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Effect of a longitudinal electric field on helicon wave propagation in InSb

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The dispersion equation accounting for the effect of a parallel electric field in an n-type semiconductor plasma has been derived. The propagation of helicon waves through InSb at low temperature has been studied. The parallel electric field is found to defocus the propagating helicon waves in InSb. The propagation of helicon waves in InSb depicts the onset of electric-field-induced instability. This shows that the helicon waves under suitable conditions would grow while propagating through the bulk specimen. The diagnostic role of helicon-wave propagation has been discussed.

I. INTRODUCTION

The dispersion equation for helicon-wave propagation in a one-component solid state plasma has been derived by many workers.[1–3] The propagational features of whistler waves in ionized plasma, helicon waves through solid-state plasma, and their focussing along the static magnetic field as they propagate have been extensively studied.[4–6] However, the simultaneous effect of an electrostatic field parallel to the magnetic field has not been rigorously studied so far. For studying the helicon-wave propagation under the simultaneous effect of parallel electrostatic and magnetostatic fields we have chosen n-type InSb. The majority of current carriers in this specimen is responsible for the helicon-wave propagation. The presence of a parallel electrostatic field in the specimen increases the drift velocity of the electrons and thus produces hot electrons.[7] The presence of hot electrons is likely to change the helicon wave propagation features. Recently, Gupta and Singh[8] made use of the dispersion equation derived by Hsieh[9] and studied the effect of a parallel electrostatic field on whistler-mode propagation. In the present paper, following the work of Hsieh, we have derived the appropriate dispersion equation for helicon wave propagation through a nondegenerate plasma in the presence of an electrostatic field parallel to the magnetic field. With a view to elucidating the anisotropic behavior of the plasma, we have derived an expression relating the ray direction and the wave-normal direction in the presence of a static electric field directed parallel to the impressed static magnetic field.

Using the dispersion equation thus obtained, we have constructed the refractive-index surfaces for helicon-wave propagation and have shown the effect of varying plasma and field parameters. The changes in the points of inflexion induced by the parallel electric field in n-type InSb give rise to focussing or defocussing of propagating helicon waves. The focussing of a helicon wave occurs when the ray direction coincides with the magnetic field direction. The variation of the angle α between the ray direction and the wave normal has been computed for different values of the angle θ between the wave normal and static magnetic field. It is shown that the presence of a parallel electric field significantly changes the direction of helicon energy flow. It is seen that the parallel electric field produces a significant variation in the angle $\theta-\alpha$ between the ray direction and the static magnetic field, with varying θ values. As a result of this variation the focussing condition $\theta=\alpha$ is never met. On the contrary $\theta-\alpha$ approaches a minimum value for certain θ values and thereafter increases monotonically. The ratio of plasma frequency to cyclotron frequency is seen to control the nature of the variation of $\theta-\alpha$ with θ. The detailed study of the deformation of refractive-index surfaces induced by various plasma parameters in the presence of a parallel electric field depicts the defocussing of helicon waves. The observed features of helicon-wave–transmitted-amplitude spectra through a given specimen can be explained in terms of the detailed focussing mechanism. The dispersion equation has been analyzed for k real and ω complex. The growth rate induced by a parallel electric field has been computed, and it is shown that the growth rate increases with increasing E_0 value. It is argued that the precise measurement of the transmitted helicon wave amplitude and its detailed analysis can provide valuable diagnostic information on the chosen specimen.

II. BASIC THEORY

The response of semiconductors and metals to electrostatic fields is well known. The semiconductors and metals in the presence of static magnetic field support a new mode of wave propagation known as helicons. Apart from magnetic field, the propagation features of these waves are significantly controlled by the conductivity and dielectric properties of the specimen. The role of a parallel electric field on the dispersion properties of whistler waves was explored by Hsieh. We have followed procedures similar to those of Hsieh and have obtained the dispersion equation valid for a solid-state plasma. In the case of a solid-state plasma, the following assumptions have been made to simplify the mathematics without loss of generality: (i) The solid state plasma is assumed to be nondegenerate and conforms to a Maxwellian distribution. The field and plasma parameters satisfy the condition $\omega_c\tau \gg 1$, where ω_c and τ are the gyrofrequency and relaxation time of the conduction electrons, respectively. (ii) The electric neutrality of the plasma is satisfied. (iii) Small-amplitude helicon waves are propagating at an angle to the magnetic field direction. (iv) The drift velocity of electrons due to the low parallel electrostatic field E_0 is small and has been ignored in comparison to the phase velocity of the helicon waves.

