Quantum theory of magnetomicrowave conductivity in semiconductors

Vijay K. Arora
Department of Physics, Western Michigan University, Kalamazoo, Michigan 49008
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A quantum theory using a density matrix for microwave electric conductivity in the presence of a magnetic field of arbitrary strength, within the effective-mass approximation, is described. No *ad hoc* introduction of relaxation time or transition probabilities as in the solution of the Boltzmann transport equation is made. The frequency-dependent relaxation time is included in the formalism in a natural way by means of a formal manipulation of Liouville's equation for the density matrix and then making approximations for the case of electron scattering via acoustic and optic phonons in the nonpolar model. The components of magnetoconductivity are obtained in a form suitable for numerical computation.

I. INTRODUCTION

The microwave techniques are being extensively used because of their potential to give useful information about the scattering parameters.[1-6] For example, the group of Srivastava *et al.*[1-4] has investigated low-field magnetomicrowave effects in magnetotransmission to estimate the relative contribution of various scattering mechanisms. Jain and Srivastava[1] have estimated the relative importance of electron scattering via acoustic and optic phonons, and impurity atoms by making comparison of the semiclassical theory based on the solution of the Boltzmann transport equation with their experimental results on magnetomicrowave transmission on n-type germanium. The intervalley scattering was neglected and the relaxation time was assumed to be independent of frequency. The effect of intervalley scattering was included in a later work[2] to interpret the magnetomicrowave Faraday effect in n-GaSb to estimate the value of the intervalley scattering parameter. The effect of piezoelectric potential scattering on the microwave Faraday effect in n-CdS was studied by the same group.[3]

Recently, Dorschner and Vernon[5] studied the Voigt-type microwave Kerr effect in semiconductors with spherical constant-energy surfaces by using a plane-wave analysis with the Boltzmann transport equation. This analysis was then used for the parabolic model of n-InSb to interpret experimental results. They claim to be in fundamental disagreement with the formulations of Srivastava and co-workers.[4]

In a more recent work, Nag and Dutta[6] have used an iteration method to solve the Boltzmann equation to investigate galvanomagnetic and microwave transport coefficients. The effects of nonparabolicity, overlap integrals, and the effect of electron scattering were included in their description. The results obtained are shown to agree well with experiments on InSb at 77 °K for an acoustic-phonon deformation-potential constant of about 20 eV for magnetic fields of up to 0.1 Wb/m^2.

All the above works are based on the magnetoconductivity tensor obtained from the semiclassical Boltzmann transport equation, where an *ad hoc* introduction of frequency-independent and magnetic-field-independent relaxation time is made. It has been explained by the author[7-10] that this semiclassical picture may break down when the radius of the cyclotron orbit of an electron in a magnetic field becomes comparable to its mean free path or de Broglie wavelength. Brodwin and Burgess[11] observed the Faraday rotation in Ge and Si at room temperature with pulsed magnetic fields of up to 15.0 Wb/m^2. When compared with the theoretical model, they found a significant departure of the experimental results from the theoretical results at higher fields. Jain and Srivastava[1] feel that this anomaly is due to the fact that impurity scattering was neglected. The small values of impurity-scattering parameters obtained by Jain and Srivastava[1] show that impurity scattering could not be that important especially at room temperature in semiconductors. At magnetic fields of the order of 15.0 Wb/m^2 the quantum effects[7-10] may become quite important.

In spite of a large amount of theoretical and experimental work,[1-6] the quantum theory has not yet been fully exploited to interpret magnetomicrowave experiments. A part of the reason may be the complexity of the quantum treatments. Kawabata[12] obtains a complex expression for line shape, which depends upon the frequency and magnetic field. Ito *et al.*[13] obtained expressions for transverse magnetoconductivity with frequency-independent relaxation time by using a diagrammatic technique. They applied these expressions to estimate the deformation-potential constants. Since the frequency dependence of the relaxation

time is not important near the cyclotron resonance,[12] their results were in reasonable agreement with those obtained by other means. Argyres and Sigel[14] review and criticize these and other quantum-mechanical treatments.

A quantum theory for microwave conductivity with no magnetic field, that of using the density matrix, was quite recently worked out by the author.[15] No *ad hoc* introduction of the relaxation time or transition probabilities was made in this work. Starting from the formal manipulation of Liouville's equation, approximations were made to obtain expressions for microwave conductivity with frequency-dependent relaxation time. A shift in microwave frequency was also observed.

A relatively simple density-matrix approach for magnetoconductivity at high frequencies based on the scattering dynamics of Arora and others[7-10] was reported earlier.[16] In this work, we describe in detail the quantum theory in the framework of the Arora-Peterson[9] density-matrix formalism to evaluate the components of magnetomicrowave conductivity. Since microwave experiments are usually done at room temperature, we will consider only the electron scattering via the acoustic and optic phonons in the nonpolar model. Other refinements such as including the nonparabolicity,[15] multiple valleys,[8] polar interactions, etc., could be done in a straightforward manner when needed and will not be considered in the present work. Our aim is simply to present a first-principles theory for microwave conductivity in a magnetic field when quantum effects are important.

