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Relaxation-time anisotropy in p-type lead telluride

S. C. Gupta, K. N. S. Rajwanshi, and A. K. Sreedhar*
Solid State Physics Laboratory, Delhi, India

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The galvanomagnetic effects in p-type lead telluride having carrier concentrations in the range from 7×10^{17} to 1.3×10^{19} cm^{-3} have been measured at 77 and at 300 K. From these measurements the mobility anisotropy has been evaluated as a function of carrier concentration. These values have been compared with the effective-mass anisotropy reported by other workers and from this the anisotropy of the relaxation time has been calculated as a function of carrier concentration. The relaxation-time anisotropy at 77 K has been found to decrease with increase of concentration as expected from the theory of Gryaznov and Ravich, if the effective-mass anisotropy values of Cuff and others are taken to be correct. If, however, a constant effective-mass anisotropy as indicated by the work of Burke and of Schilz is taken as correct, the relaxation-time anisotropy shows a trend contrary to the conclusions of Gryaznov and Ravich.

I. INTRODUCTION

The effective-mass anisotropy in p-type lead telluride which has band extrema at the L points of the Brillouin zone has been determined by a number of workers.[1] As pointed out by Gryaznov and Ravich,[2] these experiments fall into two groups. Studies on cyclotron resonance, Shubnikov–de Haas and de Haas–van Alphen effects, oscillatory ultrasonic, and optical absorptions in a magnetic field, etc., give directly the effective-mass anisotropy factor $K \equiv m_l/m_t$, where m_l and m_t are, respectively, the longitudinal and transverse components of the effective-mass tensor. The mobility anisotropy, defined as $K' \equiv \mu_t/\mu_l$, where μ_t and μ_l are, respectively, the mobilities perpendicular and parallel to the symmetry axis of the energy ellipsoid, can be determined by the measurement of transport properties such as the weak-field magnetoresistance and Hall effect. These two anisotropies are related by $K' = K(\tau_t/\tau_l)$, where τ_t and τ_l are the transverse and longitudinal components of the relaxation-time tensor. In general, K and K' are not equal and their ratio depends upon the anisotropy of scattering. The relaxation-time anisotropy in the case of scattering by optic and acoustic phonons has been theoretically considered by Gryaznov and Ravich.[2] Their conclusion is that the relaxation time is highly anisotropic when optical-phonon scattering is the predominant mobility limiting process, while for purely acoustic-phonon scattering the relaxation time is isotropic.

Experimental determination of this anisotropy as a function of carrier concentration at 77 and at 300 K has been lacking. Shogenji[3] and Allgaier[4] determined the mobility anisotropy for two and six samples, respectively, of p-type lead telluride all at one carrier concentration of approximately 3×10^{18} cm^{-3}. Since the predominant scattering mechanism in p-type lead telluride at higher temperatures is expected to change from optic to acoustic phonon scattering as the carrier concentration increases from about 10^{18} cm^{-3} to 10^{20} cm^{-3} and above, it is of interest to determine the relaxation-time anisotropy as a function of carrier concentration.

We have therefore measured the mobility anisotropy for series of samples of lead telluride in the carrier concentration range from 7×10^{17} cm^{-3} to 1.3×10^{19} cm^{-3}. The results have been analyzed to obtain the relaxation-time anisotropy.

II. EXPERIMENTAL DETAILS

p-type lead telluride was synthesized from spectroscopically pure lead and tellurium (Johnson and Matthey, U.K.). Single crystals were grown by the vertical Bridgman method. The crystals were found to have an etch pit density of about 10^5 cm^{-2}. The crystals were subjected to heat treatment as discussed by Scanlon[5] to obtain various hole concentrations in the range 7×10^{17} cm^{-3} to 1.3×10^{19} cm^{-3}. Electrical current leads and potential probes were of platinum wires of size 0.003 in. attached to the sample by condenser discharge. The samples were rectangular paralellepipeds of dimensions approximately $0.5 \times 1.0 \times 5$ mm^3. The samples were cut along the cubic axes of the crystal so that the direction of the current was along the [100] direction, and the direction of the magnetic field could be along any one of the [100], [010], or [001] directions. Standard dc methods[6] were used to measure the various galvanomagnetic properties, viz., the longitudinal and transverse magnetoresistance and the planar and normal Hall effects.

