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Strong-coupling correction to the jump in the quasiparticle current of a superconducting tunnel junction

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Calculated values of the strong-coupling correction to the jump in the quasiparticle current of superconductor-insulator-superconductor tunnel junctions are given for a number of elements, amorphous materials, and alloys. It is shown that there is a simple empirical relation between the size of jump in the quasiparticle current and the effective electron-electron coupling parameter $(\lambda - \mu^*)$.

In the preceding paper, Ginsberg, Harris, and Dynes[1] have evaluated the strong-coupling correction to the critical current of Josephson junctions made of a variety of materials. They have also shown that there exists a simple, empirical relation between this correction and the effective electron-electron coupling parameter $(\lambda - \mu^*)$. Here λ is McMillan's[2] electron-phonon coupling parameter and μ^* is the Coulomb pseudopotential.[3] We now show that there also exists a simple relation between the jump at the energy gap voltage in the quasiparticle current $\Delta I_{\rm qp}$ of a superconducting tunnel junction and $(\lambda - \mu^*)$. In addition it is shown how the ratio of the critical current I_c to $\Delta I_{\rm qp}$ provides a convenient way of determining whether the measured critical Josephson current in a junction has its maximum possible value.

We consider a tunnel junction formed from two superconductors separated by a thin insulating barrier. The superconductors are assumed to be made of the same material and to be at absolute zero. We consider that measurements are made by biasing the junction with a source having negligible impedance. Although this kind of source is effectively impossible to achieve, except at low frequencies, the intrinsic capacitance of thin-film tunnel junctions is usually sufficiently large that its capacitive impedance is much lower than that due to the tunneling process. Thus the junction sees a low ac impedance regardless of the actual impedance of the source itself.

When one measures the dc current-voltage characteristic of a junction biased as described, one finds the following: at zero voltage one observes the critical Josephson current (the current of ground-state electron pairs); as the voltage is increased one observes the quasiparticle cur-

TABLE I. Values of $\Delta I_{\rm qp}/\Delta I_{\rm qp,w}$ and $I_c/\Delta I_{\rm qp}$ for different materials.

Material [a]	$\Delta I_{\rm qp}/\Delta I_{\rm qp,w}$ [b]	$I_c/\Delta I_{\rm qp}$ [b]	Material [a]	$\Delta I_{\rm qp}/\Delta I_{\rm qp,w}$ [b]	$I_c/\Delta I_{\rm qp}$ [b]
Sn	1.009	0.907	$In_{0.9}Tl_{0.1}$	1.016	0.880
Tl	1.011	0.895	$In_{0.73}Tl_{0.27}$	1.021	0.861
In	1.014	0.888	$In_{0.67}Tl_{0.33}$	1.019	0.869
Pb	1.056	0.755	$In_{0.5}Tl_{0.5}$	1.014	0.887
Hg [c]	1.075	0.735	$Tl_{0.9}Bi_{0.1}$	1.012	0.880
β Ga	1.027	0.844	$Pb_{0.8}Tl_{0.2}$	1.064	0.744
Amorphous Ga	1.078	0.743	$Pb_{0.8}Bi_{0.2}$	1.093	0.694
Amorphous Bi	1.102	0.691	$Pb_{0.7}Bi_{0.3}$	1.103	0.678
			Amorphous $Pb_{0.45}Bi_{0.55}$	1.135	0.649

[a] Except as noted these data are from J. M. Rowell, W. L. McMillan, and R. C. Dynes, J. Phys. Chem. Ref. Data (to be published).
[b] The accuracy of these quantities is not known well. However, the uncertainty introduced by the present calculations is small. Larger uncertainties arise from limitations in the accuracy of the original tunneling data and from the spacing of the points for which the complex gap parameter was obtained when the tunneling data were inverted.
[c] W. N. Hubin and D. M. Ginsberg, Phys. Rev. 188, 716 (1969).

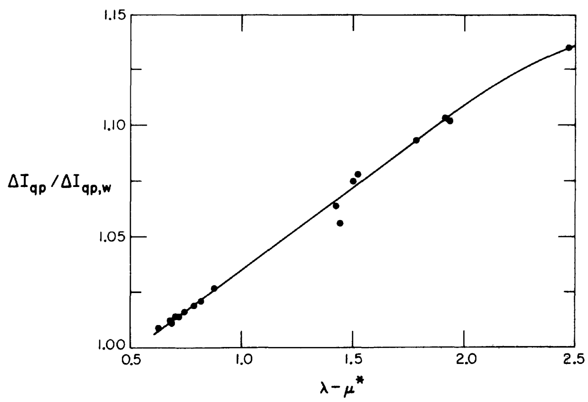

FIG. 1. Jump ΔI_{qp} in the quasiparticle current as a function of $\lambda - \mu^*$. ΔI_{qp} is normalized to its weak-coupling value $\Delta I_{qp,w}$. The line is drawn approximately through the points and is not based on any theory.

