Vortex generation in modulated superfluid ^{4}He flow through a pinhole*

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Sabo and Zimmermann found that the amplitude of resonant ac superfluid flow through the 10-μm orifice of a Helmholtz resonator is limited by sudden collapse of the oscillation, rather than by saturation predicted by the theory of thermal nucleation of vortices. This paper reports measurements on superimposed ac and dc flow. Saturation at a peak velocity essentially equal to the (intrinsic) dc critical velocity is observed when the dc velocity is larger than the peak ac velocity. In the opposite case collapse occurs and except near the transition the peak velocity preceding collapse is somewhat less than the dc critical velocity. The Helmholtz resonator is shown to be a sensitive device for detecting vortex generation in the saturation regime.

I. INTRODUCTION

The superfluid component of liquid ^{4}He flows without friction up to a fairly well-defined critical velocity, at which dissipation rapidly sets in due to generation of quantized vortices. Small pinholes (of order 10-μm diam) under suitable conditions exhibit a large temperature-dependent "intrinsic" critical velocity[1,2] otherwise observed only in submicrometer channels,[3] in experiments done with multiple parallel paths. This intrinsic critical velocity agrees tolerably well with the onset of measurable dissipation calculated by Iordanskii[4] and Langer and Fisher[5] for homogeneous thermal nucleation of vortex rings.

Several years ago Sabo and Zimmermann[6] used a pinhole orifice as the inertial element of a double Helmholtz resonator and studied oscillatory superfluid flow. Any theory such as Iordanskii-Langer-Fisher in which the dissipation is a (strongly increasing) function of velocity predicts that the resonant amplitude will saturate in the neighborhood of the critical velocity, where the number of vortices nucleated per cycle increases rapidly to absorb any increase in drive power. In the experiment, however, the oscillation would build up a critical velocity of intrinsic magnitude, from which it would collapse in a few milliseconds to near zero. This paper presents further study of the collapse phenomenon and extends the work of Sabo and Zimmermann by measuring also dc gravitational flow through the orifice, and by studying oscillatory flow with a superimposed dc flow.

II. THEORETICAL REMARKS

Anderson[7] showed that a chemical potential difference $\Delta\mu$ between the ends of a superfluid flow channel in steady state must be supported by vortices of circulation $\kappa = h/m$ (h is Planck's constant, m is the mass of a ^{4}He atom) crossing the channel at a frequency $\nu = \Delta\mu/\kappa$. Huggins[8] extended this result in a model calculation to show that a vortex which moves in the channel (cross section a) so as to cut a fraction ψ of the mass current, delivers an impulse $\rho_s \kappa a \psi$ to the main flow and supports an instantaneous chemical potential difference $\Delta\mu = \kappa(d\psi/dt)$. Then if one vortex completely crosses the inertial flow channel of a Helmholtz resonator in a time very short compared to the oscillation period, the resulting impulse will change the velocity in the channel by $\Delta v_s = \kappa/l$, where l is the effective length of the channel.[9] In our experiments $\kappa/l = 0.8$ cm/sec, so that several hundred vortex crossings typically would be required to reduce the resonator amplitude from its maximum value to zero. An estimate of the vortex crossing time in the interior of a long channel of width d is $\tau \approx d/\alpha v_s$, where α is the drift tangent defined by Campbell.[10] For $T = 1.44$ K ($\alpha = 0.06$), $d = 10^{-3}$ cm, $v_s = 440$ cm/sec, this gives $\tau \approx 0.04$ msec. If a vortex ring is created at the exit of an orifice in which the velocity greatly exceeds the Feynman critical velocity,[11] the ring will expand greatly and require a much longer time to completely cut the flow[8,12] (i.e., many seconds, for the above parameters). The form of the vortex trajectories is thus of some interest, but is not known.

III. APPARATUS AND PROCEDURE

The apparatus (Fig. 1) consists of a double Helmholtz resonator (HR)[13] $C1$-L-$C2$ connected to the bottom of a gravitational flow cryostat by powder-packed capillary tubes $L1$ and $L2$ (0.058 cm i.d., 11.4 cm long). These superleaks have considerably larger inertance than the orifice L, and thus isolate the HR from the liquid free surfaces in the reservoirs. They also have much larger critical superfluid mass flow rate than the orifice (except close to T_λ), allowing gravitational flow measurements on the latter. The differential pressure between the HR chambers is measured by the change in capacitance between a flexible diaphragm (usual-