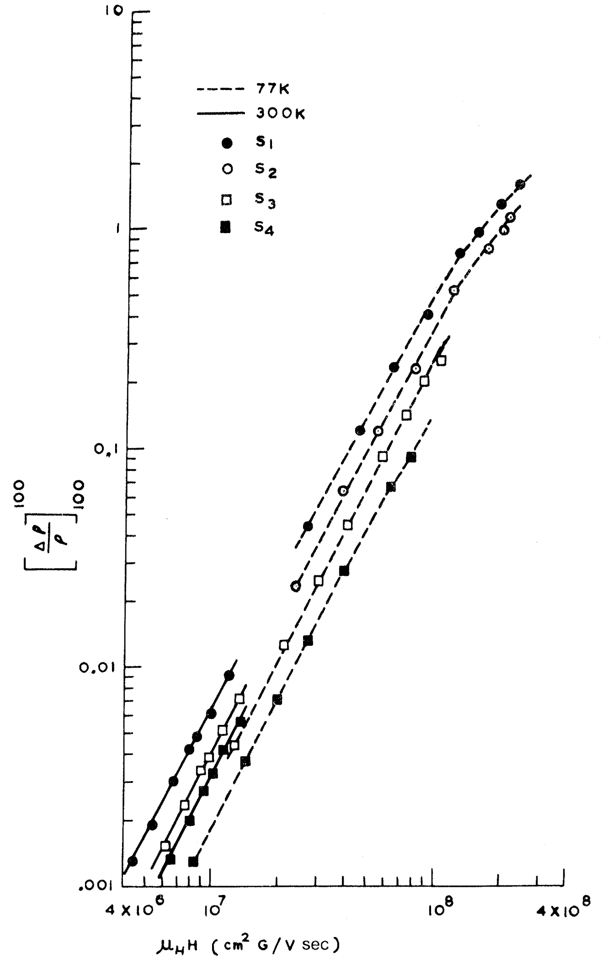

FIG. 1. Longitudinal magnetoresistance as a function of the product of the mobility by the magnetic field strength for samples S_1, S_2, S_3, and S_4 with various hole concentrations as mentioned in text.

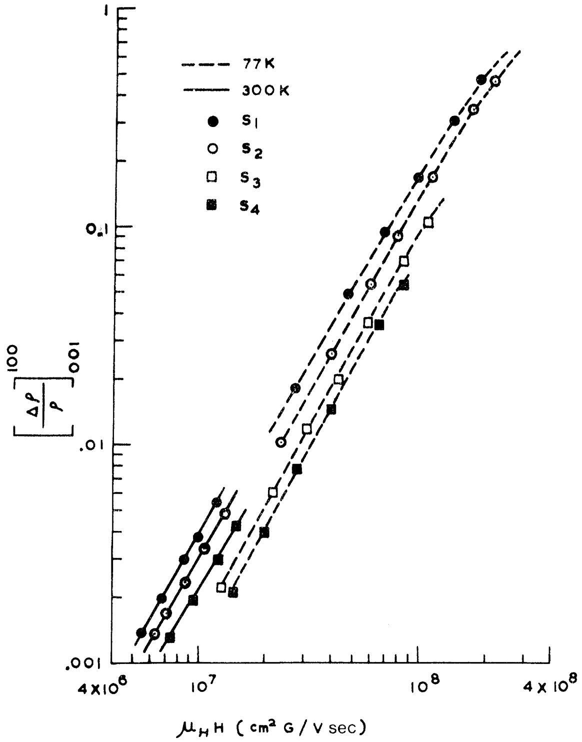

FIG. 2. Transverse magnetoresistance as a function of the product of the mobility by the magnetic field strength for samples S_1, S_2, S_3, and S_4 with various hole concentrations as mentioned in text.

