i=l+1}^{n}\frac{\omega_i}{r_i}, \quad \omega(\alpha) = \sum_{i=1}^{\alpha}\omega_i. \quad (3.13)$$

Together with $\partial p_k/\partial t + p_k r_k = p\omega_k$ for $k = 1, \ldots, \nu$, Eq. (3.12) may now be solved using Laplace transforms.

IV. APPROXIMATE SOLUTION AND TRANSIT TIME

If $\tilde{p}(x,s)$ again denotes the Laplace transform of $p(x,t)$, Eqs. (2.4) and (3.12) now give

$$\frac{\partial\tilde{p}}{\partial x} + b(s)\tilde{p} = \lambda\delta(x), \quad (4.1)$$

with

$$b(s)=\gamma s+\omega(\alpha)-\sum_{i=1}^{\nu}\frac{\omega_i r_i}{s+r_i} . \quad (4.2)$$

Hence, $\tilde{p}(x,s)=\lambda e^{-b(s)x}$ and, by (2.7),

$$\tilde{I}(s)=\lambda\left(\frac{1}{b(s)}-\frac{e^{-b(s)}}{b(s)}\right) . \quad (4.3)$$

The first term above may again be treated using the partial fraction expansion

$$\frac{1}{b(s)}=\sum_{j=0}^{\nu}\frac{B_j}{s+u_j} , \quad (4.4)$$

giving

$$\phi(t)=\mathcal{L}^{-1}\left(\frac{\lambda}{b(s)}\right)=\lambda\sum_{j=0}^{\nu}B_j e^{-u_j t} . \quad (4.5)$$

Inversion of the second term in (4.3) to determine the exit function $\psi(t)$ now involves $\nu+1$ functions of the form

$$\mathcal{L}^{-1}\left(\frac{e^{-b(s)}}{s+u_j}\right)=e^{-u_j(t-\gamma)-\omega(\alpha)} \times\int_0^t \mathcal{L}^{-1}\left[\exp\left(-\gamma s+\sum_{i=1}^{\nu}\frac{r_i\omega_i}{s+r_i-u_i}\right)\right]dt .$$

If

$$F_j(t)=e^{u_j t}\mathcal{L}^{-1}\left[\exp\left(\sum_{i=1}^{\nu}\frac{r_i\omega_i}{s+r_i}\right)\right] , \quad (4.6)$$

then, as $\mathcal{L}^{-1}(e^{-\gamma s})=\delta(t-\gamma)$,

$$\mathcal{L}^{-1}\left(\frac{e^{-b(s)}}{s+u_j}\right)=\begin{cases}F(t-\gamma) & \text{for } t\geq\gamma ,\\ 0 & \text{for } t<\gamma .\end{cases} \quad (4.7)$$

The function $\psi(t)$ is thus identically zero for $t<\gamma$. As befits its interpretation as an exit function, $\psi(t)$ begins to contribute to $I(t)$ at the start of the tail in the current trace. Hence, $t=\gamma$ or, in dimensional variables,

$$t_m^*=\gamma T_0=T_0\left(1+\sum_{i=l+1}^{N}\frac{\omega_i^*}{r_i^*}\right) , \quad (4.8)$$

provides an approximation to the mean transit time in the presence of traps. The major contribution to t_m^* thus comes from traps with ω_i large but r_i at most order one. To lowest order, the current $I(t)$ is now

$$I(t)=\phi(t)-e^{-\gamma_1}\psi(t-\gamma) , \quad (4.9)$$

where

$$\psi(t)=\phi(t)*(\delta_1+g_1)*\cdots*(\delta_\nu+g_1) \quad (4.10)$$

and δ_j, g_j, and γ_1 are as in (2.16) and (3.11).

For a given set of values ω_i, r_1, the expressions (4.9)–(4.10) may be evaluated numerically. Results show good qualitative agreement with experimental current traces.

V. INVERSE PROBLEM

This section examines the problem of numerically determining the types of traps which are required to produce a given current trace $I(t)$. Now, not only the values ω_i and r_i associated with each type of trap, but also the number of types n are unknowns. The general model equations predict that $\tilde{I}(s)$ will have the form (2.10) so

$$I(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\tilde{I}(s)e^{st}ds . \quad (5.1)$$

Because $\tilde{I}(s)$ is holomorphic for $\mathrm{Re}(s)>0$ and regular at zero, c may be taken as zero leading to

$$\tilde{I}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}I(i\nu)e^{i\nu t}d\nu . \quad (5.2)$$

However, for large v, $|\tilde{I}(iv)|=O(|v|^{-1})$. Thus, the convergence of the integral (5.2) is too slow to allow practical numerical evaluation in connection with a standard fitting routine, which may require several hundred evaluations of $I(t)$ and its gradients with respect to ω_i, r_i. To remove this difficulty with evaluation and facilitate rapid determination of n, ω_i, and r_i, $I(t)$ will be approximated here by a partial sum of its Fourier series with respect to an appropriate orthonormal sequence $\{\psi_n(t)\}$ on $(0,\infty)$.

