PHYSICAL REVIEW B VOLUME 15, NUMBER 1 1 JANUARY 1977

Failure of the Rayleigh hysteresis law in low magnetic fields

John A. Baldwin, Jr. and Jacques Revol
College of Engineering and Quantum Institute, University of California, Santa Barbara, California 93106

Frederick Milstein*
Weizmann Institute of Science, Rehovot, Israel
(Received 6 November 1975)

The Rayleigh expression for minor hysteresis loops gives the remanent flux density B_r to be proportional to the square of the amplitude of the applied field H_1, i.e., $B_r \propto H_1^2$. By use of a phase-sensitive detector, it has been found that this is valid only over a limited range of H_1 and that, in particular, $B_r \propto H_1$ for smaller values of H_1. All measurements were made at a frequency of 10 Hz.

INTRODUCTION

The purpose of this paper is to present experimental results which show that the "well-known" Rayleigh law,[1] which is generally thought to describe with reasonable accuracy ferromagnetic hysteresis in weak oscillating fields, is apparently approximately valid only over a limited range of field strengths, and fails, qualitatively as well as quantitatively, to describe adequately magnetic hysteresis at very low fields. These results are unexpected and surprising because the condition for Rayleigh behavior to exist is that the applied field should be small compared with the coercive force, and apparently there has been no prior indication that the Rayleigh law does not hold for arbitrarily small fields. We initially found qualitative departures from Rayleigh behavior in a study[2] of ferromagnetic hysteresis in gadolinium metal at 283 °K (which is not far from the Curie temperature of about 290 °K); we initially assumed that these departures were anomalous and were peculiar to Gd near its Curie temperature, or perhaps peculiar to hysteresis, generally, near a Curie temperature. However, we have found similar departures from Rayleigh behavior in the present series of experiments which were carried out on plastically deformed and fully annealed (or recrystallized) Gd at 196, 260, and 283 °K, and 50-50 NiFe at 295 °K. The present results suggest that such "anomalous" behavior is the rule rather than the exception.

EXPERIMENTS AND RESULTS

The Rayleigh law states that the magnetic induction B is given by

$$B = \mu_0[(\mu + aH_1)H \pm \tfrac{1}{2}a(H_1^2 - H^2)] , \quad (1)$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m, H_1 is the amplitude of the applied field H, μ and a are presumed to be properties of the ferromagnetic substance and hence independent of the field amplitude H_1, and the "±" refers to the two branches of the hysteresis loop. For purposes of analysis, it is convenient to consider a more general formula for hysteresis which does not contain any material parameters, viz.,

$$B = (B_1/H_1)H \pm B_r[1 - (H/H_1)^2] , \quad (2)$$

where the remanent induction $B_r = B(H=0)$; $B_1 = B(H=H_1)$; and the energy loss per cycle resulting from hysteresis is proportional to H_1B_r. Thus, the Rayleigh relation predicts that B_r is proportional to H_1^2.

If H is varied sinusoidally as $H(t) = H_1\cos(\omega t)$, Fourier analysis of Eq. (2) yields

$$B(t) = B_1\cos(\omega t) + B_r[(8/3\pi)\sin(\omega t) - (8/15\pi)\sin(3\omega t) \cdots] . \quad (3)$$

Thus, the component of dB/dt which is in phase with H and the third-harmonic component are both proportional to B_r; these components are, respectively, proportional to the in-phase voltage V_i and the third-harmonic voltage V_3 which appear across secondary windings which are wound about a ferromagnetic specimen of toroidal configuration. In the present series of experiments, the voltage components V_i and V_3 were measured with the aid of a phase sensitive detector. The experimental details were discussed in Ref. 2, and will be presented more fully in a complete journal article which is currently being prepared. Measurements of the quadrature voltage V_q (which is proportional to B_1) were also made; however, in the interest of brevity, the V_q data will be presented and discussed in the complete article, rather than herein. The sample preparation and characterization of the Gd specimens is discussed in Ref. 3; the NiFe is characterized in Ref. 4. All measurements were made at a frequency of 10 Hz.

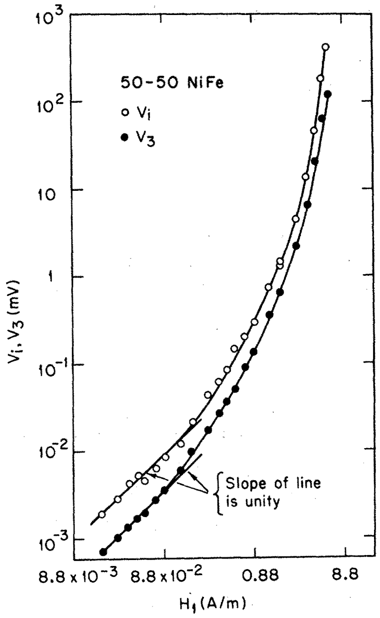

FIG. 1. In-phase and third-harmonic voltages, V_1 and V_3, vs field amplitude H_1 for grain oriented 50-50 NiFe at 295 °K.

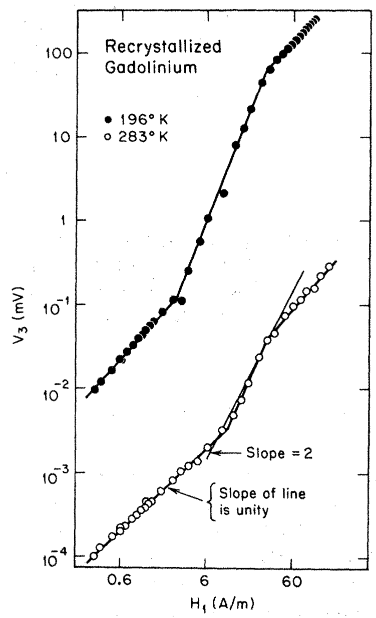

FIG. 3. Third harmonic voltage V_3 vs field amplitude H_1 for recrystallized Gd at 196 and 283 °K.

