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Theory of Rayleigh-wave amplification in piezoelectric semiconductor films—quantum theoretical approach

Shin-ichiro Tamura and Tetsuro Sakuma
Department of Engineering Science, Hokkaido University, Sapporo 060, Japan

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A quantum theory is presented for the interaction of the Rayleigh waves with electrons in piezoelectric semiconductor films. Our aim is to investigate the amplification of the Rayleigh waves with frequencies higher than 1 GHz. The effect of the finite relaxation time of electronic states is properly considered in the range of frequencies satisfying $\omega\tau < 1$. A numerical example is developed for an n-type GaAs epitaxial layer on a semi-insulating GaAs substrate taking the relaxation time into account phenomenologically.

I. INTRODUCTION

Recently, there has been an accumulation of theoretical topics on the dynamical behavior of elastic surface waves in an elastic medium: scattering by the surface irregularities[1] or mass defects,[2] anharmonic attenuation in solids,[3] interaction with conduction electrons,[4] interaction with surface modes of elementary excitations,[5] and the broadening of surface states of carriers due to the scattering with surface modes of thermal phonons.[6] Above all, the interaction of low-frequency surface waves with carriers in semiconductors has been extensively investigated both theoretically[4] and experimentally.[7] The amplification of surface waves in piezoelectric semiconductors with frequencies higher than 1 GHz is especially important from the viewpoint of applications to microwave electronic devices. However, for surface waves with frequencies above 1 GHz, it has not been fully analyzed, partly owing to the difficulty of generating such high-frequency surface waves.

In this paper we present a quantum theory of the Rayleigh-wave amplification with frequencies of GHz range in a piezoelectric semiconductor layer epitaxially grown on an insulating substrate with same elastic properties. An epitaxial film of n-type GaAs on a semiinsulating GaAs substrate may be a good example of such a structure. It should be noted that the Rayleigh wave cannot exist as an eigenmode in the structure with different elastic properties between the layer and the substrate. The thickness a of the layer is assumed to be 1 μm, which corresponds to the penetration depth from the surface of a Rayleigh wave of 1 GHz frequency.

Throughout this paper, the medium is assumed to be isotropic in its elastic properties, since we only wish to survey the general features of the amplification coefficient and to obtain some qualitative results which may be observed by the experimental investigations. We suppose that several complications due to the crystal anisotropy, possible existence of an oxidized thin layer on the surface, surface roughness and so on, are irrelevant insofar as the qualitative features of the results are concerned. We further assume that some of the physical parameters (elastic stiffness constants, piezoelectric constants, etc.) have the same values as those obtained in bulk measurements. These parameters could be different from those of the bulk region near the surface. All these complications, however, probably just modify by some modest amount the overall magnitude of the effect.

Now we should treat the whole problem quantum mechanically. This is a consequence of the discreteness of the electronic states in the amplifier, as will be shown in the following section. We calculate the loss and gain of phonons due to their interaction with the equilibrium electron distribution using a quasifree description of the conduction electrons and regarding the Rayleigh waves as highly excited phonon modes. Perhaps at this point one should note the value of the products ql and $\omega\tau$, where q and $\omega/2\pi$ $(=\nu)$ are the wave number and frequency of the Rayleigh waves, and l and τ are the mean free path and relaxation time of the carriers, respectively. The value of ql is estimated to be unity at 1.19- and 3.45-GHz frequency for appropriate carrier concentrations $N = 1.73\times10^{15}/\text{cm}^3$ (case I) and $N = 1.07\times10^{16}/\text{cm}^3$ (case II), respectively, at $T = 77$ K.[8] This is another reason why we should formulate the theory quantum mechanically. In the calculation of the amplification coefficient, the Born approximation is valid only in the region $\omega\tau > 1$. The typical values of τ at $T = 77$ K, obtained from mobility measurements for n-type GaAs, are

$$\tau = 2.0\times10^{-12}\ \text{sec} \quad \text{for case I}$$

and

$$\tau = 6.84\times10^{-13}\ \text{sec} \quad \text{for case II.}$$

Hence, for Rayleigh waves with frequencies of the order of 1 GHz, we have $\omega\tau < 1$. We must, therefore, go beyond the Born approximation,[9] that is, it is important to take into account the finite relaxation time of carriers in our calculation.