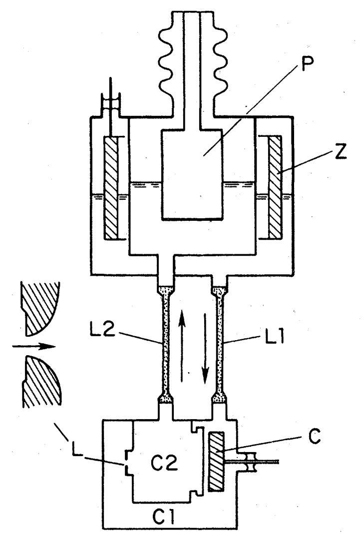

FIG. 1. Schematic diagram of the apparatus. A double Helmholtz resonator, consisting of chambers $C1$ and $C2$ joined by orifice L, is connected through capillary superleaks $L1$ and $L2$ to the two reservoirs of a coaxial gravitational flow apparatus. Gravitational dc flow is initiated by displacing the plunger P in the inner reservoir and measured by the capacitive level sensor Z in the outer reservoir. Resonant oscillatory flow is driven thermomechanically by a heater in $C2$ or by a piezoelectric disk (not shown) which deflects an outside wall of $C1$. The differential pressure sensor C is a flexible diaphragm which constitutes one plate of a capacitor. Arrows indicate the sign convention for positive dc flow. The inset shows the approximate profile of the orifice.

ly 0.013 cm brass) and fixed counterelectrode C, which shifts the frequency f_c ($\approx$ 19 MHz) of a tunnel diode oscillator (not shown). The sensitivity $\Delta f_c/\Delta p$ is calibrated against the thermomechanical effect of a measured applied temperature difference or the pressure head created by a large plunger displacement. This sensitivity is found to be independent of temperature in the He II range but may change from run to run. The FM signal f_c is mixed down to 100 kHz and demodulated by a phase-locked loop. The demodulated signal representing Δp is fed to an oscilloscope, ac voltmeter, and a signal averager used as a transient recorder. The (rms) pressure at collapse is taken to be the maximum voltmeter reading reproducibly attained with the drive adjusted for a slow buildup. Occasional collapses occur at a somewhat lower level.

Attached to the center of each superleak $L1$, $L2$ is a heater (not shown) encapsulated in epoxy. A power of 5 mW or less to one of the heaters raises a section of that superleak above the λ point, so that it appears to the superfluid as a closed valve.[14] When both thermal valves are heated, the HR is completely isolated. Under these conditions the resonant angular frequency is given to a good approximation[15] by

$$\omega_1^2=(\rho_s/\rho)u_1^2\frac{a}{l(V_r+V_c)}\equiv\frac{\rho u_1^2}{L(V_r+V_c)}\,, \tag{1}$$

where ρ_s/ρ is the superfluid fraction, u_1 is the first sound velocity, a and l are the orifice cross-section area and length, $V_r^{-1}=V_1^{-1}+V_2^{-1}$, V_1 and V_2 are the volumes of chambers $C1$ and $C2$, and V_c is the volume equivalent of the compliance of the diaphragm. (That is, $V_c=\rho u_1^2C=\beta^{-1}C$, where β is the adiabatic compressibility of liquid helium and C is the diaphragm compliance.) $L=(\rho^2/\rho_s)l/a$ will be called the orifice inertance.[16] The resonator Q is approximately 1000 at 1.2 K, 400 at 1.4 K, and 30 at 2.08 K.

The ac velocity in the orifice v_s is related to the measured differential pressure Δp by

$$\Delta p=\rho l\omega v_s=(\rho_s/\rho)La\omega v_s \tag{2}$$

apart from a small thermomechanical correction; so that l, or equivalently L or V_c, must be determined. (The parameters $V_1=3.36$ cm^3, $V_2=0.95$ cm^3, and $a=(6.8\pm0.7)\times10^{-3}$ cm^2 are computed from measured linear dimensions.) There are eight other resonant modes of the system which obtain with one or both of the superleaks conducting. The frequencies of these modes can be calculated from an extended lumped-element model including inertances L_1 and L_2 (The compliance of the free surfaces is so large that they are effectively a ground, in this frequency range); then these measured frequencies give values for L/L_1, L/L_2, and V_c/V_r. This determines all of the lumped-element parameters, including L and V_c, to sufficient precision that they do not affect the precision of the velocity calibration. We find that L, L_1, and L_2 are constant within a few percent from run to run, but V_c does change. Most of the data presented below were obtained in runs in which $V_c\approx 4$ cm^3.

