Effect of magnon-phonon thermal relaxation on heat transport by magnons*

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(Received 24 June 1976)

The experimental evidence for heat transport by magnons in magnetic insulators is reviewed. It is noted that in a thermally isolated system, the magnon temperature comes to equilibrium with the phonon temperature with a finite relaxation time τ_{mp}. Since a conventional thermal-conductivity experiment is inherently a nonequilibrium situation, the steady-state magnon temperature gradient will differ from that of the phonons. A calculation is presented to show how the experimentally measured conductivity depends on τ_{mp}, as well as on the intrinsic magnon and phonon conductivities K_m and K_p. In the limit of very long relaxation times, only K_p is experimentally observed, regardless of the magnitude of K_m. The calculation is illustrated for the ferrimagnet YIG and the antiferromagnet MnF_2, using magnon-phonon relaxation times measured by magnetic-resonance experiments. It is shown that τ_{mp} for YIG is short enough to allow magnon heat transport to be observed, which is in agreement with experimental results. It is also shown that the long relaxation time for MnF_2 may be responsible for the absence of magnon conductivity in this material. This general explanation may also apply to many other magnetic systems.

I. INTRODUCTION

In a magnetically ordered crystal the propagating excitations of the spin system, known as magnons, can transport heat in the same manner as the more familiar lattice excitations, or phonons. In 1955, Sato[1] pointed out that at liquid-helium temperatures the magnon system can have a specific heat equal to, or greater than, that of the phonons. The velocities of the two excitations are comparable, and at sufficiently low temperatures their mean free paths are equal, since both are limited only by the boundaries of the sample. Therefore, at low temperatures a large fraction of the total thermal conductivity of the crystal may be due to the magnons.

There has been considerable interest in finding experimental evidence for magnon heat transport. These experiments have been confined to dielectric magnetic materials, because the thermal conductivity of a metal is dominated by the conduction electrons. Magnon conductivity can be recognized by a departure of the thermal conductivity from the T^3 behavior expected for boundary-limited phonons. Another useful technique is the measurement of the conductivity in an external magnetic field. For a ferromagnet or a ferrimagnet, the applied field raises the magnon energies, which decreases their thermal population, and thus lowers the magnon component of the conductivity. In an antiferromagnet, one magnon branch is lowered, causing a net increase in the magnon contribution.

The earliest example of heat conduction by magnons was reported in 1962 for the ferrimagnet yttrium iron garnet (YIG).[2-4] The thermal conductivity of YIG exhibited the expected dependences on temperature and on magnetic field. By measuring the conductivity in fields large enough to completely remove the magnon contribution, it was estimated that about two-thirds of the zero-field conductivity was due to the magnons.[5] The experimental results for YIG have also been quantitatively explained in terms of theoretical predictions for the magnon and phonon components.[6,7]

Experimental evidence for magnon conductivity has also been reported for the ferromagnets EuS (Ref. 8) and[9] $CuCl_2(CH_3NH_3Cl)$; the ferrimagnet Li ferrite[5]; and the antiferromagnets $CoCl_2\cdot 6H_2O$,[10] $Co[(NH_2)_2CS]_4\cdot Cl_2$,[11] and $GdVO_4$.[12] However, compared to YIG, the evidence for these materials is not as conclusive. In some cases [EuS and $CuCl_2$-(CH_3NH_3Cl)] it is based only on the temperature dependence of the conductivity, and in other cases ($CoCl_2\cdot 6H_2O$, Li ferrite, and $GdVO_4$) only on the magnetic field dependence. Also, in most cases there has been no attempt to compare the results quantitatively with theoretical predictions.

On the other hand, many magnetic materials have been investigated without finding evidence of magnon conductivity. A partial list includes the ferromagnets $GdCl_3$ (Refs. 5 and 13) and[14] $CuK_2Cl_4\cdot 2H_2O$; the ferrimagnets[15] $MnFe_2O_4$, $Co(Zn)Fe_2O_4$, and $Mn(Zn)Fe_2O_4$; and the antiferromagnets $CuCl_2\cdot 2H_2O$,[10] $MnCl_2\cdot 4H_2O$,[13,16] $FeCl_2$,[17] $RbMnF_3$,[18] and MnF_2.[18,19] Therefore, while the theoretical prediction of heat transport by magnons has long been established, experimental confirmation of this prediction has been somewhat elusive. In fact, in most magnetic systems, the experimental evidence indicates that magnons do not contribute to the conductivity.