The magnetostatic and the electrostatic fields in the specimen are applied along the Z axis. The helicon waves propagating along the Z axis and the induced plasma perturbations are assumed to be harmonic of the form $\exp[i(\omega t - kz)]$. We denote the time-independent wave and plasma parameters by the subscript 0 and the time-dependent ones by 1. The magnetic flux density $\vec{B}$, electric field intensity $\vec{E}$ and the electron velocity distribution function f are decomposed as

$$\vec{B} = \vec{B}_0(z) + \vec{B}_1(z, t) \ , \tag{1}$$

$$\vec{E} = \vec{E}_0(z) + \vec{E}_1(z, t) \ , \tag{2}$$

and

$$f = f_0 + f_1(r, v, t) \ . \tag{3}$$

The microscopic behavior of the solid-state specimen subjected to the parallel electrostatic and magnetic fields changes due to the resulting redistribution in the electron velocity distribution function. Under these conditions the transport equation for the electron gas is completely expressible by the coupled Maxwell-Boltzmann equation

$$\frac{\partial f}{\partial t} + \nabla \cdot \frac{\partial \vec{f}}{\partial r} - \frac{e}{m_e^*}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right) \cdot \frac{\partial \vec{f}}{\partial r} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} \ . \tag{4}$$

The nondegenerate plasma is assumed to be characterized by a simple collisional term

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \frac{f - f_0}{\tau} \ . \tag{5}$$

The right-handed circularly polarized helicon-wave electric and magnetic field vectors are written

$$E_+ = \tfrac{1}{2}(E_{1x} + iE_{1y}) \ , \quad B_+ = \tfrac{1}{2}(B_{1x} + iB_{1y}) \ , \tag{6}$$

and the longitudinal component of the electric field is given by

$$E_{1z} = \frac{4\pi i e}{\omega} \int v_z f_1 \, d^3v \ . \tag{7}$$

The perturbation in the velocity distribution function of the electron gas under the influence of the fields of the electromagnetic waves is written in terms of the perpendicular and parallel velocity components

$$f_1(z, t, v_\perp, v_\parallel, \psi) = f_+(z, t, v_\perp, v_\parallel)\, e^{-i\psi} + g(z, t, v_\perp, v_\parallel) \ , \tag{8}$$

where f_+ and g represent the changes in the distribution function arising from the propagation of right-hand circularly polarized wave and the longitudinal electrostatic field, respectively. The velocity components are defined as

$$v_\perp = v \sin\psi \ , \quad v_\parallel = v\cos\psi \ ,$$

where ψ is the angle made by the instantaneous velocity vector with the magnetic field. Substituting this perturbation scheme in Eq. (4) and carrying out the analysis,[9] we obtain the dispersion equation for low-frequency ($\omega < \omega_c$) helicon waves propagating at an angle θ with the magnetic field

$$\frac{c^2k^2}{\omega^2} = \epsilon_L + \frac{\omega_p^2}{\omega(\omega_c\cos\theta - \omega + i\nu)} + \frac{\omega_p^2 v_{\text{th}}^2 (k + i\, eE_0/k_B T_e)^2}{2\omega(\omega_c\cos\theta - \omega + i\nu)^3} \ , \tag{9}$$

where ϵ_L is the lattice contribution to the dielectric constant of the solid specimen, ω_p is the electron plasma frequency, ν is the electron collision frequency, v_{th} is the thermal velocity of the electron, T_e is the electron temperature, and k_B is the Boltzmann constant.

This is a fairly general dispersion equation and includes the effect of collisions between the current carriers and the lattice ions. The collisions in general are strongly temperature dependent, and in solid-state experiments the role of collisions is minimized by subjecting the specimen to very low temperatures. Therefore, the losses due to collisions in certain cases can be ignored, and we write the complex refractive index $n = \mu + i\chi$. Using Eq. (9), we have obtained an expression for the real part of the refractive index assuming $\chi/\mu \ll 1$. In the rationalized form, we write[4]

$$\mu^2 = \epsilon_L + \frac{\delta^2}{\Omega(\cos\theta - \Omega)} - \frac{\delta^2 v_{\text{th}}^2 e^2 E_0^2}{2\Omega\omega_c^2 k_B^2 T_e^2(\cos\theta - \Omega)^3} \ , \tag{10}$$

where

$$\delta = \omega_p/\omega_c \ , \quad \Omega = \omega/\omega_c \ .$$

It is obvious from Eq. (10) that the net effect of the longitudinal electric field is to reduce the magnitude of the refractive index.