In Sec. II we describe the configuration of the layered system of a piezoelectric semiconductor and an insulator which we shall use for determining the amplification characteristics and specify the wave function of the electrons. In Sec. III, a brief report on a quantum version of the Rayleigh wave is presented. In Sec. IV we write down the electron–Rayleigh-wave interaction in a quantized form. Section V is devoted to the derivation of the amplification coefficients. In Sec. VI, the numerical analysis is developed for the epitaxial layer of n-type GaAs grown on a semiinsulating GaAs substrate. The amplification coefficients are calculated both in the Born approximation and in the phenomenological approximation where we take the width (or, equivalently, relaxation time) of the electronic states into account phenomenologically. A summary and a discussion will be given in Sec. VII.

II. ELECTRONIC STATES IN A SEMICONDUCTOR LAYER

The configuration of the amplifier we prepared for Rayleigh-wave amplification is depicted in Fig. 1. A thin layer (thickness a) of a piezoelectric semiconductor is grown epitaxially on an insulating substrate with the same elastic properties as the semiconductor layer. A Rayleigh wave with frequency of 1 GHz can penetrate about 1 μm from the surface of the medium, so we take as a typical value of the thickness a, 1 μm. We fix the Cartesian coordinates so that the material occupies the half-space $z \geq 0$ and has the stress-free surface parallel to the x-y plane (see Fig. 1).

In this configuration, the motions of electrons parallel to the surface may be described by plane waves and those perpendicular to the surface will be described by some kind of standing waves depending on the structure of the potential. In the first approximation, we might assume the potential along the z axis as a square well which has infinitely high potential barriers at $z = 0$ and $z = a$. Of course, there might be depletion layers where electrons are absent on both sides near $z = 0$ and $z = a$. The thickness of these depletion layers becomes narrower by increasing the concentration of donor impurities or by making an epitaxial film with layered structure in which impurities are more highly doped in the layers near $z = 0$ and $z = a$. As for the height of the potential barriers, one may doubt whether the height on the side adjacent to the insulating materials should be considered infinite. However, we shall see in the example of Sec. VI that this assumption is justified. Under these approximations, we can easily obtain the wave function $\Psi(\vec{r})$ of the conduction electrons as

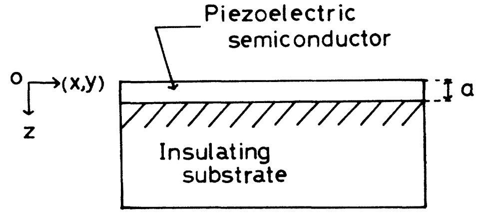

FIG. 1. Schematic drawing of the amplifier consisting of a layered structure of a piezoelectric semiconductor and an insulating material. Rayleigh waves propagate parallel to the surface of the film (x-y plane).

$$\Psi(\vec{r}) = \frac{1}{\sqrt{S}} \sum_{n=1}^{\infty} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \phi_n(z) \ , \qquad (1)$$

with

$$\phi_n(z) = (2/a)^{1/2} \sin(n\pi/a)z \quad (n = 1, 2, 3, \ldots) \ , \qquad (2)$$

where $\vec{r} = (\vec{x}, z) = (x, y, z)$, $\vec{k} = (k_x, k_y)$, and S is a surface area. The energies of the electronic states specified by $\vec{k}$ and n are given by

$$E_n(\vec{k}) = \hbar^2 \vec{k}^2/2m + \epsilon_n \ , \qquad (3)$$

with

$$\epsilon_n = (\pi^2\hbar^2/2ma^2)n^2 \ ,$$

where m is the effective mass of the conduction electrons (assuming a spherical constant-energy surface). We note that the energy difference of the two lowest quantized levels, $\epsilon_2 - \epsilon_1$, is 1.61 $\times 10^{-5}$ eV for $a = 1$ μm and $m = 0.07m_0$ (m_0 being the mass of the free electron), and the phonon energy at a frequency of 1 GHz is 4.14×10^{-6} eV. Thus the discreteness of the electronic energy levels due to the quantized motion along the z direction is important for such energetic phonons. This point will be discussed again in Sec. V. Now the field operator $\psi(\vec{r})$ of the electrons is written, in the second quantized form, as

$$\psi(\vec{r}) = \frac{1}{\sqrt{S}} \sum_{n=1}^{\infty} \sum_{\vec{k}} b_{\vec{k},n} e^{i\vec{k}\cdot\vec{x}} \phi_n(z) \ , \qquad (4)$$

where $b_{\vec{k},n}$ and its Hermitian conjugate $b^{\dagger}_{\vec{k},n}$ are annihilation and creation operators of the electron field which satisfy commutation relations of Fermi type.