III. RESULTS

Figures 1 and 2 show the variation of longitudinal and transverse magnetoresistance as a function of the product $\mu_H H$ of the Hall mobility μ_H and the magnetic field strength at 77 and 300 K for various hole concentrations in the range 7×10^{17} cm^{-3} to 1.3×10^{19} cm^{-3}. The Hall mobility of our samples is given in Table I. The longitudinal magnetoresistance is greater than the transverse magnetoresistance. Both of these vary as the square of the magnetic field up to $\mu_H H/10^8 = 0.47$ and satisfy the weak field approximation. For $\mu_H H/10^8 > 0.47$ the magnetoresistance varies less strongly than the square of the magnetic field. From these measurements and the measurements of planar Hall effect, we have calculated the Seitz symmetry coefficients in the low-field limit and these are also given in Table I. The interrelationship between these coefficients confirms the ⟨111⟩ band model, as predicted by various workers,[1] up to hole concentrations of 1.3×10^{19} cm^{-3}.

Using the relation

TABLE I. Experimental Hall mobility and Seitz-coefficient values of p-type lead telluride samples.

Sample No.	Hole concentration (cm^{-3})	Hall mobility (cm^2/V sec) 77 K	300 K	b 77 K	300 K	c 77 K	300 K	$b+c+d$ 77 K	300 K	$b+c$ 77 K	300 K
S_1	7.05×10^{17}	23 100	849	0.227	0.430	−0.225	−0.421	0.554	0.605	0.002	0.009
S_2	2.00×10^{18}	19 800	872	0.179	0.355	−0.180	−0.345	0.417	0.522	−0.001	0.010
S_3	6.95×10^{18}	10 500	910	0.131	0.301	−0.129	−0.294	0.249	0.409	0.002	0.007
S_4	1.30×10^{19}	7020	898	0.110	0.258	−0.109	−0.251	0.187	0.345	0.001	0.007

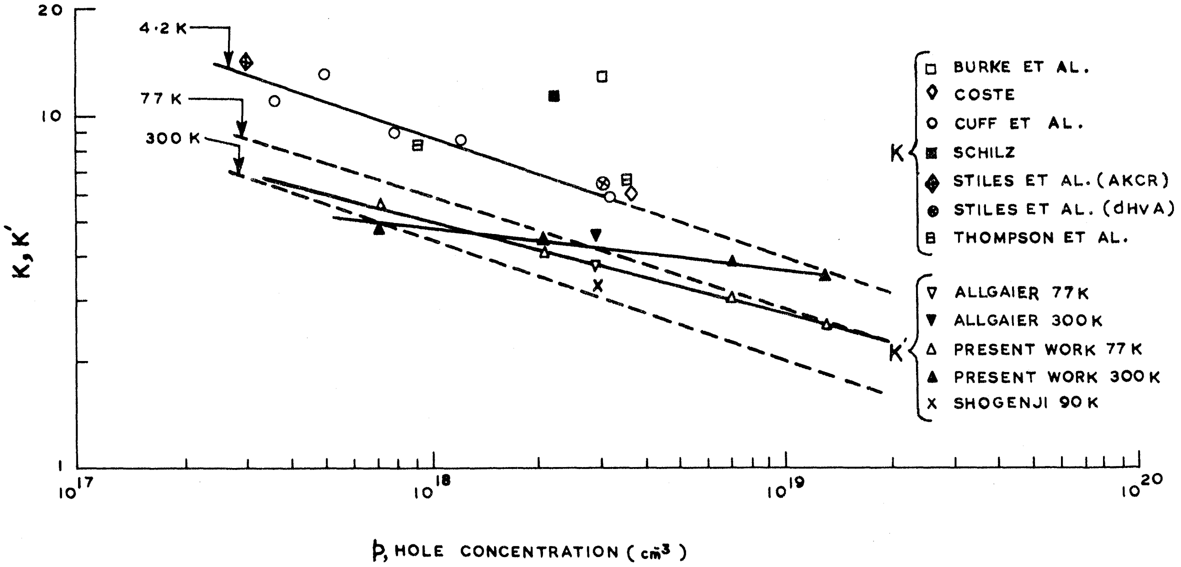

FIG. 3. Effective-mass and mobility anisotropies as a function of hole concentration.

$$\left(\frac{\Delta\rho}{\rho}\right)^{100}_{100}\Big/\left[\left(\frac{\Delta\rho}{\rho}\right)^{100}_{001}+\left(\frac{\mu_H H}{10^8}\right)^2\right]=\frac{2(K'-1)^2}{(2K'+1)(K'+2)} \qquad (1)$$

(where $\Delta\rho/\rho$ is the magnetoresistance, the superscripts and subscripts denoting, respectively, the direction of magnetic field and the direction of electric current), K' was calculated from the longitudinal and transverse magnetoresistances both at 77 and 300 K as a function of hole concentration. The results are shown in Fig. 3. Equation (1) gives two values of K', $K' < 1$ and $K' > 1$, for each value of magnetoresistance. However, the large ratio of longitudinal to transverse magnetoresistance as well as the theoretical models favor $K' > 1$.