One possible reason for this is that the magnon conductivity may be much less than its boundary limit because of other magnon scattering process-

es. If impurities with different spins or exchange constants substitute for magnetic ions in the crystal, these magnetic impurities can strongly scatter the magnons. This has been analyzed by Callaway[20,21] for the case of ferromagnets. It was shown that magnetic impurity scattering can cause a large reduction in the magnon conductivity from its boundary-limited value. However, it would seem to be an unfortunate coincidence if the magnons are scattered so much more strongly than the phonons by residual defects in so many materials.

The magnons can also be scattered by phonons. The interaction is strongest at the point where the dispersion curves cross, since at this point energy and wave vector can be conserved in a one-phonon–one-magnon interaction. At the point of intersection a gap is introduced into the dispersion curves, and near the gap the excitations are neither magnons nor phonons, but coupled magnetoelastic modes. The size of the gap is determined by the magnitude of the magnetoelastic coupling constant. If this is large, it can cause a significant reduction in the magnon conductivity. This has been shown to be the case for YIG.[7]

The magnon-phonon interaction can also affect the phonon conductivity. Because of the gap, a band of phonons is effectively removed from the heat-carrier spectrum. Therefore, the magnons can act like a resonant phonon scatterer, and produce resonance dips in the temperature dependence, or magnetic field dependence of the conductivity. This type of behavior has been found in $GdCl_3$,[5,13] $MnCl_2 \cdot 4H_2O$,[13,16] and $FeCl_2$.[17]

Higher-order interactions such as one-phonon–two-magnon processes are also possible.[22] Unlike the resonant magnon-phonon interaction, these are not limited to a narrow range of frequency and wave vector, but they also can cause mutual scattering of the magnons and phonons.[1]

Therefore, in searching for evidence of magnon conductivity, it would seem reasonable to investigate systems with weak magnon-phonon coupling in order to minimize these effects. However, a fact which has not been universally recognized is that some degree of interaction between the magnons and phonons is necessary for any magnon heat transport to be observed in a conventional thermal-conductivity experiment. Since only phonons are generated by the heater at one end of the sample, and absorbed at the cold end, this heat can enter or leave the magnon system only via magnon-phonon interactions. Therefore, the heat flux in the magnon system depends on the magnon-phonon coupling, as well as on the magnon conductivity. If the coupling is too small, there will be no magnon heat flux, and only the phonon conductivity will be experimentally observed, even though the intrinsic magnon conductivity may be large.

These ideas are quantitatively developed in Sec. II for a simple model. In Sec. III, the results of the calculation are illustrated for two well-known magnetic systems, YIG and MnF_2.

II. CALCULATION

Expressing the total conductivity as a simple sum of the magnon and phonon conductivities implicitly assumes that the temperature gradient in the magnon system is the same as that in the phonon system. Since thermal transport is inherently a nonequilibrium phenomenon, this is not necessarily the case. In a thermally isolated system, the difference between the magnon and phonon temperatures, T_m and T_p, would decay exponentially. The magnon-phonon relaxation time τ_{mp} is defined by[23]

$$\frac{d}{dt}\Delta T = -\frac{\Delta T}{\tau_{mp}}, \tag{1}$$

where $\Delta T = T_p - T_m$. It is easy to show from Eq. (1) that T_m and T_p approach each other according to the expressions

$$\frac{dT_p}{dt} = \frac{C_m}{C_T}\frac{T_m - T_p}{\tau_{mp}} \tag{2a}$$

and

$$\frac{dT_m}{dt} = \frac{C_p}{C_T}\frac{T_p - T_m}{\tau_{mp}}, \tag{2b}$$

where C_m and C_p are the specific heats of the magnon and phonon systems, and $C_T = C_p + C_m$.