III. SURFACE PHONON–A QUANTUM OF RAYLEIGH WAVE

It is well known that there exist five eigenmodes of elastic waves in an isotropic elastic medium occupying a half space with stress-free plane boundary. In this paper, we call the Rayleigh wave which is a member of the above-mentioned eigenmodes elastic surface wave or simply surface wave. Hereafter, we refer to a quantum of elastic surface wave as surface phonon.

In the configuration shown in Fig. 1 in which the isotropic elastic continuum occupies the half-space $z \geqslant 0$ and has a stress-free plane boundary at $z=0$, the surface-phonon field operator is written, using well-known eigenfunctions for the Rayleigh wave, as

$$\vec{u}(\vec{r})=\sum_{\vec{q}}\left(\frac{\hbar}{2\rho\omega S}\right)^{1/2} a_{\vec{q}}\vec{u}_{\vec{q}}(z)e^{i\vec{q}\cdot\vec{x}}+\text{H.c.} , \tag{5}$$

where ρ is the mass density of the medium, and $a_{\vec{q}}$ and its Hermitian conjugate $a_{\vec{q}}^{\dagger}$ are the annihilation and creation operators of the surface-phonon field satisfying commutation relations of Bose type. The explicit expressions for the wave functions $\vec{u}_{\vec{q}}(z)$ are

$$u_{\vec{q}}^{j}(z)=i(q_j/q)(q/J)^{1/2}\{e^{-\gamma qz}-[2\gamma\eta/(1+\eta^2)]e^{-\eta qz}\};$$
$$j=(x,y) , \tag{6}$$
$$u_{\vec{q}}^{z}(z)=-\gamma(q/J)^{1/2}\{e^{-\gamma qz}-[2/(1+\eta^2)]e^{-\eta qz}\} .$$

The wave vector $\vec{q}$ which is parallel to the surface is related to the angular frequency ω as $\omega=c_r|\vec{q}|=c_r q$ and c_r is the velocity of the Rayleigh wave. γ, η and J are constants defined by the velocities of the longitudinal sound wave (c_l) and the transverse sound wave (c_t) as $\gamma^2=1-(c_r/c_l)^2$, $\eta^2=1-(c_r/c_t)^2$, and $J=(\gamma-\eta)(\gamma-\eta+2\gamma\eta^2)/2\gamma\eta^2$.

In the presence of piezoelectric coupling, the sound velocity c is modified to c' as a result of "piezoelectric stiffening,"[10]

$$c'^2=c^2(1+K^2) , \tag{7}$$

if the conductivity of the medium is small. Here K^2 is the electromechanical coupling constant defined by

$$K^2=4\pi e_P^2/c^E\epsilon_0 . \tag{8}$$

e_P, c^E, and ϵ_0 are the material's piezoelectric, elastic and static dielectric constants along an appropriate crystal axis, respectively. We quote a typical value of K^2 for GaAs, $4\pi e_{14}^2/c_{44}^E\epsilon_0=3.6\times10^{-3}$. Therefore the correction for piezoelectric coupling to the sound velocity is small in this example.

IV. SURFACE-PHONON-ELECTRON INTERACTION

There are two mechanisms of phonon-electron interaction in piezoelectric semiconductors. The lattice distortion produces a change in the potential energy of a conduction electron which is proportional to the strain, i.e., deformation-potential coupling, and the electric polarization associated with the acoustic modes of vibration may lead to a local charge accumulation and a periodic electric potential, i.e., piezoelectric coupling. It is the induced electric field that is proportional to the strain in the latter case.