IV. DISCUSSION

From the magnetoresistance measurements we have determined the mobility anisotropy K'. In order to calculate the anisotropy in the relaxation time, a knowledge of the variation of mass anisotropy K with carrier concentration at 77 and at 300 K is required. Unfortunately no experimental data on the mass anisotropy as a function of concentration at these temperatures are available. The only extensive data available are at 4.2 K. These are due to Cuff, Ellet and Kuglin,[7] Burke, Houston, and Savage,[8] Schilz,[9] Stiles, Burstein, and Langenberg,[10,11] Coste,[12] and Thompson, Aron, Chandrasekhar, and Langenberg.[13] The published values of mass anisotropy obtained by them as a function of carrier concentration are shown in Fig. 3. It is seen that the results of Cuff *et al.*, Thompson *et al.*, Stiles *et al.*, and Coste are consistent with one another. These results give a mass anisotropy $K = 13$ at a low carrier concentration of 3×10^{17} cm^{-3}, decreasing to a value of approximately 6.5 at a higher concentration of about 3×10^{18} cm^{-3}. On the other hand, Burke *et al.* obtained a value $K = 13$ at the concentration 3×10^{18} cm^{-3} and Schilz a value $K = 11.2$ at the concentration 2.3×10^{18} cm^{-3}. Burke *et al.* concluded that their measurements of $K = 13$ are more reliable than that of Cuff *et al.*, because of better resolution that could be obtained by them with the use of Fourier analysis and larger magnetic fields. They also pointed out that Cuff's measurements of K at higher concentrations could not account for all the carriers as compared with the carrier count measured by high-field Hall coefficient. Since the value of K at concentration of 3×10^{17} cm^{-3} obtained by Cuff *et al.*, was also 13, at which concentration the carrier count agreed with Hall-effect data, Burke *et al.* concluded that K should be independent of carrier concentration in the range 10^{17} to 3×10^{18} cm^{-3}. It should also be noted that the angular variation of Burke's data is complete, permitting a direct determination of K without the need to fit the data to a separate measurement of carrier concentration. The differing results of Burke *et al.* and of Schilz as compared with those of other workers have been discussed by Thompson *et al.* who concluded on the basis of their magnetoelastic and magnetostriction amplitude coefficients that the lower values of K obtained by them are more nearly correct.

If we accept the results of Cuff *et al.*, Thompson *et al.*, and of Stiles *et al.*, the mass anisotropy decreases by a factor of about 2 when the carrier concentration increases from about 3×10^{17} cm^{-3} to about 3×10^{18} cm^{-3}. These results are in conformity with Cohen's[14] model. According to this model, m_l remains constant and m_t increases with the increase of both the concentration and temperature. The variation of m_t with temperature is related to the change in band gap with temperature.

m_t at any temperature T is given by[15, 16]

$$(m_t)_T=(m_t)_0[(E_g)_T/(E_g)_0], \qquad (2)$$

where

$$(E_g)_T=(E_g)_0+T\left(\frac{\partial E_g}{\partial T}\right). \qquad (3)$$

The subscript "0" denotes the values at 0 K, and $\partial E_g/\partial T$ is the temperature variation of the band gap. Using the reported values by Dixon and Riedl,[15] $(E_g)_0=0.14$ eV and $\partial E_g/\partial T=4.9\times10^{-4}$ eV/°K, we obtain m_t at 77 and 300 K from (2) and (3) as follows:

$$(m_t)_{77\ \mathrm{K}}=1.3(m_t)_0, \quad (m_t)_{300\ \mathrm{K}}=2.0(m_t)_0.$$

Applying these temperature corrections to the experimental data at 4.2 K, we obtain values of K at 77 and 300 K plotted in Fig. 3 by dashed lines. The experimental values of Cuff *et al.* and Thompson *et al.* are available only up to a carrier concentration of about 3.5×10^{18} cm^{-3}. In Fig. 3 we have made a smooth extrapolation of Cuff's data up to concentration 1.3×10^{19} cm^{-3}. The ratio K/K' determines the relaxation-time anisotropy K_τ.

