Interaction of a magnetic monopole with a ferromagnetic domain*

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We calculate the interaction energy of a magnetic monopole with a single ferromagnetic domain, taking into account the ferromagnetic exchange interaction in a linear approximation. In vacuum at 400 Å from the surface a monopole of strength $137e/2$ is bound by 35 eV in magnetite, and for iron at 300 Å from the surface the binding energy is 50 eV. We expect the binding energy to increase at smaller distances. The attractive force on a slow monopole approaching the surface of a ferromagnet from the vacuum differs, at these large distances, only slightly from that computed by simple classical image methods treating the magnet as a medium with an isotropic and wavelength-independent permeability equal to the long-wavelength transverse permeability of the ferromagnetic material. We consider the apparent contradictions with energy and momentum conservation in the problem of a monopole in the field of an electron. The exclusion of s-wave scattering largely resolves the contradictions. The effective field on a monopole in a ferromagnet is $\vec{H}$ and not $\vec{B}$.

Goto[1] pointed out the existence of an attractive interaction between ferromagnetic substances and magnetic monopoles, and he made an estimate of the trapping energy that is a consequence of the interaction. He also estimated the value of the external magnetic field intensity that would be necessary in order to extract monopoles that have been trapped by magnetic materials. Eberhard and Ross[2] showed that ferromagnetic materials will trap monopoles regardless of the details of the interactions within the material. Our object is to calculate the interaction energy of a monopole with a single ferromagnetic domain, as in iron or in magnetite. The interest of the calculation lies in its bearing on the numerous experimental monopole searches[3] that depend on both trapping and extraction of monopoles from magnetic materials.

ANISOTROPIC IMAGE PROBLEM WITH EXCHANGE

We solve for the interaction of a magnetic monopole in vacuum at $z<0$ with a magnetic domain that fills the half-space $z<0$. We suppose that in the absence of the pole the domain magnetization is directed along the $+x$ axis, an easy axis of the magnetocrystalline anisotropy energy. The single-domain diagonal permeability tensor for long-wavelength perturbations is $\mu=(1,\mu_0,\mu_0)$, where at room temperature $\mu_0=46$ for iron and 16.1 for magnetite.

We write the energy density in the ferromagnet as

$$W_{\text{exch}}+W_{\text{anis}}+W_{\text{demag}}+W_{pm}\,, \tag{1}$$

where the exchange energy density is

$$W_{\text{exch}}=\tfrac{1}{2}C\left[(\nabla\alpha)^2+(\nabla\beta)^2+(\nabla\gamma)^2\right], \tag{2}$$

with α,β,γ as the direction cosines of the magnetization. The anisotropy energy density is

$$W_{\text{anis}}=K_1(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)\,. \tag{3}$$

The demagnetization energy density is

$$W_{\text{demag}}=-\tfrac{1}{2}\vec{M}\cdot\vec{H}_1\,, \tag{4}$$

where $\vec{H}_1$ is the field caused by the magnetization itself; and the interaction energy density of the pole with the magnetization is

$$W_{pm}=-\vec{M}\cdot\vec{H}_0\,, \tag{5}$$

that is, the interaction energy of the magnetization with the bare monopole field

$$\vec{H}_0=g\,\frac{\vec{r}-z_0\hat{z}}{|\vec{r}-z_0\hat{z}|^3}\,. \tag{6}$$

Let $\vec{m}$ be a unit vector in the direction of the local magnetization; then the following equation expresses the condition that the total energy be an extremum:

$$\vec{m}\times\left(C\nabla^2\vec{m}-\frac{\partial}{\partial\vec{m}}W_{\text{anis}}+M_s(\vec{H}_0+\vec{H}_1)\right)=0\,. \tag{7}$$

This is the condition that the torque be zero. We linearize the equation by neglecting β^2 and γ^2. This approximation limits the region of validity of the results to distances of the pole from the surface that are greater than the Bloch-wall thickness parameter Λ, as discussed below. The linearized form of (7) is

$$\begin{aligned} &C\nabla^2 m_y-2K_1m_y+M_sH_y=0\,,\\ &C\nabla^2 m_z-2K_1m_z+M_sH_z=0\,, \end{aligned} \tag{8}$$

where $\vec{H}=\vec{H}_0+\vec{H}_1$. Here C is the Landau-Lifshitz exchange constant, K_1 is the first anisotropy constant, and M_s is the saturation magnetization.

