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Nonlinear acoustoelectric effects in semiconductors

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The theory of parametric amplification, with particular reference to subharmonic and second-harmonic generation, is developed by using the Boltzmann transport equation with the constant-relaxation-time ansatz. Subharmonic and second-harmonic generation in the absence of external fields is discussed with reference to: (a) piezoelectric semiconductors in which piezoelectric coupling dominates the interaction, (b) nonpiezoelectric extrinsic semiconductors in which majority carriers dominate the interaction, and (c) intrinsic semiconductors, semimetals, and nonpiezoelectric extrinsic semiconductors in which minority carriers dominate the interaction. At very high frequencies, extrinsic semiconductors in which majority carriers dominate the interaction are found to be most effective for parametric amplification.

I. INTRODUCTION

Ultrasonic-second-harmonic generation due to acoustoelectric interactions was first observed in photoconducting CdS by Tell.[1] Recently, there has been a revival of interest, both experimental[2-8] and theoretical,[9-15] in frequency-mixing effects, mainly because of the role they play in the growth of domains of acoustic flux under conditions of acoustic amplification. In recent papers, Conwell and Ganguly[10] and Spector[14] have discussed frequency-mixing effects due to the acoustoelectric interaction of the acoustic waves with the conduction electrons in piezoelectric semiconductors. They used a phenomenological theory to obtain the ac current induced by the acoustic wave. This approach limits the validity of the calculations to the region in which the phonon wavelength, $2\pi/q$ is much larger than the electron mean free path, i.e., $ql \ll 1$. This condition is met in photoconducting CdS over a wide range of frequencies of interest. However, amplification of acoustic flux has been observed in high-mobility semiconductors such as n-InSb,[16] GaAs,[4,17] and Ge,[18] where the phonon wavelength may be much smaller than the electron mean free path. Therefore, it is of interest to use a transport-equation formalism, which is not limited in its validity to the long-wavelength region, to calculate the ac current induced by the acoustic wave. This approach would extend the validity of the calculation to the short-wavelength regime, $ql \gg 1$. At very high frequencies, deformation potential becomes the dominant mode of acoustoelectric interaction even for piezoelectric semiconductors. Therefore, it is also of interest to include deformation-potential coupling in our discussion of the frequency-mixing processes.

In Sec. II the theory of parametric amplification is developed in terms of the linear and the nonlinear electronic conductivity tensors for semiconductors in which both electrons and holes contribute to the interaction. In Sec. III, the linear and nonlinear conductivity tensors are calculated through the use of the Boltzmann transport equation. In Sec. IV, subharmonic and second-harmonic generation is discussed in extrinsic semiconductors, intrinsic semiconductors and semimetals, and in piezoelectric semiconductors. Finally, in Sec. V we present a discussion of our results.

II. THEORY

We shall consider, as our model, a semiconductor or semimetal with both electrons and holes present. The acoustoelectric interaction of the ultrasound with the charge carriers can be described by the equation of motion of the lattice, the equation of state of the material, Maxwell's equations and an equation for the electronic current induced by the ultrasound. For both deformation-potential coupling and piezoelectric coupling, the interaction is only appreciable if the electric fields induced are longitudinal.[19] Therefore, we need only Poisson's equation and the equation of continuity to determine the electric displacement in terms of the electronic current induced by the wave. The relevant set of equations are

$$\rho \frac{\partial^2 \xi_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j} , \quad (2.1)$$

$$T_{ij} = c_{ijkl} S_{kl} - nC^e_{ij} - pC^p_{ij} - \beta_{ijk} E_k , \quad (2.2)$$

$$D_i = \epsilon E_i + 4\pi \beta_{ijk} S_{jk} , \quad (2.3)$$

$$S_{ij} = \tfrac{1}{2}\left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i}\right), \quad (2.4)$$

$$\vec{\nabla} \cdot \vec{D} = -4\pi e(n - p), \quad (2.5)$$

$$-e\frac{\partial(n-p)}{\partial t} = \vec{\nabla} \cdot \vec{j} . \quad (2.6)$$

Here ξ_i is the displacement due to the sound wave, T_{ij} is the stress tensor, S_{ij} is the strain tensor, c_{ijkl} are the elastic constants, C^e_{ij} and C^p_{ij} are the deformation coupling constants for the electrons and holes, respectively, β_{ijk} is the piezoelectric tensor, n and p are the respective electron and hole densities, $\vec{E}$ is the electric field and $\vec{D}$ the electric displacement, and ϵ is the static dielectric constant.

This set of equations has to be supplemented by an equation for the electronic current density $\vec{j}$. We shall now limit ourselves to the case where we have three collinear waves present where the highest frequency $\omega_3 = \omega_1 + \omega_2$. As we shall show in Sec. III, the current densities induced by these waves can be written

$$j_{1,2i} = \sigma_{ij}(\omega_{1,2})E'_{1,2j} + \Lambda_{ijk}(\omega_3, -\omega_{1,2})E'_{3j}E'^{*}_{2,1k}\ , \tag{2.7}$$

$$j_{3i} = \sigma_{ij}(\omega_3)E'_{3j} + \Lambda_{ijk}(\omega_1, \omega_2)E'_{1j}E'_{2k}\ . \tag{2.8}$$

Here $\vec{E}'_n = \vec{E}_n - \vec{q}_n\vec{q}_n \cdot \vec{C} \cdot \vec{\xi}_n/e$ is the effective electric field arising from the passage of the sound wave of frequency ω_n. Also, we take the electric fields and the current densities to be parallel to the wave vectors of the three waves. Therefore, in the following we shall drop the subscripts i, j, k, l and use a one-dimensional model, with c, $C_{e,p}$, and β as the appropriate components of the elastic, deformation potential and piezoelectric constants, respectively. We shall now treat separately the two cases, in which either deformation-potential coupling or piezoelectric coupling is the dominant mode of interaction of the waves with the carriers.

A. Deformation-potential coupling

When deformation-potential coupling dominates the interaction, we can neglect the terms involving β in Eqs. (2.1)–(2.6). In the absence of recombination and intervalley scattering, we get the following relation between electron and hole densities and currents:

$$n_i = -j_{ei}/ev_s, \quad p_i = j_{pi}/ev_s\ , \tag{2.9}$$

where n_i and j_{ei} are the electron number density and current density, respectively, induced by the acoustic wave of frequency ω_i and p_i and j_{pi} are the corresponding parameters for holes.

Also, from (2.3) and (2.5) we get

$$E_i = -4\pi(n_i - p_i)e/i\,q_i\epsilon\ . \tag{2.10}$$

By substituting from (2.7), (2.8), and (2.10) in (2.9) we get the contribution of the three waves to the electron and hole densities up to terms second order in the strain amplitude S:

$$\begin{aligned} n_1 = &-\frac{iq_1\sigma_e(\omega_1)C_eS_1}{e^2v_s\Gamma(\omega_1)} + \frac{4\pi\sigma_e(\omega_1)\sigma_p(\omega_1)(C_p+C_e)S_1}{e^2v_s^2\epsilon\Gamma(\omega_1)} \\ &+\Bigg(-\frac{\Lambda_e(\omega_3,-\omega_2)}{ev_s\Gamma(\omega_1)}[\Gamma_p(\omega_1)]\frac{q_2q_3}{e^2\Gamma(-\omega_2)\Gamma(\omega_3)}[(C_p+C_e)\Gamma_p(-\omega_2)-C_p][(C_p+C_e)\Gamma_p(\omega_3)-C_p] \\ &-\frac{\Lambda_p(\omega_3,-\omega_2)}{ev_s\Gamma(\omega_1)}[1-\Gamma_e(\omega_1)]\frac{q_2q_3}{e^2\Gamma(-\omega_2)\Gamma(\omega_3)}[(C_p+C_e)\Gamma_e(-\omega_2)-C_e][(C_p+C_e)\Gamma_e(\omega_3)-C_e]\Bigg)S_2^*S_3e^{i\Delta x}\ , \end{aligned} \tag{2.11}$$