The precision of the ac velocity calibration is limited by the measurement of the orifice area a under an optical microscope (±10%) and the transducer calibration df_c/dp (±5%). dc velocities are measured by the rate of liquid level rise, dz/dt in capacitor Z as measured by the rate of frequency shift of the associated tunnel diode oscillator df_z/dt:

$$v_{s\,\mathrm{dc}}=\frac{\rho}{\rho_s}\frac{A_p}{a}\frac{A_0}{A_I+A_0}\frac{\Delta Z_p}{\Delta f_z}\frac{df_z}{dt}\,. \tag{3}$$

Here A_I and A_0 are the free-surface areas in the inner and outer reservoirs ($A_I\gg A_0$), A_p is the plunger cross-section area, and Δf_z is the equilibrium frequency change produced by plunger displacement ΔZ_p. All of these factors besides a are known to good precision except A_0, which is uncertain by ±10%. For a lack of a precise direct measurement of A_0, we use a value derived from HR parameters and the frequency of U-tube oscillations at the lowest temperatures:

$$\omega_U^{-2} = (L + L_1 + L_2)(g\rho)^{-1} A_I A_0/(A_I + A_0) . \qquad (4)$$

Of most relevance to the present experiment is the relative calibration of ac and dc velocities, which is independent of the orifice area a. The precision of this relative calibration, determined primarily by df_c/dp, is about ±5%.

In order to make measurements with superimposed ac and dc flow in the orifice, we heat one of the capillary superleaks (usually $L2$) to a temperature slightly below T_λ. By fine adjustment of the heater current it is possible to adjust the critical current of the superleak to maintain any subcritical velocity in the orifice. This "series-limited" flow shows excellent stability so long as a large level difference between the reservoirs maintains a large chemical potential difference across the heated superleak. A small ac flow in the unheated superleak produces a minor perturbation from the isolated HR mode. There is a second normal mode, involving flow through the unheated superleak and in-phase pressure oscillation in $C1$ and $C2$, which could in principle be excited by the (nonlinear) collapse of the primary mode oscillation. In practice this is undetectable.[17]

The orifice used in these experiments is an Ealing utility optical pinhole and is slightly out of round, with major axes of 9.9 and 8.8 ± 0.5 μm. This type of pinhole has an asymmetric profile, approximately as shown in the inset of Fig. 1. The magnitude and temperature dependence of the dc critical velocity for this orifice agrees closely with data reported previously for similar pinholes.[1,18]

The random error, indicated by short term reproducibility, is usually about ±5 cm/sec for both ac and dc velocity.

IV. EXPERIMENTAL RESULTS

A moderately weak driving force at the resonant frequency is applied to the isolated HR, causing an oscillation to build up until collapse occurs. Figure 2 shows some typical waveforms in the neighborhood of the collapse event. These traces were recorded by manually triggering a single sweep of a 200 channel signal averager. Trace (a) illustrates some common characteristics, previously reported by Sabo[6]: The main dissipative event occurs near a pressure zero-crossing, or velocity maximum. The preceding pressure peak has reduced magnitude, indicating that some vortex

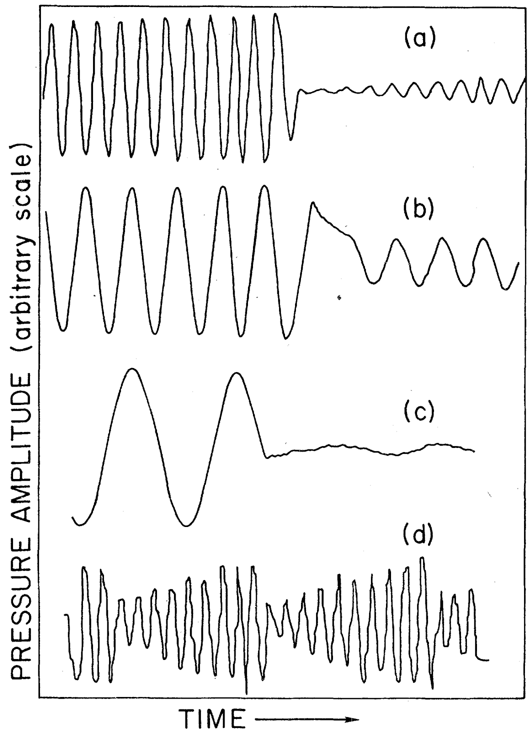

FIG. 2. Helmholtz resonator waveforms with a drive sufficient to reach critical velocity. The slope of the traces is proportional to the superfluid velocity in the orifice. (a) 1-sec sweep, T =1.39 K, max $v_s \approx 380$ cm/sec; (b) 0.5-sec sweep, T =1.39 K; (c) 0.1-sec sweep, T =1.18 K; (d) 1-sec sweep, T =2.077 K, max $v_s \approx 120$ cm/sec.

