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Spin-flip electron scattering in the zero-gap semiconductor $Hg_{0.86}Cd_{0.14}Te$

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A 0^- maximum in the longitudinal magnetoresistance of $Hg_{0.86}Cd_{0.14}Te$ has been observed at 10°K, providing evidence for spin-flip electron scattering in the material. Theoretical analysis indicates that the spin-flip processes are caused by acoustic phonons through the deformation-potential interaction.

I. INTRODUCTION

The Shubnikov–de Haas oscillations of magnetoresistance, after the first observations in semiconductors by Frederikse and Hosler[1] and Sladek,[2] have served as a powerful tool in the investigations of band structure and its behavior in a magnetic field. The effect is particularly useful in strongly doped materials, allowing for penetration into high electron energies, not accessible by other methods. On the other hand, it has not been effectively used until present to identify dominant scattering mechanisms.

Of special interest is the quantum limit of higher magnetic fields, where only a few lowest Landau levels are populated with electrons. In this region there are striking differences of behavior in transverse and longitudinal configurations. Efros[3] has shown theoretically that in the longitudinal magnetoresistance one should not observe the 0^- oscillation if the spin-flip scattering between 0^- and 0^+ subbands is forbidden.[4] Keiter and Hajdu[5] showed subsequently in a model calculation that, conversely, if the spin-flip scattering exists, the 0^- maximum will appear. One can interpret these conclusions observing that the spin-conserving scattering processes (*a* and *b* in Fig. 1) are nonresonant, whereas the spin-flip transitions (*c* and *d*) to the state 0^- (with the singular density of states around $k_z = 0$), if not forbidden, will dramatically lower the relaxation time producing a peak in magnetoresistance.

Experimentally, the situation is not quite clear. A semblance of 0^- oscillation in the longitudinal magnetoresistance has been observed in InSb and InAs by Amirkhanov and Bashirov,[6] and Zakiev,[7] in GaSb by Ponomarev and Tsidilkovskii,[8] and in GaAs by Askénazy *et al.*[9] All authors conclude that the 0^- maximum is seen better at *higher* temperatures, contrary to all other oscillations. However, observations at liquid-helium temperature have been also reported.[6,9]

II. RESULTS AND DISCUSSION

We have performed measurements of the longitudinal magnetoresistance on a $Hg_{0.86}Cd_{0.14}Te$ mixed crystal, which for this composition is a zero-gap semiconductor. The band structure of the material can be well described by a three-level model of the degenerate Γ_8-symmetry level separated by the energy $\epsilon_0 = 45$ meV from the Γ_6 level and by the spin-orbit energy Δ from the Γ_7 level.[10,11] The $\vec{k}\cdot\vec{p}$ interaction between these levels is taken into account exactly and all other bands neglected. In the approximation $\Delta \gg \epsilon_0$ the spin splitting is almost exactly half of the orbital splitting. Nonparabolic electron energies λ are to be determined from the equation

$$D_n^{\pm} = (n + \tfrac{1}{2} \mp \tfrac{1}{4})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*} = \lambda\left(1 + \frac{\lambda}{\epsilon_0}\right), \tag{1}$$

where $\omega_c = eH/m^*c$ is the cyclotron frequency. On the other hand the concentration N of the degenerate electron gas in the nonparabolic conduction band of the same form is

$$N = \frac{1}{3\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} \lambda_F^{3/2}\left(1 + \frac{\lambda_F}{\epsilon_0}\right)^{3/2}, \tag{2}$$

where λ_F is the Fermi energy. Comparing Eqs. (1) and (2) the condition for Shubnikov–de Haas is obtained as,

$$(n + \tfrac{1}{2} \mp \tfrac{1}{4})eH/\hbar c = \tfrac{1}{2}(3\pi^2 N)^{2/3}, \tag{3}$$

where plus and minus signs correspond to the two possible spin orientations. At $T = 1.7$ °K [$N = 1.90 \times 10^{15}$ cm^{-3}, mobility $\mu = 1.11 \times 10^5$ cm^2/V sec, Fig. 2(a)] the two maxima correspond to unresolved spin doublets for $n = 2$ and 1, and the 0^- maximum is not observed. On the other hand, at $T = 10$°K [$N = 2.33 \times 10^{15}$ cm^{-3}, $\mu = 0.81 \times 10^5$ cm^2/V sec, Fig. 2(b)] the 0^- maximum clearly appears, providing evidence for the spin-flip scattering in the material. The temperature of 10°K was found optimal for this observation. Our assignment of the max-