$$\begin{aligned} n_3 = &-\frac{iq_3\sigma_e(\omega_3)C_eS_3}{e^2v_s\Gamma(\omega_3)} + \frac{4\pi\sigma_e(\omega_3)\sigma_p(\omega_3)(C_p+C_e)S_3}{e^2v_s^2\epsilon\Gamma(\omega_3)} \\ &+\Bigg(\frac{\Lambda_e(\omega_1,\omega_2)}{ev_s\Gamma(\omega_3)}[\Gamma_p(\omega_3)]\frac{q_1q_2}{e^2\Gamma(\omega_1)\Gamma(\omega_2)}[(C_p+C_e)\Gamma_p(\omega_1)-C_p][(C_p+C_e)\Gamma_p(\omega_2)-C_p] \\ &+\frac{\Lambda_p(\omega_1,\omega_2)}{ev_s\Gamma(\omega_3)}[1-\Gamma_e(\omega_3)]\frac{q_1q_2}{e^2\Gamma(\omega_1)\Gamma(\omega_2)}[(C_p+C_e)\Gamma_e(\omega_1)-C_e][(C_p+C_e)\Gamma_e(\omega_2)-C_e]\Bigg)S_1S_2e^{-i\Delta x}\ , \end{aligned} \tag{2.12}$$

and n_2 is identical to n_1 with the subscripts 1 and 2 interchanged. The expressions for the hole densities are identical to those for the electron densities with C_e, $\sigma_e(\omega_i)$, $\Gamma_e(\omega_i)$, and $\Lambda_e(\omega_j, \omega_k)$ replaced by C_p, $\sigma_p(\omega_i)$, $\Gamma_p(\omega_i)$, and $\Lambda_p(\omega_j, \omega_k)$ and a change in the sign of e. In the above

$$\begin{aligned} &\Delta = (q_3 - q_2 - q_1), \\ &\Gamma_{e,p}(\omega_i) = 1 - 4\pi\sigma_{e,p}(\omega_i)/i\omega_i\epsilon\ , \end{aligned} \tag{2.13}$$

and

$$\Gamma(\omega_i) = 1 - (4\pi/i\omega_i\epsilon)[\sigma_e(\omega_i) + \sigma_p(\omega_i)]\ . \tag{2.14}$$

The electric fields induced can now be calculated using (2.10). However, in this case it is more convenient to work with the induced electron and hole densities.

B. Piezoelectric coupling

When piezoelectric coupling dominates the interaction, we can drop the terms involving C_p and C_e in Eqs. (2.1)–(2.6). Also, in this case we shall consider only one type of carrier (electrons) to contribute to the interaction since the presence of holes would lead to negligible effects as long as the hole density is much smaller than the electron density. This is indeed the case in most piezoelectric semiconductors of interest.

By using Eqs. (2.3), (2.7), and (2.8) together with Maxwell's equations for a nonmagnetic medium, we find that the electric fields induced by the ultrasound are

$$E_1 = -\frac{4\pi\beta/\epsilon}{\Gamma(\omega_1)} S_1 - \frac{4i\pi}{\Gamma(\omega_1)\epsilon\omega_1^2}\Lambda(\omega_3, -\omega_2) \times (\omega_3-\omega_2)\frac{(4\pi\beta/\epsilon)^2}{\Gamma(\omega_3)\Gamma(-\omega_2)} S_2^* S_3 e^{i\Delta x} , \quad (2.15)$$

$$E_2 = E_1(1 \rightarrow 2), \quad (2.16)$$

$$E_3 = -\frac{4\pi\beta/\epsilon}{\Gamma(\omega_3)} S_3 - \frac{4i\pi}{\Gamma(\omega_3)\epsilon\omega_3^2}\Lambda(\omega_1, \omega_2) \times (\omega_1+\omega_2)\frac{(4\pi\beta/\epsilon)^2}{\Gamma(\omega_1)\Gamma(\omega_2)} S_1 S_2 e^{-i\Delta x} . \quad (2.17)$$

We are now in a position to solve the equations of motion of the lattice (2.1) and (2.2). Following the same procedure as in Ref. 10, we write the sound wave amplitude as

$$\xi_i = u_i(x)\exp i(q_i x - \omega_i t). \quad (2.18)$$

Because of the smallness of the electromechanical coupling constants, the amplitude will change very little over the distance of a wavelength, and using this fact we obtain the following set of equations for u_1, u_2, and u_3:

$$\frac{du_1}{dx} = -\alpha_1 u_1 - \eta_1 u_2^* u_3 e^{i\Delta x} , \quad (2.19a)$$

$$\frac{du_2}{dx} = -\alpha_2 u_2 - \eta_2 u_1^* u_3 e^{i\Delta x} , \quad (2.19b)$$

$$\frac{du_3}{dx} = -\alpha_3 u_3 + \eta_3 u_1 u_2 e^{-i\Delta x} , \quad (2.19c)$$

where, when deformation-potential coupling dominates

$$\alpha_i = \frac{q_i}{2c}\,\mathrm{Im}\left(\frac{iq_i}{e^2 v_s \Gamma(\omega_i)}[\sigma_e(\omega_i)C_e^2 + \sigma_p(\omega_i)C_p^2] - \frac{4\pi\sigma_e(\omega_i)\sigma_p(\omega_i)}{\epsilon e^2 v_s^2 \Gamma(\omega_i)}(C_p + C_e)^2\right) , \quad (2.20)$$

$$\begin{aligned}\eta_1 = {} & \frac{q_2^2 q_3^2 (q_3 - q_2)}{2q_1 c e^3 v_s \Gamma(\omega_1)\Gamma(-\omega_2)\Gamma(\omega_3)} \\ & \times\{\Lambda_e(\omega_3, -\omega_2)[(C_e + C_p)\Gamma_p(\omega_1) - C_p][(C_e + C_p)\Gamma_p(-\omega_2) - C_p][(C_e + C_p)\Gamma_p(\omega_3) - C_p] \\ & - \Lambda_p(\omega_3, -\omega_2)[(C_p + C_e)\Gamma_e(\omega_1) - C_e][(C_p + C_e)\Gamma_e(-\omega_2) - C_e][(C_p + C_e)\Gamma_e(\omega_3) - C_e]\} ,\end{aligned} \quad (2.21)$$

$$\begin{aligned}\eta_3 = {} & -\frac{q_1^2 q_2^2 (q_1 + q_2)}{2q_3 c e^3 v_s \Gamma(\omega_1)\Gamma(\omega_2)\Gamma(\omega_3)} \\ & \times\{\Lambda_e(\omega_1, \omega_2)[(C_e + C_p)\Gamma_p(\omega_1) - C_p][(C_e + C_p)\Gamma_p(\omega_2) - C_p][(C_e + C_p)\Gamma_p(\omega_3) - C_p] \\ & - \Lambda_p(\omega_1, \omega_2)[(C_p + C_e)\Gamma_e(\omega_1) - C_e][(C_p + C_e)\Gamma_e(\omega_2) - C_e][(C_p + C_e)\Gamma_e(\omega_3) - C_e]\} .\end{aligned} \quad (2.22)$$

The expression for η_2 is identical to that for η_1 with subscripts 1 and 2 interchanged. When piezoelectric coupling dominates, we have

$$\alpha_i = -\frac{q_i}{2c}\,\mathrm{Im}\frac{(4\pi\beta^2/\epsilon)}{\Gamma(\omega_i)} , \quad (2.23)$$

$$\eta_1 = \frac{q_2 q_3 (q_3 - q_2)}{2q_1 c}\;\frac{i(\omega_3 - \omega_2)(4\pi\beta/\epsilon)^3}{\omega_1^2 \Gamma(\omega_1)\Gamma(-\omega_2)\Gamma(\omega_3)}\Lambda(\omega_3, -\omega_2), \quad (2.24)$$

$$\eta_3 = \frac{q_1 q_2 (q_1 + q_2)}{2q_3 c}\;\frac{i(\omega_1 + \omega_2)(4\pi\beta/\epsilon)^3}{\omega_3^2 \Gamma(\omega_1)\Gamma(\omega_2)\Gamma(\omega_3)}\Lambda(\omega_1, \omega_2). \quad (2.25)$$

Again the expression for η_2 is identical to that for η_1 with subscripts 1 and 2 interchanged.

