Comments and Addenda

The section Comments and Addenda is for short communications which are not appropriate for regular articles. It includes only the following types of communications: (1) Comments on papers previously published in **The Physical Review** *or* **Physical Review Letters**. *(2) Addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters**, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section must be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts follow the same publication schedule as articles in this journal, and page proofs are sent to authors.*

Reply to comments on "Qualitative explanation of Pellam's helium paradox"

Robert V. Penney
39410 Koppernick Road, Plymouth, Michigan 48170
(Received 10 April 1975)

A recent paper of Lynch is found valid. The heat-exchange torque vanishes for uniform heating provided that we go to the infinitely thin disk.

Recently, Lynch[1] claimed to show that my conclusions about the temperature dependence and sense of the heat-exchange torque on a Rayleigh disk, when it is heated were invalid. He was correct, in fact.

To see this, consider the expression developed by the author[2] for the heat-exchange torque $\vec{\tau}_{\text{heat}}$,

$$\vec{\tau}_{\text{heat}} = \int \frac{\vec{r} \times \rho_n \kappa (\vec{v}_n - \vec{v}_S) dS}{\rho S T + \rho_n \vec{v}_n \cdot (\vec{v}_n - \vec{v}_S)} \tag{1}$$

and apply it to a two-dimensional flow problem (infinitely long plate); the superfluid is modeled by potential flow.

The boundary conditions are twofold; one is the mass current density satisfying the equation

$$\vec{J} \cdot d\vec{S} = (\rho_n \vec{v}_n + \rho_s \vec{v}_s) \cdot d\vec{S} = 0 \tag{2}$$

on the surface of the disk.

The other condition arises from the heat flow from the disk κ, which is taken to satisfy the equation

$$\begin{aligned} \kappa\, d\vec{S} = \vec{Q} \cdot d\vec{S} &= \{(\mu + \tfrac{1}{2}v_s^2)\vec{J} + \rho S T \vec{v}_n \\ &\quad + \rho_n \vec{v}_n [\vec{v}_n \cdot (\vec{v}_n - \vec{v}_s)]\} \cdot d\vec{S} \\ &= \{\rho S T + \rho_n [\vec{v}_n \cdot (\vec{v}_n - \vec{v}_s)]\} \vec{v}_n \cdot d\vec{S} \end{aligned} \tag{3}$$

on the surface of the disk.

The torque becomes[3-5] per unit length,

$$\begin{aligned} \vec{\tau}_{\text{heat}} &= \rho_s \oint (\vec{r} \times \vec{v}_{s\parallel})(\vec{v}_{s\perp} \cdot d\vec{S}) \\ &\quad + (\rho_s \rho / \rho_n) \oint (\vec{r} \times \vec{v}_{s\perp})(\vec{v}_{s\perp} \cdot d\vec{S}), \end{aligned} \tag{4}$$

where $\vec{v}_{s\parallel}$ and $\vec{v}_{s\perp}$ are the components of the superfluid flow parallel and perpendicular to the surface.

Specifically, we apply the torque formulation to a cylinder with an elliptical cross section, such as

$$X^2/a^2 + Y^2/b^2 = 1. \tag{5}$$

We have calculated the torque per unit length, and it reads

$$\begin{aligned} \tau_{\text{heat}} &= \frac{V_0 (a+b) ab}{s_n T} \oint \frac{\kappa(\theta) \sin(\theta - \alpha) d\theta}{r_0} \\ &\quad + \frac{\rho}{\rho_n \rho_s^2} \frac{a^2 - b^2}{(s_n T)^2} \oint \kappa^2(\theta) \sin\theta \cos\theta \, d\theta. \end{aligned} \tag{6}$$

V_0 is the uniform velocity at $r = \infty$, $r_0^2 = a^2 \sin^2\theta + b^2 \cos^2\theta$.

Now, when we go to the limit of $b \to 0$, an infinitely thin disk; the first term vanishes, leaving Lynch's result.[1]

To repeat, Lynch has shown that my estimation of the torque of a Rayleigh disk due to heat-exchange forces[5] was incorrect.

---

[1] R. Lynch, Phys. Rev. A 10, 1435 (1974).
[2] R. Penney, Phys. Rev. Lett. 25, 138 (1970).
[3] L. M. Milne-Thomson, *Theoretical Hydrodynamics*, 2nd ed. (Macmillan, New York, 1950), p. 163.
[4] R. Penney, Phys. Fluids 10, 2147 (1967).
[5] Unless there is trapped circulation around the elliptic cylinder. See, for example, R. Penney and A. W. Overhauser, Phys. Rev. 164, 268 (1967).