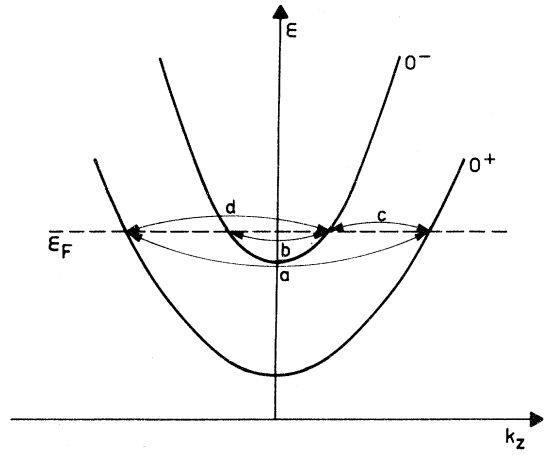

FIG. 1. Elastic scattering transition for electrons in the two lowest magnetic subbands. *a* and *b* denote intrasubband nonresonant processes, *c* and *d* denote resonant spin-flip processes.

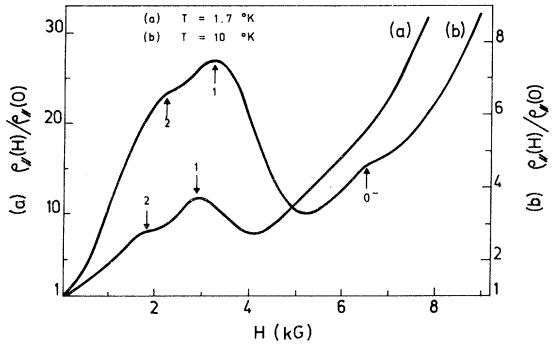

FIG. 2. Relative change of the longitudinal resistivity of $Hg_{0.86}Cd_{0.14}Te$ as a function of magnetic field at $T = 1.7$ °K (left scale) and $T = 10$ °K (right scale). 0^- oscillation clearly appears at 10 °K, but is not observed at 1.7 °K.

ima has been confirmed by supplementary investigations of transverse magnetoresistance and hydrostatic pressure experiments in which the energy ϵ_0 controlling the effective-mass value m^* could be changed. The above mobility values indicate that at 10°K the acoustic phonons contribute appreciably to electron scattering.

Trying to explain this typical but rather strange temperature behavior of the spin-flip scattering, we employ the above three-level model solved for the presence of a magnetic field. Choosing the Landau gauge for the vector potential of the magnetic field: $\vec{A} = (-Hy, 0, 0)$, the solutions for the two projections of the total angular momentum j on the magnetic field direction are obtained in the form (we take $\Delta \gg \epsilon_0$, which is a very good approximation for the case in question),

$$\Psi_{n,k_x,k_z,-} = \exp(ik_x x + ik_z z)\Big\{ia_n^- \phi_n S\downarrow - b_n^-\Big[\frac{3}{2}\Big(\frac{\hbar\omega_c(n+1)}{D_n^-}\Big)^{1/2}\phi_{n+1}R_- + \frac{1}{2}\Big(\frac{\hbar\omega_c n}{D_n^-}\Big)^{1/2}\phi_{n-1}R_+ + \sqrt{2}\Big(\frac{\hbar^2/2m^*}{D_n^-}\Big)^{1/2}k_z\phi_n Z\Big]\downarrow$$
$$- b_n^-\Big[\Big(\frac{\hbar^2/2m^*}{D_n^-}\Big)^{1/2}k_z\phi_n R_- - \frac{1}{\sqrt{2}}\Big(\frac{\hbar\omega_c n}{D_n^-}\Big)^{1/2}\phi_{n-1}Z\Big]\uparrow\Big\}, \quad (4)$$

$$\Psi_{n,k_x,k_z,+} = \exp(ik_x x + ik_z z)\Big\{ia_n^+ \phi_n S\uparrow + b_n^+\Big[\frac{1}{2}\Big(\frac{\hbar\omega_c(n+1)}{D_n^+}\Big)^{1/2}\phi_{n+1}R_- + \frac{3}{2}\Big(\frac{\hbar\omega_c n}{D_n^+}\Big)^{1/2}\phi_{n-1}R_+ + \sqrt{2}\Big(\frac{\hbar^2/2m^*}{D_n^+}\Big)^{1/2}k_z\phi_n Z\Big]\uparrow$$
$$+ b_n^+\Big[\Big(\frac{\hbar^2/2m^*}{D_n^+}\Big)^{1/2}k_z\phi_n R_+ - \frac{1}{\sqrt{2}}\Big(\frac{\hbar\omega_c(n+1)}{D_n^+}\Big)^{1/2}\phi_{n+1}Z\Big]\downarrow\Big\}, \quad (5)$$