K_τ is nearly equal to unity at 77 K and at a carrier concentration 1.3×10^{19} cm^{-3}. It increases with decrease in carrier concentration. The value $K_\tau=1$ at high concentration is as expected because at this carrier concentration the scattering, which is mainly due to acoustic phonons, is isotropic. At the lower concentrations (less than 10^{18} cm^{-3}), scattering by polar optical phonons becomes increasingly important and the relaxation time is expected to be anisotropic. The theoretical calculations of Gryaznov and Ravich give a value K_τ between 2 and 3 for optical-phonon scattering. The experimental value at the lowest concentration measured, viz. at 7×10^{17} cm^{-3} is about 1.4. This difference between the experimental and theoretical values of the anisotropy may be due to (i) ignoring the effect of screening by free carriers in the theoretical derivation of K_τ; (ii) the temperature dependence of m_t being smaller than that used above in arriving at the value of K at 77 K; (iii) m_l also being a function of temperature and concentration, though a weaker function than that for m_t, i.e., the Cohen model may not be strictly applicable.

At 300 K, it is seen that K_τ varies from unity at carrier concentration 7×10^{17} cm^{-3} to about 0.53 at a concentration of 1.3×10^{19} cm^{-3}. Since the relative importance of acoustic scattering increases with carrier concentration and temperature, K_τ should tend to unity at higher concentrations. The reasons for this disagreement may be due to unjustified extrapolation of experimental values of K measured at 4.2 K and low carrier concentrations to 300 K and much higher carrier concentrations based on the Cohen model. The applicability of the Cohen model to p-type lead telluride at room temperature and above and for higher carrier concentrations may need revision, as has been discussed by Ravich.[1] Incidentally, such low values of K_τ have been observed in n-type silicon by Long and Myers.[17]

If we assume that K is independent of carrier concentration as suggested by Burke *et al.* and further assume that m_t varies with temperature as given by Eq. (2), we obtain K equal to 10 and 6.5, respectively, at 77 and 300 K. We see that at 77 K, K_τ varies from a value of 1.8 at carrier concentrations 7×10^{17} cm^{-3} to a value of about 3.8 at a concentration of 1.3×10^{19} cm^{-3}. At 300 K, K_τ varies from 1.3 to 1.8 for these concentrations, respectively. Thus on this basis, the relaxation time becomes increasingly anisotropic as the carrier concentration increases, i.e., as the acoustic scattering becomes more important as compared with the optical-phonon scattering. This trend is contrary to the conclusion of Gryaznov and Ravich.

V. CONCLUSION

The longitudinal and transverse magnetoresistance for several samples of p-type PbTe in the carrier concentration range 7×10^{17} cm^{-3} to 1.3×10^{19} cm^{-3} have been experimentally measured. From these measurements the mobility anisotropy for p-type PbTe has been calculated as a function of carrier concentration both at 77 and at 300 K. These results have been compared with the values of mass anisotropy of PbTe at 4.2 K to obtain the relaxation-time anisotropy. An analysis of our results taking into account the conclusions of Burke and others shows that the relaxation-time anisotropy at 77 and 300 K increases with increase of carrier concentration if we apply temperature correction to Burke's results on the basis of Cohen's model, but assume that the effective-mass anisotropy is independent of temperature. This conclusion is contrary to the theoretical analysis of Gryaznov and Ravich.

The relaxation-time anisotropy obtained on the basis of the results of Cuff and others shows the expected behavior at 77 K, viz. the relaxation time is anisotropic at low carrier concentration where polar-optical-model scattering is the dominant mobility limiting process. At higher carrier concentrations the relaxation time tends to be isotropic as is expected for the case of acoustic-phonon scattering limiting the mobility. The results at 300 K do not agree well with the extrapolation based on Cohen's model of mass anisotropy measurements at 4.2 K. The possible reasons for this have been briefly discussed. However, a more detailed analysis is possible only when experimental values of mass anisotropy as a function of carrier concentration become available at 77 and at 300 K.

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*Now at the Defense Science Laboratory, Delhi, India.

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