Probing the phase-dependent conductance and nonequilibrium properties of Josephson junctions by means of flux entry into weakly closed loops*

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Numerical simulations have shown that the relative number of flux quanta which penetrate a weakly closed superconducting loop depends upon the magnitude and sign of the phase-dependent conductance. We examine the possible use of this dependence as an experimental probe of the $\cos\varphi$ term and nonequilibrium properties of Josephson junctions.

INTRODUCTION

When an external magnetic field is applied to a multiply connected superconductor, screening currents flow so as to maintain an integral number of flux quanta within the "holes." Fluxoid quantization operates differently in *weakly closed* loops as was discussed by Silver and Zimmerman.[1] When a superconducting ring is interrupted by a Josephson device (point contact, microbridge, or thin-film junction), the magnetic flux inside the ring can be either a continuous or a discontinuous function of the applied field. The exact behavior depends on Li_c/ϕ_0, where L is the loop inductance, i_c is the weak-link critical current, and ϕ_0 is the flux quantum (2.07×10^{-15} Wb).

Previously[2] we studied the dynamics of flux entry into loops which satisfied $Li_c \gg \phi_0$ and for which the junction capacitance C was large. Under this latter condition the damping is light and flux enters the ring in bundles.

The total current in a Josephson junction is given by[3]

$$i = i_c \sin\varphi + \sigma_1 V \cos\varphi + \sigma_0 V, \tag{1}$$

where φ is the order-parameter phase difference, V is the instantaneous voltage appearing across the junction, σ_0 is the normal-state junction conductance, and $\sigma_1 \cos\varphi$ is the phase-dependent conductance. In our previous calculation the $\cos\varphi$ term was not included. However that calculation showed that the oscillating voltage could be relatively large and it is under just this condition that the $\cos\varphi$ term is expected to play an important role. Inclusion of this factor modifies the differential equation which governs the flux dynamics as follows:

$$\frac{d^2\Phi}{dt^2} + \beta\left[1 + \left(\frac{\sigma_1}{\sigma_0}\right)\cos(2\pi\Phi)\right]\frac{d\Phi}{dt} + \Phi + \gamma\sin(2\pi\Phi) = \Phi_x, \tag{2}$$

where Φ is the flux in the ring measured in units of ϕ_0, $\gamma \equiv Li_c/\phi_0$, $\beta \equiv R^{-1}(L/C)^{1/2}$ is the damping coefficient, Φ_x is the normalized applied flux, and time is measured in units of $(LC)^{1/2}$.

Gayley and Wang[4] have numerically solved this equation and showed that at fixed values of L, R, C, and i_c the number of flux quanta which enter the loop Φ_{enter} is significantly different for the sample choices $\sigma_1/\sigma_0 = -1, 0, +1$. They also pointed out that the magnitude and sign of this ratio is still in some dispute and proposed this type of experiment as a probe of the $\cos\varphi$ amplitude. We have considered this suggestion and report here a comparison of the theory with experimental data.

DISCUSSION

From computer solutions of Eq. (2) we have generated the universal curves of Fig. 1 in which Φ_{enter}/γ is plotted vs β/β_c. The critical damping parameter β_c is defined as the maximum value of β for which *multiple* quantum transitions will occur. From numerical results obtained in the range $10^2 < \gamma < 10^5$ we have deduced empirically that all Φ_{enter}/γ-vs-β curves scale as $(\gamma)^{1/2}$. In Fig. 2 we have plotted the dependence of $(\gamma)^{1/2}/\beta_c$ on σ_1/σ_0. The five points derived from simulation studies at $\gamma = 1000$ are well represented by the empirical linear relationship $(\gamma)^{1/2}/\beta_c = 0.20\,(\sigma_1/\sigma_0) + 0.33$.