Now consider a thermal conductivity sample of length L and cross-sectional area A, with heat flow along the x direction. A total heat flux Q is supplied at $x = -\frac{1}{2}L$, and absorbed at $x = \frac{1}{2}L$. In general, the temperature of the magnon system $T_m(x)$ at any point x will differ from that of the phonons $T_p(x)$. If there were no heat flow down the sample through the magnon system, the magnon temperature at each point would come into equilibrium with the phonon temperature, as in Eq. (2b). Therefore, in volume element $A\,dx$ of the sample, the amount of heat per unit time flowing into the magnon system from the phonon system would be

$$dP_m(x) = C_m \frac{dT_m(x)}{dt} A\,dx$$

$$= \frac{C_p C_m}{C_T}\frac{T_p(x) - T_m(x)}{\tau_{mp}} A\,dx, \tag{3}$$

where the specific heat is now explicitly the heat capacity per unit volume.

However, if the magnon system has a finite conductivity K_m, the heat which flows into it will be

conducted along the sample and a steady state will be reached. The contribution of a small length dx to the heat flux in the magnon system is

$$dQ_m(x) = \frac{dP_m(x)}{A} = \frac{C_p C_m}{C_T} \frac{T_p(x) - T_m(x)}{\tau_{mp}} dx . \quad (4)$$

Therefore, the total magnon heat flux at any point x is given by

$$Q_m(x) = \frac{C_p C_m}{C_T} \frac{1}{\tau_{mp}} \int_{-L/2}^{x} [T_p(x') - T_m(x')]\, dx' . \quad (5)$$

But, by definition of the magnon conductivity,

$$Q_m(x) = -K_m \frac{dT_m(x)}{dx} . \quad (6)$$

Combining Eqs. (5) and (6), and differentiating yields a second-order differential equation for the magnon temperature

$$\frac{d^2 T_m(x)}{dx^2} + \frac{C_p C_m}{C_T} \frac{1}{K_m \tau_{mp}} [T_p(x) - T_m(x)] = 0 . \quad (7)$$

The total heat flux Q is divided between the magnons and the phonons according to

$$Q = Q_p(x) + Q_m(x) = -K_p \frac{dT_p(x)}{dx} - K_m \frac{dT_m(x)}{dx} . \quad (8)$$

Therefore, $T_p(x)$ is related to $T_m(x)$ by

$$\frac{dT_p(x)}{dx} = -\frac{Q}{K_p} - \frac{K_m}{K_p} \frac{dT_m(x)}{dx} , \quad (9)$$

with the boundary condition that at the center of the sample, the phonon and magnon temperatures are both equal to the average sample temperature T_0; i.e.,

$$T_m(0) = T_p(0) = T_0 . \quad (10)$$

Solving Eq. (9) using (10), it is found that

$$T_p(x) = \frac{K_T}{K_p} T_0 - \frac{Q}{K_p} x - \frac{K_m}{K_p} T_m(x) , \quad (11)$$

where $K_T = K_p + K_m$. Substituting (11) into Eq. (7), the differential equation for $T_m(x)$ becomes

$$\frac{d^2 T_m(x)}{dx^2} + A^2 \left(T_0 - \frac{Q}{K_T} x - T_m(x) \right) = 0 , \quad (12)$$

where

$$A^2 = \frac{C_p C_m}{C_T} \frac{K_T}{K_p K_m} \frac{1}{\tau_{mp}} . \quad (13)$$

Since heat enters or leaves the sample at each end only via the phonon system, the magnon heat flux must be zero at both ends. Therefore, from Eq. (6),

$$\left. \frac{dT_m}{dx} \right|_{L/2} = \left. \frac{dT_m}{dx} \right|_{-L/2} = 0 . \quad (14)$$

Solving Eq. (12) with the boundary conditions (10) and (14), the magnon temperature is found to be

$$T_m(x) = T_0 - \frac{Q}{K_T} \left(x - \frac{\sinh Ax}{A \cosh \frac{1}{2} AL} \right) . \quad (15)$$

By substituting (15) into Eq. (11), the phonon temperature can be expressed

$$T_p(x) = T_0 - \frac{Q}{K_T} \left(x + \frac{K_m}{K_p} \frac{\sinh Ax}{A \cosh \frac{1}{2} AL} \right) . \quad (16)$$

Typical solutions for $T_p(x)$ and $T_m(x)$ are illustrated in Fig. 1.