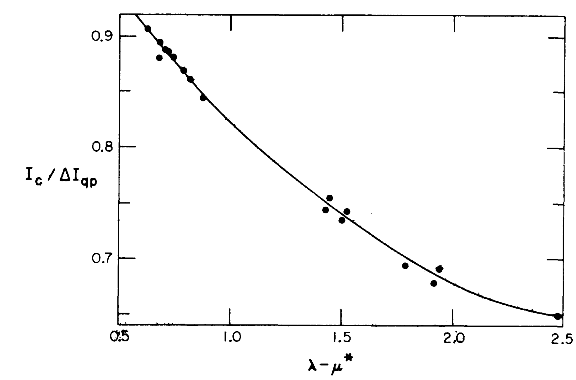

FIG. 2. Ratio $I_c/\Delta I_{qp}$ of the critical Josephson current to the jump in the quasiparticle current as a function of $\lambda - \mu^*$. The line is drawn approximately through the points and is not based on any theory.

rent (due to excitations), which remains nearly zero up to the energy-gap voltage $2\Delta_0/e$ where it jumps abruptly by ΔI_{qp} to a value somewhat less than the normal state current; with further increases in voltage the current gradually approaches the normal-state current.

In a real junction the jump in the quasiparticle current is spread over a range of voltages. A graphical construction for deriving both the energy-gap voltage and the current jump from experimental data has been described by McMillan and Rowell.[4] However, most of the superconductors described in this paper are sufficiently dirty ($l \ll \xi_0$) that the jump will be quite sharp. The mean free path is l and the coherence length is ξ_0.

In order to compare an experimentally determined critical Josephson current with the theoretical maximum, using only the results of Ref. 1, one must also measure the normal-state resistance of the junction. This resistance measurement requires either a bias voltage substantially higher than the energy-gap voltage, or a magnetic field strong enough to quench the superconductivity (not just the critical current) in the junction. The former can introduce changes in the normal state resistance due to heating and the latter is difficult because of the high magnetic fields required.

One can avoid both of these difficulties by using another method to compare measured critical currents with the theoretical maximum. This approach requires measurement of the jump in the quasiparticle current, and a comparison of that jump with the critical current. Since both I_c and ΔI_{qp} are inversely proportional to R_N, comparison of their measured ratio with a theoretical value does not require measurement of R_N. In the present paper we therefore give values of ΔI_{qp}, and they are for the same materials for which I_c is given in Ref. 1.

For weakly coupled materials the jump $\Delta I_{qp,w}$ in the quasiparticle current is identical to the critical Josephson current I_{cw}:

$$\Delta I_{qp,w} = I_{cw} = \pi\Delta/2eR_N . \tag{1}$$

For strongly coupled materials the jump is given by McMillan and Rowell[4]:

$$\Delta I_{qp} = \Delta I_{qp,w} \times \left(1 + \frac{1}{2}\,\frac{d\Delta}{d\omega}\bigg|_{\omega_g}\right)^2 . \tag{2}$$

Here $\Delta(\omega)$ is the complex-valued energy-dependent gap parameter and ω_g is the energy gap. Our values of ΔI_{qp} are calculated from Eq. (2).

Table I gives values of $\Delta I_{qp}/\Delta I_{qp,w}$ for the same materials considered in Ref. 1. These values are plotted in Fig. 1 as a function of $\lambda - \mu^*$. The figure illustrates the simple relation between the two variables.

Because the ratio $I_c/\Delta I_{qp}$ is useful for experimental determinations of the critical current, we list values of it in Table I and have plotted it in Fig. 2, also as a function of $\lambda - \mu^*$. The simple dependence of this ratio on $\lambda - \mu^*$ is apparent.

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[1] D. M. Ginsberg, R. E. Harris, and R. C. Dynes, preceding paper, Phys. Rev. B 14, 990 (1976).

[2] W. L. McMillan, Phys. Rev. 167, 331 (1968).

[3] We have used $\mu^*(\omega_{ph})$ as discussed by P. B. Allen and R. C. Dynes, Phys. Rev. B 12, 905 (1975).

[4] W. L. McMillan and J. M. Rowell, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), Chap. 11.