We assume, therefore, that the interaction Hamiltonian of the surface-phonon–electron system may be written as in the bulk case

$$H_I=H_D+H_P , \tag{9}$$

$$H_D=C\int\psi^{\dagger}(\vec{r})\vec{\nabla}\cdot\vec{u}(\vec{r})\psi(\vec{r})\,d\vec{r}$$
$$=\frac{C}{\sqrt{S}}\sum_{i,j}\sum_{\vec{k},\vec{q}}b_{\vec{k}+\vec{q},j}^{\dagger}b_{\vec{k},i}a_{\vec{q}}\Phi_{ij}^{D}(q)+\text{H.c.} , \tag{10}$$

$$H_P=\frac{4\pi e}{\epsilon_0}\int\psi^{\dagger}(\vec{r})\left[\frac{1}{2}\sum_{i,j,l=1}^{3}\int dr_i\,e_{ijl}\right.$$
$$\left.\times\left(\frac{\partial u_j}{\partial r_l}+\frac{\partial u_l}{\partial r_j}\right)\right]\psi(\vec{r})\,d\vec{r}$$
$$=\frac{4\pi e e_P}{\epsilon_0\sqrt{S}}\sum_{i,j}\sum_{\vec{k},\vec{q}}b_{\vec{k}+\vec{q},j}^{\dagger}b_{\vec{k},i}a_{\vec{q}}\Phi_{ij}^{P}(q)+\text{H.c.} , \tag{11}$$

where

$$\Phi_{ij}^{D}(q)=-\left(\frac{\hbar q^2}{2\rho c_r J}\right)^{1/2}\left(\frac{c_r}{c_l}\right)^2\int_0^a\phi_j^*(z)e^{-\gamma qz}\phi_i(z)\,dz \tag{12}$$

and

$$\Phi_{ij}^{P}(q)=-\left(\frac{\hbar}{2\rho c_r J}\right)^{1/2}\int_0^a\phi_j^*(z)\left(\frac{1-4\gamma^2}{\gamma}e^{-\gamma qz}+\frac{2\gamma(1+2\eta^2)}{1+\eta^2}e^{-\eta qz}\right)\phi_i(z)\,dz . \tag{13}$$

In the case of piezoelectric coupling (H_P), we have considered semiconductors with zincblende crystal structure which have only three nonvanishing components of the piezoelectric tensor e_{ijk}, i.e., $e_{14}=e_{25}=e_{36}=e_P$. We have also fixed the direction of the wave vector $\vec{q}$ along the [110] crystal axis (the easiest direction in which the Rayleigh wave is accompanied by piezoelectric effects). We see that electrons couple only with the longitudinal wave in deformation-potential coupling (H_D), but couple with both longitudinal and transverse waves in piezoelectric coupling. Unlike the case of bulk-phonon–electron interaction, the relative phase between the matrix elements for scattering by those two mechanisms is not fixed to $\frac{1}{2}\pi$ and depends on the crystal structure of the semiconductors in question.[11] Consequently, we cannot treat the two mechanisms independently. However, for a value of the deformation-potential coupling constant C of the order of 10 eV, the strength of the deformation-potential coupling is considerably smaller than that of the piezoelectric coupling for phonons of low frequency. A typical example is

$$qC/(4\pi e e_P/\epsilon_0) \simeq 1.8\times 10^{-12}\omega \ ,$$

where we have tentatively taken the following set of parameters: $C = 7$ eV, $e_P = 0.15$ C/m^2, $\epsilon_0 = 13$, and $c_r = 2.9\times 10^5$ cm/sec. Then, two effects become comparable at a surface-phonon frequency of about 100 GHz and hereafter we can safely neglect the contribution of H_D. In this paper we also neglect the effects of surface corrugation which causes some change in the electronic wave functions proportional to the magnitude of the surface displacement along the z direction.

V. CALCULATION OF THE AMPLIFICATION COEFFICIENT

To calculate the surface-phonon amplification coefficient, we conveniently employ the Green's-function method. The one-particle electron Green's function is written

$$G_l(\vec{p},\zeta_\nu) = \left(\zeta_\nu - E_l(\vec{p}) - \Sigma_l(\vec{p},\zeta_\nu)\right)^{-1} \ , \tag{14}$$

where l denotes the levels associated with the quantized motions in the z direction and $\zeta_\nu \equiv -(2\nu+1)\pi/i\beta + \mu$ ($\beta = 1/k_BT$ and μ is chemical potential). $\Sigma_l(\vec{p},\zeta_\nu)$ is the electron self-energy function. We further introduce the spectral representations for both the Green's function and the self-energy function,

$$G_l(\vec{p},\zeta_\nu) = \int \frac{d\omega'}{2\pi} \frac{A_l(\vec{p},\omega')}{\zeta_\nu - \omega'} \tag{15}$$

and

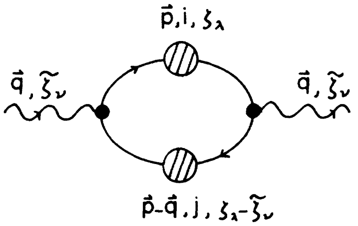

FIG. 2. Surface-phonon self-energy diagram. The internal lines represent the complete electron propagators (shaded circles denote the proper electron self-energy parts). Vertex corrections are neglected apart from electronic screening effects.