The absorption coefficient defined by equations (2.20) and (2.23) takes into account only the electronic losses. However, the linear absorption coefficient, α_i, appearing in Eq. (2.23) includes both the electronic and lattice losses and therefore our results are valid only when the nonelectronic lattice losses are small. In situations for which the above condition does not hold, our results can be easily modified by using the total, electronic and lattice, linear absorption coefficients that appear in the parameters which determine the frequency mixing processes. In n-InSb[20] the lattice losses have been determined to be less than 2 cm^{-1} at a frequency of 3.8 gigahertz. However, at higher frequencies they can become considerably larger.[21]

By using the set of coupled equations (2.19), we can now discuss the various parametric processes. When the amplitude of the acoustic wave is large at the highest of the three frequencies ω_3 (pump) while the amplitudes at both ω_1 (signal) and ω_2 (idler) are small, we can neglect the second term on the right hand side of Eq. (2.23) as compared to the first. This neglects the depletion of the pump and is therefore only valid for small x. As a result, the second term on the right hand side of (2.19a) and (2.19b) acts as a source for waves at the idler and signal frequencies thereby leading to nonlinear gain at these lower frequencies. The solutions for the case of down-conversion have been worked out and discussed in great detail in Refs. 10 and 14. It is found that if the real part of m, where

$$m=\{[\tfrac{1}{2}(\alpha_2-\alpha_1-i\Delta)]^2+\eta_1\eta_2^*\,|u_3(0)|^2\}^{1/2}$$

is larger than the imaginary part, then for large enough x with $\alpha_3 x \ll 1$

$$u_1(x)\simeq Ce^{\alpha_{NL}x}, \tag{2.26}$$

where the nonlinear gain coefficient is

$$\alpha_{NL}=-\tfrac{1}{2}(\alpha_1+\alpha_2)+\mathrm{Re}\,m\,. \tag{2.27}$$

In particular, for the degenerate case $\omega_1=\omega_2=\omega$

$$\alpha_{NL}=-\alpha_1+|\eta||S_3(0)|\,, \tag{2.28}$$

where $\eta=\eta_1/q_3$.

In the presence of an external drift field we can have $\alpha_1<0$. In this situation, the nonlinear correction to the gain coefficient causes the signal (subharmonic) to be amplified at a greater rate than that predicted by the linear theory. In the absence of a drift field, when $\alpha_1>0$, there will be a net gain at the subharmonic only if $|\eta||S_3(0)|>\alpha_1$. Therefore, the threshold pump strain above which the signal at the subharmonic would be amplified is

$$|S_{\rm th}(0)|=\alpha_1/|\eta|\,. \tag{2.29}$$

If initially there are waves present at frequencies ω_1 and ω_2, then a third wave will be generated at the frequency ω_3. At small enough x where the amplitude of the wave at frequency ω_3 is still small, we can neglect the terms containing u_3 in (2.19a) and (2.19b) and solve for u_1 and u_2. The solutions for the case of upconversion have also been discussed in detail in Refs. 10 and 14. For the degenerate case where $\omega_3=2\omega=2\omega_1=2\omega_2$ we have second harmonic generation. The ratio of the acoustic flux at the second harmonic to the initial flux in the fundamental is

$$P_2/P_1^2(0)=A[e^{-2\alpha_2 x}+e^{-4\alpha_1 x}-2e^{-(2\alpha_1+\alpha_2)x}\cos(2q_1-q_2)x]\,, \tag{2.30}$$

where the acoustic flux at a frequency ω_i is

$$P(\omega_i)=\tfrac{1}{2}\rho\omega_i^2|\xi_i|^2v_s \tag{2.31}$$

and

$$A=8|\eta_2|^2/|(\alpha_2-2\alpha_1)-i(q_2-2q_1)|^2\rho v_s\omega^2\,. \tag{2.32}$$

In (2.30) and (2.32) the subscripts 1 and 2 refer to the fundamental and the second harmonic, respectively.

III. CALCULATION OF CONDUCTIVITY TENSORS

The Boltzmann equation for electrons interacting with acoustic waves of frequencies ω_1, ω_2, and $\omega_3=\omega_1+\omega_2$ and wave vectors q_1, q_2, and q_3 is[23]

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial\vec{r}}-\sum_{i=1}^{3}\frac{e}{m}\vec{E}_i'\cdot\frac{\partial f}{\partial\vec{v}}=-\frac{f-f_s}{\tau}\,, \tag{3.1}$$

where $\vec{E}_i'$ is the effective electric field induced by the acoustic wave, τ is the carrier relaxation time, and f_s is the distribution to which the carriers relax in the presence of the acoustic wave. This distribution is[23]

$$f_s(\vec{v})\approx f_0(\vec{v})+\sum_{i=1}^{3}\left(n_i\frac{df_0}{dn_0}\right). \tag{3.2}$$

Here $f_0(\vec{v})$ is the equilibrium distribution function, $\vec{\xi}_i$ is the amplitude of the acoustic wave, and n_i is the carrier density induced by the wave. In most materials of interest the carrier density is low enough so that we use classical Maxwell-Boltzmann statistics. Therefore, we shall take $f_0(\vec{v})$ to be the Maxwell-Boltzmann distribution function.

The solution to Eq. (3.1) can be written

$$f(\vec{v})=f_0(\vec{v})+\sum_{i=1}^{3}q_i(\vec{v})\exp i(q_i x-\omega_i t)+\text{c.c.} \tag{3.3}$$

Here we have taken the three waves to be collinear with their wave vectors lying along the x axis. Substituting from (3.3) in (3.1), we get the following equations:

$$[\tau^{-1}+i(q_1v_x-\omega_1)]g_1(v)=-\frac{2ev_x}{mv_0^2}E_1f_0+\frac{n_1}{n_0\tau}f_0$$
$$+\frac{e}{m}\left\{\frac{f_0}{1-i(q_2v_x-\omega_2)\tau}\left[\frac{2e}{mv_0^2}E_2^*E_3\left(-1+\frac{2v_x^2}{v_0^2}-\frac{iq_2v_x\tau}{1-i(q_2v_x-\omega_2)\tau}\right)\right.\right.$$
$$\left.-\frac{n_2^*}{n_0\tau}E_3\left(\frac{2v_x}{v_0^2}-\frac{iq_2\tau}{1-i(q_2v_x-\omega_2)\tau}\right)\right]$$
$$+\frac{f_0}{1+i(q_3v_x-\omega_3)\tau}\left[\frac{2e}{mv_0^2}E_2^*E_3\left(-1+\frac{2v_x^2}{v_0^2}+\frac{iq_3v_x\tau}{1+i(q_3v_x-\omega_3)\tau}\right)\right.$$
$$\left.\left.-\frac{n_3}{n_0\tau}E_2^*\left(\frac{2v_x}{v_0^2}+\frac{iq_3\tau}{1+i(q_3v_x-\omega_3)\tau}\right)\right]\right\}. \quad (3.4)$$

g_2 obeys an equation identical to (3.4) with subscripts 1 and 2 interchanged.

$$[\tau^{-1}+i(q_3v_x-\omega_3)]g_3(v)=-\frac{2ev_x}{mv_0^2}E_3f_0+\frac{n_3}{n_0\tau}f_0$$
$$+\frac{e}{m}\left\{\frac{f_0}{1+i(q_1v_x-\omega_1)\tau}\left[\frac{2e}{mv_0^2}E_1E_2\left(-1+\frac{2v_x^2}{v_0^2}+\frac{iq_1v_x\tau}{1+i(q_1v_x-\omega_1)\tau}\right)\right.\right.$$
$$\left.-\frac{n_1E_2}{n_0\tau}\left(\frac{2v_x}{v_0^2}+\frac{iq_1\tau}{1+i(q_1v_x-\omega_1)\tau}\right)\right]$$
$$+\frac{f_0}{1+i(q_2v_x-\omega_2)\tau}\left[\frac{2e}{mv_0^2}E_1E_2\left(-1+\frac{2v_x^2}{v_0^2}+\frac{iq_2v_x\tau}{1+i(q_2v_x-\omega_2)\tau}\right)\right.$$
$$\left.\left.-\frac{n_2E_1}{n_0\tau}\left(\frac{2v_x}{v_0^2}+\frac{iq_2\tau}{1+i(q_2v_x-\omega_2)\tau}\right)\right]\right\}. \quad (3.5)$$

The ac current induced by the ultrasound is

$$\vec{j}_i=-e\int d\vec{v}\,\vec{v}g_i(\vec{v}),\quad i=1,2,3. \quad (3.6)$$

Here the charge of the carriers is taken to be $-e$.