We want to find a diagonal-permeability tensor

$$\tilde{\mu}(k_x,k_y)=(1,\mu(k_x,k_y),\ \mu(k_x,k_y)) \tag{9}$$

consistent with (8) and having the property that in the medium $\text{div}\vec{B}=0$, or, in a mixed representation,

$$\text{div}\left(\hat{x}\ \frac{\partial\phi_i}{\partial x}+\hat{y}\mu(k_x,k_y)\ \frac{\partial\phi_i}{\partial y}+\hat{z}\mu(k_x,k_y)\ \frac{\partial\phi_i}{\partial z}\right)=0 \tag{10}$$

for $z<0$. This will be satisfied if ϕ_i has the spatial dependence

$$\exp[(\mu^{-1}k_x^2+k_y^2)^{1/2}z]\exp[i(k_x x+k_y y)], \tag{11}$$

with μ being $\mu(k_x,k_y)$ here and hereafter. Then (8) and (11) give

$$\mu(k_x,k_y)=\frac{\mu_0 k_s^2+2k_x^2+[\mu_0^2 k_s^4+4(\mu_0-1)k_x^2k_s^2]^{1/2}}{2(k_x^2+k_s^2)}, \tag{12}$$

where $k_s^2\equiv 2K_1/C$ and $\mu_0\equiv 1+4\pi M_s^2/2K_1$, the single-domain permeability. This definition of μ_0 follows from (8), written for a uniaxial crystal or for a cubic crystal, both with K_1 positive. For a cubic crystal with K_1 negative, one should replace K_1 by $\frac{8}{9}|K_1|$. The length Λ is associated with the thickness of a Bloch wall and is defined by $\Lambda=2/k_s$.

In the medium the potential is

$$\phi_i(\vec{r})=\iint dk_x\,dk_y\,\phi_i(k_x,k_y)\times\exp[(\mu^{-1}k_x^2+k_y^2)^{1/2}z]\times\exp[i(k_x x+k_y y)], \tag{13}$$

and outside the medium the potential is

$$\phi(r)=\iint dk_x\,dk_y\left[\phi(k_x,k_y)\exp[-(k_x^2+k_y^2)^{1/2}z]+\left(\frac{g}{2\pi(k_x^2+k_y^2)^{1/2}}\right)\exp[-(k_x^2+k_y^2)^{1/2}|z-z_0|]\right]\exp[i(k_x x+k_y y)]. \tag{14}$$

The usual boundary conditions on $\vec{B}$ and $\vec{H}$ at the interface $z=0$ determine $\phi_i(k_x,k_y)$ and $\phi(k_x,k_y)$ in terms of z_0 and g.

The magnetic field at the pole due to the magnetization of the domain is

$$H_z=-\left(\frac{g}{2\pi}\right)A\int_0^\infty Ke^{-2Kz_0}dK,$$

where

$$A\equiv\int_0^{2\pi}\frac{(\mu\cos^2\phi+\mu^2\sin^2\phi)^{1/2}-1}{(\mu\cos^2\phi+\mu^2\sin^2\phi)^{1/2}+1}d\phi, \tag{15}$$

and the potential energy of the pole, referred to zero at infinity, is

$$W(z_0)=-\left(\frac{g^2}{4\pi}\right)A\int_0^\infty e^{-2Kz_0}dK. \tag{16}$$

We have carried out numerical integration of this integral as a function of z_0, with results given in Table I for iron and in Table II for magnetite. We tabulate also values of the direction cosine γ (or m_z) at the point $z=0$. These values are useful as a measure of the distortion of the magnetization within a domain. The larger is γ, the greater the distortion from the initial parallel configuration of the domain. We cannot expect the linearized equations (8) to be very good for $\gamma>0.3$, say.