The parameter A is related to the coupling between the magnons and phonons. If they are perfectly coupled, $\tau_{mp} \to 0$, so $A \to \infty$. In this limit the magnon and phonon temperatures are the same; i.e.,

$$T_p(x) \to T_m(x) \to T_0 - (Q/K_T)x . \quad (17)$$

On the other hand, if there were no coupling between the two systems, $\tau_{mp} \to \infty$, so $A \to 0$. In this case the solutions are

$$T_p(x) \to T_0 - (Q/K_p)x$$

and (18)

$$T_m(x) \to T_0 ,$$

so there would be no gradient at all in the magnon temperature. These limiting cases are also shown in Fig. 1.

In a thermal-conductivity experiment, thermometers are attached to the sample which measure the difference between the temperature of the phonon system at either end

$$\Delta T_p = T_p(\tfrac{1}{2}L) - T_p(-\tfrac{1}{2}L) . \quad (19)$$

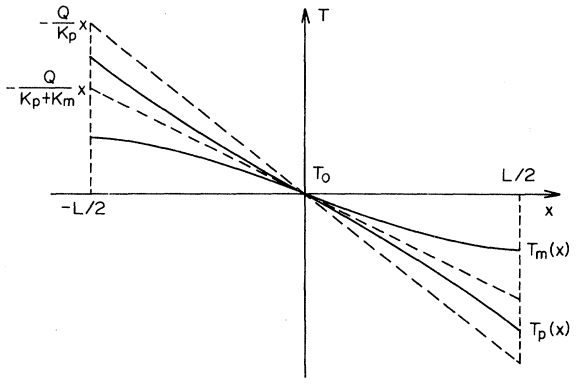

FIG. 1. Temperature profiles in a thermal conductivity sample. The sample length is L, and its average temperature is T_0. Typical solutions to Eqs. (15) and (16) for the magnon temperature $T_m(x)$, and the phonon temperature $T_p(x)$, respectively, are shown by the solid curves. The limiting cases given by Eqs. (17) and (18) for $\tau_{mp} \to 0$ and ∞, respectively, are shown by the dashed lines.

Knowing the heat flux Q supplied by the heater, the effective thermal conductivity is then calculated from

$$K_{\text{eff}} = -QL/\Delta T_p . \tag{20}$$

Therefore, the presence of magnon heat transport can be detected only in so far as its affects the phonon temperature gradient. From Eq. (16), ΔT_p is given by

$$\Delta T_p = -\frac{Q}{K_T} L\left(1 + \frac{K_m}{K_p} \frac{\tanh \frac{1}{2}AL}{\frac{1}{2}AL}\right). \tag{21}$$

Therefore, the effective conductivity is

$$K_{\text{eff}} = K_T\left(1 + \frac{K_m}{K_p} \frac{\tanh \frac{1}{2}AL}{\frac{1}{2}AL}\right)^{-1}. \tag{22}$$

If the magnon-phonon relaxation time is very short, $A \to \infty$ and $K_{\text{eff}} \to K_T$. Therefore, only in this limit is the experimentally measured conductivity the sum of the magnon and phonon conductivities. If the relaxation time is very long, $A \to 0$ so from Eq. (22), $K_{\text{eff}} \to K_T\,(1 + K_m/K_p)^{-1} = K_p$. Therefore, in this limit only the phonon conductivity can be measured, regardless of the magnitude of the intrinsic magnon conductivity.