Substituting from (3.4) and (3.5) in (3.6) we can write the ac current density as

$$j_{1x}=\sigma_{xx}(\omega_1)E_1-R_x(\omega_1)n_1ev_s+[\tau_{xxx}(\omega_3,\omega_1-\omega_3)+\tau_{xxx}(-\omega_2,\omega_1+\omega_2)]E_2^*E_3$$
$$-S_{xx}(\omega_3,\omega_1-\omega_3)\,ev_sn_3E_2^*-S_{xx}(-\omega_2,\omega_1+\omega_2)\,ev_sn_2^*E_3. \quad (3.7a)$$

The expression for j_{2x} is identical to that for j_{1x} with subscripts 1 and 2 interchanged while

$$j_{3x}=\sigma_{xx}(\omega_3)E_3-R_x(\omega_3)n_3ev_s+[\tau_{xxx}(\omega_1,\omega_3-\omega_1)+\tau_{xxx}(\omega_2,\omega_3-\omega_2)]E_1E_2$$
$$-S_{xx}(\omega_1,\omega_3-\omega_1)ev_sn_1E_2-S_{xx}(\omega_2,\omega_3-\omega_2)ev_sn_2E_1\,. \quad (3.7b)$$

Here[24]

$$\sigma_{xx}(\omega_i)=\frac{2\sigma_0\pi^{1/2}}{q_il}\left[\frac{a_i^2}{(q_il)^2}W\left(-\frac{a_i}{iq_il}\right)-\frac{a_i}{\pi^{1/2}q_il}\right], \quad (3.8)$$

$$R_x(\omega_i)=\frac{v_0}{v_siq_il}\left[1-\frac{a_i}{iq_il}i\pi^{1/2}W\left(\frac{-a_i}{iq_il}\right)\right], \quad (3.9)$$

$$\tau_{xxx}(\omega_1, \omega_3-\omega_1) = \frac{2\sigma_0\mu}{\pi^{1/2}v_0}\left\{\frac{-\pi}{q_1l(a_3-\lambda_{31}a_1)}\left[\frac{a_1}{iq_1l}W\left(\frac{-a_1}{iq_1l}\right)-\frac{a_3}{iq_3l}W\left(\frac{-a_3}{iq_3l}\right)\right]\right.$$
$$-\frac{2\pi}{a_3-\lambda_{31}a_1}\left[\frac{a_1^3}{i(q_1l)^3}W\left(\frac{-a_1}{iq_1l}\right)-\frac{a_3^3}{i(q_3l)^3}\times W\left(\frac{-a_3}{iq_3l}\right)-\frac{a_1^2}{i\pi^{1/2}(q_1l)^2}+\frac{a_3^2}{i\pi^{1/2}(q_3l)^2}\right]$$
$$-\frac{i\pi\lambda_{31}}{(a_3-\lambda_{31}a_1)^2}\times\left[\frac{a_1^2}{(q_1l)^2}W\left(\frac{-a_1}{iq_1l}\right)-\frac{a_3^2}{(q_3l)^2}W\left(\frac{-a_3}{iq_3l}\right)-\frac{a_1}{\pi^{1/2}q_1l}+\frac{a_3}{\pi^{1/2}q_3l}\right]$$
$$\left.-\frac{i\pi}{a_3-\lambda_{31}a_1}\left[\frac{2a_1}{(q_1l)^2}W\left(\frac{-a_1}{iq_1l}\right)-\frac{1}{\pi^{1/2}q_1l}+\frac{2a_1^3}{(q_1l)^4}\times W\left(\frac{-a_1}{iq_1l}\right)-\frac{2a_1^2}{\pi^{1/2}(q_1l)^3}\right]\right\}, \tag{3.10}$$

and

$$S_{xx}(\omega_1, \omega_3-\omega_1) = \frac{-\mu}{\pi^{1/2}v_s}\left\{\frac{-2\pi}{q_1l(a_3-\lambda_{31}a_1)}\left[\frac{a_1^2}{(q_1l)^2}W\left(\frac{-a_1}{iq_1l}\right)-\frac{a_3^2}{(q_3l)^2}W\left(\frac{-a_3}{iq_3l}\right)-\frac{a_1}{\pi^{1/2}q_1l}+\frac{a_3}{\pi^{1/2}q_3l}\right]\right.$$
$$+\frac{\pi\lambda_{31}}{(a_3-\lambda_{31}a_1)^2}\left[\frac{a_1}{q_1l}W\left(\frac{-a_1}{iq_1l}\right)-\frac{a_3}{q_3l}W\left(\frac{-a_3}{iq_3l}\right)\right]$$
$$\left.+\frac{\pi}{(a_3-\lambda_{31}a_1)}\left[\frac{1}{q_1l}W\left(\frac{-a_1}{iq_1l}\right)+\frac{2a_1^2}{(q_1l)^3}W\left(\frac{-a_1}{iq_1l}\right)-\frac{2a_1}{\pi^{1/2}(q_1l)^2}\right]\right\}, \tag{3.11}$$

where $W(z)=e^{-z^2}\operatorname{erfc}(-iz)$, σ_0 is the dc conductivity of the electrons, μ is the mobility of the electrons, $a_i=1-i\omega_i\tau$, $\lambda_{nk}=\omega_n/\omega_k$, l is the electron mean free path, $v_0=(2k_BT/m)^{1/2}$ is the mean thermal velocity and v_s is the velocity of sound. By using the equation of continuity we can rewrite (3.7) as

$$j_{1x}=\sigma'_{xx}(\omega_1)E_1+\Lambda_{xxx}(\omega_3,-\omega_2)E_2^*E_3, \tag{3.12a}$$
$$j_{2x}=\sigma'_{xx}(\omega_2)E_2+\Lambda_{xxx}(\omega_3,-\omega_1)E_1^*E_3, \tag{3.12b}$$
$$j_{3x}=\sigma'_{xx}(\omega_3)E_3+\Lambda_{xxx}(\omega_1,\omega_2)E_1E_2, \tag{3.12c}$$

where

$$\sigma'_{xx}(\omega_i)=\sigma_{xx}(\omega_i)/[1-R_x(\omega_i)], \tag{3.13}$$

$$\Lambda_{xxx}(\omega_3,-\omega_2)=\frac{\tau_{xxx}(\omega_3,\omega_1-\omega_3)+\tau_{xxx}(-\omega_2,\omega_1+\omega_2)}{1-R_x(\omega_1)}+\frac{S_{xx}(\omega_3,\omega_1-\omega_3)\sigma'_{xx}(\omega_3)+S_{xx}(-\omega_2,\omega_1+\omega_2)\sigma'_{xx}(-\omega_2)}{1-R_x(\omega_1)}, \tag{3.14}$$

$$\Lambda_{xxx}(\omega_1,\omega_2)=\frac{\tau_{xxx}(\omega_1,\omega_3-\omega_1)+\tau_{xxx}(\omega_2,\omega_3-\omega_2)}{1-R_x(\omega_3)}+\frac{S_{xx}(\omega_1,\omega_3-\omega_1)\sigma'_{xx}(\omega_1)+S_{xx}(\omega_2,\omega_3-\omega_2)\sigma'_{xx}(\omega_2)}{1-R_x(\omega_3)}, \tag{3.15}$$

We note that for the degenerate case $\omega_1=\omega_2=\omega$, the nonlinear conductivity $\Lambda(\omega_1,\omega_2)$ is smaller by a factor of 2 than the expression obtained by using Eqs. (3.10), (3.11), and (3.15).

In the long-wavelength limit, $q_il\ll 1$, we can use the asymptotic form of $W(z)$,[25]

$$W(z)=(i/\pi^{1/2}z)(1+1/2z^2),$$

and the conductivity tensors reduce to

$$\sigma'_{xx}(\omega_i)=\epsilon\omega_C/4\pi(1+i\omega_i/\omega_D), \tag{3.16}$$

$$\Lambda_{xxx}(\omega_3,-\omega_2)=(\epsilon\omega_c\mu/4\pi v_s)(1+i\omega_1/\omega_D)^{-1}\times[(1+i\omega_3/\omega_D)^{-1}+(1-i\omega_2/\omega_D)^{-1}], \tag{3.17}$$

$$\Lambda_{xxx}(\omega_1,\omega_2)=(\epsilon\omega_c\mu/4\pi v_s)(1+i\omega_3/\omega_D)^{-1}\times[(1+i\omega_1/\omega_D)^{-1}+(1+i\omega_2/\omega_D)^{-1}], \tag{3.18}$$

where ω_c is the dielectric relaxation frequency, ϵ is the static dielectric constant and $\omega_D=v_s^2/D$, where D is the diffusion coefficient for the electrons.