$$\Sigma_l(\vec{p},\zeta_\nu) = \int \frac{d\omega'}{2\pi} \frac{\Gamma_l^{(e)}(\vec{p},\omega')}{\zeta_\nu - \omega'} \ . \tag{16}$$

The spectral function $A_l(\vec{p},\omega)$ $[\Gamma_l^{(e)}(\vec{p},\omega)]$ is given by the discontinuity of $G_l(\vec{p},\zeta_\nu)$ $[\Sigma_l(\vec{p},\zeta_\nu)]$ across the real axis of the complex ζ_ν plane. In terms of $\Gamma_l^{(e)}$, A_l is explicitly written

$$A_l(\vec{p},\omega) = \frac{\Gamma_l^{(e)}(\vec{p},\omega)}{[\omega - E_l(\vec{p}) - \Delta_l(\vec{p},\omega)]^2 + [\frac{1}{2}\Gamma_l^{(e)}(\vec{p},\omega)]^2} \ , \tag{17}$$

with

$$\Delta_l(\vec{p},\omega) = \mathrm{Re}\,\Sigma_l(\vec{p},\omega)$$

$$= \mathrm{P}\int \frac{d\omega'}{2\pi} \frac{\Gamma_l^{(e)}(\vec{p},\omega')}{\omega - \omega'} \ . \tag{18}$$

Thus, we see that the spectral function $\Gamma^{(e)}$ of the self-energy gives us the width of the single-electron excited states.

To derive the surface-phonon amplification (or attenuation) coefficient α, we calculate the surface-phonon self-energy function Π (Fig. 2). Neglecting vertex corrections other than the screening effect of the electrons, we obtain

$$\Pi(\vec{q},\tilde{\zeta}_\nu) = \frac{2}{\beta} g(q)^2 \sum_{ij} |\Phi_{ij}(q)|^2 \sum_\lambda \int \frac{d\vec{p}}{(2\pi)^2} \int\int \frac{d\omega'\,d\omega''}{(2\pi)^2} \frac{A_i(\vec{p},\omega')A_j(\vec{p}-\vec{q},\omega'')}{(\zeta_\lambda - \omega')(\zeta_\lambda - \tilde{\zeta}_\nu - \omega'')}$$

$$= 2g(q)^2 \sum_{ij} |\Phi_{ij}(q)|^2 \int \frac{d\vec{p}}{(2\pi)^2} \int\int \frac{d\omega'd\omega''}{(2\pi)^2} \frac{f(\omega'') - f(\omega')}{\tilde{\zeta}_\nu + \omega'' - \omega'} A_i(\vec{p},\omega')A_j(\vec{p}-\vec{q},\omega'') \ , \tag{19}$$

with

$$\tilde{\zeta}_\nu = -2\nu\pi/i\beta + \mu \ ,$$

where $\Phi_{ij} \equiv \Phi_{ij}^{\ell}$, $f(\omega) = (e^{\beta(\omega-\mu)}+1)^{-1}$, and $g(q)$ is the coupling constant involving the correction due to the screening effect which is induced by the redistribution of electrons, i.e., $g(q) = 4\pi e e_P/\epsilon_0\epsilon(q)$ with electronic screening factor $\epsilon(q) = 1 + (4\pi Ne^2/q^2\epsilon_0 k_BT)$. N is the electron concentration. As in the preceding discussion, the width $\Gamma^{(s)}$ of the surface phonon is given by the discontinuity of Π,

$$\Gamma^{(s)}(\vec{q},\hbar\omega) = 2g(q)^2 \sum_{i,j} |\Phi_{ij}(q)|^2$$

$$\times \int \frac{d\vec{p}}{(2\pi)^2} \int \frac{d\omega'}{2\pi} [f(\omega' - \hbar\omega) - f(\omega')]$$

$$\times A_i(\vec{p},\omega')A_j(\vec{p}-\vec{q},\omega' - \hbar\omega) \ . \tag{20}$$