In the presence of magnetic field we find that the refractive index becomes highly anisotropic. The tensorial components parallel to the static magnetic field are affected by magnitude and direction of the electrostatic field. The direction of flow of helicon-wave energy markedly differs from the direction of the wave normal. Schematically, we have shown the ray and wave-normal directions in Fig. 1. The expression for the angle between the ray direction and wave-normal direction, in the presence of parallel electrostatic field, is obtained from Eq. (10):

$$\tan\alpha = \frac{1}{\mu}\frac{d\mu}{d\theta} = \tfrac{1}{2}\sin\theta\left(\frac{(\cos\theta-\Omega)^2 - 3\,e^2E_0^2/m_e^* k_B T_e \omega_c^2}{\Omega\epsilon_L(\cos\theta-\Omega)^4/\delta^2 + (\cos\theta-\Omega)\left((\cos\theta-\Omega)^2 - e^2E_0^2/m_e^* k_B T_e\omega_c^2\right)}\right) . \tag{11}$$

This expression shows that the ray direction depends upon the direction and magnitude of the impressed electrostatic and magnetostatic fields and also upon the electron-gas parameters. The geometry shown in Fig. 1, shows that the tangent to the surface at an inflexion point P makes an angle ϕ with the normal to the $\mu(\theta)$ curve. At the inflexion point and in its vicinity the angle ϕ can be written

$$\tan\phi = \frac{1}{\mu}\frac{d\mu}{d\theta} . \tag{12}$$

Thus at the inflexion point we find that $\phi = \alpha$ and the ray direction is perpendicular to the surface.

III. REFRACTIVE-INDEX SURFACE, RAY, AND WAVE-NORMAL DIRECTIONS

Using Eq. (10) we have constructed the refractive-index surfaces for the InSb sample for various values of parallel electric fields. The specimen is assumed to be at a temperature of 4.2 °K. The adopted parameters of the n-type InSb specimen are given in Table I. The refractive-index surfaces for low values of the parallel electric fields are shown in Fig. 2. The presence of a parallel electric field is seen to distort the refractive-index surfaces. The position of the inflexion point becomes a function of wave frequency and the parallel electric field. These features are depicted in Fig. 2. We find that in the presence of the parallel electric field alone, the refractive-index surfaces are deformed. The effect of collisions and parallel electrostatic field is to constrain the gyroradius of the charged particles. Therefore, the deformations in the refractive-index surfaces arising from collisions and electrostatic field are similar. For very low electric field the inflexion point in the refractive-index surface is almost unaffected. The inflexion point in the refractive-index surface is characterized by least absorption for propagation of helicon waves along the steady magnetic field. Thus the precise measurements of helicon wave absorption in InSb samples at different temperature and small parallel electric field permit us to explore the effect of the electron gas in focussing the helicon waves. As the strength of the parallel electric field increases, the refractive-index surface deforms in such a way that the inflexion point loses its significance. We find that the changing values of parallel electric fields produce corresponding changes in the surfaces. Thus, we find that the helicon phase velocity changes by changing the magnitude of the parallel electric field. This may result in interference effects which are often recorded.[13–15] The refractive-index surfaces indicate that there is a critical value of the parallel electric field at which the helicon modes are cut off and beyond which no resonances could be observed in the transmitted frequency spectra. This value of the electric field for InSb at 4.2 °K is found to be 17.95 V/cm for the wave frequency characterized by $\Omega = 0.05$. This cutoff value of the parallel electric field may be affected by the coupling of the helicon mode with the ordinary wave which also begins to propagate above $E_0 = 12.25$ V/cm, but we have ignored these details in the present paper.

Using Eq. (11), we have computed the variation of $\theta - \alpha$ with θ. Figure 3 shows the effect of the

TABLE I. Parameters of n-type InSb at 4.2 °K.