In the short-wavelength limit, $ql\gg 1$, we can use the small argument expansion of $W(z)$,[26]

$$W(z)=\sum_{n=0}\frac{(iz)^n}{(n/2)!},$$

and we get

$$\sigma'_{xx}(\omega_i) = -(i\omega_i\epsilon/4\pi)(q_d/q_i)^2(1+i\pi^{1/2}v_s/v_0), \tag{3.19}$$

$$\Lambda_{xxx}(\omega_3, -\omega_2) = \frac{\epsilon}{4\pi}\,\frac{e v_s}{m v_0^2}\left[\left(\frac{q_d}{q_3}\right)^2 \frac{1}{(1-\lambda_{13})}\left(2(1-\lambda_{31}) - 2i\pi^{1/2}\frac{v_s}{v_0}(1-2\lambda_{31})\right) + \left(\frac{q_d}{q_2}\right)^2 \frac{1}{(1+\lambda_{12})}\left(2(1+\lambda_{21}) - 2i\pi^{1/2}\frac{v_s}{v_0}(1+2\lambda_{21})\right)\right], \tag{3.20}$$

$$\Lambda_{xxx}(\omega_1, \omega_2) = \frac{\epsilon}{4\pi}\,\frac{e v_s}{m v_0^2}\left[\left(\frac{q_d}{q_1}\right)^2 \frac{1}{(1-\lambda_{31})}\left(2(1-\lambda_{13}) - 2i\pi^{1/2}\frac{v_s}{v_0}(1-2\lambda_{13})\right)\right] + \left(\frac{q_d}{q_2}\right)^2 \frac{1}{(1-\lambda_{32})}\left(2(1-\lambda_{23}) - 2i\pi^{1/2}\frac{v_s}{v_0}(1-2\lambda_{23})\right)\Bigg], \tag{3.21}$$

where $q_d = (4\pi n_0 e^2/\epsilon k_B T)^{1/2}$ is the electron Debye wave vector.

The results of this section are also valid for the contribution of holes to the current density if we replace the electron parameters in the expressions for the conductivity tensors with the corresponding hole parameters.

IV. APPLICATIONS

We shall now use the results of Secs. II and III to discuss subharmonic and second-harmonic generation in extrinsic and intrinsic semiconductors, and in semimetals where the deformation-potential coupling dominates the interaction and also in semiconductors where piezoelectric coupling dominates the interaction.

A. Extrinsic semiconductors with majority carriers dominating the interaction

As a specific example of such an extrinsic semiconductor, we shall consider a n-type material. The results for a p-type material will follow in an analogous fashion. For the majority carriers to dominate the electron-phonon interaction, the ac conductivity of the minority carriers must satisfy the condition $4\pi\sigma_p(\omega_i)/\epsilon \ll \omega_i$. This condition will be satisfied if there are no minority carriers and will be satisfied at low frequencies if the density of minority carriers is small. Therefore, neglecting the minority carriers, we find that

$$\alpha(\omega_i) = \frac{q_i^2}{2cv_s}\,\frac{(C_e/e)^2\sigma_0}{1+(\omega_c/\omega_i+\omega_i/\omega_D)^2} \tag{4.1}$$

when $q_i l \ll 1$, and

$$\alpha(\omega_i) = \frac{q_i}{2c}\,\frac{\epsilon}{4\pi}\,\frac{(C_e/e)^2 q_d^2(\pi^{1/2}v_s/v_0)}{[1+(q_d/q_i)^2]^2} \tag{4.2}$$

when $q_i l \gg 1$.

For subharmonic generation at the subharmonic frequency ω, we get

$$|\eta| = \frac{2q^3}{v_s^2}\,\frac{\mu\sigma_0}{c}\left(\frac{C_e}{e}\right)^3 \frac{1}{1+(\omega_c/\omega+\omega/\omega_D)^2} \times \frac{[1+(\omega/2\omega_D)^2]^{1/2}}{[1+(\omega_c/2\omega+2\omega/\omega_D)^2]^{1/2}}, \tag{4.3}$$

$$S_{\text{th}} = \frac{v_s^2}{4\omega\mu}\,\frac{e}{C_e}\,\frac{[1+(\omega_c/2\omega+2\omega/\omega_D)^2]^{1/2}}{[1+(\omega/2\omega_D)^2]^{1/2}} \tag{4.4}$$

when $ql \ll 1$, and

$$|\eta| = \frac{q^3}{c}\left(\frac{\epsilon e}{4\pi m v_0^2}\right)\left(\frac{q_d}{q}\right)^2 \frac{(C_e/e)^3}{[1+(q_d/q)^2]^2[1+(q_d/2q)^2]}, \tag{4.5}$$

$$S_{\text{th}} = (1/2C_e)\, m v_0^2 \pi^{1/2}(v_s/v_0)[1+(q_d/2q)^2] \tag{4.6}$$

when $ql \gg 1$.

In Figs. 1 and 2 we plot $|\eta|$ when $ql \ll 1$ and when $ql \gg 1$ respectively and in Figs. 3 and 4 we plot S_{th} in the regions $ql \ll 1$ and $ql \gg 1$, respectively. The parameters used for n-Ge and are given in Table I. The ordinate in Figs. 1 and 2 is in arbitrary units. We note that $|\eta|$ is a monotonically increasing function of frequency in both of the regimes $ql \ll 1$ and $ql \gg 1$. Also, for $ql \ll 1$, S_{th} is quite large ($>10^{-1}$) up to a frequency of 1 GHz. Therefore, in the absence of external drift fields, subharmonic generation will be negligible below 1 GHz. Above a frequency of 10 GHz, the threshold pump strain for nonlinear gain at the subharmonic becomes comparable to the one obtained by Conwell, for piezoelectric CdS.[10] But here we are almost at the boundary of the regime $ql \ll 1$. For $ql \gg 1$ the threshold strain decreases rapidly with frequency until it reaches a constant value at a phonon wave vector equal to twice the electron Debye wave vector q_d. Therefore, the deformation-potential coupling should become a more important mechanism for subharmonic generation than piezoelectric coupling at very high frequencies. However, the phenomenological model used by Conwell[10] is not valid at very high frequencies when $ql \gg 1$. We shall discuss the case when piezoelectric coupling dom-

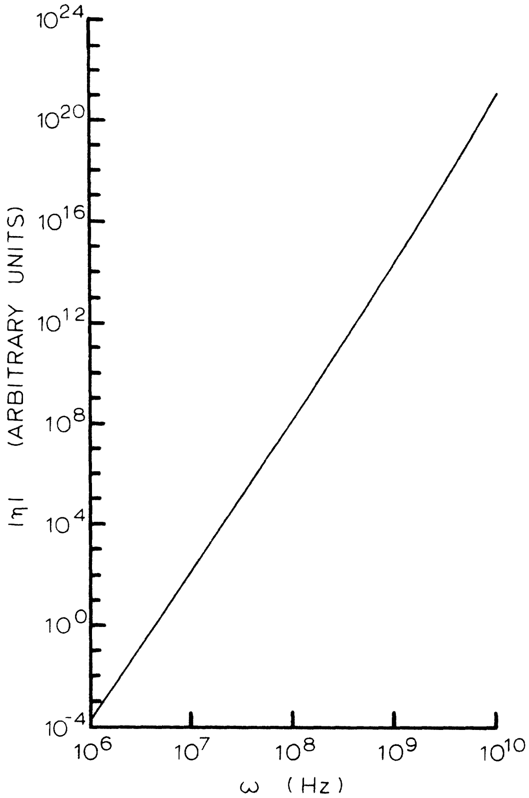

FIG. 1. $|\eta|$ shown as a function of frequency in n-Ge in the region $ql \ll 1$.

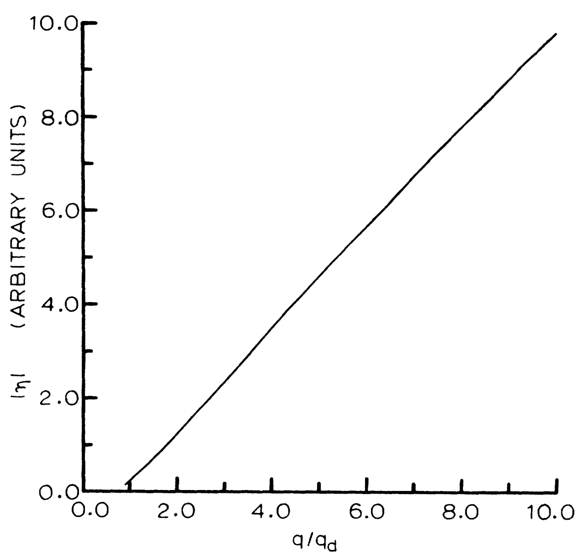

FIG. 2. $|\eta|$ shown as a function of wave vector in n-Ge in the region $ql \gg 1$.