So far, no account has been taken of the fact that the electrons have a drifted distribution in the presence of the applied electric field. In a situation where the electrons have a drift velocity v in the direction of the surface-phonon wave vector $\vec{q}$, we must replace $\hbar\omega$ by $-\hbar\omega x$ with $x = v/c_r - 1$, the drift parameter. Noticing that $\beta\hbar\omega x \ll 1$ in the range of surface-phonon frequency and x in which we are interested and that α is related to $\Gamma^{(s)}$ by $\Gamma^{(s)} = -\alpha\hbar c_r$, we get the final expression for α as

$$\alpha = \frac{\beta\omega x}{2c_r} g(q)^2 \sum_{i,j} |\Phi_{ij}(q)|^2 \times \int \frac{d\vec{p}}{(2\pi)^2} \int \frac{d\omega'}{2\pi} \operatorname{sech}^2\left[\frac{\beta}{2}(\omega' - \mu)\right] \times A_i(\vec{p}, \omega') A_j(\vec{p} - \vec{q}, \omega' + \hbar\omega x) \; . \tag{21}$$

The amplification coefficient α in the Born approximation is obtained by taking the limit $\Gamma^{(e)} \to 0$ in the spectral function A, or simply replacing A by the δ function:

$$A_i(\vec{p}, \omega) = 2\pi\delta(\omega - \hbar^2\vec{p}^2/2m - \epsilon_i) \; . \tag{22}$$

Here, the educated reader will probably ask to what kind of physical quantity the real part of the phonon self-energy function corresponds. The real part of the surface-phonon self-energy function gives the frequency shift or the velocity shift of the surface phonon, i.e.,

$$\tilde{\omega} = \omega + \operatorname{Re}\Pi(\vec{q}, -\hbar\omega x)/\hbar = q c_r(1 + \sigma) = q\tilde{c}_r \; ,$$

where $\tilde{\omega}$ and $\tilde{c}_r$ are the renormalized angular frequency and velocity of the Rayleigh waves, respectively. In the numerical example of Sec. VI, we shall see that σ is negligibly small (of order 10^{-4}–10^{-5}). Finally, we write the explicit expression of $\Phi_{ij}(q)$,

$$\Phi_{ij}(q) = \left(\frac{\hbar}{2\rho c_r J}\right)^{1/2} 4\pi^2 q a i j \left(\frac{2\gamma(1 + 2\eta^2)}{1 + \eta^2} \frac{\eta[1 - (-1)^{i\;\;j} e^{-\eta q a}]}{[(\eta q a)^2 + \pi^2(i+j)^2][(\eta q a)^2 + \pi^2(i-j)^2]} + \frac{1 - 4\gamma^2}{\gamma} \frac{\gamma[1 - (-1)^{i+j} e^{-\gamma q a}]}{[(\gamma q a)^2 + \pi^2(i+j)^2][(\gamma q a)^2 + \pi^2(i-j)^2]}\right) \; . \tag{23}$$

For a Rayleigh wave with a frequency of 1 GHz $\gamma q a \simeq 2\eta q a \simeq 1$, so the number of electronic energy levels which contribute to α is 2 or 3 around a fixed level denoted by i.

VI. NUMERICAL EXAMPLE

In this section, a numerical example is developed for an n-type GaAs layer grown epitaxially on a semiinsulating GaAs substrate. We consider the Rayleigh wave propagating along the [110] crystal axis. The following set of parameters are taken: $T = 77$ K, $e_P = 4.7 \times 10^4$ esu/cm^2, $m = 0.07 m_0$ (m_0 being the mass of the free electron), $\rho = 5.32$ g/cm^3, $\epsilon_0 = 12.9$, and $c_r = 2.9 \times 10^5$ cm/sec. As for the electron concentration, we take two typical values, $N = 1.73 \times 10^{15}$/cm^3 (case I) and $N = 1.07 \times 10^{16}$/cm^3 (case II), as stated in Sec. I. The energy gap E_g of an intrinsic GaAs is 1.51 eV at 77 K. The number of energy levels which contribute to the summation in the expression for α [Eq. (21)] is about 50 and $\epsilon_{50} = 1.35 \times 10^{-2}$ eV $\simeq 0.018 \times \frac{1}{2} E_g$. Therefore, it is a good approximation to regard the height of the potential barrier as infinity.

