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Theory of a Fermi glass*

Robert Freedman and J. A. Hertz†
The James Franck Institute and The Department of Physics, The University of Chicago, Chicago, Illinois 60637
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We construct a theory for localized electrons in disordered solids similar in spirit and intent to the Fermi-liquid theory of Landau. That is, for low temperatures and excitation energies small compared to the Fermi energy, the electrons may be viewed as forming a gas of localized quasiparticles. The phenomenological theory is presented and applications are made to both equilibrium and time-dependent properties of the glass. A microscopic justification of the quasiparticle picture is made by an examination of the single-particle Green's function of the interacting system. The collective properties of the glass are examined by studing the Bethe-Salpeter equation satisfied by the four-point vertex for particle-hole scattering. This leads to the identification of the phenomenological effective interaction between quasiparticles with a certain limit of the four-point vertex and to a microscopic justification of the phenomenological transport equation used to study time-dependent phenomena.

I. INTRODUCTION

A great deal of theoretical effort has been directed toward elucidating the features of the electronic wave functions and energy bands in the presence of a random potential, leading to a qualitative understanding of many of the electronic properties of noncrystalline solids.[1] However, the effects of electron-electron interactions have received little attention. In this paper we try to fill part of this gap by formulating a Fermi-liquid theory for electrons in disordered solids. It is a generalization of the Landau theory of translationally invariant Fermi liquids.[2]

The existence of such a theory was first conjectured by Anderson.[3] He suggested that phase-space restrictions on scattering processes near the Fermi level are relevant in the disordered system as they are in the uniform one, with the consequence that a quasiparticle description of low-lying excitations is still valid. The difference is that the noninteracting system to which one makes reference in defining quasiparticle excitations is a disordered one, so the quasiparticle wavefunctions have no long-range phase coherence and may even be localized. He called this description a Fermi-glass theory. We construct such a theory here, with special emphasis on the case where the quasiparticles near the Fermi level are localized.

Anderson's quasiparticle picture depends on a one-to-one correspondence, close enough to the Fermi level, between the eigenstates of the interacting system and those of a noninteracting one. We therefore require knowledge of the single-particle states of this disordered system. Although we do not know the explicit form of these states the way we do in the crystalline case (where they are Bloch states), we do have a reasonable knowledge of their general properties.[1] For example, we know that they may be extended or localized, in the sense that the correlation function $\langle |\Psi_E(x)|^2 |\Psi_E(0)|^2 \rangle$ has infinite or finite range, and that localized and extended states cannot coexist at a given energy E. Critical energies E_c separate a band into localized and extended parts; their positions are functions of the degree of disorder in the potential. For sufficient disorder, all states will be localized.[4] On the basis of these established features, we shall assume here that we know adequately any statistical properties of the states near the Fermi energy which are necessary to evaluate physically measurable quantities.

Despite our ignorance of the detailed features of the one-electron eigenstates of a particular member of the statistical ensemble of systems, we will sometimes find it useful to have a labeling scheme for localized states. Although this scheme is incomplete and has no rigorous basis, it is consistent with what we know about these states from previous work and is useful for understanding physically the expressions we write later for various quantities. Consider a particular energy E; eigenfunctions of this energy have a characteristic extent in space, the localization length $l(E)$, if they are localized. Part of specifying a state, then, is saying where it is localized, which we do by specifying a position $\vec{R}$. No $\vec{R}$, of course, is defined much more precisely than within a localization length. There are many states localized near a given $\vec{R}$, and since they are orthogonal to each other, they must all have different numbers of oscillations in amplitude within the localization region. We express these oscillatory features (admittedly imprecisely) by a generalized local

wave vector $\vec{K}$. We therefore write the general eigenstate $|n\rangle$ as $|\vec{R},\vec{K}\rangle$. The number of values of $\vec{K}$ at a given $\vec{R}$ depends on the localization length and thus on E; the limit on the precision of definition of $\vec{K}$ can be obtained through the uncertainty principle: $\Delta K \approx 1/l(E)$. Hence there are $\nu(E)[l(E)/L]^d$ values of $\vec{K}$ per unit energy characterizing states with energy E localized near a given point $\vec{R}$ where $\nu(E)$ is the density of states, L the size of the system and d the dimensionality. Note that $\vec{K}$ is not really a wave vector, and thinking of it as one too literally can lead to nonsensical results. It is just a way of specifying the typcial number of wiggles in the wave function.

It is appropriate here to review the basic features of Landau's Fermi-liquid theory for the uniform system. Its aim is to account for the effects of interparticle interactions on the properties of a system of fermions at low temperatures. Landau assumed that the excitations of the Fermi liquid, like those of the Fermi gas, could be labeled by their momenta or wave vector k. That is, in the adiabatic generation of the Fermi liquid from the Fermi gas, no phase transition which breaks translational invariance occurs. Landau argued further that because of Pauli-principle restrictions on the scattering in and out of these eigenstates near the Fermi surface, their collisional lifetimes should vary like $(E-E_F)^{-2}$, thus justifying the quasiparticle picture for low-lying excitations. This argument assumes that no broken symmetry which alters or destroys the Fermi surface, such as magnetic order, Mott transition, or superconductivity occurs. The condition that no such catastrophe occur serves to define a *normal* Fermi system to which Landau's theory applies. In this paper we shall also restrict our attention to systems which are normal in an analogous sense.

A second consequence of the Fermi-surface restrictions on scattering processes is the fact that quasiparticle-quasiparticle interactions can be described by a self-consistent field picture. The interaction is characterized by the phenomenological scattering function $f^{\alpha\beta\gamma\delta}(\hat{k}\cdot\hat{k}')$ in terms of which the energy $\tilde{\epsilon}_k^{\alpha\beta}$ of a quasiparticle in a state $|k\rangle$ is given by

$$\tilde{\epsilon}_k^{\alpha\beta}=\epsilon_k^{\alpha\beta}+\sum_{\gamma\delta}\sum_{k'} f^{\alpha\beta\gamma\delta}(\hat{k}\cdot\hat{k}')\delta n^{\delta\gamma}(k')\,. \tag{1.1}$$

The Greek indices are spin indices which assume the values $\alpha=1,2$. In the absence of an external magnetic field $\epsilon_k^{\alpha\beta}=(k^2/2m^*)\delta_{\alpha\beta}$ (m^* is the quasiparticle mass) is the energy of a *single* excited quasiparticle. The second term in (1.1) describes the modifications of the quasiparticle energy resulting from interactions of a quasiparticle in a state $|k\rangle$ with the other excited quasiparticles in states $|k'\rangle$. The degree of excitation of the system is described by $\delta n^{\alpha\beta}(k)=n^{\alpha\beta}(k)-n^0(\epsilon_k)\delta_{\alpha\beta}$, the deviation of the quasiparticle distribution functions $n^{\alpha\beta}(k)$ from its equilibrium value $n^0(\epsilon_k)$ (a Fermi function). A knowledge of $f^{\alpha\beta\gamma\delta}(\hat{k}\cdot\hat{k}')$ together with m^* [or equivalently the quasiparticle density of states at the Fermi energy $\nu(\epsilon_F)=m^*k_F/\pi^2$] provides a complete description of all of the low-temperature properties of an interacting Fermi system.

The Landau scattering function may be written

$$f^{\alpha\beta\gamma\delta}(\hat{k}\cdot\hat{k}')=A(\hat{k}\cdot\hat{k}')\delta_{\alpha\beta}\delta_{\gamma\delta}+B(\hat{k}\cdot\hat{k}')\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\gamma\delta}\,, \tag{1.2}$$

where $A(\hat{k}\cdot\hat{k}')$ describes the spin-independent part of the quasiparticle interaction while $B(\hat{k}\cdot\hat{k}')$ describes the exchange interaction between quasiparticles. The functions $A(\hat{k}\cdot\hat{k}')$ and $B(\hat{k}\cdot\hat{k}')$ depend only on the angle between $\vec{k}$ and $\vec{k}'$ since for quasiparticles on the Fermi surface $|\vec{k}|=|\vec{k}'|=k_F$. The expansion of these functions in a series of Legendre polynomials defines a set of dimensionless parameters A_l and B_l. The effects of interactions on the low-temperature properties of the system are then expressed in terms of these parameters.

The Landau theory is applicable to both equilibrium and nonequilibrium phenomena provided that $(q,\omega)\ll(k_F,\epsilon_F)$, where q and ω are the wave number and frequency of an external disturbance. To study nonequilibrium phenomena Landau derived a Boltzmann-like transport equation for the semiclassical phase-space distribution function $n^{\alpha\beta}(k,r,t)$ by using the quasiparticle energy in (1.1) as a classical Hamiltonian in the classical Liouville equation.

In the phenomenological Fermi-glass theory we describe below, we shall proceed in as close analogy as possible with Landau's theory. The quasiparticle state label $\vec{k}$ will have to be replaced by the formal label n of the exact eigenstates [or sometimes for illustrative purposes, by our informal labels $(\vec{R},\vec{K})$], and we shall have to use a full density matrix in place of the semiclassical distribution function $n^{\alpha\beta}(k,r,t)$, but the basic structure will be the same.

Our paper is organized in the following manner. In Sec. II we introduce the basic assumptions underlying the theory and proceed to develop the phenomenological theory. We obtain a phenomenological Hamiltonian for quasiparticles in terms of an effective interaction (analogous to the Landau scattering function). This effective Hamiltonian is used to obtain a quantum-mechanical transport equation (analogous to the Landau-Boltzmann

equation) which may be used to study the effects of interactions on nonequilibrium properties of the Fermi glass. Section III is devoted to some applications of the phenomenological theory. In the first part of this section we consider some static properties of the glass (i.e., specific heat and magnetic susceptibility). The second part of this section considers time-dependent properties of the glass. In particular we calculate the transverse dynamical susceptibility $\chi_+(\omega)$ (which is measured in an electron-spin resonance experiment) in the presence of a static uniform magnetic field. We also calculate the ac conductivity $\sigma(\omega)$ of the glass. Section IV considers the microscopic description of the Fermi glass at $T=0$ and is intended in part to justify the phenomenological theory. The first part of this section is devoted to a study of the single-particle properties of the Fermi glass. We study the single-particle Green's function and show that for energies near the Fermi energy the notion of well-defined quasiparticle excitations is valid in the glass. In the second part of this section we study the collective properties of the Fermi glass. We study the Bethe-Salpeter equation satisfied by the four-point vertex for particle-hole scattering. From a consideration of the poles in this function for small energy transfer ω we establish the validity of our phenomenological transport equation and identify our effective interaction with a certain limit of the four-point vertex.