III. EXAMPLES AND DISCUSSION

The order of magnitude of the magnon-phonon relaxation time which can suppress the magnon conductivity may be illustrated by considering two examples: The ferrimagnet YIG and the antiferromagnet MnF_2. As pointed out in Sec. I, it is well established that YIG exhibits a magnon component in its thermal conductivity.[2-7] On the other hand, it has been demonstrated that MnF_2 shows no evidence of magnon conduction in either the temperature dependence or the magnetic field dependence of its conductivity.[18,19] Moreover, this result is in disagreement with calculations of the boundary-limited magnon conductivity[19] which predict a large magnon contribution. These two materials are chosen because both are very well studied magnetic systems, and some information about magnon-phonon relaxation times is available from magnetic-resonance studies.

A. YIG

In order to calculate K_{eff}, it is necessary to have expressions for K_p, K_m, C_p, and C_m. It has been shown[2-4] that the thermal conductivity of YIG in zero magnetic field can be reasonably well explained by assuming boundary-limited conductivities for both the magnons and the phonons, but with a boundary-scattering mean free path approximately an order of magnitude smaller than the smallest dimension of the sample. Friedberg and Harris[2] have suggested that these internal boundaries are thin layers, rich in Fe^{++}, which are produced during the crystal-growing process.

K_{eff} was calculated for a hypothetical thermal-conductivity sample with the typical dimensions of 3.0 cm long and 0.5 cm in cross-sectional diameter. Therefore, the boundary limited mean free path for YIG is taken to be $\Lambda_b = 0.05$ cm. The phonon and magnon conductivities can then be calculated from[2,6]

$$K_p = \frac{2\pi^2}{15} \frac{k_B^4 T^3}{\hbar^3 V_p^2} \Lambda_b \tag{23a}$$

and

$$K_m = \frac{\zeta(3)}{\pi^2} \frac{k_B^3 T^2}{\hbar D} \Lambda_b . \tag{23b}$$

In (23a), V_p is an average sound velocity. For YIG, the velocities of transverse and longitudinal phonons are $V_T = 3.87 \times 10^5$ and $V_L = 7.17 \times 10^5$ cm/sec.[24] In (23b), ζ is the Riemann ζ function, and D is defined by the magnon-dispersion relation $\hbar\omega = Dq^2$. For YIG, $D = 8.3 \times 10^{-36}$ J cm^2.[25] Therefore, $K_p = 0.0104T^3$ and $K_m = 0.0183T^2$ W cm^{-1}K^{-1}.

Simple expressions can also be obtained for the phonon and magnon specific heats[25]; i.e.,

$$C_p = \frac{2\pi^2}{5} \frac{k_B^4 T^3}{\hbar^3 V_p^3} \tag{24a}$$

and

$$C_m = \frac{15}{32} \frac{\zeta(\frac{5}{2})}{\pi^{3/2}} \frac{k_B^{5/2} T^{3/2}}{D^{3/2}} . \tag{24b}$$

Therefore, $C_p = 1.52 \times 10^{-6} T^3$ and $C_m = 3.34 \times 10^{-6} T^{3/2}$ J K^{-1} cm^{-3}.

Using Eqs. (23) and (24), K_{eff} for YIG was calculated from Eq. (22) as a function of τ_{mp}. The results for various temperatures are shown in Fig. 2 which gives K_{eff} in terms of K_p. Since K_m has a T^2 temperature dependence, whereas K_p varies as T^3, the magnon contribution to the total conductivity increases with decreasing temperature. However, it can be seen from the diagram that if the magnon-phonon relaxation time is greater than 10^{-3} sec, K_{eff} for any temperature differs from K_p by less than 5%. Therefore, only the phonon conductivity could be observed experimentally. This is shown as $\tau_{\max}$ in Fig. 2.

On the other hand, if τ_{mp} is shorter than the mean free time for boundary scattering, the magnon-phonon interaction could cause significant mutual scattering of the magnons and phonons which would reduce both K_m and K_p from their boundary limits. This is shown as $\tau_b = \Lambda_b/V_p = 10^{-7}$ sec in Fig. 2. Therefore, there is a range of relaxation times, $\tau_b < \tau_{mp} < \tau_{\max}$, over which one could hope to observe magnon heat transport.