Serial No.	Parameters		References	Remarks
1	ω_p	4.6×10^{12} rad sec^{-1}	10	$\epsilon_L = 19.7$
2	ω_c	3.066×10^{12} rad sec^{-1}	...	$\Omega = 0.05$
3	ν	0.90×10^{10} rad sec^{-1}	11	$Z = 0.0029$
4	m_e^*	$0.013\ m_0$	12	$m_0 = 0.911\times10^{-27}$ g

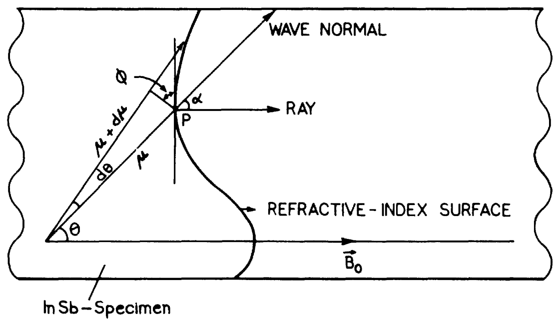

FIG. 1. Schematic diagram showing the helicon-wave propagation through an anisotropic solid state plasma.

parallel electric field on the propagation of fixed-frequency helicon waves. The increasing magnitude of the electric field changes the direction of energy flow, which is in contrast to $\theta = \alpha$ in the absence of electric field, as shown in Fig. 3. The effect of the electric field is to violate the condition $\theta = \alpha$ and to shift the position of the minimum towards lower value of θ for the increasing values of the electric field. This depicts that the flow of energy in the presence of a parallel electric field is not restricted to be along the magnetic field and instead propagates at an angle to the magnetic field. The condition which governs the position of the minima in the $\theta - \alpha$ vs θ curves can be ob-

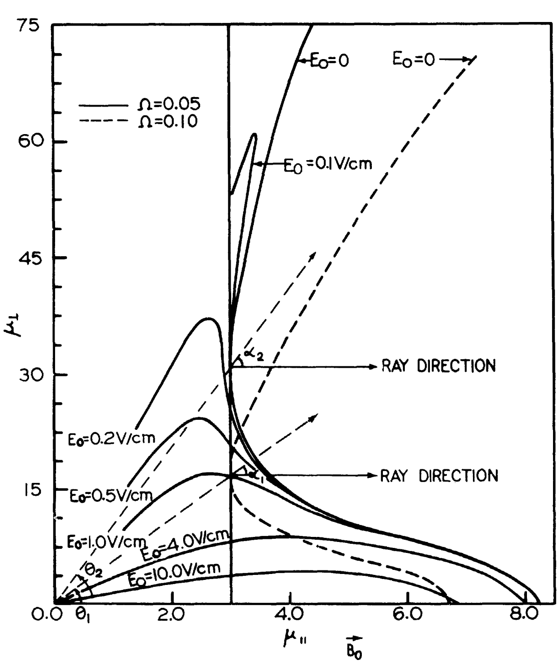

FIG. 2. Effect of parallel electric fields on the refractive-index surfaces in n-type InSb at 4.2 °K. $\mu_\parallel$ and $\mu_\perp$ are the parallel and perpendicular components of the refractive index.

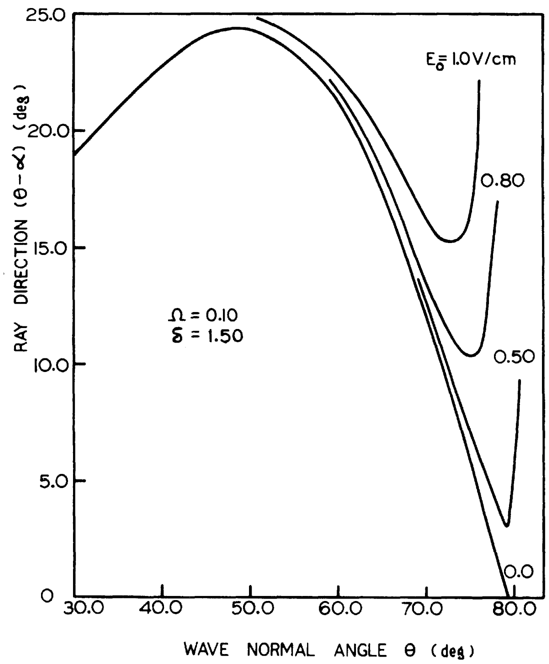

FIG. 3. Effect of parallel electric fields on the variation of ray direction with wave-normal angle for n-type InSb at 4.2 °K.