Measurements of Φ_{enter}/γ were made for both tin and lead thin-film Josephson tunnel junctions using previously reported experimental techniques.[2] For these experimental data the normalized damping factor β/β_c is determined from the expression $\beta/\beta_c = [(\gamma)^{1/2}/\beta_c]R^{-1}(\phi_0/Ci_c)^{1/2}$, where $(\gamma)^{1/2}/\beta_c$ depends on the choice for σ_1/σ_0 as indicated in Fig. 2. The normalized experimental values of β/β_c are plotted in Fig. 1. The choice $\sigma_1/\sigma_0 = +1$ seems to yield closest agreement between experiment and
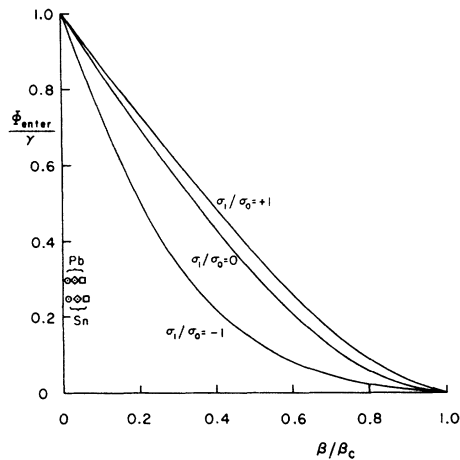
FIG. 1. Plot of the normalized number of flux quanta
Φ_{enter}/γ, which enter a superconducting loop broken by a
weak link as a function of the relative damping β/β_c for
three values of the conductance ratio σ_1/σ_0. These curves
were derived from numerical solutions of Eq. (2) in the
text. Experimental points obtained from a superconduct-
ing loop interrupted by a tin or lead Josephson junction
are also plotted. For either case three points are shown,
corresponding to the normalizing appropriate to the
possible choices $\sigma_1/\sigma_0 = -1$ (⊙), 0 (◇), +1 (▣).

FIG. 2. Plot of $(\gamma)^{1/2}/\beta_c$ vs the conductance ratio σ_1/σ_0.
The points are obtained from computer simulations; the
straight line is a least-squares fit.

theory but even this extreme value of σ_1/σ_0 does
not produce a really satisfactory fit. The "ob-
served" value of β/β_c is consistently much smaller
than the theoretically expected value appropriate
to the measured Φ_{enter}/γ.

The question then arises as to the origin of such
quantitative discrepancies. Let us now consider
the various factors appearing in the expression
for β/β_c.

There is no problem with i_c because it is mea-
sured directly. On the other hand, C is *calculated*
using the parallel plate formula with dielectric
constant $\epsilon = 4$, area $A = 1.7 \times 10^{-8}$ m^2, and spacing
$d = 25 \times 10^{-10}$ m. This yields $C \cong 2.4 \times 10^{-10}$ F. It
is unlikely that the uncertainty exceeds $\pm 100\%$.
Whether this parallel plate formula is applicable
at the high frequencies ($\sim 10^{10}$ Hz) associated with
flux entry is a problem that has been analyzed by
Bassaviah and Greiner[5] who examined the self-re-
sonant peaks in a Josephson junction. They con-
cluded that the formula was indeed correct in this
type of situation. Finally the resistance R must be
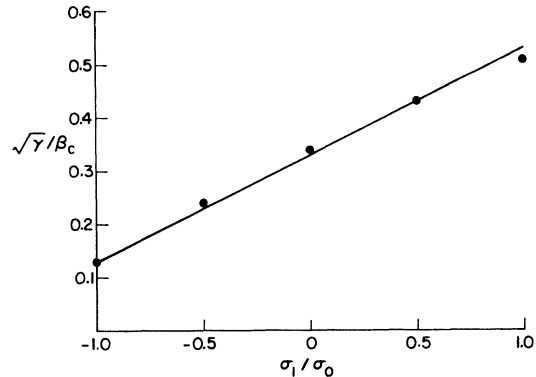considered. Here there is little justification for

using the dc junction resistance since the flux flow
mechanism leads to very high-frequency normal
currents. To achieve agreement between our ex-
periment and theory we would need to *decrease R*
by a factor of approximately 10.

We note in this connection the recent papers by
Højgard Jensen and Lindelof[6] and by Clark and
Lindelof[7] which treat the effect of the finite relaxa-
tion time for normal electron excitations on the
current voltage characteristic of resistively shunt-
ed Josephson junctions. It was concluded that the
effective shunt resistance in the equivalent circuit
for a microbridge could be treated as

$$1/R = 1/R_N(1 + \tfrac{1}{15}\Gamma),$$

where R_N is the normal-state resistance. The di-
mensionless parameter Γ was estimated for their
samples in the range 50–100.

For our thin-film junctions agreement between
the experimental points and the $\sigma_1 = 0$ curve in Fig.
1 can be achieved with such an equivalent resist-
ance and $\Gamma = 115$.

These experiments are more sensitive to the
effects of finite lifetime than to the magnitude of
the phase-dependent conductance. If σ_1/σ_0 can be
independently determined then it will be possible
to experimentally probe the nonequilibrium proper-
ties by means of the flux-entry process.

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