where $R_\pm = (X \pm iY)/\sqrt{2}$ and the symbols $\uparrow$ and $\downarrow$ denote the spin-up and spin-down functions, respectively. S and X, Y, Z are periodic functions which transform like atomic s and p functions under operation of the tetrahedral group at the point Γ. $\phi_n((y - y_0)/L)$ are harmonic oscillator functions, in which $y_0 = k_x L^2$ with $L = (\hbar c/eH)^{1/2}$. The coefficients a and b are given by the relations

$$(a_n^\pm)^2 = \frac{\lambda_n^\pm}{\epsilon_0 + 2\lambda_n^\pm}, \quad (b_n^\pm)^2 = \frac{1}{3}\,\frac{\epsilon_0 + \lambda_n^\pm}{\epsilon_0 + 2\lambda_n^\pm}. \quad (6)$$

Owing to strong spin-orbit interaction the above electron wave functions corresponding to the two projections of the total angular momentum j_z (which hitherto and hereafter we call for brevity "spin") are mixtures of spin-up and spin-down components. We now use the wave functions to calculate probabilities of spin-flip transitions induced by ionized impurities and acoustic phonons, the main scattering modes at the temperatures of interest. Interaction of electrons with ionized impurities is described by the screened Coulomb potential $V_{ii}(r)$. This potential is a slowly-varying function of coordinates and it acts only on the slowly-varying parts of the electron wave functions. The square of the matrix element for transitions between $|0, k_x, k_z, -\rangle$ and $|0, k_x', k_z', +\rangle$ states is

$$|\langle 0+|V_{ii}|0-\rangle|^2 = \frac{9}{4}(b_0^- b_0^+)^2 \frac{\hbar\omega_c}{D_0^+} \frac{\hbar^2 k_z^2/2m^*}{D_0^-} \times |(\phi_1, V_{ii}\phi_0)|^2, \quad (7)$$

where

$$(\phi_1, V_{ii}\phi_0) = \frac{1}{L}\int \exp(i\Delta k_x x + i\Delta k_z z)\, V_{ii}(r) \times \phi_1\left(\frac{y-y_0'}{L}\right)\phi_0\left(\frac{y-y_0}{L}\right) d^3r, \quad (8)$$

with

$$\Delta k_x = k_x - k_x'; \quad \Delta k_z = k_z - k_z';$$

$$y_0 = k_x L^2, \quad \text{and} \quad y_0' = k_x' L^2.$$

It can be seen that the matrix element squared is proportional to k_z^2, so that just in the region of energies where the density of states associated with the upper spin level becomes singular, there is no spin-flip scattering due to ionized impurities.

To calculate spin-flip transitions caused by acoustic phonons[12] we employ a procedure used in Ref. 13. The Hamiltonian of electron-phonon interaction contains also a quickly varying factor which will act on periodic amplitudes of the wave functions. Owing to predominantly p-like symmetry of the latter, also transverse phonon modes can interact with electrons. We denote the directional cosines of the phonon wave vector $\vec{q}$ as $\vec{e}$. Then the polarization of longitudinal branch is $\vec{e}_{\parallel} = (e_1, e_2, e_3)$ and for the two transverse branches we take

$$\vec{e}_1^{(1)} = (e_1^2 + e_2^2)^{-1/2}(-e_2, e_1, 0)$$

and

$$\vec{e}_1^{(2)} = (e_1^2 + e_2^2)^{-1/2}(-e_3 e_1, -e_2 e_1, e_1^2 + e_2^2),$$

respectively. In the calculation we assume that the three deformation potentials related to the p-symmetry levels are equal to E. The deformation potential related to the Γ_6 level does not contribute to the spin-flip processes. Since for acoustic coupling the probability of spin-flip transitions *does not* vanish for $k_z = 0$, we consider namely this case. The squares of the matrix elements between the $|0, k_x, 0, -\rangle$ and $|0, k_x', k_z', +\rangle$ states due to the three acoustic modes in question are obtained as (after integration over the azimuthal angle in the phonon space):

$$|\langle 0+|V_{ac}^{\parallel}|0-\rangle|^2 = 4C_l[(1-\zeta)^2 + \zeta^2](q_\perp^2 q_z^2/q^4), \quad (9)$$

$$|\langle 0+|V_{ac}^{\perp(1)}|0-\rangle|^2 = C_t[(1-\zeta)^2 + 4\zeta^2](q_z^2/q^2), \quad (10)$$

$$|\langle 0+|V_{ac}^{\perp(2)}|0-\rangle|^2 = C_t[(q_\perp^2 - q_z^2)^2(1-\zeta)^2 + 4q_z^4\zeta^2] \times (1/q^4), \quad (11)$$

where

$$C_\nu = \frac{k_0 TE^2}{4\rho v_\nu^2}(b_0^+ b_0^-)^2 e^{-\xi} \quad (12)$$

with v_ν denoting the sound velocity for the respective mode, ρ is the crystal density, and $2\zeta = q_\perp^2 / L^2$. The selection rules $k_x' = k_x \pm q_x$ and $k_z' = k_z \pm q_z$ have been omitted for brevity. It can be seen that all three acoustic branches give nonvanishing contributions to spin-flip scattering at $k_z = 0$. The magnetoconductivity can now be calculated following procedures of Argyres[14] and Efros[3]. The amplitudes of oscillations will depend on damping, which smears out singularities in the density of states.