tained by setting the denominator of Eq. (11) equal to zero. Thus we obtain

$$\delta^2 = \frac{(\cos\theta - \Omega)^3}{e^2 E_0^2/\Omega\epsilon_L m_e^* k_B T_e \omega_c^2 - (\cos\theta - \Omega)^2/\Omega\epsilon_L} \; . \tag{13}$$

We find from Eq. (13) that the minimum value of $\theta - \alpha$ changes with changing the value of δ^2. The variation of $\theta - \alpha$ with θ for different values of δ^2 is shown in Fig. 4. These curves are drawn for a fixed value of electric field and a fixed value of plasma frequency and a fixed value of the wave frequency. The variations in $\theta - \alpha$ with θ shown in the curves depict the effect of varying ω_c. The analytical study of this equation shows that for fixed helicon waves, the increasing value of the parallel electric field and the decreasing value of the magnetic field produce a similar shift in the position of the minima. The shift in the position of the maxima is less pronounced as compared to the shift in the position of the minima depicted in Fig. 4.

IV. HELICON-WAVE INSTABILITY

Whenever the dispersion equation becomes complex, its solution corresponds to complex frequency (k being real) or complex wave vector (frequency being real). Depending upon whether the imaginary value of ω is positive or negative, there exists an instability which results in either decay or growth of the propagating waves. The instability analyses of helicon waves under various conditions have been reported by various workers.[16–18] Here we have carried out a theoretical study of electric-field-induced absolute instability of helicon waves in an InSb sample at low temperatures. The lattice contribution to the dielectric constant for an InSb specimen in the low-frequency region becomes small and can be ignored. Thus the dispersion equation for the low-frequency helicon wave can be written

$$\frac{c^2k^2}{\omega^2}=\frac{\omega_p^2}{\omega(\omega_c+i\nu)}+\frac{\omega_p^2 v_{\rm th}^2(k+i\,eE_0/k_BT_e)^2}{2\omega(\omega_c+i\nu)^3}\,. \tag{14}$$

The above dispersion equation has been solved for complex ω and real k in order to study the absolute instability. Substituting $\omega=\omega_r+i\omega_i$ in the dispersion equation (14) and solving for ω_r and ω_i, we get the expressions in the rationalized form:

$$\frac{\omega_r}{\omega_i}=\frac{\tilde{k}^2}{\delta^2}\left(\frac{1+k_BT_e/m_e^*c^2-3z^2k^2-e^2E_0^2/m_e^*\omega_c^2+z^2(2+3\,e^2E_0^2/m_e^*\omega_c^2k_BT_e+6z\,eE_0\tilde{k}/cm_e^*\omega_c)}{\{1-z^2+(v_{\rm th}^2/2\omega_c^2)(\omega_c^2k^2/c^2-e^2E_0^2/k_B^2T_e^2)\}^2+(2z+v_{\rm th}^2\,eE_0\tilde{k}/c\omega_ck_BT_e)^2}\right) \tag{15}$$

and

$$\frac{\omega_i}{\omega_c}=\frac{3\tilde{k}^2z\,(1+k_BT_e\tilde{k}^2/m_e^*c^2-e^2E_0^2/m_e^*k_BT_e\omega_c^2)-2\tilde{k}^2(z+eE_0\tilde{k}/cm_e^*\omega_c)}{\delta^2[(1+k_BT_e\tilde{k}^2/m_e^*c^2-e^2E_0^2/m_e^*k_BT_e\omega_c^2)^2+4(z+eE_0\tilde{k}/cm_e^*\omega_c)^2]}\;, \tag{16}$$

where $\tilde{k}=ck/\omega_c$ and $z=\nu/\omega_c$. Equation (15) gives the propagating helicon mode for $\omega_r>0$ and Eq. (16) gives the growth or decay according to the sign of Im(ω). If Im(ω) < 0, then one can study the growth rate of helicon wave induced by the absolute instability and Im(ω) > 0 gives the decay of the helicon wave. The condition for the onset of the absolute instability can be obtained by putting Im(ω) equal to zero. So we get the condition

$$3z\left(1+\frac{k_BT_e\tilde{k}^2}{m_e^*c^2}-\frac{e^2E_0^2}{m_e^*k_BT_e\omega_c^2}\right)\leq 2\left(z+\frac{eE_0\tilde{k}}{cm_e^*\omega_c}\right). \tag{17}$$