II. THEORY: BASIC ASSUMPTIONS AND PHENOMENOLOGY

The basic assumptions of our theory are similar to those of the Landau theory of Fermi liquids and we briefly state them here:

(i) In the adiabatic transition from the noninteracting disordered system to the Fermi glass there remains a one-to-one correspondence between the low-lying excitations of the noninteracting system and those of the interacting system. An excited state of the Fermi glass corresponds to the occupation of quasiparticle states $|n\rangle$ with energies $\epsilon_n > \epsilon_F$ and the occupation of an equal number of quasihole states $|m\rangle$ with energies $\epsilon_m < \epsilon_F$. These particle-hole excitations differ from those in the noninteracting system because of their interactions. The interactions lead to a damping of these excitations and, as we shall see, the notion of a quasiparticle is only valid for energies ϵ_n near the Fermi energy ϵ_F.

(ii) Our second assumption is that the state of the Fermi glass is completely determined by a knowledge of the quasiparticle density matrix n_{nm}. It is a 2×2 matrix in spin space (in the following analysis we shall not write the spin indices explicitly as we did in Sec. I).

It follows from the above assumptions that the total energy of the glass is some functional of the quasiparticle density matrix which for small departures δn_{nm} from equilibrium may be expanded in a Taylor series

$$E[n_{nm}] = E_0 + \sum_{mn} \epsilon_{mn}\,\delta n_{nm} + \tfrac{1}{2}\sum_{mnrs} \Phi_{mnrs}\delta n_{nm}\delta n_{sr}\,, \tag{2.1}$$

where in both of the above summations a trace over all spin indices must be understood. We have defined the matrices

$$\epsilon_{mn} = (\delta E/\delta n_{mn})_{n=n^0} \tag{2.2}$$

and

$$\Phi_{mnrs} = (\delta^2 E/\delta n_{mn}\delta n_{rs})_{n=n^0} \tag{2.3}$$

(here n^0 is the equilibrium density matrix, a Fermi function). We note from (2.3) (by interchanging the order of differentiation) that

$$\Phi_{mnrs} = \Phi_{rsmn}\,. \tag{2.3′}$$

The first term in (2.1) is the ground-state energy of the glass. If we define $\delta E = E - E_0$ then we can write

$$\delta E = \mathrm{tr}(h + \tfrac{1}{2}\delta h)\delta n\,, \tag{2.4}$$

where we have used (2.1)–(2.4) to define a single-particle Hamiltonian for quasiparticles $\tilde{h} = h + \delta h$ which has matrix elements in the quasiparticle state basis $|m\rangle$ given by

$$\tilde{h}_{mn} = \epsilon_{mn} + \sum_{rs} \Phi_{mnrs}\delta n_{sr}\,. \tag{2.5}$$

In the quasiparticle Hamiltonian $\tilde{h}$, the first term h is the Hamiltonian for a *single* quasiparticle while the second term δh describes the interactions between a quasiparticle and the other excited quasiparticles of the glass. If we consider a single quasiparticle, the quasiparticle Hamiltonian h will be diagonal in the quasiparticle state basis and we have $h_{mn} = \epsilon_{mn} = \epsilon_m \delta_{mn}$, where ϵ_n is the energy of a single quasiparticle in the state $|n\rangle$. In the absence of an external magnetic field or of magnetic order there is not preferred directions of spin and therefore ϵ_n will be independent of spin (i.e., proportional to the unit matrix in spin space). The effects of interactions are described by the matrix Φ_{mnrs} which is analogous to $f(\hat{k}\cdot\hat{k}')$ in Landau's theory of Fermi liquids.

The quasiparticle density of states $\nu(\epsilon)$ (for both directions of spin) is defined at finite temperatures by

$$\nu(\epsilon) = \mathrm{tr}\left(-\frac{\partial n^0}{\partial h}\right), \tag{2.6}$$

where

$$n^0(h) = (e^{\beta(h-\mu)}+1)^{-1}. \tag{2.6'}$$

At low temperatures $T \ll T_F$ the quasiparticle density of states at the Fermi energy may be written

$$\nu(\epsilon_F) = 2\sum_n \delta(\epsilon_n - \epsilon_F). \tag{2.7}$$

If we neglect interactions (i.e., $\Phi_{mnrs} = 0$) our theory describes a system of independent quasiparticles and the only modifications of the low-temperature properties of the glass is the replacement of the free-electron density of states by the quasiparticle density of states. The interesting modifications of the noninteracting system arise from the interaction part δh of the quasiparticle Hamiltonian. As we shall see, the quasiparticle density of states together with certain averages of the matrix Φ_{mnrs} (these averages define dimensionless parameters similar to the Landau parameters) over states at the Fermi energy give a complete description of the low-temperature properties of the Fermi glass.

In the absence of spin-orbit coupling the most general rotationally invariant form for Φ_{mnrs} is given by

$$\Phi_{mnrs} = A_{mnrs} + B_{mnrs}\vec{\sigma}\cdot\vec{\sigma}'. \tag{2.8}$$

A_{mnrs} describes the spin-independent part of the quasiparticle interaction and B_{mnrs} describes the exchange interaction between quasiparticles. It is convenient to write

$$\delta n_{nm} = \delta f_{nm} + \delta\vec{\Psi}_{nm}\cdot\vec{\sigma}, \tag{2.9}$$

where we have defined the matrices

$$\delta f_{nm} = \tfrac{1}{2}\,\mathrm{tr}_\sigma\,\delta n_{nm} \tag{2.10}$$

and

$$\delta\vec{\Psi}_{nm} = \tfrac{1}{2}\,\mathrm{tr}_\sigma\,\vec{\sigma}\,\delta_{nm} \tag{2.11}$$

(here tr_σ denotes a trace over spins). δf_{nm} and $\delta\vec{\Psi}_{nm}$ are the number density and spin-density matrices, respectively. One finds that the quasiparticle Hamiltonian may be written in terms of δf and $\delta\vec{\Psi}$ as

$$\tilde{h}_{mn} = \epsilon_n\delta_{nm} + 2\sum_{rs} A_{mnrs}\delta f_{sr} + 2\vec{\sigma}\cdot\sum_{rs} B_{mnrs}\delta\vec{\Psi}_{sr}. \tag{2.12}$$

In Sec. III we shall use the self-consistent single-particle Hamiltonian (2.12) to calculate various static and dynamic properties of the Fermi glass.

In general one is interested in calculating the linear response of the Fermi glass to a weak time-dependent, nonuniform external field of frequency $\omega \ll \epsilon_F$ ($\hbar = 1$ in our units) and wave vector q. The Hamiltonian for a quasiparticle in the presence of the external field is obtained from (2.12) by making the replacement $\tilde{h} \to \tilde{h} + V(r,t)$, where the interaction energy of a quasiparticle with the field may be written

$$V(r,t) = V_1(r,t) + \vec{V}_2(r,t)\cdot\vec{\sigma}. \tag{2.13}$$

The average value of a dynamical variable A (i.e., current, magnetization, etc.) is given by

$$\langle A(q',q,\omega)\rangle = \mathrm{tr}\,A^{\mathrm{op}}_{q'}\,\delta n_q(\omega), \tag{2.14}$$

where $A^{\mathrm{op}}_{q'}$ is the Fourier transform at wave vector q' of the single-particle operator $A^{\mathrm{op}}(r)$ corresponding to A, and the trace in (2.14) denotes both a trace over spins and quasiparticle states $|n\rangle$. While for a crystal $q = q'$ modulo a reciprocal-lattice vector, q and q' are in general independent in a random system. In order to calculate the statistical average in (2.14) we need to determine the deviation δn of the quasiparticle density matrix from its equilibrium value. That is, for time-dependent problems, we must solve the equation of motion of the quasiparticle density matrix:

$$i\frac{\partial n}{\partial t} = [\tilde{h}, n] \tag{2.15}$$

obeyed by the quasiparticle density matrix. Equation (2.15) in our theory is the analog of the Landau-Boltzmann equation in the Landau theory of Fermi liquids.

We observe that after linearizing (2.15) we will have an integral equation to solve for $\delta n(\omega)$ as can be seen by recalling (2.5).

It should be clear from (2.14) and (2.15) that $\langle A(q',q,\omega)\rangle$ is a functional of the parameters of our theory. That is, the statistical average in (2.14) will in general depend functionally on the quasiparticle energies ϵ_n and the interaction matrix Φ_{mnrs}. These parameters are random variables describable by a probability distribution function. In order to compare calculated quantities such as $\langle A(q',q,\omega)\rangle$ with the results of measurements we must perform a further configurational average over an ensemble of random systems. We denote such a configurational average by a subscript c. If the configurational statistics are homogeneous, off-diagonal ($q \neq q'$) elements of $\langle\langle A(q',q,\omega)\rangle\rangle_c$ will vanish.