The calculations were carried out with $g=\frac{137}{2}e$ using the following values of the physical constants: for iron, $M_s=1714$ G, $K_1=4.1\times10^5$ erg cm^{-3}, $a=2.87$ Å; and, for magnetite, $M_s=485$ G, $K_1=-1.1\times10^5$ erg cm^{-3}, $a=8.39$ Å. These values are at room temperature. The exchange parameter C is obtained from the experimental constant D in the magnon dispersion relation $\hbar\omega=Dq^2$ in the quadratic region: $D=281$ meV Å^2 for iron and 615 meV Å^2 for magnetite, both being room-temperature values.[4] Derived from these data are the values $\mu_0=46$ and 16.1; $k_s=3.29\times10^5$ cm^{-1} and 3.83×10^5 cm^{-1}; $\Lambda=610$ Å and 520 Å; $C=47.5$ meV Å^{-1} and 8.33 meV Å^{-1}, for iron and magnetite, respectively.

The successive columns of the tables give, reading from the left-hand side, the distance z_0 of the pole from the surface in dimensionless units $k_s z_0$ and in Å; the component $\gamma=M_z/M_s$ of the induced magnetization normal to the surface, as evaluated at the origin; the induced magnetic field H_1 at the pole; the binding energy of the pole at rest, as calculated from (16); and the binding energies calculated by the method of images with neglect of exchange for an isotropic permeability μ_0 and for an anisotropic permeability $(1,\mu_0,\mu_0)$.

We see from the tables that the monopole is strongly bound to a ferromagnetic domain, at least at ranges of the order of 300 Å from the surface; here the binding energy is of the order of 50 eV. The calculation becomes highly nonlinear at smaller distances, and we may expect the response of the domain to saturate, giving smaller values of the interaction field H_1, but always of the same sign. We see no reason to expect the binding en-

TABLE I. For iron: values of the γ component of the magnetization at the origin; magnetic field that acts on the pole; and the energy of the pole. For reference the image energies without exchange for the isotropic and purely anisotropic permeabilities are given.

$k_s z_0$	z_0 (Å)	γ	H_1 (G)	Calculated $\lvert W\rvert$ (eV)	Isotropic model $\lvert W\rvert$ (eV)	Anisotropic model $\lvert W\rvert$ (eV)
0.5	152	1.12[a]	3105	97.1	106.3	102.4
1.0	304	0.298	804	50.3	53.2	51.2
1.5	456	0.135	360	33.8	35.4	34.1
2.0	609	0.076	204	25.5	26.6	25.6
2.5	761	0.049	130	20.4	21.3	20.5
3.0	913	0.034	91	17.0	17.7	17.1
3.5	1065	0.025	67	14.6	15.2	14.6
4.0	1217	0.019	51	12.8	13.3	12.8
4.5	1369	0.015	40	11.4	11.8	11.4
5.0	1521	0.012	33	10.2	10.6	10.2
6.0	1826	0.008	23	8.5	8.9	8.5
7.0	2130	0.006	17	7.3	7.7	7.3
8.0	2434	0.005	13	6.4	6.6	6.4
9.0	2739	0.004	10	5.7	5.9	5.7
10.0	3043	0.003	8	5.1	5.3	5.1

[a] For $k_s z_0 = 0.5$, we see that $\gamma = 1.12$, which is not a physical solution. It is listed here as reference for the cutoff of the validity of the linearized equations.

ergy to become smaller than its maximum value in the linear region. At atomic distances the magnetic pole is liable to capture by nuclear magnetic moments[5] and, if the pole bears an electric dipole moment, it may be captured by the inhomogeneous electric field of a nucleus. It would appear conservative to set 50 eV as a lower limit on the binding energy of a Dirac monopole to a ferromagnetic domain.