The measured values of V_i and V_3 were plotted versus H_1 on a log-log scale. According to the Rayleigh law, such plots should be linear and their slopes should equal two, identically (since both V_i and V_3 should be proportional to H_1^2). Figure 1 shows V_i and V_3 for 50-50 NiFe, and Figs. 2 and 3 show V_3 vs H_1 for Gd in the plastically deformed

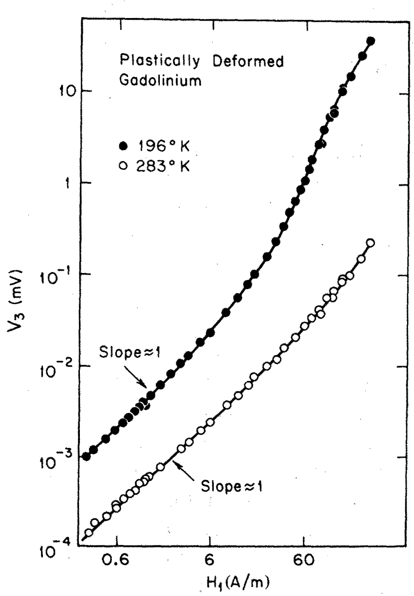

FIG. 2. Third harmonic voltage V_3 vs field amplitude H_1 for plastically deformed Gd at 196 and 283 °K.

and recrystallized states, respectively, at 196 and 283 °K; the V_i data and the data taken at 260 °K for Gd were similar to that shown in Figs. 2 and 3 and hence is not included in these figures.

Before discussing the figures, it is worthwhile to note the variety of different systems which are represented. The NiFe is a tape-wound grain-oriented specimen with a coercive force of about 8 A/m; the Gd cores are solid, with coercive forces of about 450 and 900 A/m, respectively, for the recrystallized and plastically deformed specimens at 196 °K; the crystal structure of NiFe is cubic, while Gd is hcp; at 283 °K, Gd is magnetically uniaxial, with c axis as the easy axis of magnetization,[5-8] while at 196 °K the easy direction of magnetization can be considered as the generator of a cone of easy magnetization making an angle ≈75 ° with the c axis[5-9] (hence the weak field permeability of Gd at 196 °K is quite large compared with that at 283 °K.[9,10]) Nevertheless, there are certain interesting similarities among the different curves.

First, it is noted that when the Rayleigh relation (i.e., slope = 2) is approximately valid, it is valid only over a rather limited range of fields. For the higher field values, i.e., when H_1 approaches the coercive force and the hysteresis loop begins to take on the shape of a major loop, the observed departure from Rayleigh behavior is quite understandable. However, the interesting and surprising feature of each curve is that there exists a lower range of field values for which the slope of each curve is very nearly unity. In this range of fields, B_r is proportional to H_1, the energy loss per cycle is thus proportional to H_1^2 (rather than H_1^3), and a new hysteresis law can be written

$$B=\mu_0[(\mu+aH_1)H \pm c(H_1^2-H^2)/H_1] . \quad (4)$$

Just as we are unaware of a physical basis for the original Rayleigh relation, Eq. (1), we are unaware of a physical basis for the modified relation, Eq. (4). More detailed quantitative discussion of the data as well as the determination of the material parameters μ, a, and c will be included in a full article.

Note added in proof. The authors have recently found reference[11] to some early (1948) experimental data which are consistent with the beginning of a first-power dependence of B_r on H_1 in fields down to about 0.2 A/m.

ACKNOWLEDGMENTS

The authors are grateful to the University of California at Santa Barbara Committee on Research for financial support for this work.

---

*Supported in part by a John Simon Guggenheim Memorial Fellowship, on leave from the University of California, Santa Barbara.

[1]Lord Rayleigh, Philos. Mag. 23, 225 (1887).

[2]F. Milstein, J. A. Baldwin, Jr., and M. Rizzuto, J. Appl. Phys. 46, 4002 (1975).

[3]F. Milstein, J. A. Baldwin, Jr., and T. James, J. Appl. Phys. 44, 4824 (1973).

[4]J. A. Baldwin, Jr., J. S. Bayne, and T. R. Calkins, J. Appl. Phys. 42, 1069 (1971).

[5]W. D. Corner, W. C. Roe, and K. N. R. Taylor, Proc. Phys. Soc. Lond. 80, 927 (1962).

[6]C. D. Graham, Jr., J. Phys. Soc. Jpn. 17, 1310 (1962); J. Appl. Phys. 38, 1375 (1967).

[7]G. Will, R. Nathans, and H. A. Alperin, J. Appl. Phys. 35, 1045 (1964).

[8]J. W. Cable and E. O. Wollan, Phys. Rev. 165, 733 (1968).

[9]F. Milstein and L. B. Robinson, Phys. Rev. 177, 904 (1969).

[10]L. B. Robinson, F. Milstein, and A. Jayaraman, Phys. Rev. 134, A187 (1964).

[11]E. Kneller, *Ferromagnetismus* (Springer-Verlag, Berlin, 1962), p. 606, Fig. 40.3.