III. APPLICATIONS OF THE PHENOMENOLOGICAL THEORY

A. Static properties

1. Specific heat

The specific heat at constant volume C_v is given by

$$C_v = \left(\frac{\partial F}{\partial T}\right)_\mu , \tag{3.1}$$

where $F = E - \mu \langle N \rangle$ is the free energy of the glass. If we recall (2.4) the deviation of the free energy from equilibrium is given by

$$\delta F = F - F_0 = \mathrm{tr}(h - \mu)\delta n + \tfrac{1}{2}\,\mathrm{tr}\,\delta h\,\delta n . \tag{3.2}$$

The change δn in the quasiparticle density matrix arises from the thermal excitation of quasiparticles and is given by

$$\delta n = n^0(T + \delta T) - n^0(T) = \frac{\partial n^0}{\partial T}\delta T \tag{3.3}$$

for an infinitesimal change δT in the temperature. Since

$$n^0(T) = (e^{\beta(h-\mu)} + 1)^{-1} , \tag{3.4}$$

we can write (3.3) as

$$\delta n = -\frac{\partial n^0}{\partial h}\frac{(h - \mu)}{T}\delta T . \tag{3.5}$$

If we substitute (3.5) into (3.2) we have

$$\delta F = 2\sum_n (\epsilon_n - \mu)^2 \left(-\frac{\partial n^0}{\partial \epsilon_n}\right)\frac{\delta T}{T} + O((\delta T)^2) , \tag{3.6}$$

where we have observed that the second term in (3.2) is of $O((\delta T)^2)$ since $\delta h \sim O(\delta n)$. If we recall (3.1), then we can read off from the above equation that

$$C_v = \frac{2}{T}\sum_n (\epsilon_n - \mu)^2 \left(-\frac{\partial n^0}{\partial \epsilon_n}\right), \tag{3.7}$$

which may be written

$$C_v = \frac{1}{T}\int_0^\infty d\epsilon\, \nu(\epsilon)\,(\epsilon - \mu)^2 \left(-\frac{\partial n^0}{\partial \epsilon}\right). \tag{3.8}$$

At low temperatures we may use the Sommerfeld expansion

$$-\frac{\partial n^0}{\partial \epsilon} = \delta(\epsilon - \mu) + \frac{\pi^2}{6\beta^2}\delta''(\epsilon - \mu) + O\left(\frac{T}{\mu}\right)^4 \tag{3.9}$$

in (3.8) and find

$$C_v = \frac{\pi^2}{6\beta^2}\frac{1}{T}\int_0^\infty d\epsilon \frac{d^2}{d\epsilon^2}[\nu(\epsilon)\,(\epsilon - \mu)^2]\,\delta(\epsilon - \mu) , \tag{3.10}$$

so that

$$C_v = \tfrac{1}{3}\pi^2 k_B \nu(\mu)\, T . \tag{3.11}$$

As in the uniform Fermi liquid, the low-temperature specific heat is not affected by the interactions between quasiparticles. The only modification of the noninteracting theory is the replacement of the free-electron density of states by the quasiparticle density of states.

2. Static magnetic susceptibility

If we apply a static magnetic field $\vec{H}_0 = H_0\hat{z}$ to the glass then the quasiparticle Hamiltonian may be written

$$\tilde{h} = h - \tfrac{1}{2}g\mu_0 H_0 \sigma_z + \delta h . \tag{3.12}$$

In the presence of $\vec{H}_0$ the glass will be in a state of constrained equilibrium and we may write the quasiparticle density matrix as

$$n^0(\tilde{h}) = n^0(h) + \delta n , \tag{3.13}$$

where n^0 is a Fermi function. If we combine (3.12) and (3.13), we find that the deviation δn of the quasiparticle density matrix from equilibrium may be written

$$\delta n = \frac{\partial n^0}{\partial h}(\delta h - \tfrac{1}{2}g\mu_B H_0 \sigma_z) . \tag{3.14}$$

We want to calculate the average magnetization $\langle \vec{M} \rangle = \chi \vec{H}_0$ produced by the external field. This is given by

$$\langle \vec{M} \rangle = \tfrac{1}{2}g\mu_B\, \mathrm{tr}\,\vec{\sigma}\,\delta n . \tag{3.15}$$

We note from (3.14) and (3.15) that only the spin-dependent part of δh will contribute to $\langle \vec{M} \rangle$ since the spin-independent part of δh will vanish on taking the trace over spins in (3.15). If we recall (2.11), then (3.15) becomes

$$\langle \vec{M} \rangle = g\mu_B \sum_n \delta \vec{\Psi}_{nn} . \tag{3.16}$$

On multiplying both sides of (3.14) by $\vec{\sigma}$ and taking tr_σ we find

$$\delta \vec{\Psi}_{nm} = \frac{\partial n^0}{\partial \epsilon_n}\left(2\sum_{rs} B_{nmrs}\delta \vec{\Psi}_{sr} - \tfrac{1}{2}g\mu_B \vec{H}_0 \delta_{nm}\right), \tag{3.17}$$

where we have used (2.12). Equation (3.17) is an integral equation for the spin density which in general does not have a solution in closed form. We can however obtain a series solution for $\delta \vec{\Psi}_{nm}$ by direct iteration of (3.17):

$$\delta\vec{\Psi}_{nm} = -\tfrac{1}{2}g\mu_B\vec{H}_0\frac{\partial n^0}{\partial\epsilon_n}\left(\delta_{nm}+2\sum_{l_1}B_{nml_1l_1}\frac{\partial n^0}{\partial\epsilon_{l_1}}+2\sum_{l_1l_2l_3}B_{nml_1l_2}2\frac{\partial n^0}{\partial\epsilon_{l_1}}B_{l_2l_1l_3l_3}\frac{\partial n^0}{\partial\epsilon_{l_3}}+\cdots\right). \tag{3.18}$$

If we recall (3.16), we find the static magnetic susceptibility may be written

$$\chi = \tfrac{1}{4}(g\mu_B)^2\nu\,(1-\nu\langle B\rangle+\nu^2\langle B^2\rangle-\cdots), \tag{3.19}$$

where we have used (2.6) to introduce the quasiparticle density of states and have defined the dimensionless parameters

$$\nu\langle B\rangle = \frac{1}{\nu}\sum_{n_1n_2}\left(-2\frac{\partial n^0}{\partial\epsilon_{n_1}}\right)B_{n_1n_1n_2n_2}\left(-2\frac{\partial n^0}{\partial\epsilon_{n_2}}\right), \tag{3.20a}$$

$$\nu^2\langle B\rangle = \frac{1}{\nu}\sum_{n_1n_2n_3n_4}\left(-2\frac{\partial n^0}{\partial\epsilon_{n_1}}\right)B_{n_1n_1n_2n_3}\left(-2\frac{\partial n^0}{\partial\epsilon_{n_2}}\right)\times B_{n_3n_2n_4n_4}\left(-2\frac{\partial n^0}{\partial\epsilon_{n_4}}\right), \tag{3.20b}$$

and so forth. The configurational-averaged susceptibility $\langle\chi\rangle_c$ is then obtained by replacing each term $\langle B^n\rangle$ in the series (3.19) by $\langle\langle B^n\rangle\rangle_c$.

In the absence of exchange interactions $\langle\langle B^n\rangle\rangle_c = 0$ for all n and the configurational-averaged susceptibility $\langle\chi\rangle_c = \frac{1}{4}(g\mu_B)^2\nu(\epsilon_F)$ is a Pauli-like susceptibility which is identical to the result one would obtain from Fermi-gas theory by simply replacing the free electron density of states by the quasiparticle density of states. The modifications of the susceptibility arising from the exchange interactions are given by the series in (3.19). In analogy with Fermi-liquid theory we can define a single parameter B_0 to represent the effects of the exchange interaction on the configurational-averaged susceptibility of the glass. That is from (3.19) we can write

$$\langle\chi_c\rangle = (g\mu_B)^2\nu/4(1+B_0), \tag{3.21}$$

where the parameter B_0 is defined by the series

$$(1+B_0)^{-1} = (1-\nu\langle\langle B\rangle\rangle_c+\nu^2\langle\langle B^2\rangle\rangle_c-\cdots). \tag{3.22}$$

As in Fermi-liquid theory the parameter B_0 must be determined experimentally. The nature of the exchange interactions in the glass determines the algebraic sign of B_0. If $B_0>0$ ($B_0<0$), then the exchange interactions in the glass are antiferromagnetic (ferromagnetic), leading to a reduction (enhancement) of the susceptibility. If B_0 is negative, then the stability of our assumed paramagnetic ground state requires that $B_0>-1$, since for $B_0 = -1$ the interactions lead to a ferromagnetic instability.

A necessary and sufficient condition for the stability of our theory is that the series in (3.22) converge to a positive number. The general conditions for the convergence of such a series are beyond the scope of this paper. However, we note that if

$$\lim_{n\to\infty}\left|\frac{\nu\langle\langle B^{n+1}\rangle\rangle_c}{\langle\langle B^n\rangle\rangle_c}\right|<1, \tag{3.23}$$

then the series in (3.22) is absolutely convergent.

If we recall (3.20a) we see that $\langle B\rangle$ is the average of the quasiparticle exchange interaction B_{mnrs} over states at the Fermi energy. An interesting theoretical situation prevails if the following two conditions are met: (i) the configurational average of the quasiparticle exchange interaction vanishes (i.e., $\langle\langle B\rangle\rangle_c = 0$) while there remains other nonzero terms in the series (3.22); and (ii) the series in (3.22) diverges. This situation might appropriately be called a "Fermi spin glass."[5]

The meaning of the formal expressions (3.18)–(3.20) we have derived can be made more transparent by an approximate treatment in the context of the informal $(\vec{R},\vec{K})$ representation we discussed in Sec. I. The first-order correction to χ [Eq. (3.20a)] becomes

$$\nu\langle B\rangle = \frac{1}{\nu}\sum_{R,K,R',K'}\left(-2\frac{\partial n_0}{\partial\epsilon(R,K)}\right)\times\langle\vec{R}\vec{K}|B|\vec{R}'\vec{K}'\rangle\times\left(-2\frac{\partial n_0}{\partial\epsilon(R',K')}\right). \tag{3.24}$$