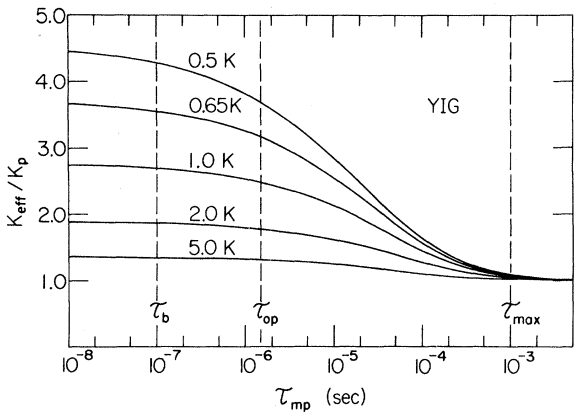

FIG. 2. Dependence of the effective conductivity of YIG on the magnon-phonon relaxation time. K_{eff} is given in terms of K_p, the boundary limited phonon conductivity. The relaxation times labeled $\tau_{\max}$, τ_b, and τ_{0p} are, respectively, the maximum relaxation time for observable magnon heat transport, the mean free time for boundary scattering, and the magnon-phonon relaxation time estimated from magnetic resonance measurements.

Magnon-phonon relaxation times for YIG have been investigated by Spencer and LeCraw.[26] By using a modulation technique, they were able to directly measure the time required for the ferromagnetic resonance mode to relax to the lattice. At their lowest temperature of 2.5 K, they found that $\tau_{0p}=1.5\times10^{-6}$ sec which is also shown in Fig. 2. Therefore, if the relaxation time of the $q=0$ magnon is representative of the magnon system as a whole, this indicates that τ_{mp} is short enough to allow almost all of the magnon conductivity to be observed in YIG. This, of course, is consistent with the experimental results.

B. MnF_2

The phonon boundary-limited conductivity K_p and the specific heat C_p for MnF_2 can be simply expressed, as in the case of YIG. For $\Lambda_b=0.5$ cm, it is found that $K_p=0.219T^3$ W cm^{-1} K^{-1},[19] and $C_p=4.95\times10^{-6}T^3$ J K^{-1} cm^{-3}.[27] However, since the magnon dispersion curves for MnF_2 have a large anisotropy gap of 12.5 K,[28] it is not possible to express K_m and C_m in simple analytical forms.

Moreover, the large anisotropy gap means that few magnon modes are thermally populated at low temperatures, so the magnon conductivity is small. However, since MnF_2 is antiferromagnetic, K_m can be increased by applying an external magnetic field. Therefore, K_{eff} was calculated for a field $H=60$ kOe, which is the largest field in which the conductivity has been measured.[19]

Since the magnon dispersion relations $\omega(\vec{q})$ are well known,[28] the magnon conductivity and specific heat can be calculated numerically from[19,27]

$$K_m=\frac{k_B}{6\pi^4}\Lambda_b\sum_j\int\left(\frac{\hbar\omega}{k_BT}\right)^2\frac{e^{\hbar\omega/k_BT}}{(e^{\hbar\omega/k_BT}-1)^2}\,|\vec{\nabla}_q\omega|\times\frac{\cos^2\theta}{\sin\theta}\,d^3q \tag{25a}$$

and

$$C_m=\frac{k_B}{(2\pi)^3}\sum_j\int\left(\frac{\hbar\omega}{k_BT}\right)^2\frac{e^{\hbar\omega/k_BT}}{(e^{\hbar\omega/k_BT}-1)^2}\,d^3q\,, \tag{25b}$$

where $\vec{\nabla}_q\omega$ is the magnon group velocity, θ is the angle between $\vec{\nabla}_q\omega$ and the heat flow direction, and

$$\hbar\omega(\vec{q},j)=\hbar\omega(\vec{q})\pm g\mu_BH\,. \tag{26}$$

The results for the calculated effective conductivity at 60 kOe as a function of τ_{mp} are shown in Fig. 3. The magnon component reaches a maximum of about 80% of K_p at a temperature of 5 K.[19] The maximum relaxation time for observable magnon heat transport is $\tau_{\max}=1.5\times10^{-4}$ sec. For the [001] direction of MnF_2, the phonon velocities are $V_T=2.85\times10^5$ and $V_L=6.65\times10^5$ cm/sec.[29] Therefore, the mean free time for boundary scattering is $\tau_b=\Lambda_b/V_p=1.2\times10^{-6}$ sec.