The essential difference between electron interactions with ionized impurities and acoustic phonons, resulting in different efficiency for spin-flip scattering, is their slow and quick spatial variation, respectively. The piezoacoustic interaction, characterized by a slow spatial variation, will act similarly to ionized impurities. As follows from the structure of the wave functions, in zero-gap semiconductors the probabilities of spin-flip and spin-conserving transitions caused by acoustic phonons through the deformation potential interaction are of the same order of magnitude. However, at low temperatures the spin-conserving processes, responsible for the Shubnikov–de Haas maxima $1^-, 1^+, 2^-, 2^+, \ldots$, are most probably caused by ionized impurities, because for the latter the spin-conserving transitions are possible: $0^+ \rightarrow 1^+$, $0^- \rightarrow 1^-$, etc., which do not have the k_z^2 dependence and are thus possible also at $k_z = 0$. On the other hand, the spin-flip processes, giving rise to the 0^- maximum, are associated with the presence of acoustic phonons, requiring somewhat higher temperatures.

III. CONCLUSIONS

It has been demonstrated that in a small-gap semiconductor in the presence of a quantizing magnetic field the ionized impurities do not provide an effective scattering mechanism for the spin-flip electron scattering. The same analysis indicates that the spin-flip scattering transitions, giving rise to the Shubnikov–de Haas 0^- maximum in the longitudinal magnetoresistance, are associated with acoustic phonons. This explains the temperature dependence of spin-flip scattering observed both in our data as well as in those of other authors.

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[1]H. P. R. Frederikse and W. R. Hosler, Phys. Rev. 108, 1136 (1957).
[2]R. J. Sladek, Phys. Rev. 110, 817 (1958).
[3]A. L. Efros, Fiz. Tverd. Tela 7, 1501 (1964) [Sov. Phys.-Solid State 7, 1206 (1965)].
[4]Due to negative g values the 0^- magnetic level is above the 0^+ one, contrary to the commonly used notation.
[5]H. Keiter and J. Hajdu, Phys. Status Solidi 38, 757 (1970).
[6]Kh. I. Amirkhanov and R. I. Bashirov, Fiz. Tverd. Tela 8, 2189 (1966) [Sov. Phys.-Solid State 8, 1739 (1967)]; Fiz. Tekh. Poluprov. 1, 667 (1967) [Sov. Phys.-Semicond. 1, 558 (1967)].
[7]Yu. E. Zakiev, Fiz. Tverd. Tela 8, 1974 (1966) [Sov. Phys.-Solid State 8, 1571 (1966)].
[8]A. I. Ponomarev and I. M. Tsidilkovskii, Fiz. Tekh. Poluprov. 1, 1656 (1967) [Sov. Phys.-Semicond. 1, 1375 (1968)].
[9]S. Askénazy, J. Léontin, and J. P. Ulmet, Solid State Commun. 7, 1355 (1969).
[10]P. Kacman and W. Zawadzki, Phys. Status Solidi B 47, 629 (1971); W. Zawadzki in *New Developments in Semiconductors*, edited by P. R. Wallace (Nordhoff, Leyden, 1973), p. 441.
[11]R. Bowers and Y. Yafet, Phys. Rev. 115, 1165 (1959).
[12]The spin-phonon interaction in this model has been first considered by Y. Yafet, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1963), Vol. 14, p. 1; cf. also S. T. Pavlov and Yu. A. Firsov, Fiz. Tverd. Tela 7, 2634 (1965); 9, 1780 (1967) [Sov. Phys.-Solid State 7, 2131 (1966); 9, 1394 (1967)]; Zh. Eksp. Teor. Fiz. 49, 1664 (1966) [Sov. Phys.-JETP 22, 1137 (1966)].
[13]W. Zawadzki and W. Szymańska, Phys. Status Solidi B 45, 415 (1971); W. Szymańska, P. Boguslawski, and W. Zawadzki, *ibid.* 65, 641 (1974).
[14]P. N. Argyres, Phys. Rev. 117, 315 (1960); 132, 1527 (1963).