Following Erdélyi,[8] let $\mu_0, \mu_1, \mu_2, \ldots$ be a sequence of distinct, positive real numbers. Then, by a theorem of Muntz,[9] if either $\mu_n\to\mu>0$, or $\mu_n\to 0$ slowly so $\sum\mu_n=\infty$, or $\mu_n\to\infty$ slowly so $\sum\mu_n^{-1}=\infty$, the set of exponentials $\{e^{-\mu_n t}\}$ has the closure property on $(0,\infty)$ that $\int_0^\infty f(t)e^{-\mu_n t}=0$ for all $n\Rightarrow f(t)=0$ almost everywhere. The Nth approximate $I_n(t)$, defined by

$$I_n(t)=\sum_{n=0}^{N}b_n e^{-\mu_n t} , \quad (5.3)$$

will thus, with appropriate values of b_n, converge to $I(t)$ almost everywhere as $N\to\infty$.

The functions $\{e^{-\mu_n t}\}$ are linearly independent and square integrable on $(0,\infty)$. Hence, they may be used to construct the required orthonormal expansion sequence $\{\psi_n(t)\}$. In particular, the Gram-Schmidt orthonormalization process gives

$$\psi_n(t)=\sum_{m=0}^{n}c_{mn}e^{-\mu_n t} ,$$

with

$$c_{mn}=(2\mu_n)^{1/2}\prod_{i=0}^{n-1}(\mu_m+\mu_i)\Big/\prod_{i\neq m}(\mu_m-\mu_i) . \quad (5.4)$$

The current $I(t)$ thus has the convergent Fourier series

$$I(t)=\sum_{n=0}^{\infty}a_n\psi_n(t) , \quad (5.5)$$

where

$$a_n = (\psi_n, I(t)) = \sum_{m=0}^{n} c_{mn} \tilde{I}(\mu_m) \tag{5.6}$$

and $\tilde{I}(\mu_m)$ is the Laplace transform of $I(t)$ evaluated at $s = \mu_m$. Further, comparison of (5.3) with (5.4) and (5.5) shows that

$$b_n = \sum_{m=n}^{N} c_{mn} a_m . \tag{5.7}$$

For the Laplace transform itself, let

$$\tilde{I}_n(s) = \mathcal{L}[I_n(t)] = \sum_{n=0}^{N} \frac{b_n}{s+\mu_n} . \tag{5.8}$$

Then, $\tilde{I}_N \to \tilde{I}$ as $N \to \infty$ for $s \geq 0$ and, for $n = 0, 1, \ldots, N$, $\tilde{I}_N(\mu_n) = \tilde{I}(\mu_n)$. Thus, $\tilde{I}_N(s)$ interpolates $\tilde{I}(s)$ on $s = \mu_n$.

In the present application, difficulties in evaluation continue to arise if the sequence $\mu_n \to \mu > 0$. In particular, the constants c_{mn} rapidly become large. Best numerical results were obtained with the sequence $\mu_n = \text{const}/2^n$, which tends to zero "quickly." (The closure condition may be retrieved by adding some small $\epsilon > 0$ to each μ_n.) With this sequence, the coefficients c_{mn} do not grow as $n \to \infty$. If data are given on the set $\{t_k\}$, the values of $\psi_n(t_k)$ and c_{mn} need be calculated only once. Derivatives of $I_n(t_k)$ with respect to $\xi = \omega_i, r_1$ are given by

$$\frac{\partial I_N(t_k)}{\partial \xi} = \sum_{n=0}^{N} \sum_{m=0}^{n} c_{mn} \frac{\partial \tilde{I}(\mu_m)}{\partial \xi} \psi_n(t_k) . \tag{5.9}$$

Having determined values of n, ω_i, and r_i from a given set of data for $I(t)$, results were checked against the forward problem of determining $I(t)$, given n, ω_i, and r_i. In typical cases, such as shown in Fig. 2, agreement to within 0.1% with the convolution solutions was obtained using $N = 16$, i.e., evaluating $\tilde{I}(s)$ only 17 times at $s = \mu_n$, $n = 0, \ldots, 16$ on the real axis.

VI. DISCUSSION

Results from the inverse problem indicate that only a small number of distinct types of traps is required to accurately reproduce given data on the transient photocurrent. Indeed, for the trace shown in Fig. 2, $n = 3$ is sufficient. Violation of the closure property by the particular sequence $\{\mu_n\}$, chosen as a base for the Laplace transform in the present case, has no practical effect on the calculation. Hence, a small number of distinct types of traps can approximate the power-law behavior of disperse photocurrent transients very well. The physical implications of this result will be discussed in a future publication.[5]

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