It can be seen from the diagram that the range of magnon-phonon relaxation times for which magnon conduction can be observed is much smaller for MnF_2 than it is in the case of YIG. Moreover, the magnon-phonon interaction is known to be much weaker in MnF_2 than in YIG. The magnetoelastic coupling constant which determines the size of the gap in the dispersion curves for YIG is $B_2=4\times10^7$ erg/cm^3 at 4.2 K.[7,30] For MnF_2, the coupling constant[31] $b=3\times10^6$ erg/cm^3 is an order of magnitude smaller, and it has been noted[19] that there is no evidence of a resonant magnon-phonon

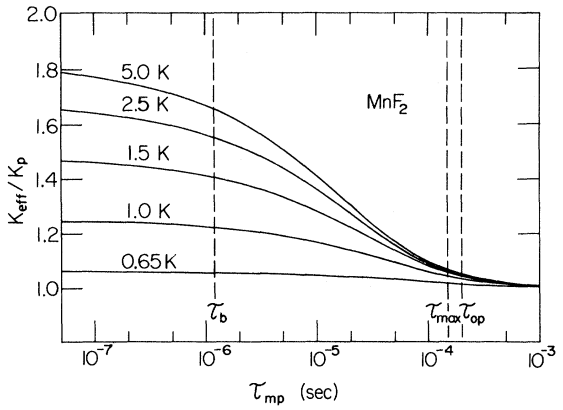

FIG. 3. Dependence of the effective conductivity of MnF_2 on the magnon-phonon relaxation time. The relaxation times labeled $\tau_{\max}$, τ_b, and τ_{0p} have the same meanings as in Fig. 2.

interaction in the thermal conductivity of MnF_2.

Unfortunately, direct measurements of the magnon-phonon relaxation time are not available for MnF_2, as they were for YIG. However, Kotthaus and Jaccarino[32] have measured the total temperature-dependent part of the antiferromagnetic resonance linewidth in a field of 83 kOe. This relaxation rate of the $q=0$ mode is the sum of its relaxation rates to the lattice and to the thermal magnons; i.e., $\tau_{0T}^{-1}=\tau_{0p}^{-1}+\tau_{0m}^{-1}$. At the lowest temperature of 5 K, they found that $\tau_{0T}^{-1}=5\times10^{5}$ sec^{-1}.

Moreover, White *et al.*[33] have shown that this relaxation rate can be quantitatively accounted for, assuming only magnon-magnon relaxation. Since these are normal magnon scattering processes, they do not in themselves contribute to thermal resistance in the magnon system.[21] If it is assumed that their calculation fits the data to within 1%, then an upper limit for the $q=0$ magnon-phonon relaxation time at 5 K is $\tau_{0p}\geq 1\%\ \tau_{0T}=2\times10^{-4}$ sec. Alternatively, it should be noted[34] that τ_{mp} varies as B_2^2 so that τ_{mp} for MnF_2 should be roughly 100 times longer than that for YIG or $\sim10^{-4}$ sec.

It can be seen from Fig. 3 that τ_{0p} is greater than $\tau_{\max}$. This indicates that the magnon-phonon relaxation time for MnF_2 may be so long as to suppress any heat transport in the magnon system. This may explain the absence of magnon conductivity in the experimental results.

C. Summary

The conductivity of a magnetic insulator, measured by a conventional thermal-conductivity experiment, depends on the magnon-phonon thermal-relaxation time. If this time is very long, it can cause heat transport in the magnon system to be suppressed. The available information for magnon-phonon relaxation times indicates that this is not the case for YIG. This is in agreement with experimental measurements of a magnon contribution to the thermal conductivity of YIG. However, the long relaxation time for MnF_2 may be the reason for the lack of magnon conduction in this material. Since this explanation could apply to any magnetic material, it may explain why magnon conductivity has been such an elusive phenomenon.

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*Research supported by the National Research Council of Canada.

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