At the distances that we have treated it would have been an adequate approximation to apply the classical method of images, with the neglect of exchange interactions. The anisotropic image method that we also tested is not a part of the standard literature as far as we know, but it follows from our method on setting $k_s = \infty$ in (12).

EFFECTIVE MAGNETIC FIELD ON A MONOPOLE

The effective magnetic field on an electron in a ferromagnet is $\vec{B} = \vec{H} + 4\pi\vec{M}$ and not $\vec{H}$. This is established by experiments[6] on the de Haas–van Alphen effect, and the theoretical limits have

TABLE II. For magnetite: values of the γ component of the magnetization at the origin; magnetic field that acts on the pole; and the energy of the pole. For reference the image energies without exchange for the isotropic and purely anisotropic permeabilities are given.

$k_s z_0$	z_0 (Å)	γ	H_1 (G)	Calculated $\lvert W\rvert$ (eV)	Isotropic model $\lvert W\rvert$ (eV)	Anisotropic model $\lvert W\rvert$ (eV)
1.0	261	1.254[a]	947	50.8	57.2	52.4
1.5	391	0.574	428	34.4	38.1	35.0
2.0	522	0.328	242	26.0	28.6	26.2
2.5	652	0.212	155	20.8	22.9	21.0
3.0	782	0.148	108	17.4	19.1	17.5
3.5	913	0.109	80	14.9	16.3	15.0
4.0	1043	0.083	61	13.1	14.3	13.1
4.5	1174	0.066	48	11.6	12.7	11.7
5.0	1304	0.054	39	10.5	11.4	10.5
6.0	1565	0.037	27	8.7	9.5	8.7
7.0	1826	0.027	20	7.5	8.2	7.5
8.0	2087	0.021	15	6.6	7.1	6.6
9.0	2347	0.017	12	5.8	6.4	5.8
10.0	2608	0.013	10	5.2	5.7	5.2

[a] Not physical.

been discussed.[7] Earlier it was shown by Wannier[8] that the field on a charged cosmic ray particle in a ferromagnet is close to $\vec{B}$, but may depart slightly from $\vec{B}$ by virtue of the recoil of the ferromagnetic atoms. The situation of a magnetic pole in a ferromagnet is drastically different, for the work done on a pole on carrying it around a closed path that passes in part through a ferromagnet must vanish if energy conservation is maintained. By a Maxwell equation the line integral of $\vec{H}$ around a closed path in a static problem is zero; the line integral of $\vec{B}$ or of $\vec{H}+\alpha\vec{M}$, where α is nonzero, does not in general vanish. Energy can be conserved[9] only if $\alpha \equiv 0$. If follows that the effective field on a pole must be exactly $\vec{H}$.

How can this happen for a magnetic pole, if it is known not to happen for an electron? The field that acts on an electron is $\vec{B}$ and not $\vec{H}$ because of the Fermi contact or s-wave part of the electron-electron interaction, the part that comes about when a charged particle passes through the "Zitterbewegung" portion of the electron orbit. What is so different about the motion of a pole in the field of an electron is that precisely the s-wave part of the relative motion is forbidden by quantum mechanics. Lipkin, Weisberger, and Peshkin[10] have, in fact, shown explicitly that for finite energy all radial wave functions vanish at the origin at least as fast as r^L, where $L > 1$ for nonvanishing allowed values of the pole strength. This result is a consequence of a $1/r^2$ term in the effective potential. With the s wave rigorously forbidden, the effective magnetic field can only be $\vec{H}$, and energy is conserved. With energy conserved, there is no cause to believe that a monopole will not be stopped by a ferromagnet. The theoretical arguments suggest that a monopole can be trapped within a ferromagnet. A parallel argument can be constructed to show that the angular momentum is conserved if there are no s-wave collisions, thereby avoiding the grave difficulty with classical orbits having zero impact parameter. The s-wave exclusion argument we have given does not resolve the question of the effective field from orbital magnetization.

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