(We have simplified notation by not writing the matrix indices of B twice.) If the quasiparticle interactions are short ranged, or, more precisely, their range is much less than the localization length, only terms with $R = R'$ contribute. Then (3.24) contains an average of the quantity $\langle\vec{R},\vec{K}|B|\vec{R},\vec{K}'\rangle$ over all localization regions $\vec{R}$, with the constraint that $\epsilon(\vec{R},\vec{K})$ and $\epsilon(R,K')$ must be within $\approx T$ of the Fermi level. Within a particular localization region, two states $|\vec{R}\vec{K}\rangle$ and $|\vec{R}\vec{K}'\rangle$ will have energy E_F only if the two wave functions have about the same number of oscillations: $|\vec{K}| = |\vec{K}'|$. This gives them about the same kinetic energy; they both have about the same potential energy because they are localized in the same region R. So for a given region $\vec{R}$, there will be a particular magnitude of $\vec{K}$ and $\vec{K}'$,

which we call $K_F(\vec{R})$, a sort of local Fermi momentum, for which $\epsilon(\vec{K},\vec{R})=\epsilon(\vec{K}',R)=E_F$. [$K_F(\vec{R})$ will be large in deep trap regions and small in shallow ones.] In this picture, the $\vec{K}$ and $\vec{K}'$ summation in (3.24) becomes an integral over a local Fermi surface. In a transparent notation,

$$\nu\langle B\rangle=\nu\langle B_0(R)\rangle_R \tag{3.25}$$

and in the same way, (3.20b) can be expressed as $\nu^2\langle B_0^2(R)\rangle_R$. Each term in the series (3.19) becomes an average over localization regions of a power of a local Landau coefficient $B_0(R)$, and

$$\chi=\tfrac{1}{4}(g\mu_B)^2\nu\langle[1+B_0(R)]^{-1}\rangle_R . \tag{3.26}$$

B. Dynamic properties

1. Dynamic susceptibility: Application to electron-spin resonance

In an electron-spin-resonance experiment one applies a small uniform rf magnetic field $\vec{b}(t)$ transverse to an applied dc field $\vec{H}_0$. If the frequency ω of the rf field is near the resonance condition $\omega\approx g\mu_B H_0$, then the rf field will induce magnetic dipole transitions between the Zeeman levels and produce a net transverse magnetization $M_+(\omega)$ proportional to the applied rf field. The linear response function for the driven spin system is the frequency-dependent transverse susceptibility $\chi_+(\omega)$, which we now calculate.

The quasiparticle Hamiltonian can be written

$$\tilde{h}=h-\tfrac{1}{2}(g\mu_B\vec{H}\cdot\vec{\sigma})+\delta h , \tag{3.27}$$

where $\vec{H}=H_0\hat{z}+\vec{b}(t)$. It is convenient to introduce the quantity $\delta\tilde{n}$ which is the deviation of the quasiparticle density matrix from local equilibrium and is given by

$$n=n^0(h)+\delta n=n^0(\tilde{h})+\delta\tilde{n} , \tag{3.28}$$

so that from the above equations we have

$$\delta n=\delta\tilde{n}+\frac{\partial n^0}{\partial h}[\delta h-\tfrac{1}{2}(g\mu_0\vec{H}\cdot\vec{\sigma})] . \tag{3.29}$$

The reason for introducing $\delta\tilde{n}$ is that it facilitates the linearization of our transport since $\delta\tilde{n}$ is proportional to the applied rf field while δn contains a piece which is zeroth order in the applied rf field. This zeroth-order piece $\delta n^{(0)}$ is proportional to the dc field $\vec{H}_0$ and already appeared in our earlier calculation of the static susceptibility.

If we substitute (3.27) into (2.15) and use (3.28) and (3.29) we obtain the linearized equation of motion

$$i\frac{\partial\delta\tilde{n}}{\partial t}=-i\frac{\partial n^0}{\partial h}\frac{\partial\delta h^{(1)}}{\partial t}-\tfrac{1}{2}g\mu_B\vec{\sigma}\cdot\frac{\partial\vec{b}}{\partial t}+[h,\delta\tilde{n}]-\tfrac{1}{2}g\mu_B H_0[\sigma_z,\delta\tilde{n}]+[\delta h^{(0)},\delta\tilde{n}] . \tag{3.30}$$

In (3.30) $\delta h^{(1)}$ and $\delta h^{(0)}$ are the parts of δh which are first and zeroth order, respectively, in the small rf field $\vec{b}(t)$.

We want to calculate the transverse magnetization $M_+(\omega)$ produced by the rf field. This is given by

$$M_+(\omega)=\tfrac{1}{2}g\mu_B\,\mathrm{tr}\sigma_+\delta n\equiv\chi_+(\omega)b_+(\omega) , \tag{3.31}$$

where $\sigma_+=\sigma_x+i\sigma_y$ and $b_+=b_x+ib_y$. If we use (3.29), then $M_+(\omega)$ may be written

$$M_+(\omega)=\tfrac{1}{2}g\mu_B\,\mathrm{tr}\sigma_+\left(\delta\tilde{n}+\frac{\partial n^0}{\partial h}[\delta h-\tfrac{1}{2}(g\mu_B\vec{H}\cdot\vec{\sigma})]\right) . \tag{3.32}$$

It is clear from (3.31) that we may write $\delta n=\delta\vec{\Psi}\cdot\vec{\sigma}$ since the δf term in (2.9) will vanish on taking the spin trace in (3.31). With this in mind we may write the interaction part of the quasiparticle Hamiltonian as follows:

$$\delta h_{nm}=2\sum_{rs}B_{nmrs}\delta\tilde{n}_{sr}+2\sum_{rs}B_{nmrs}\frac{\partial n^0}{\partial\epsilon_s}\delta h_{sr}-\tfrac{1}{2}(g\mu_B\vec{H}\cdot\vec{\sigma})\,2\sum_r B_{nmrr}\frac{\partial n^0}{\partial\epsilon_r} , \tag{3.33}$$

where we have made use of (3.29). From (3.33) we see that $\delta h^{(0)}$ and $\delta h^{(1)}$ satisfy the following integral equations:

$$\delta h^{(0)}_{nm}=2\sum_{rs}B_{nmrs}\frac{\partial n^0}{\partial\epsilon_s}\delta h^{(0)}_{sr}-\tfrac{1}{2}(g\mu_B H_0\sigma_z)\,2\sum_r B_{nmrr}\frac{\partial n^0}{\partial\epsilon_r} \tag{3.34}$$

and

$$\delta h^{(1)}_{nm}=2\sum_{rs}B_{nmrs}\delta\tilde{n}_{sr}+2\sum_{rs}B_{nmrs}\frac{\partial n^0}{\partial\epsilon_s}\delta h^{(1)}_{rs}-\tfrac{1}{2}(g\mu_B\vec{b}\cdot\vec{\sigma})\,2\sum_r B_{nmrr}\frac{\partial n^0}{\partial\epsilon_r} . \tag{3.35}$$

In order to calculate $\delta\tilde{n}$ we need to solve (3.30), (3.34), and (3.35) self-consistently. We look for a solution of the form

$$\delta\tilde{n}_{nm}(\omega)=A(\omega)\frac{\partial n^0}{\partial\epsilon_n}[-\tfrac{1}{2}(g\mu_B\vec{b}\cdot\vec{\sigma})]\delta_{nm} , \tag{3.36}$$

where $A(\omega)$ is to be determined. If we substitute the ansatz (3.36) into (3.35) we find

$$\delta h^{(1)}_{nm}(\omega)=-\tfrac{1}{2}(g\mu_B\vec{b}\cdot\vec{\sigma})[A(\omega)+1]\,2\sum_r B_{nmrr}\frac{\partial n^0}{\partial\epsilon_r}+2\sum_{rs}B_{nmrs}\frac{\partial n^0}{\partial\epsilon_s}\delta h^{(1)}_{sr} . \tag{3.37}$$

If we iterate (3.37) we obtain the following series solution:

$$\delta h^{(1)}_{nm}(\omega) = -\tfrac{1}{2}(g\mu_B \vec{b}\cdot\vec{\sigma})[A(\omega)+1]\left(2\sum_r B_{nmrr}\frac{\partial n^0}{\partial \epsilon_r}+\sum_{rst} 2B_{nmrs}\frac{\partial n^0}{\partial \epsilon_s}2B_{srtt}\frac{\partial n^0}{\partial \epsilon_t}+\cdots\right). \tag{3.38}$$

In the following analysis it will also be useful to have a series solution for $\delta h^{(0)}$. Iterations of (3.34) produces the following equation:

$$\delta h^{(0)}_{nm} = -\tfrac{1}{2}(g\mu_B H_0\sigma_z)\left(2\sum_r B_{nmrr}\frac{\partial n^0}{\partial \epsilon_r}+\sum_{rst} 2B_{nmrs}\frac{\partial n^0}{\partial \epsilon_s}2B_{srtt}\frac{\partial n^0}{\partial \epsilon_t}+\cdots\right). \tag{3.39}$$

If we substitute the ansatz (3.36) into our time Fourier-transformed equation of motion (3.30) multiply both sides of the resulting equation by σ_+ and take the trace over spins we find on using Eqs. (3.38) and (3.39) that $A(\omega)$ satisfies

$$[A(\omega)(\omega - g\mu_B H_0)+\omega]S_n = 0\,, \tag{3.40}$$

where we have defined

$$S_n = 1+\sum_r 2B_{nmrr}\frac{\partial n^0}{\partial \epsilon_r} + \sum_{rst} 2B_{nmrs}\frac{\partial n^0}{\partial \epsilon_s}B_{srtt}\frac{\partial n^0}{\partial \epsilon_t}+\cdots\,. \tag{3.41}$$

For $S_n \neq 0$ we have from (3.40) that

$$A(\omega) = -\omega/(\omega - g\mu_B H_0)\,. \tag{3.42}$$

If we combine (3.42) and (3.36), we can write the deviation from local equilibrium as follows:

$$\delta\vec{n}_{nm}(\omega) = \frac{\omega}{\omega - g\mu_B H_0}\frac{\partial n^0}{\partial \epsilon_n}\left(\frac{g\mu_B \vec{b}\cdot\vec{\sigma}}{2}\right)\delta_{nm}\,. \tag{3.43}$$

The many-body effects which are contained in the terms involving $\delta h^{(1)}$ and $\delta h^{(0)}$ in (3.30) have not appeared in the solution (3.43) and the function $\delta\vec{n}_{nm}(\omega)$ is resonant at the free-electron Larmor frequency $\omega_L = g\mu_B H_0$. The remainder of the calculation is straightforward. We substitute (3.43) into (3.32) and make use of (3.38). After a little algebra one finds that the configurational-averaged transverse susceptibility $\langle\chi_+(\omega)\rangle_c$ may be written

$$\langle\chi_+(\omega)\rangle_c = -\langle\chi\rangle_c g\mu_B H_0/(\omega - g\mu_B H_0)\,, \tag{3.44}$$

where $\langle\chi\rangle_c$ is given by (3.21). We see from (3.44) that all of the many-body effects in $\langle\chi_+(\omega)\rangle_c$ are contained in the static susceptibility. In particular we note that the position of the electron-spin resonance occurs at the free-electron Larmor frequency $\omega_L = g\mu_B H_0$ and is not affected by the exchange interactions. We could have anticipated this result since we have not included spin-orbit coupling in our theory. The reason is that the electron spin resonance frequency is the rate of precession of the total magnetization of the quasiparticles which is a constant of the motion (i.e., commutes with the microscopic Hamiltonian including Coulomb interactions in the absence of spin-orbit interactions).