The calculation of α in the Born approximation is straightforward with no reference to the width of electronic states or the relaxation time of carriers. On the other hand, the exact calculation of Eq. (21) requires the knowledge of the relaxation time. The finite relaxation time of conduction electrons is caused by several collision processes such as acoustic-phonon scattering, piezoelectric scattering, optical- and polar-optical-phonon scattering, intervalley-phonon scattering, impurity-atom (both ionized and neutral) scattering, and carrier-carrier scattering.[12] For electrons in bulk III-V compound semiconductors such as n-type InSb and n-type GaAs, the predominant scattering mechanism is thought to be scattering by the polar optical phonons within room-temperature range and down to about 30 K. A detailed fit of experimental results to theory cannot, however, be complete due to the lack of accurate information on the values of the coupling constants, particularly the values for the intervalley and the polar optical-phonon scattering. According to our scheme, we should evaluate the relaxation time of the electrons from the microscopic Hamiltonians which cause the several scattering mechanisms mentioned above. In this paper, however, we wish only to discuss the general features of the theory and the effect of the finite relaxation time qualitatively, and then we incorporate the width of the electronic states phenomenologically, that is we replace, irrespective of energy levels, the $\Gamma_i^{(e)}$ by the constant $\hbar/\tau$ where τ is the relaxation time obtained from Hall-mobility measurements. The values of $\hbar/\tau$ are 5.28×10^{-16} erg for case I and 1.54×10^{-15} erg for case II, and both are of order $\frac{1}{10} k_B T$. We also neglect the energy shifts

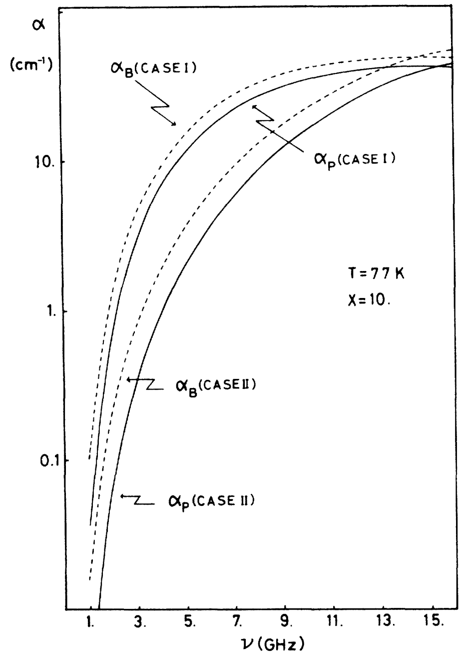

FIG. 3. Amplification coefficients of Rayleigh waves versus frequency at $T=77$ K and $x=v/c_r-1=10$. For case I ($N=1.73\times10^{15}$ cm^{-3}), both α_B and α_P reach maximum values at a frequency of about 15 GHz, $\alpha_B^{\max}=49$ (cm^{-1}) and $\alpha_P^{\max}=43$ (cm^{-1}). For case II ($N=1.07\times 10^{16}$ cm^{-3}), α continues to increase in this frequency range.

Δ_i, because they are of the same order of magnitude as the broadening of the electronic states and are incorporated into the shift of the chemical potential which is of order k_BT.

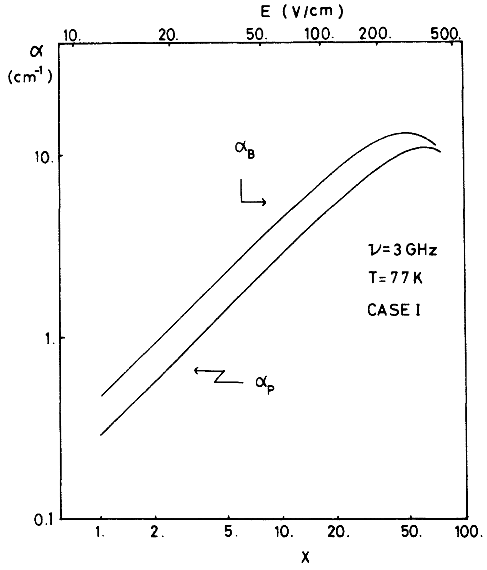

FIG. 4. Amplification coefficients vs drift parameter $x=v/c_r-1$ or applied electric field E for case I at $\nu=3$ GHz and $T=77$ K.