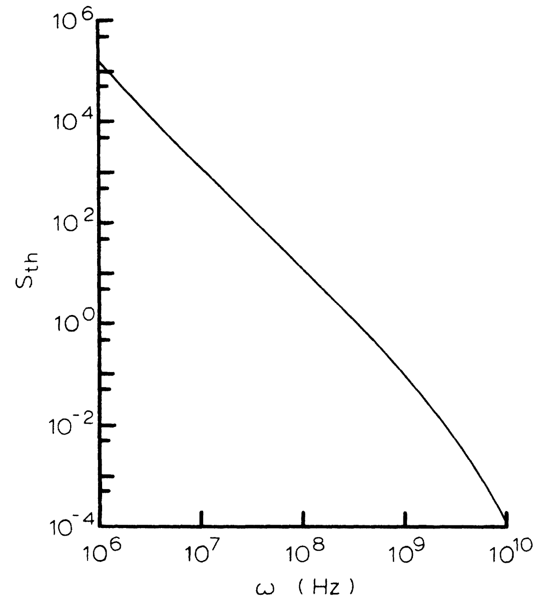

FIG. 3. Threshold pump strain shown as a function of frequency in n-Ge when $ql \ll 1$.

inates later on in this section.

For second-harmonic generation at the second-harmonic frequency 2ω, we get

$$\eta_2 = -\frac{q^4}{2c}\frac{\epsilon}{4\pi}\frac{e}{mv_0^2}\left(\frac{q_d}{q}\right)^2 \frac{(C_e/e)^3}{[1+(q_d/q)^2]^2[1+(q_d/2q)^2]}\,, \tag{4.7}$$

$$A = \frac{8}{9\rho v_s^3}\frac{e^2}{(mv_0^2)^2}(C_e/e)^2\frac{(q/q_d)^4}{[1+(q_d/q)^2]^2} \tag{4.8}$$

for $ql \gg 1$.

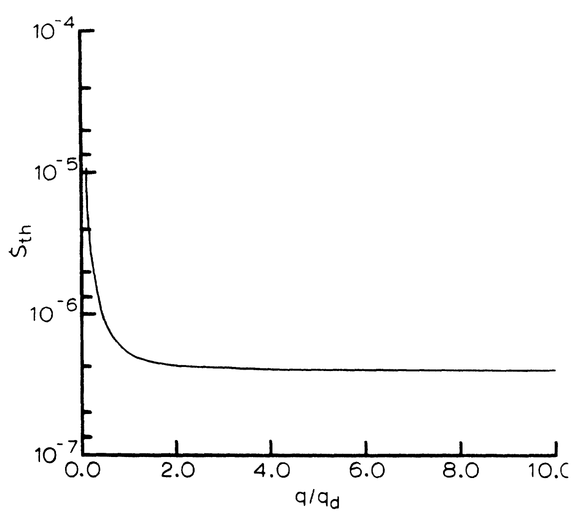

FIG. 4. Threshold pump strain shown as a function of wave vector in n-Ge when $ql \gg 1$.

TABLE I. Physical parameters for *n*-InSb, *n*-Ge, and Bi.

Parameter	*n*-InSb (77 K)	*n*-Ge (10 K)	Bi
m_e^*	$0.013m_0$	$0.082m_0$	$0.01m_0$
m_p^*	...	...	m_0
v_s (cm sec^{-1})	4.0×10^5	10^5	2.0×10^5
v_0 (cm sec^{-1})	4.2×10^7	10^6	...
v_F (cm sec^{-1})	...	...	10^7
τ_e (sec)	10^{-12}	10^{-12}	10^{-10}
τ_p (sec)	...	...	10^{-10}
n_0,p_0 (cm^{-3})	2.5×10^{14}	10^{16}	5×10^{17}
ϵ	18	16	10
ρ (g cm^{-3})	5.8	3.0	9.74
β (esu cm^{-2})	1.8×10^4	...	...
C_e (eV)	6.0	10	2
C_p (eV)	...	...	8

When $ql \ll 1$ our results reduce to those of Spector.[15] In Figure 5 we plot the second-harmonic flux A versus the phonon wave vector for $ql \gg 1$. The flux at the second harmonic increases with frequency. For phonon wave vectors much above the electron Debye wave vectors, the frequency dependence of A is approximately ω^4. Therefore, deformation-potential coupling should become the dominant mechanism for harmonic generation at very high frequencies even for piezoelectric semiconductors. The harmonic generation due to this mechanism should be completely negligible at frequencies below the dielectric relaxation frequency of the majority carriers.[15]

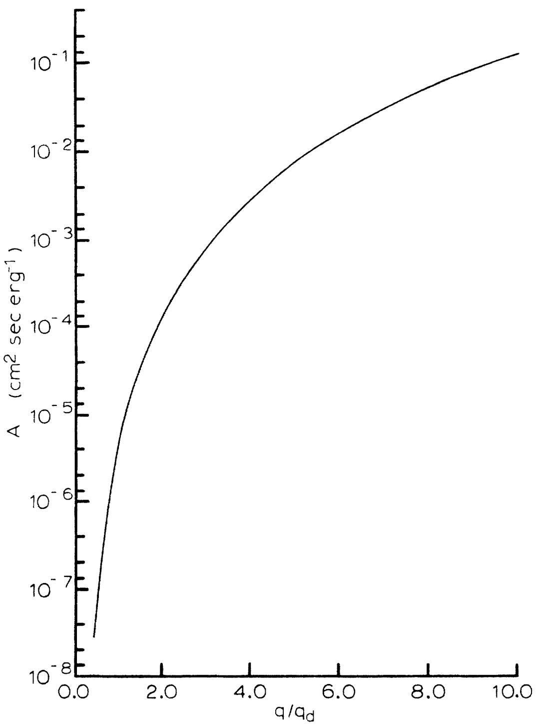

FIG. 5. Second-harmonic flux as a function of wave vector shown for *n*-Ge when $ql \gg 1$.

B. Intrinsic semiconductors and semimetals

In intrinsic semiconductors and semimetals, there are equal numbers of electrons and holes. Therefore, the ac electron and hole conductivities are of the same order of magnitude. When the inequalities $(4\pi/\epsilon)\sigma_e(\omega_i) \gg \omega_i$ and $(4\pi/\epsilon)\sigma_p(\omega_i) \gg \omega_i$ hold, we have space-charge neutrality and no electric fields are induced by the sound wave. Since the density of electrons and holes is equal and can be quite high (of the order of 10^{18} cm^{-3} in a semimetal like Bi), it is possible for the condition of space-charge neutrality to hold up to very high frequencies. In fact, for Bi, the conditions for space-charge neutrality hold to the highest frequencies now available. Under these conditions, where charge neutrality holds, we get

$$\alpha(\omega_i) = \frac{q_i^2}{2cv_s}\frac{[(C_p+C_e)/e]^2}{[1+(\omega_i/\omega_D)^2]}\,n_0 e\mu \ , \tag{4.9}$$

when $ql \ll 1$, and

$$\alpha(\omega_i) = \pi^{1/2}\frac{q_i v_s \epsilon}{16\pi c v_0}\,q_d^2[(C_p+C_e)/e]^2, \tag{4.10}$$

when $ql \gg 1$, where $\mu = \mu_e\mu_p/(\mu_e+\mu_p)$ and $D = (\mu_e D_p + \mu_p D_e)/(\mu_e+\mu_p)$ and $\omega_D = v_s^2/D$ and the Debye wave vector q_d is the same for both electrons and holes.

For subharmonic generation at the subharmonic frequency ω, we get

$$|\eta| = \frac{2q^3}{cv_s^2}\,\mu\sigma_0\left(\frac{C_p+C_e}{e}\right)^3\frac{1}{[1+(\omega/\omega_D)^2]} \times\frac{[1+(\omega/2\omega_D)^2]^{1/2}}{[1+(2\omega/\omega_D)^2]^{1/2}}\ , \tag{4.11}$$

$$S_{\text{th}} = \frac{v_s^2}{4\omega\mu}\,\frac{e}{(C_p+C_e)}\,\frac{[1+(2\omega/\omega_D)^2]^{1/2}}{[1+(\omega/2\omega_D)^2]^{1/2}}\ , \tag{4.12}$$

when $ql \ll 1$, and

$$|\eta| = \frac{q^3}{4c}\,\frac{\epsilon e}{4\pi m v_0^2}\left(\frac{q_d}{q}\right)^2\left(\frac{C_p+C_e}{e}\right)^3\ , \tag{4.13}$$

$$S_{\text{th}} = [mv_0^2/(C_p+C_e)](\pi^{1/2}v_s/v_0), \tag{4.14}$$

when $ql \gg 1$.

The above results are valid for nondegenerate semiconductors. In semimetals, where the electrons and holes are degenerate, we have to replace the factors k_BT in the above equations by $\frac{2}{3}E_F$, E_F being the Fermi energy of the degenerate carriers.