2. *ac conductivity of localized quasiparticles*

In order to calculate the ac conductivity $\sigma(\omega)$ of the Fermi glass we must first obtain an expression for the current produced when a time-dependent external electric field is applied to the glass. For a noninteracting system one simply solves the linearized Liouville equation for $\delta n(\omega)$ and calculates the induced current from the standard expression

$$\vec{J}(\omega) = e\,\mathrm{tr}\,\vec{V}\delta n(\omega)\,, \tag{3.45}$$

where the velocity operator is given by

$$i\vec{V} = [\vec{r}, h] \tag{3.45'}$$

(h is the single-particle Hamiltonian). In the Fermi glass the change δn in the quasiparticle density matrix, produced by the external field, produces a change δh in the quasiparticle Hamiltonian and therefore a first order (proportional to the applied field) change $\delta\vec{V}$ in the velocity operator given by

$$i\delta\vec{V} = [\vec{r}, \delta h]\,. \tag{3.46}$$

If we include this contribution the induced current becomes

$$\vec{J}(\omega) = e\,\mathrm{tr}\,n^0(h)\delta\vec{V}(\omega) + e\,\mathrm{tr}\vec{V}\delta n(\omega)\,. \tag{3.47}$$

If we apply a time-dependent electric field $\vec{E} = E_0\hat{x}e^{-i\omega t}$ ($\omega = \omega + i0$) to the glass the quasiparticle Hamiltonian may be written

$$\tilde{h} = h + \delta h + V(t)\,, \tag{3.48}$$

where

$$V(t) = -eE_0 x e^{-i\omega t}\,. \tag{3.49}$$

The linearized Liouville equation becomes

$$i\frac{\partial \delta n}{\partial t}=[h,\delta n]+[\delta h,n^0(h)]+[V(t),n^0(h)]. \quad (3.50)$$

In order to solve (3.50) it is convenient to introduce the auxiliary function $\delta\hat{n}$ defined by

$$\delta n = e^{-iht}\delta\hat{n}e^{iht}, \quad (3.51)$$

which when substituted into (3.50) yields the following equation:

$$i\frac{\partial \delta\hat{n}}{\partial t}=[\delta\hat{h},n^0(h)]+[\hat{V}(t),n^0(h)], \quad (3.52)$$

where

$$\delta\hat{h}(t)=e^{iht}\delta h(t)e^{-iht} \quad (3.53)$$

and similarly for $\hat{V}(t)$. We easily solve (3.52) obtaining

$$\delta\hat{n}_{mn}(t)=-i\int_{-\infty}^{t}dt'\,e^{-i(\omega-\epsilon_m+\epsilon_n)t'}\times[\delta h_{mn}(\omega)-eE_0X_{mn}][n^0(\epsilon_n)-n^0(\epsilon_m)]. \quad (3.54)$$

If we observe that

$$\delta\hat{n}_{mn}(t)=e^{-i(\omega-\epsilon_m+\epsilon_n)t}\delta n_{mn}(\omega), \quad (3.55)$$

then we find from (3.54) that

$$\delta n_{mn}(\omega)=(\delta h_{mn}-eE_0X_{mn})L_{mn}(\omega), \quad (3.56)$$

where we have defined the function

$$L_{mn}(\omega)=[n^0(\epsilon_n)-n^0(\epsilon_m)]/(\omega-\epsilon_m+\epsilon_n+i\eta). \quad (3.57)$$

On taking the tr_σ of both sides of (3.56) we find that the spin-independent part of the quasiparticle density matrix satisfies the equation

$$\delta f_{mn}(\omega)=\left(2\sum_{rs}A_{mnrs}\delta f_{sr}-eE_0X_{mn}\right)L_{mn}(\omega). \quad (3.58)$$

We obtain by iteration the series solution of (3.58)

$$\delta f_{mn}(\omega)=-eE_0\left(X_{mn}+\sum_{rs}2A_{mnrs}X_{sr}L_{sr}(\omega)+\sum_{rstv}2A_{mnrs}L_{sr}(\omega)2A_{srtv}X_{vt}L_{vt}(\omega)+\cdots\right)L_{mn}(\omega). \quad (3.59)$$

By making use of Eqs. (3.45′), (3.46), and (3.47) it is easy to show that the current may be written in the form

$$\vec{J}(\omega)=2e\sum_{mn}\vec{V}_{nm}\left(\delta f_{mn}(\omega)-\sum_{rs}2A_{mnrs}\delta f_{sr}(\omega)L_{mn}(0)\right), \quad (3.60)$$

where the second term in the above expression is the analog of the "backflow current" in the Landau theory of Fermi liquids.[6] If we make use of (3.58) then we can write (3.60) in the form

$$\vec{J}(\omega)=2e\sum_{mn}\vec{V}_{nm}\left(\sum_{rs}2A_{mnrs}\tilde{L}_{mn}(\omega)\delta f_{sr}(\omega)-eE_0X_{mn}L_{mn}(\omega)\right), \quad (3.61)$$

where we have defined the function

$$\tilde{L}_{mn}(\omega)=L_{mn}(\omega)-L_{mn}(0). \quad (3.62)$$

We note that when $A_{mnrs}=0$ Eq. (3.61) reduces to the Kubo-Greenwood formula for noninteracting particles.[7] Since the interaction (3.61) always appears multiplied by $\tilde{L}_{mn}(\omega)$ we can immediately deduce that there are no interaction effects at zero frequency. The real part of the dc conductivity vanishes as in the noninteracting case[1] since $\tilde{L}_{mn}(0)=0$. If we observe that

$$\tilde{L}_{mn}(\omega)=\omega L_{mn}(\omega)/(\epsilon_m-\epsilon_n), \quad (3.63)$$

then on substituting (3.59) into (3.61) we find that the ac conductivity is given by

$$\sigma(\omega)=2ie^2\left(\sum_{mn}X_{nm}L_{mn}(\omega)X_{mn}(\epsilon_m-\epsilon_n)+\omega\sum_{mnrs}X_{nm}L_{mn}(\omega)2A_{mnrs}L_{sr}(\omega)X_{sr}+\omega\sum_{mnrstv}X_{nm}L_{mn}(\omega)2A_{mnrs}L_{sr}(\omega)2A_{srvt}L_{tv}(\omega)X_{tv}+\cdots\right). \quad (3.64)$$

From (3.64) we obtain an expression for the real part of the conductivity

$$\mathrm{Re}\sigma(\omega) = -2e^2\omega\,\mathrm{Im}\underline{x}\cdot\hat{L}\sum_{n=0}^{\infty}(2A\hat{L})^n\cdot\underline{x} = -2e^2\omega\,\mathrm{Im}\underline{x}\cdot\hat{L}(1-2A\hat{L})^{-1}\cdot\underline{x}, \tag{3.65}$$

where A and $\hat{L}$ are matrices with elements

$$\langle l_1 l_2|A|l_3 l_4\rangle = A_{l_1 l_2 l_3 l_4}, \quad \langle l_1 l_2|\hat{L}|l_3 l_4\rangle = L_{l_1 l_2}\delta_{l_1 l_4}\delta_{l_2 l_3}, \tag{3.66}$$

and $\underline{x}$ is a vector with components X_{mn}. Explicitly

$$\underline{x}\cdot\hat{L}(2A\hat{L})^n\cdot\underline{x} = \sum_{l_1\cdots l_{2n+2}}(x_{l_1 l_2}L_{l_1 l_2}(\omega)\,2A_{l_2 l_1 l_3 l_4}\cdots\ 2A_{l_{2n}\,l_{2n-1}\,l_{2n+1}\,l_{2n+2}}L_{l_{2n+2},\,l_{2n+1}}(\omega)X_{l_{2n+2}\,l_{2n+1}}). \tag{3.67}$$

Since $\hat{L}$ is the only complex quantity in these expressions. The imaginary part of the nth order term in (3.65) is proportional to

$$\mathrm{Im}\underline{x}\cdot\hat{L}(2A\hat{L})^n\cdot\underline{x} = \sum_{l=1}^{n+1}\underline{x}\cdot(2\hat{L}^*A)^{l-1} \times \mathrm{Im}\hat{L}(2A\hat{L})^{n+1-l}\cdot\underline{x}, \tag{3.68}$$

in the same matrix notation. Hence

$$\mathrm{Re}\sigma(\omega) = -2e^2\omega\,\underline{x}\cdot\sum_{n=0}^{\infty}(2\hat{L}^*A)^n\,\mathrm{Im}\hat{L}\sum_{n'=0}^{\infty}(2A\hat{L})^{n'}\cdot\underline{x}$$
$$= -2e^2\omega\,\underline{x}\cdot(1-2\hat{L}^*A)^{-1}\,\mathrm{Im}\hat{L}(1-2A\hat{L})^{-1}\cdot\underline{x}. \tag{3.69}$$

Thus, except for the vertex correction to the position operator

$$\underline{x}\to\underline{x}' = (1-2A\hat{L})^{-1}x \tag{3.70}$$

(3.70) is simply the Kubo-Greenwood formula valid for noninteracting quasiparticles. Mott's argument for the resulting frequency dependence

$$\mathrm{Re}\sigma(\omega)\propto\omega^2(\ln\omega)^4 \tag{3.71}$$

then goes through in essentially its original form. The frequency dependence remains of the form (3.71); only the magnitude of σ is affected by the vertex corrections.