generation took place in the preceding half cycle. It is impossible to say whether one or two vortices may have been generated in an earlier half cycle; ten vortex crossings would reduce the amplitude by 2% and would be marginally perceptible. The main collapse often occurs earlier, even shortly after the velocity reversal, as in the next trace. Trace (b) has an extended segment (~35 msec) during which the dissipative process supports a pressure head of 5 to 3 dyne/cm^2 at nearly constant velocity of 25 cm/sec. This is suggestive of the extrinsic dissipation sometimes seen in dc flow through this type of orifice.[19] The opposite limit is illustrated by trace (c), in which an abrupt change of velocity requiring about 580 vortex crossings occurs in not more than 1 msec. Collapse events are initiated from either direction of flow, not necessarily with equal probability (our instrumentation is not adequate for a systematic study). A much stronger drive was used with trace (d), so that the collapse threshold is reached several times per second despite reduced Q at this higher temperature. However, the behavior is

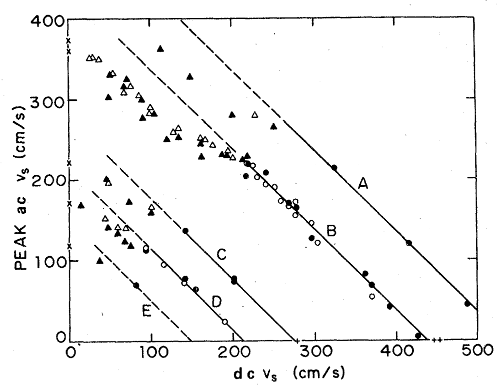

FIG. 3. Critical velocities: Locus of saturation (circles) or collapse (triangles and crosses) of the Helmholtz oscillation as a function of dc velocity at several temperatures. (A) 1.22 K, (B) 1.435 K (piezoelectric drive) and 1.432 K, (C) 1.834 K, (D) 1.954 K, (E) 2.077 K. The drive is thermomechanical except as noted. Saturated amplitude is measured at a dissipative pressure of 0.01 to 0.1 dyne/cm^2. Open symbols denote positive and solid symbols negative dc flow, while the symbol + indicates gravitational dc flow at 1 dyne/cm^2. The diagonal lines have slope -1 and thus correspond to constant peak velocity.

generally similar to that at lower temperatures.

In order to clarify the relation of this collapse phenomenon to the intrinsic dissipation found for dc flow, we have studied oscillatory flow in the presence of a wide range of superimposed dc velocities. The main results are presented in Fig. 3, where the maximum peak ac velocity is plotted against dc velocity. So long as the dc velocity is greater than one-half of its critical value, the ac amplitude is limited by the intrinsic dissipation process when the combined peak velocity reaches a critical value, essentially equal to the dc critical velocity. Collapse, rather than limiting, occurs when the dc velocity is less than one-half of its critical value (i.e., when the flow reverses direction part of the cycle), at least at 1.43 K. At 1.95 K the regime of limiting is slightly more extended. Away from the transition, collapse occurs at a roughly constant peak velocity which is appreciable lower than the intrinsic critical velocity. The collapse velocity with no dc flow seems to be slightly lower yet. Data from two runs two weeks apart, the first using thermomechanical drive and the second piezoelectric drive, are plotted together in (B). The intrinsic critical velocity appeared to be 5 cm/sec lower in the second run, and this contributes to the apparent scatter. There was no difference attributable to the method of drive.

Figure 4 shows some examples of the HR ac pressure amplitude as a function of drive, with several different dc velocities. Each trace here is represented by a point in Fig. 3. Traces (a) and (b) exhibit collapse; the rest show limiting. The traces for negative dc flow exhibit small collapse events, corresponding to the crossing of 5 to 10 vortices at a time, near threshold only. This is qualitatively distinct from the major collapse phenomenon in traces (a) and (b), which continues in the same form to large drive (these traces have been cut off). The smooth traces for positive dc flow in the limiting regime suggest that vortices are nucleated individually.