We shall now apply these results to Bi the pertinent parameters for which are given in Table I. In

Figs. 6 and 7 we plot $|\eta|$ in arbitrary units versus frequency and S_{th} versus frequency, respectively, in the region $ql \ll 1$. In the region $ql \ll 1$, $|\eta|$ increases with frequency. This dependence on frequency becomes linear in the region $ql \gg 1$. Therefore, nonlinear gain at the subharmonic will continue to increase with frequency even in the region $ql \gg 1$. However, in this region, the linear loss also monotonically increases with frequency. The threshold pump strain for net gain at the subharmonic falls from 10^{-2} at 1 MHz to 10^{-4} at 0.1 GHz. It must then go through a minimum because when $ql \gg 1$, S_{th} becomes independent of frequency and has a relatively large value (5×10^{-2}). Therefore, in Bi, in the absence of external drift fields subharmonic generation should be negligible above a frequency of 0.1 GHz.

For second harmonic generation at the second harmonic frequency 2ω, our results agree with those of Spector[15] when $ql \ll 1$. The behavior of the second-harmonic flux with particular reference to Bi is discussed in that paper in the region $ql \ll 1$. To calculate the second-harmonic flux when $ql \gg 1$ we note that for an intrinsic semiconductor or semimetal the denominator on the right-hand side of equation (2.40) becomes vanishingly small if we only take into account linear electronic loss or gain. In that limit the ratio of the flux at the second harmonic to the square of the flux at the fundamental is

$$P_2/P_1^2(0) = Be^{-2\alpha_2 x} x^2 \ , \qquad (4.15)$$

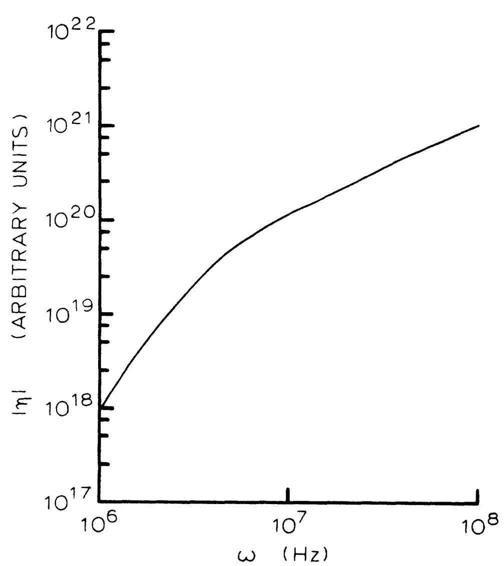

FIG. 6. $|\eta|$ as a function of frequency in Bi shown in the region $ql \ll 1$.

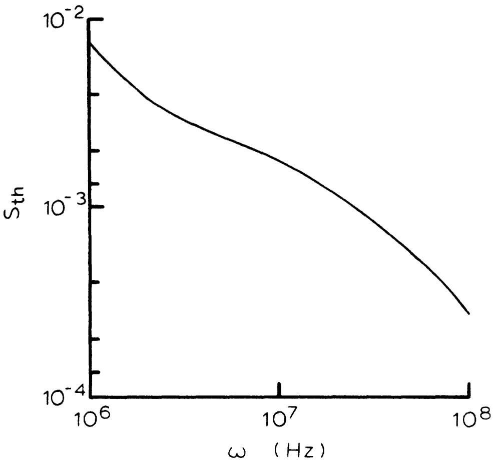

FIG. 7. Threshold pump strain as a function of frequency shown in Bi for $ql \ll 1$.

where

$$B = (8/\rho v_s \omega^2)|\eta_2|^2 \ . \qquad (4.16)$$

Therefore, for a intrinsic semiconductor we get

$$B = \frac{q^2}{8\rho v_s^3}\left(\frac{\epsilon e q_d^2}{8\pi k_B T \rho v_s^2}\right)^2 \left(\frac{C_p + C_e}{e}\right)^6 \ . \qquad (4.17)$$

For a semimetal such as Bi, we have to replace the factor $k_B T$ by $\frac{2}{3}E_F$ in Eq. (4.17). The coefficient B which determines the second-harmonic flux increases quadratically with frequency. At frequencies in the gigahertz range, B has a value of about 10^{-4}. Therefore second-harmonic generation should be quite important in Bi in the region $ql \gg 1$.

The above results for second-harmonic generation in an intrinsic semiconductor or semimetal when $ql \gg 1$ neglect linear lattice losses. However, because the frequency dependence of the linear lattice losses is the same as the electron losses in the Landau-Rumer regime where $\omega\tau_p \gg 1$ and τ_p is the lifetime of thermal phonons,[21] (4.17) is still valid even when lattice losses are taken into account.

It is interesting to note that for an extrinsic semiconductor where minority carriers dominate the interaction all the results of this section are applicable, the parameters σ_0, μ, and ω_D being the ones for minority carriers. However, for minority carriers to dominate the interaction in an extrinsic semiconductor, the condition $4\pi\sigma_p(\omega_i)/\epsilon \gg \omega_i$ must be met. This puts an upper limit on the frequency up to which the results of our calculations would be valid.

C. Piezoelectric semiconductors

Here we shall consider the carriers to be electrons and the dominant mode of the interaction to be via the piezoelectric coupling. For such a material we get

$$\alpha(\omega_i) = \frac{q_i}{2c}\,\frac{(4\pi\beta^2/\epsilon)\omega_c/\omega_i}{1+(\omega_c/\omega_i+\omega_i/\omega_D)^2]}\,, \tag{4.18}$$

when $q_i l \ll 1$, and

$$\alpha(\omega_i) = \frac{q_i}{2c}\,\frac{(4\pi\beta^2/\epsilon)\pi^{1/2}v_s/v_0}{[1+(q_d/q_i)^2]^2}\left(\frac{q_d}{q_i}\right)^2, \tag{4.19}$$

when $q_i l \gg 1$.

For subharmonic generation at the subharmonic frequency ω, we get

$$|\eta| = \frac{\sigma_0\mu}{c v_s^2}\,\frac{(4\pi\beta/\epsilon)^3}{[1+(\omega_c/\omega+\omega/\omega_D)^2]}\times\frac{[1+(\omega/2\omega_D)^2]^{1/2}}{[1+(\omega_c/2\omega+2\omega/\omega_D)^2]^{1/2}} \tag{4.20}$$

and

$$S_{\rm th} = \frac{v_s}{2\mu}\left(\frac{\epsilon}{4\pi\beta}\right)\frac{[1+(\omega_c/2\omega+2\omega/\omega_D)^2]^{1/2}}{[1+(\omega/2\omega_D)^2]^{1/2}}\,, \tag{4.21}$$

when $ql \ll 1$, and

$$|\eta| = \frac{1}{2c}\left(\frac{\epsilon e}{4\pi m v_0^2}\right)\left(\frac{q_d}{q}\right)^2\times\frac{(4\pi\beta/\epsilon)^3}{[1+(q_d/q)^2]^2[1+(q_d/2q)^2]} \tag{4.22}$$

and

$$S_{\rm th} = \frac{\epsilon}{4\pi\beta}\left(\frac{mv_0^2}{e}\,q_d\right)\left(\frac{q}{q_d}\right)\pi^{1/2}\,\frac{v_s}{v_0}\left[1+\left(\frac{q_d}{2q}\right)^2\right], \tag{4.23}$$

when $ql \gg 1$.

In the long-wavelength region, $ql \ll 1$, our results are the same as those obtained by Conwell and Ganguly[10] using a phenomenological approach. In Figs. 8 and 9 we plot $|\eta|$ in arbitrary units, when $ql \ll 1$ and when $ql \gg 1$, respectively. The parameters used are for n-InSb and are given in Table I. In the region $ql \ll 1$, $|\eta|$ increases with frequency as long as $\omega_C\,\omega_D \gg \omega^2$. This condition is satisfied in n-InSb. In the region $ql \gg 1$, $|\eta|$ has a maximum at a phonon wave vector equal to the electron Debye wave vector and falls off rapidly above that. In Figs. 10 and 11 we plot $S_{\rm th}$ versus frequency when $ql \ll 1$ and when $ql \gg 1$, respectively. When $ql \ll 1$, $S_{\rm th}$ falls off from a relatively large value ($>10^{-1}$) at 1 MGz to 10^{-4} at 1 GHz. In the short-wavelength region, $ql \gg 1$, $S_{\rm th}$ has a minimum ($\sim 10^{-5}$) at a phonon wave vector of the subharmonic equal to the electron Debye wave vector and then increases for wave vectors above q_d. At very high frequen-

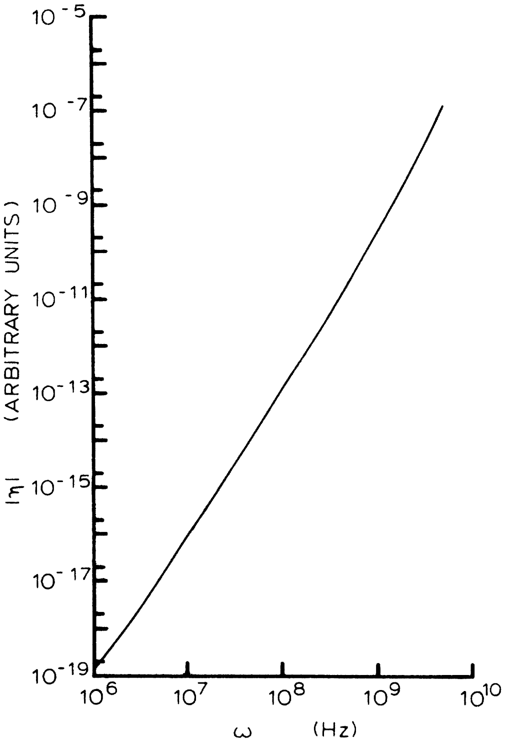

FIG. 8. $|\eta|$ as a function of frequency shown for n-InSb in the region $ql \ll 1$.