IV. MICROSCOPIC THEORY OF THE FERMI GLASS AT $T=0$

A. Single-particle properties

The existence of the Fermi-glass theory we have constructed depends, like the Fermi-liquid theory of Landau, on the notion of well defined quasiparticle excitations near the Fermi energy. The purpose of this subsection is to demonstrate that the same phase space restrictions on electron-electron scattering that exist in the Fermi liquid are also operative in the Fermi glass. As in the liquid, the physical basis for those restrictions lie in the Pauli exclusion principle which prohibits an electron from scattering into an occupied state.

In this section we will work in a basis of states $|n\rangle$ in which the exact single-particle Green's function G is diagonal. In this basis G satisfies the usual Dyson equation

$$G_m^{-1}(E) = E - \epsilon_m^{(0)} - \Sigma_m(E). \tag{4.1}$$

In order to prove Anderson's conjecture that there exist well defined quasiparticle excitations in the Fermi glass we need to examine the irreducible self-energy $\Sigma_m(E)$ in Eq. (4.1). We will examine $\Sigma_m(E)$ by using self-consistent perturbation theory, in which the fermion lines in all diagrams represent exact propagators $G_m(E)$, and we omit all diagrams with dressings on the propagator lines. The first-order contributions to $\Sigma_m(E)$ correspond to the standard Hartree-Fock diagrams. These diagrams give a real, energy-independent con-

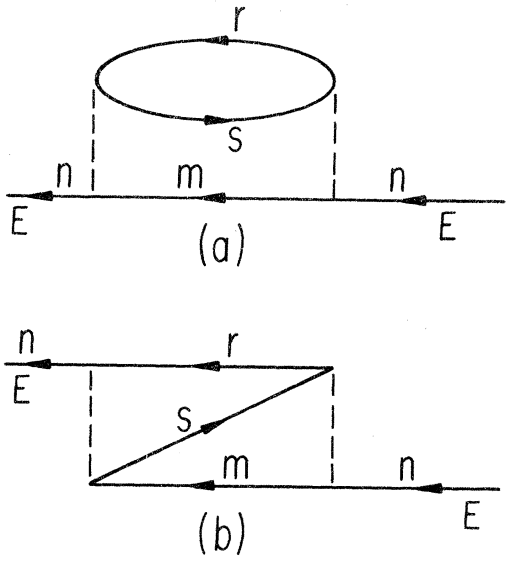

FIG. 1. Two second-order diagrams contributing to $\mathrm{Im}\,\Sigma_n(E)$. The solid lines are exact single-particle propagators and the dashed lines represent the electron-electron interaction.

tribution to $\Sigma_m(E)$ which corrects the single-particle energy $\epsilon_m^{(0)}$. The first contribution to $\mathrm{Im}\Sigma_m(E)$ (and therefore to the quasiparticle lifetime) comes from the two second-order diagrams in Figs. 1(a) and 1(b). We will consider in detail the diagram shown in Fig. 1(a) [the diagram in Fig. 1(b) leads to similar results].

The contribution of the diagram in Fig. 1(a) is

$$\Sigma_n(E) = \sum_{mrs} V_{rmns} V_{smnr} \int_{-\infty}^{\infty} \frac{dE'\, dE''}{(2\pi)^2} \quad G_m(E') G_s(E'') G_r(E + E'' - E') . \tag{4.2}$$

To proceed, we introduce the zero-temperature spectral representation for $G_n(E)$

$$G_n(E) = \int_{-\infty}^{\infty} dx A_n(x) \left(\frac{f^-(x)}{E - x - i\eta} + \frac{f^+(x)}{E - x + i\eta} \right), \tag{4.3}$$

where $A_n(x)$ is the spectral weight function, $f^{\pm}(x) = \Theta(\pm x)$ [$\Theta(x)$ is a step function] and $\eta = 0^+$. If we substitute (4.3) into (4.2), we find

$$\Sigma_n(E) = \sum_{mrs} V_{rmns} V_{smnr} \int_{-\infty}^{\infty} dx_1 dx_2 dx_3 A_m(x_1) A_s(x_2) A_r(x_3) \left(\frac{f^-(x_1) f^+(x_2) f^-(x_3)}{E - x_1 - x_3 + x_2 - i\eta} + \frac{f^+(x_1) f^-(x_2) f^+(x_3)}{E + x_2 - x_3 - x_1 + i\eta} \right). \tag{4.4}$$

Since the spectral weight functions are by definition real we have

$$\mathrm{Im}\Sigma_n(E) = -\pi \sum_{mrs} V_{rmns} V_{smnr} \int_{-\infty}^{\infty} dx_1 dx_2 \{ A_m(x_1) A_s(x_2) A_r(x_1 + x_2 - E) [f^+(x_1) f^+(x_2) f^-(x_1 + x_2 - E) - f^-(x_1) f^-(x_2) f^+(x_1 + x_2 - E)] \} . \tag{4.5}$$

There are two cases to be considered in (4.5): (i) $E \gtrless 0$ and (ii) $E < 0$. We shall only consider the case $E \gtrless 0$ since $E < 0$ can be similarly analyzed. For $E \gtrless 0$ the term proportional to $f^-(x_1) f^-(x_2) f^+(x_1 + x_2 - E)$ vanishes due to an incompatibility of the step functions. If we consider the limit $E \to 0^+$ in the remaining term, then because of the step functions the only contribution from the integral comes from $x_1, x_2 \to 0^+$ for $x_1 + x_2 < E$. Since all energies in (4.5) are then near the Fermi energy ($\epsilon_F = 0$) we make the quasiparticle ansatz for the spectral weight function

$$A_n(x_1) = Z_n \delta(x_1 - \epsilon_n) . \tag{4.6}$$

To show the self-consistency of this ansatz is the aim of this subsection. If we substitute (4.6) into (4.5), then to $O(E^2)$ we find that $\mathrm{Im}\Sigma_n(E)$ has the form

$$\mathrm{Im}\Sigma_n(E) = -\frac{\pi}{2} \sum_{mrs} E^2 V_{rmns} V_{smnr} Z_m Z_r Z_s \times \delta(\epsilon_m - \eta^+) \delta(\epsilon_s - \eta^+) \delta(E - \epsilon_m - \epsilon_s + \epsilon_r) . \tag{4.7}$$

We find the same proportionality to E^2 that characterizes the quasiparticle decay rate in the normal uniform Fermi liquid. It arises because of the restrictions on the region of integration in (4.5) imposed by the Fermi functions $f^{\pm}$; $x_1, x_2 > 0$ and $(x_1 + x_2) < E$. The part of $\mathrm{Im}\Sigma_n$ coming from Fig. 1(b) gives a contribution of the same form, that is, a series of δ-function spikes modulated by a factor E^2.

In general we can write a spectral representation for $\Sigma_n(E)$ which has the form

$$\Sigma_n(E) = \sum_m \frac{C(E_m^{(n)})}{E - E_m^{(n)} + i\eta E_m^{(n)}} . \tag{4.8}$$

That is, Σ has a set of poles $E = E_m^{(n)} + i\eta$ with residues $C(E_m^{(n)}) \propto E_m^{(n)2}$.

This form is valid in a finite system (of arbitrary size). For a uniform system, as the limit of infinite size is taken, the number of poles grows like the size of the system while the residues of the individual poles scale like the inverse of the size. Hence Σ (like G) acquires a branch cut singularity. When the states of relevance (i.e., those near E_F in this case) are localized, however, the analytic structure of Σ in the infinite-volume limit is rather different, as Anderson[3] has pointed out for the case of the Green's function in the non-

interacting system. As the size of the system grows, Σ acquires more and more poles, but their residues will generally be smaller and smaller. The system can always be approximated arbitrarily well (in the sense that the sum of the residues of the omitted poles can be made arbitrarily small) by a finite one, with a finite number of poles in Σ. (The correct qualitative features of Σ can be obtained just by choosing the size of the system larger than the localization length.) In order to discuss the problem we have here, therefore, we will always use the discrete-pole form of Σ (4.8), and only take the limit of large volume at the end.

The single particle spectrum is given by the poles in $G_n(E)$, that is by the solutions of

$$\epsilon_n^i - \epsilon_n^{(0)} = \mathrm{Re}\,\Sigma_n(\epsilon_n^i)\,. \tag{4.9}$$

The residue Z_n^i corresponding to the pole ϵ_n^i is given by

$$Z_n^i = \left[1 - \left(\frac{\partial\Sigma}{\partial E}\right)_{\epsilon_n^i}\right]^{-1}, \tag{4.10}$$

so that $G_n(E)$ has the form

$$G_n(E) = \sum_i \frac{Z_n^i}{E - \epsilon_n^i + i\eta}\,. \tag{4.11}$$

Figure 2(a) shows how the roots ϵ_n^i are interleaved between the poles of $\Sigma_n(E)$. The residues Z_n^i will be large when the line $E - \epsilon_n^{(0)}$ crosses $\Sigma_n(E)$ at a point where the slope of Σ is small. That is, Z_n^i will tend to be large when the intersection lies about halfway between two successive $E_i^{(n)}$, and small when the intersection lies near an $E_i^{(n)}$ since $|\partial\Sigma/\partial E|$ blows up at the $E_i^{(n)}$. Figure 2(b) shows the resulting structure in

$$|\,\mathrm{Im}\,G_n(E)| = \pi \sum_i Z_n^i \delta(E - \epsilon_n^i)\,.$$

The question we want to ask in ascertaining the validity of the quasiparticle hypothesis (4.6) is how fast the residues fall off as we move away in energy from the largest one. If they fall off fast enough, we can ignore all poles but the one with the largest Z_n^i, and the self-consistency of (4.6) will be proved. To show this, we pretend for the moment that all the $E_i^{(n)}$ are equally spaced, and that in the energy range in which we are looking, the strengths $C(E_i^{(n)})$ of all the poles in $\Sigma_n(E)$ are all the same value, C. This fictional situation will be sufficient to establish the typical way that the Z_n^i fall off, although it will miss fluctuations in the Z_n^i such as appear in the more realistic picture of Fig. 2.