The ordinate of Fig. 4 can be converted from (reactive) pressure to velocity by Eq. (2). The abscissa can be converted to dissipative pressure as follows: Along the common subcritical line the driving force is balanced by linear dissipative processes, such as heat conduction or laminar normal fluid flow. In this regime the Q of the HR, measured by the width of the resonance, is 290; this is also the ratio of reactive to dissipative pressure, and thus gives a calibration of the drive pressure. The dashed slant lines parallel to the subcritical line indicate excess dissipative pressure due to vortex generation of 0.01 and 0.1 dyne/cm^2. ac measurements can only determine the Fourier component of pressure in phase with the ac velocity. However, under the plausible assumptions that vortex nucleation occurs predominately near peak velocity and crossing is rapid, the dc dissipative pressure can be calculated and is half the ac amplitude computed above. Note that the generation of one vortex per HR cycle ($\approx$24 msec) corresponds to a dc pressure head of 0.006 dyne/cm^2, which is resolved by this technique.

Some examples of collapse waveforms with sub-

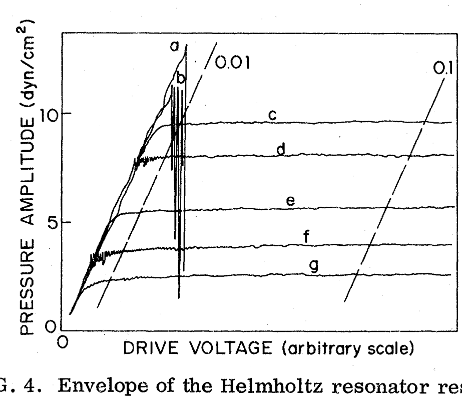

FIG. 4. Envelope of the Helmholtz resonator response with various values of superimposed dc current. The peak value of the ac pressure is plotted against driving voltage at the HR frequency to the piezoelectric transducer. The dc velocity through the orifice in cm/sec is (a) -106, (b) $+203$, (c) $+230$, (d) -268, (e) $+304$, (f) -361, (g) $+370$. The dashed lines are lines of constant excess dissipative pressure as deseribed in the text. T = 1.435 K.

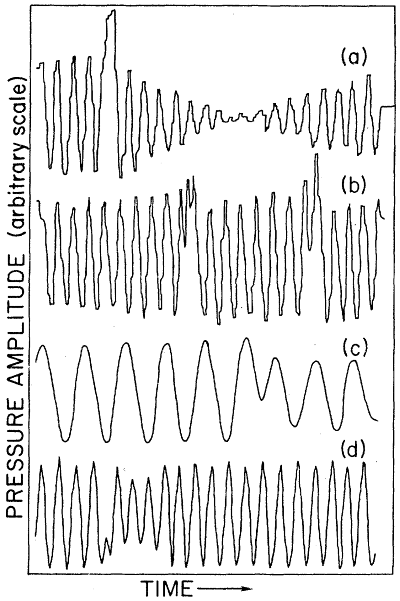

FIG. 5. Helmholtz resonator waveforms with a drive sufficient to reach critical velocity in the presence of a fixed dc bias current. The ac contribution to the velocity is proportional to the negative of the slope of the trace. For traces (a) and (b) T =1.22 K, v_{sdc}=+240 cm/sec; (c) 1.435 K, +201 cm/sec; (d) 1.435 K, −217 cm/sec.

stantial dc flow are given in Fig. 5. In trace (c) the *total* velocity is suddenly reduced from near peak value to near zero, an event similar to trace (c) of Fig. 2. In the present case, however, the effect is a reversal of the ac component of velocity and only a slight reduction in amplitude. If the dissipative event occurs earlier in the cycle, then the pressure amplitude is driven above its normal maximum, as in traces (b) and (a). When this happens the critical velocity will be exceeded again one-half cycle later. Apparently, this can produce another spike, as in the first event in (b), or the velocity can simply be trimmed back to a limiting value. In trace (a) the dissipative event delayed the oscillation by almost one-half period, leaving it out of phase with the drive, which subsequently carried the amplitude through zero. The conditions of trace (d) are very close to the transition between limiting and collapse. There is a strong drive and, apparently, dissipation of the limiting type during the constant-amplitude portions of the trace, as indicated by the rapid recovery to saturation after the velocity-reversal event.