This equation clearly shows the time growth of the field for a parallel electric field and decay of

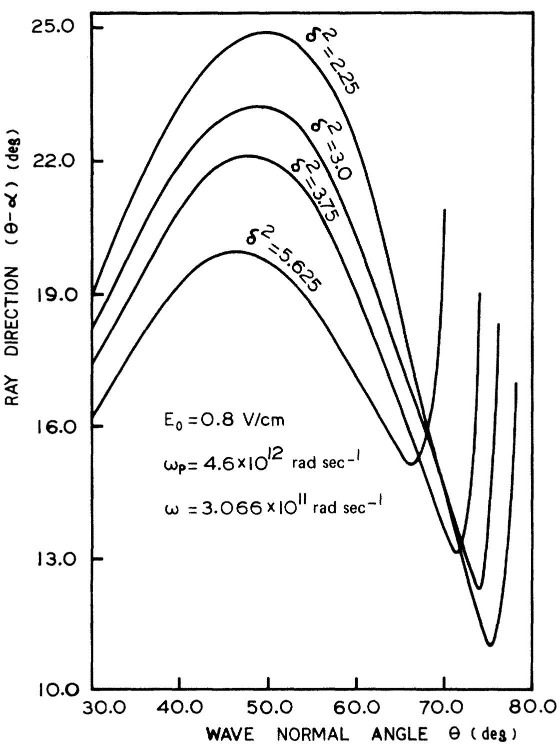

FIG. 4. Ray direction variation with wave-normal angle for different values of impressed static magnetic field and a fixed value of parallel electric field.

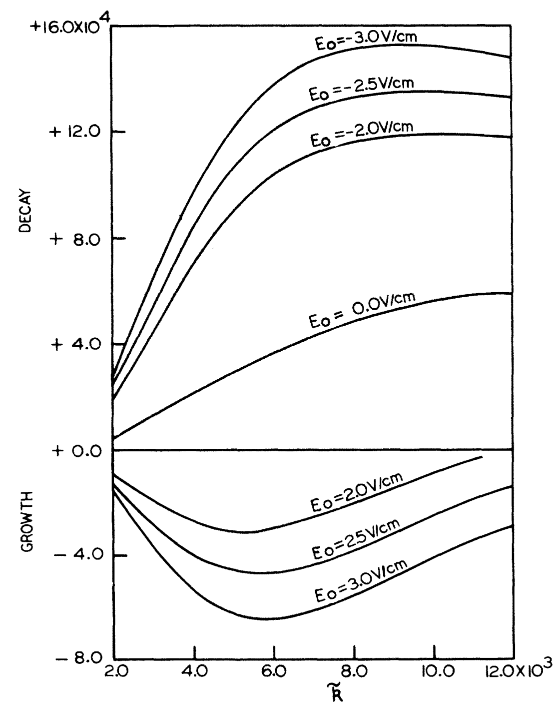

FIG. 5. Effect of parallel and antiparallel electric fields on the growth and decay rates of helicon waves in n-type InSb at 4.2 °K.

the field for an antiparallel electric field. The effect of a parallel electric field on the growth rate has been computed and shown in Fig. 5. As the strength of the parallel electric field increases, the growth rate is found to increase. The effect of the parallel electric field is large in the region of short wavelength and decreases as the wavelength increases. The instability analysis presented in this paper accounts for the effects of a parallel electric field and very low collision frequency. We see from Eq. (16) that for $z=0$, the growth rate is still finite. The role of the collision frequency of electrons is to modify the instability set-in due to the presence of a parallel electric field. The instability set-in into the specimen may, under certain conditions, amplify the propagating helicon waves. The amplification can be made significant for helicon waves propagating through a bulk specimen. When the group velocity of the wave becomes small, the propagation time and hence the amplification becomes significant. Choosing appropriate parameters, the helicon waves can be amplified with the help of a parallel electric field.

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