Consider the graphical solution of Eq. (4.9), as shown in Fig. 2. We let δ be the spacing between

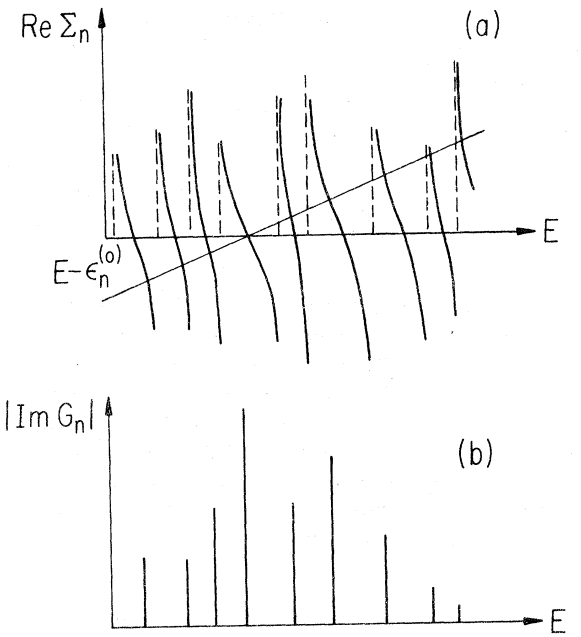

FIG. 2. (a) Schematic graphical solution of Eq. (4.8). The solid lines are the $\mathrm{Re}\Sigma_n(E)$. The dashed lines are the spikes in $|\,\mathrm{Im}\Sigma_n(E)|$ at the energies $E_m^{(n)}$ in Eq. (4.8). (b) Shows the resulting behavior in $|\,\mathrm{Im}G_n(E)|$. Note that Figs. 2(a) and 2(b) are drawn with a common energy scale E.

the $E_m^{(n)}$ in (4.8). The largest $Z_n^{(i)}$ is clearly obtained when the line $E - E_n^{(0)}$ crosses $\Sigma_n(E)$ at $\Sigma = 0$. We number our roots ϵ_n^i so that this $i = 1$. In the following, we take $\epsilon_n^{(1)}$ as the origin of energy. In the first interval we can write

$$\Sigma_n(E) \approx C\left[(E + \tfrac{1}{2}\delta)^{-1} + (E - \tfrac{1}{2}\delta)^{-1}\right], \tag{4.12}$$

where we have approximated the sum in Eq. (4.8) by only keeping the energies $E_m^{(n)}$ corresponding to the end points of the interval. This is a good approximation, in a small region of energy where C is slowly varying, since these terms have the smallest denominators. The derivative of $\Sigma_n(E)$ evaluated at $E = 0$ is

$$\left(\frac{\partial\Sigma}{\partial E}\right)_{E=0} = -\frac{8C}{\delta^2}\,. \tag{4.13}$$

In the second interval $\Sigma_n(E)$ crosses zero with the same slope (4.13); elementary algebra then yields the position of the second root, $\epsilon_n^{(2)}$, as

$$\epsilon_n^{(2)} = \delta - \delta^3/8C + O(\delta^3)\,. \tag{4.14}$$

The position of the second root has therefore been shifted by $\delta^3/8C$ towards the left-hand end point of the interval. In each subsequent interval the root is shifted closer to the left-hand end point of the interval. Since $|\partial\Sigma/\partial E|$ increases in each in-

terval as the end point is approached the corresponding residues decrease in successive intervals. In an interval for which the shift is $\frac{1}{4}\delta$ the residue will be decreased by roughly a factor of 2 relative to the residue at the root in the first interval. Therefore let the shift in the Nth interval be $\frac{1}{4}\delta$. Then from (4.14) N is given by $N\delta^3/8C \approx \frac{1}{4}\delta$. If we define the width in energy $\Gamma = N\delta$, then

$$\Gamma = 2C/\delta . \tag{4.15}$$

This is the desired result, since C is proportional to the square of the energy (measured from the Fermi energy). Thus the behavior of the *envelope* of the spikes in the spectral weight function for the Fermi glass is the same as that of the spectral weight function itself for a uniform normal Fermi liquid. In either case $\mathrm{Im}G$ can be approximated as well as one wants by a single spike at the quasiparticle energy, provided that the energy is close enough to the Fermi level.

Higher order diagrams should not change this result. They can be analyzed using the reduced graph expansion of Langer[8] and Ambegaokar[9] and (to leading order in E) only affect the magnitude of (4.7), not its E^2 dependence.

B. Collective properties

The intent of this section is to give a microscopic basis to the phenomenological transport equation introduced in Eq. (2.15). In the course of this analysis we shall also relate the phenomenological effective interaction Φ_{mnrs} [see Eq. (2.1)] between quasiparticles to a quantity which can in principle be calculated microscopically. In writing this section we have followed as closely as possible the original work of Landau.[10]

The four-point vertex for particle-hole scattering in the glass satisfies the Bethe-Salpeter equation

$$\Gamma_{mnsr}(\epsilon,\epsilon',\omega) = \Gamma^{(1)}_{mnsr}(\epsilon,\epsilon',\omega) - i\int_{-\infty}^{\infty}\frac{dE}{2\pi}\sum_{\bar{m}\bar{s}}\Gamma^{(1)}_{mn\bar{s}\bar{m}}(\epsilon,E,\omega)\,G_{\bar{m}}(E+\omega)\,G_{\bar{s}}(E)\,\Gamma_{\bar{m}\bar{s}sr}(E,\omega,\epsilon') . \tag{4.16}$$

In (4.16) $\Gamma^{(1)}_{mnrs}(\epsilon,\epsilon',\omega)$ is the irreducible four-point vertex for particle-hole scattering. The Bethe-Salpeter equation (4.16) is shown graphically in Fig. 3.

We are interested in $\Gamma_{mnsr}(\epsilon,\epsilon',\omega)$ for energies near the Fermi energy $\epsilon_F = 0$. By virtue of the results of Sec. IV A the exact Green's functions in (4.16) may be written at $T = 0$ in the form

$$G_s(E) = Z_s f_s^-/(E-\epsilon_s - i\eta) + Z_s f_s^+/(E-\epsilon_s + i\eta) , \tag{4.17}$$

where all quantities appearing in (4.17) have been previously defined.

The poles in the four-point vertex $\Gamma_{mnrs}(\epsilon_1,\epsilon_2,\omega)$ as a function of the energy transfer ω in a scattering correspond to the collective modes in the system. In (4.16) we expect some singular behavior in $\Gamma_{mnrs}(\epsilon,\epsilon',\omega)$ for $\omega \to 0$ since for $\epsilon_{\bar{m}} \approx \epsilon_{\bar{s}} \approx 0$ the poles of the two internal propagators in (4.16) coincide as $\omega \to 0$ and $\omega_{\bar{m}\bar{s}} = \epsilon_{\bar{m}} - \epsilon_{\bar{s}} \to 0$. A generalization of the argument given by Landau[10] leads to the result

$$\lim_{\substack{\omega\to 0\\ \omega_{\bar{m}\bar{s}}\to 0}} G_{\bar{m}}(E+\omega)\,G_{\bar{s}}(E) = G_{\bar{s}}^2(E) + iR_{\bar{m}\bar{s}}(E,\omega,\omega_{\bar{m}\bar{s}}) , \tag{4.18}$$

where we have defined the quantity

$$R_{\bar{m}\bar{s}}(E,\omega,\omega_{\bar{m}\bar{s}}) = \frac{2\pi Z_{\bar{s}} Z_{\bar{m}}\,\delta(E)\,\delta(\epsilon_{\bar{s}})\,\omega_{\bar{m}\bar{s}}}{\omega - \omega_{\bar{m}\bar{s}} + i\alpha} , \tag{4.19}$$

with $\alpha = \eta^+$. We note that $\lim R_{\bar{m}\bar{s}}$ for $\omega \to 0$ and $\omega_{\bar{m}\bar{s}} \to 0$ is not well defined since it depends on the order in which the two limits are taken. We define the limit $\Gamma^{\omega}(\epsilon,\epsilon')$ of $\Gamma(\epsilon,\epsilon',\omega)$ by first performing the sum over states $\epsilon_{\bar{m}} \approx \epsilon_{\bar{s}}$ in (4.16) keeping ω finite and then letting $\omega \to 0$. If we use (4.18) we find that

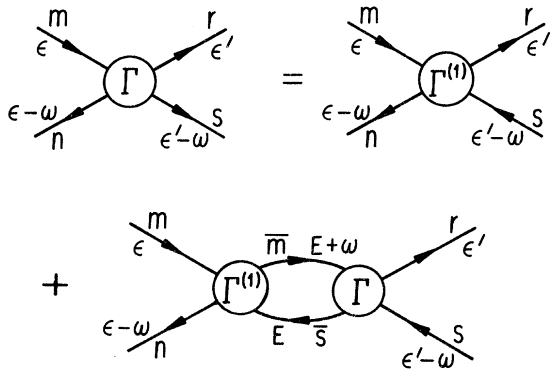

FIG. 3. Graphical representation of the Bethe-Salpeter equation (4.16).

$$\Gamma^{\omega}_{mnrs}(\epsilon,\epsilon') = \Gamma^{(1)}_{mnsr}(\epsilon,\epsilon') - i\sum_{\bar{m}\bar{s}} \int_{-\infty}^{\infty} \frac{dE}{2\pi}\, \Gamma^{(1)}_{mn\,\bar{s}\bar{m}}(\epsilon,\epsilon')\, G^{2}_{\bar{m}}(E)\, \Gamma^{\omega}_{\bar{m}\bar{s}\,sr}(E,\epsilon') . \tag{4.20}$$

If we take the limit $\omega \to 0$ in (4.16) and use (4.18), we have

$$\Gamma_{mnsr}(\epsilon,\epsilon',\omega) = \Gamma^{(1)}_{mnsr}(\epsilon,\epsilon') - i\sum_{\bar{m}\bar{s}} \int_{-\infty}^{\infty} \frac{dE}{2\pi} \Gamma^{(1)}_{mn\,\bar{s}\bar{m}}\ (\epsilon,E)\left(G^{2}_{\bar{m}}(E) + \frac{2\pi i Z_{\bar{s}} Z_{\bar{m}}\, \delta(E)\, \delta(\epsilon_{\bar{s}})\omega_{\bar{m}\bar{s}}}{\omega - \omega_{\bar{m}\bar{s}} + i\alpha}\right) . \tag{4.21}$$