V. DISCUSSION

It could be argued that the transition from collapse to limiting has nothing to do with an intrinsic dissipation mechanism: A model in which collapse to zero velocity is regarded as the basic dissipative mechanism would predict a transition to a limit cycle when the dc velocity exceeds the peak ac velocity, because such a collapse would then increase, rather than decrease, the magnitude of the ac velocity. Neither experimental observations nor theoretical expectations support this interpretation. A limit cycle would involve strong distortion of the oscillatory waveform, a substantial frequency shift, and a large dc pressure differential, none of which is observed. A more supportable point of view is that there is an intrinsic vortex nucleation mechanism which always operates to generate vortices at a well defined rate dependent on v_s, as in the Iordanskii-Langer-Fisher calculation.[4,5] There is also an extrinsic process whereby vortices already present (due to the intrinsic process or otherwise) can generate further vorticity, given suitable conditions of vortex configuration and flow velocity. This extrinsic process might involve vortex pinning at the orifice surface, as in the vortex mill model,[20] or it might be a homogeneous process such as is supposed to operate in counterflow.[21]

If the limiting regime represents an intrinsic dissipation process involving thermal nucleation of vortices, then the peak velocity should depend logarithmically on dc pressure head. The following data for 1.435 K are relevant: For gravitational flow the pressure-head dependence is qualitatively correct but asymmetric; $dv_s/d(\log_{10}\Delta p)$ = 23 cm/sec per decade for positive and 33 for negative flow, in the range 0.5 to 8 dyne/cm^2. For ac flow, data such as the traces in Fig. 4 give 5 ± 2 cm/sec per decade.[22] This discrepancy is not understood. It is possible that interaction with previously generated vortices is already significant at $\Delta p\approx 1$ dyne/cm^2 or less and determines this characteristic in gravitational flow. Gravitational flow at 0.5 dyne/cm^2 is 2 ± 5 cm/sec faster than the peak velocity at a dc pressure head of 0.2 dyne/cm^2, estimated from ac dissipation, but a correction must be made for the fraction of time near peak velocity in the ac case. Thus agreement is reasonably good.

The demonstrated sensitivity of the HR to the onset of vortex generation in ac-modulated dc flow is likely to have further applications. A number of experiments have sought to demonstrate the analog of the ac Josephson effect by a synchronization technique.[23] If a region of synchronization existed in which exactly one vortex was generated per ac cycle, this would induce a slanted step in the traces of Fig. 4 (or in similar traces with fixed ac drive and swept dc velocity). No such synchronization is seen or expected with the intrinsic dissipation process, but it might occur in an orifice with a reproducible low critical velocity.

Collapse occurs at essentially the same peak velocity as limiting near the transition, which suggests that the collapse is triggered by thermally nucleated vortices. For smaller dc velocities, collapse occurs at an appreciably lower peak velocity, about 55 cm/sec lower at 1.43 K. If the velocity dependence of nucleation rate derived from ac limiting is extrapolated, no thermally nucleated vortices are expected at this speed. However, this conclusion is uncertain in view of the discrepancy with gravitational flow. Other possibilities are very short (≈300 Å) pinned vortex lines or preexisting free vorticity. The velocity preceding collapse is still much larger than typical extrinsic critical velocities, although a similarity *within* the collapse event has already been pointed out. The fact that collapse does not occur when the dc velocity is sufficiently large that the flow does not reverse (except for the small jumps for one direction of dc flow) suggests that sweeping away of vorticity is important in suppressing the extrinsic dissipative mechanism.

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[9]This is another way of saying that the quantum-mechanical phase difference changes by 2π. Huggins's formulation is useful in more general cases. In a nonuniform channel, l depends on the definition of the average velocity. We define $v_s =$ (mass current)$/\rho_s a$ where a is the *minimum* cross section of the channel. The effective lengths for inertia, measured by Helmholtz frequency, and phase difference are identical.

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[14]A thermal valve of this type was described by W. M. van Alphen, R. de Bruyn Ouboter, K. W. Taconis, and E. van Spronsen, Physica 39, 109 (1968).

[15]If V_c is small, the thermal contribution to $\Delta\mu = \Delta p/\rho - s\Delta T$ (s is the specific entropy) across the orifice is at most 1%. The thermal effect may be much larger if $V_c >> V_r$ because the diaphragm compliance shunts only $\Delta p/\rho$, not $s\Delta T$. We routinely include this effect in our analysis, but it is not important for the present data.

[16]It is convenient to include in the definition of inertance the factor ρ/ρ_s arising from the approximate relation for fourth sound velocity, $u_4^2 = (\rho_s/\rho)u_1^2$.

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