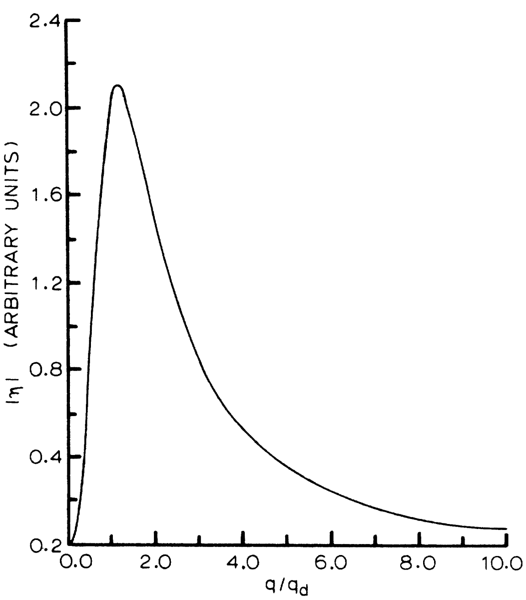

FIG. 9. $|\eta|$ shown as a function of wave vector in n-InSb when $ql \gg 1$.

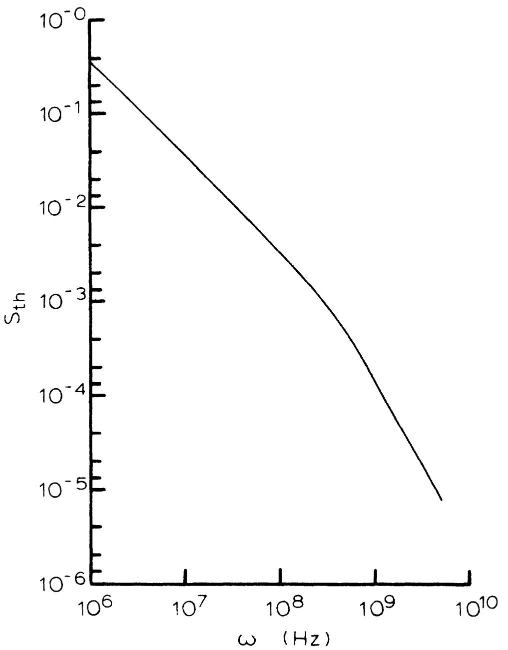

FIG. 10. Threshold pump strain shown as a function of frequency in n-InSb when $ql \ll 1$.

cies, $q \gg q_d$, even for piezoelectric semiconductors the dominant mode of interaction is via the deformation-potential coupling because of its stronger frequency dependence.

For second-harmonic generation our results reduce to those derived in an earlier paper[23] and of Spector and Wu.[11] The behavior of the second-harmonic flux is discussed in detail in Refs. 11 and 23 for both $ql \ll 1$ and $ql \gg 1$ regions. The second-harmonic flux also falls off rapidly for phonon wave vectors above the electron Debye wave vectors in semiconductors in which piezoelectric coupling dominates the interaction.

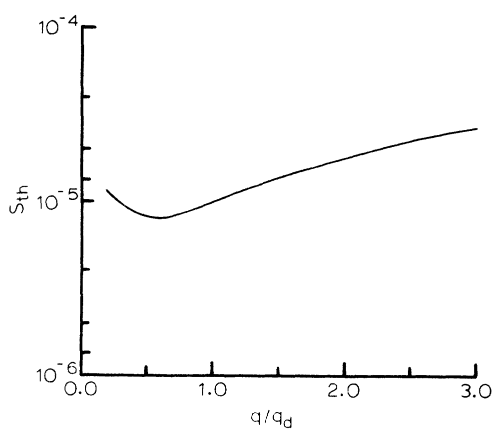

FIG. 11. Threshold pump strain as a function of wave-vector in n-InSb shown in the region $ql \gg 1$.

V. DISCUSSION

In this paper, we have studied the nonlinear parametric processes which can occur when ultrasound propagates in a semiconducting material with particular emphasis on subharmonic and second-harmonic generation. Although the calculations presented here were done in the absence of a dc electric field, the presence of such a field would not greatly modify our results for the nonlinear parameters η and A which determine subharmonic and second-harmonic generation as long as the drift velocity of the carriers is much smaller than their mean thermal velocity.[24] However, the presence of such an electric field would have a drastic effect on the linear absorption coefficient changing a linear loss to a linear gain when the drift velocity of the carriers in the electric field exceeds the sound velocity. Also, even in the absence of a dc electric field, both electronic and lattice losses in high mobility semiconductors such as InSb and GaAs are low enough so that second-harmonic generation should still be detectable in samples of reasonable length.[20]

Our main results have already been given in Sec. IV. These results indicate that deformation-potential coupling will become the dominant mechanism determining both second-harmonic and subharmonic generation at wave vectors above the carrier Debye wave vector even in piezoelectric semiconductors. For carrier concentrations in the range 10^{14}–10^{16} cm^{-3} this corresponds to frequencies between 1 and 100 GHz. On the other hand, for acoustic wave vectors much smaller than the carrier Debye wave vector, parametric processes via deformation-potential coupling should be completely negligible except in intrinsic semiconductors and semimetals. The reasons for this are twofold. First of all, the coupling coefficient for deformation-potential coupling has a much stronger frequency dependence than that for piezoelectric coupling. Therefore at high enough frequencies, deformation-potential coupling should dominate piezoelectric coupling just because of the frequency dependence of the coupling coefficients. In addition, the effects of screening seem to reduce harmonic generation due to deformation coupling to a greater extent than they do when piezoelectric coupling is important. However, in intrinsic semiconductors and semimetals, screening of the deformation potential will be absent due to the existence of space-charge neutrality which results from the equality of the electron and hole densities. Thus in these materials, because of the absence of screening, deformation-potential coupling can be an important mechanism for parametric amplification processes even in the megahertz frequency range.

In addition to taking the effects of deformation coupling into account in considering nonlinear frequency mixing effects and showing that it is the dominant interaction mechanism at frequencies in the high-gigahertz regime, our treatment using the Boltzmann transport equation has extended the previous treatments of ultrasonic parametric effects[9-10,13-15] into the frequency range where $ql \gg 1$. Our results agree with these previous treatments in the long-wavelength limit $ql \ll 1$ but do yield new results for both subharmonic generation and second-harmonic generation. For example, for piezoelectric coupling, the threshold strain for subharmonic generation reaches a smaller value when $ql \gg 1$ then it does when $ql \ll 1$ in n-InSb as shown in Figs. 10 and 11. Also in this same frequency regime, the threshold strain for deformation-potential coupling reaches a constant value for wave vectors $q > q_d$ that is much smaller than the minimum value of the threshold strain for piezoelectric coupling.

The theory developed using the Boltzmann transport equation is valid for ultrasonic frequencies such that the sound wavelength is much longer than the de Broglie wavelength of the carriers. This condition may be violated in the microwave frequency region at low temperatures. Mosekilde *et al.*[27] have extended the quantum treatment of Wu and Spector[11] to treat second-harmonic generation due to the interaction of ultrasound with electrons of arbitrary degeneracy. Their treatment agrees with that of Johri and Spector[23] for the case of nondegenerate electrons when the sound wavelength is smaller than the electron de Broglie wavelength but they find a cutoff when the wavelength of the fundamental is less than half a de Broglie wavelength. We expect that a quantum treatment of parametric processes will yield the same kind of high-frequency cutoff for other frequency-mixing effects.

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