In (4.21) we have set $\omega = 0$ in $\Gamma^{(1)}$ since it is a regular function of ω. If we write (4.20) in an obvious matrix notation, we have

$$\Gamma^{\omega} = \Gamma^{(1)} - i\Gamma^{(1)} G^{2} \Gamma^{\omega} , \tag{4.22}$$

which may be formally solved for Γ^{ω} to give

$$\Gamma^{\omega} = (1 + \Gamma^{(1)} G^{2})^{-1} \Gamma^{(1)} . \tag{4.23}$$

On writing a similar equation for (4.21) we have

$$\Gamma = \Gamma^{(1)} - i\Gamma^{(1)}(iR + G^{2})\Gamma . \tag{4.24}$$

We can eliminate $\Gamma^{(1)}$ from the above equation by combining (4.24) and (4.23). We find that

$$\Gamma = \Gamma^{\omega} + \Gamma^{\omega} R \Gamma . \tag{4.25}$$

[Equations (4.22)–(4.25) are analogous in form to equations found by Landau for the uniform Fermi liquid.] In the limit $\omega \to 0$ (i.e., near the pole of Γ) $\Gamma \gg \Gamma^{\omega}$ so that (4.25) may be written as the following homogeneous integral equation:

$$\Gamma_{mnsr}(\epsilon,\epsilon',\omega) = \sum_{\bar{m}\bar{s}} \Gamma^{\omega}_{mn\,\bar{s}\bar{m}}(\epsilon) \frac{Z_{\bar{m}} Z_{\bar{s}}\, \delta(\epsilon_{\bar{s}})\omega_{\bar{m}\bar{s}}}{\omega - \omega_{\bar{m}\bar{s}} + i\alpha} \Gamma_{\bar{m}\bar{s}\,sr}(\epsilon',\omega) . \tag{4.26}$$

We observe that the indices s and r and the variable ϵ' play the role of parameters in the above equation. Therefore we look for a solution of the form

$$\Gamma_{mnsr}(\epsilon,\epsilon',\omega) = g_{mn}(\epsilon,\omega) h_{sr}(\epsilon',\omega) . \tag{4.27}$$

If we substitute (4.27) into (4.26) and divide both sides of the resulting equation by the common factor $h_{sr}(\epsilon',\omega)$ we obtain the equation

$$g_{mn}(\epsilon,\omega) = \sum_{\bar{m}\bar{s}} \Gamma^{\omega}_{mn\,\bar{s}\bar{m}}(\epsilon) \frac{Z_{\bar{m}} Z_{\bar{s}}\, \delta(\epsilon_{s})\omega_{\bar{m}\bar{s}}}{\omega - \omega_{\bar{m}\bar{s}} + i\alpha} g_{\bar{m}\bar{s}}(\omega) . \tag{4.28}$$

In order to make further progress towards relating (4.28) to our phenomenological transport equation we write

$$\delta\bar{n}_{ms}(\omega) = [\omega_{ms}/(\omega - \omega_{ms} + i\alpha)]\, g^{(\omega)}_{ms} , \tag{4.29}$$

which leads to the equation

$$(\omega - \omega_{mn})\delta\bar{n}_{mn}(\omega) = \omega_{mn} \sum_{\bar{m}\bar{s}} \Gamma^{\omega}_{mn\,\bar{s}\bar{m}}(\epsilon) Z_{\bar{m}} Z_{\bar{s}}\, \delta(\epsilon_{\bar{s}})\delta\bar{n}_{\bar{m}\bar{s}}(\omega) . \tag{4.30}$$

We recall that our phenomenological transport equation [see (2.15)] in its linearized form is given by

$$i\frac{\partial \delta n}{\partial t} = [h, \delta n] + [\delta h, n^{0}(h)] , \tag{4.31}$$

or more explicitly by

$$(\omega - \omega_{mn})\delta n_{mn}(\omega) = \delta h_{mn}[n^{0}(\epsilon_{n}) - n^{0}(\epsilon_{m})] , \tag{4.32}$$

where at $T = 0$ the quasiparticle equilibrium distribution function is a step function $n^{0}(\epsilon_{n}) = \Theta(-\epsilon_{n})$. Therefore at $T = 0$ we may write

$$n^{0}(\epsilon_{n}) - n^{0}(\epsilon_{m}) = \omega_{mn}\delta(\epsilon_{n}) . \tag{4.33}$$

If we recall that δh_{mn} is related to the effective interaction Φ_{mnrs} by the equation

$$\delta h_{mn} = \sum_{rs} \Phi_{mnrs}\delta n_{sr} , \tag{4.34}$$

then we may write (4.32) as follows:

$$(\omega - \omega_{mn})\delta n_{mn}(\omega) = \omega_{mn}\delta(\epsilon_{n}) \sum_{\bar{m}\bar{s}} \Phi_{mn\,\bar{s}\bar{m}}\delta\bar{n}_{\bar{m}\bar{s}}(\omega) , \tag{4.35}$$

or upon writing

$$\delta n_{mn}(\omega) = \delta\bar{n}_{mn}\delta(\epsilon_{n}) , \tag{4.36}$$

we find that (4.35) becomes

$$(\omega - \omega_{mn})\delta\bar{n}_{mn}(\omega) = \omega_{mn} \sum_{\bar{m}\bar{s}} \Phi_{mn\,\bar{s}\bar{m}}\delta\bar{n}_{\bar{m}\bar{s}}(\omega)\delta(\epsilon_{\bar{s}}) , \tag{4.37}$$

so that our homogeneous phenomenological transport equation is identical in form to (4.30) which was obtained by an analysis of the Bethe-Salpeter equation and which determines the collective modes of the glass. By a comparison of (4.37) and (4.30) we find that the effective interaction Φ_{mnrs} between quasiparticles is related to their scattering by the

equation

$$\Phi_{mnrs} = Z_m Z_s \Gamma^{\omega}_{mnrs}, \tag{4.38}$$

which is completely analogous to a result obtained by Landau[10] for the uniform Fermi liquid.

V. SUMMARY AND CONCLUSIONS

We have constructed a theory to account for the effects of the electron-electron interaction on the low-temperature ($T \ll T_F$) electronic properties of disordered solids. Although we have placed special emphasis on the case where the single-particle states at the Fermi energy are localized, the theory we have constructed is easily adapted to the case of extended states. The low-lying excitations of the system are viewed as quasiparticles. Each quasiparticle is then viewed as a single entity moving in the self-consistent field of all the other quasiparticles. The resulting theory has the form of a mean-field theory for the quasiparticles.

We have shown that the low-temperature electronic contribution to the specific heat is unaffected by interactions; the only modification of the free-electron theory is the replacement of the free-electron density of states by the quasiparticle density of states. The static spin susceptibility has been calculated and shown to be either enhanced or reduced by the interactions. Conditions under which the series for the susceptibility may diverge and a phase transition into a "Fermi spin glass" state can occur have been briefly discussed. The transverse dynamic susceptibility observed in an electron-spin-resonance experiment has also been calculated. The position of the resonance is unaffected by either the disorder or interactions and occurs at the free-electron Larmor frequency. We have calculated the ac conductivity of localized quasiparticles and shown that Mott's result[1] $\mathrm{Re}\sigma(\omega) \propto \omega^2(\ln\omega)^4$ is still valid for interacting electrons; only the magnitude of $\mathrm{Re}\sigma(\omega)$ is affected by the interaction. We have studied the single-particle Green's function $G(E)$ for the interacting system and proven the self-consistency of the quasiparticle picture by showing that close enough to the Fermi energy one can approximate the spectral weight function arbitrarily well by a single spike at the quasiparticle energy. Finally, we have studied the Bethe-Salpeter equation satisfied by the four-point vertex for particle-hole scattering and have obtained a microscopic justification of the phenomenological transport equation used to study time-dependent phenomena.

As we have noted, our theory applies only to what we call "normal" Fermi systems. A condensation into an abnormal state would be signaled by the divergence of the power-series expression for some response function. The modification of this theoretical framework to accommodate abnormal ground states is under study.

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†Alfed P. Sloan Foundation Fellow.

[1]N. F. Mott, Adv. Phys. 16, 49 (1967); N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Clarendon, Oxford, 1971); E. N. Economou, M. H. Cohen, K. F. Freed, and E. S. Kirkpatrick, in *Amorphous and Liquid Semiconductors*, edited by J. Tauc (Plenum, New York, 1972); D. J. Thouless, Phys. Rep. 13C, 93 (1974).

[2]L. D. Landau, Zh. Eksp. Teor. Fiz. 30, 1058 (1956); 32, 59 (1957); 35, 97 (1958) [Sov. Phys.-JETP 3, 920 (1957); 5, 101 (1957) 35, 70 (1959)].

[3]P. W. Anderson, Comments Solid State Phys. 2, 193 (1970).

[4]P. W. Anderson, Phys. Rev. 109, 1492 (1958); E. N. Economou and Morrel H. Cohen, Phys. Rev. B 5, 2931 (1971).

[5]S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975); D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. 35, 1792 (1975).

[6]P. Nozières, *Theory of Inteacting Fermi Systems* (Benjamin, New York, 1964).

[7]D. A. Greenwood, Proc. Phys. Soc. Lond. 71, 585 (1958); Ryogo Kubo, J. Phys. Soc. Jpn. 12, 570 (1957).

[8]J. S. Langer, Phys. Rev. 124, 997 (1961).

[9]V. Ambegaokar, *Astrophysics and the Many-Body Problem*, edited by K. W. Ford (Benjamin, New York, 1963), p. 322.

[10]The relevant reference here is the third paper in the trilogy of papers by Landau quoted in Ref. 2. Other useful works are Ref. 6 and A. A. Abrikosov, L. P. Gorkov, and I. Ye. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon, New York, 1965).