Under these approximations, the results are shown in Figs. 3 and 4. Figure 3 shows the amplification coefficient α versus frequency ν as given in Eq. (21) for two typical values of electron concentration. The discrepancy between the amplification coefficient obtained in the Born approximation (α_B) and that obtained in the approximation including the effect of finite relaxation time phenomenologically (α_P) varies with frequency as follows: the ratio α_B/α_P decreases from 2.78 to 1.20 for case I and from 5.94 to 1.37 for case II as the frequency varies from 1 to 10 GHz. Therefore, one finds that the effect of the finite relaxation time of the electronic state reduces the coefficient α considerably at relatively low frequency. This may be qualitatively understood by the fact that a freely propagating electron ($\tau=\infty$) can be accelerated more easily than an interacting one (τ finite). The amplification coefficient α_P for case I has a maximum value of 43 cm at $\nu\simeq 15$ GHz and for case II reaches its maximum at the frequency beyond 20 GHz. Near a surface-phonon frequency of 1 GHz, α_P is approximately expressed by the power formulas $\alpha_P\sim\omega^{4.3}$ for case I and $\alpha_P\sim\omega^{4.7}$ for case II. These frequency dependences seem to be considerably different from that of bulk phonons ($\alpha_{\text{BULK}}\sim\omega^2$).[13] Figure 4 shows the amplification coefficient α versus the drift parameter x or applied electric field E for case I and at $\nu=3$ GHz. The amplification coefficients α have linear dependence on x up to $x=10$ (corresponding to $E=64$ V/cm for case I) and then begin to saturate.

Finally we estimate the magnitude of the ratio σ of the velocity shift to the unperturbed velocity of Rayleigh waves. We may take ReΠ as approximately of the same order of magnitude as ImΠ $=\Gamma^{(s)}$. Then $\sigma=\text{Re}\Pi(\vec{q}, -\hbar\omega x)/\hbar c_r q$ is of order 10^{-5} at $\nu=1$ GHz and 10^{-4} at $\nu=10$ GHz for case I with $x=10$. Therefore the velocity shift is extremely small in this example.

VII. SUMMARY AND DISCUSSIONS

We have developed a quantum-theoretical approach to the amplification of surface phonons in piezoelectric semiconductor films. As a numerical example, the amplification coefficient α has been calculated for an epitaxial layer of n-type GaAs on a semi-insulating GaAs substrate. This analysis has revealed that the frequency dependence of α_P near $\nu=1$ GHz is well described by the power formulas $\alpha_P\sim\omega^{4.3}$ for case I and $\alpha_P\sim\omega^{4.7}$ for case II, which are considerably different from that of bulk phonons ($\alpha_{\text{BULK}}\sim\omega^2$). The effect of the finite relaxation time of the electronic states is very important in the frequency region around 1 GHz and it reduces the coefficient about 64% at $\nu = 1$ GHz and about 17% at $\nu = 10$ GHz for case I. This can be qualitatively understood by the fact that freely propagating noninteracting electrons can be accelerated more easily than interacting ones, and that the effect of a proper phonon field surrounding the drifted electrons becomes smaller as the surface-phonon frequency becomes higher. Consequently, the amplification coefficient α_B approaches α_P from upside as the surface-phonon frequency ν becomes higher.

Strictly speaking, the width and the energy shift of the electronic state should be evaluated from the basic Hamiltonian which causes several scattering mechanisms. As we have taken the finite width into account only phenomenologically, our numerical estimate is not complete in a quantitative sense. In addition, the electronic states we have used hitherto are also valid approximately due to the possible existence of depletion layers at the film boundaries. However, we believe that our results reflect the qualitatively correct features of surface-phonon amplification.

Another interesting subject relevant to the amplification of surface phonons is the reflection of very-high-frequency acoustic plane waves with finite incident angle from the stress-free plane boundary of a piezoelectric semiconductor layer. If we consider the sufficiently high-frequency phonons for which the relation $\omega\tau > 1$ holds, we may safely neglect the effect of the relaxation time of the electronic states. We can, therefore, calculate the amplification coefficient for various surface modes of phonons with no reference to the relaxation time, and consequently obtain quantitatively reliable results using the Born approximation.

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