II. DENSITY MATRIX

The Hamiltonian of the electron-phonon system in the presence of a magnetic field with magnetic potential $\vec{A}=(0, Bx, 0)$ in the Landau gauge and a weak alternating electric field $\vec{\mathcal{E}}=\vec{\mathcal{E}}_0 e^{i\omega t}=(\mathcal{E}_{0x}, \mathcal{E}_{0y}, \mathcal{E}_{0z})e^{i\omega t}$ could be written as a sum of an unperturbed part and a perturbation:

$$\mathcal{H}(t)=\mathcal{H}_0+\mathcal{H}'(t)\,, \tag{1}$$

with

$$\mathcal{H}_0=\mathcal{H}_e+\mathcal{H}_L\,, \tag{2}$$

$$\mathcal{H}_e=[p_x^2+(p_y+m^*\omega_c x)^2+p_z^2]/2m^*\,, \tag{3}$$

$$\mathcal{H}_L=\sum_q (N_q+\tfrac{1}{2})\hbar\omega_q\,, \tag{4}$$

$$\mathcal{H}'=V+Fe^{(s+i\omega)t}=Ve\vec{\mathcal{E}}_0\cdot\vec{r}e^{(s+i\omega)t}\,, \tag{5}$$

where $\omega_c=eB/m^*c$ is the cyclotron frequency of the electron of effective mass m^* in a magnetic field of strength B. N_q is the occupation number and ω_q the angular frequency for phonons of wave vector $\vec{q}$. The factor e^{st}, where s is a small number, describes the slow time development of the switching on of the applied electric field.[7-9] At $t=-\infty$, the electric field is turned on with zero initial amplitude. Our aim is to describe the steady-state behavior of the system at $t=0$, which could be done by taking the limit $s\to 0^+$. The inclusion of the electric field interaction in the perturbation part gives us the convenience of choosing a uniform initial steady-state density matrix at $t=-\infty$. The eigenvalue solution of $\mathcal{H}_e$ of Eq. (3) is well-known[7,9] with eigenfunctions

$$|\alpha\rangle=|nk\rangle=e^{i(k_y y+k_z z)}\varphi_n((x-x_k)/\lambda)\,, \tag{6}$$

where φ_n is the harmonic-oscillator wave function which in terms of Hermite polynomials is

$$\varphi_n((x-x_k)/\lambda)=(1/\sqrt{\pi}\,2^n n!\lambda)^{1/2}\times H_n((x-x_k)/\lambda)e^{-(x-x_k)^2/2\lambda^2}\,, \tag{7}$$

with

$$x_k=-\lambda^2 k_y\,,\quad \lambda=(\hbar c/eB)^{1/2}\,. \tag{8}$$

The corresponding eigenvalues of $\mathcal{H}_e$ are Landau levels:

$$\epsilon_\alpha=\epsilon_{nk}=(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*\,, \tag{9}$$

where k stands for (k_y, k_z).

In the absence of perturbation ($\mathcal{H}'=0$), the quantum-mechanical state of an electron is well represented by a wave function of the type given by Eq. (6). The perturbation introduces the uncertainty in the quantum-mechanical state of the system, necessitating an expansion of the wave function ψ^i of the ith electron in terms of an orthonormal set of Eq. (6):

$$\psi^i(t)=\sum_\alpha a_\alpha^i(t)|\alpha\rangle\,. \tag{10}$$

An ensemble average of the current for n_e electrons in the system can then be described by

$$\langle\vec{J}\rangle=\frac{1}{n_e}\sum_i\langle\psi^i|\vec{J}_{op}|\psi^i\rangle=\mathrm{Tr}(\rho\vec{J}_{op})\,, \tag{11}$$

where

$$\langle\alpha|\rho|\alpha'\rangle\equiv\frac{1}{n_e}\sum_i a_\alpha^i(t)a_{\alpha'}^{i*}(t)$$

satisfies Liouville's equation[7-9,15]

$$i\hbar\frac{d\rho}{dt}=[\mathcal{H},\rho]\,. \tag{12}$$

To solve this equation, we will describe the time development of the density matrix as

$$\rho=\rho_0+\rho' e^{(s+i\omega)t}\,, \tag{13}$$

where ρ_0 is the uniform density matrix independent

of an electric field whose diagonal elements are the Fermi-Dirac distribution functions[9] f_α, and $\rho' e^{(s+i\omega)t}$ a time varying part driven by the electric field. Substitution of Eqs. (5) and (13) in Eq. (12) results in a coupled equation for matrix elements of ρ':

$$[\epsilon_{\alpha\alpha'} - i\hbar(s+i\omega)]\langle\alpha|\rho'|\alpha'\rangle = f_{\alpha\alpha'}\langle\alpha|V|\alpha'\rangle e^{-(s+i\omega)t} + \langle\alpha|[\rho_0, F]|\alpha'\rangle + \langle\alpha|[\rho', V]|\alpha'\rangle + \langle\alpha|[\rho', F]|\alpha'\rangle e^{(s+i\omega)t}, \quad (14)$$

with

$$\epsilon_{\alpha\alpha'} = \epsilon_\alpha - \epsilon_{\alpha'}, \quad (15)$$

$$f_{\alpha\alpha'} = f_\alpha - f_{\alpha'}. \quad (16)$$

Since we are not interested in this work in the generation of higher harmonics, we will neglect the last term of Eq. (14). In the solution of a coupled equation such as Eq. (14), a linearization procedure is usually adopted, where the $[\rho', V]$ term containing higher powers of V is neglected.[17] The linearized value of $\langle\alpha|\rho'|\alpha'\rangle$ is then used in the commutator $[\rho', V]$ term to generate higher-order terms. As has been discussed earlier in detail,[7,9] this will result in divergent results as the series expansion does not converge. We, therefore, solve Eq. (14) formally

$$\langle\alpha|\rho'|\alpha'\rangle = \frac{f_{\alpha\alpha'}\langle\alpha|V|\alpha'\rangle e^{-(s+i\omega)t} + \langle\alpha|[\rho_0, F]|\alpha'\rangle + \langle\alpha|[\rho', V]|\alpha'\rangle}{\epsilon_{\alpha\alpha'} + \hbar\omega - i\hbar s}. \quad (17)$$

This formal solution is then used in the $[\rho', V]$ term of Eq. (14) to get

$$(\epsilon_{\alpha\alpha'} + \hbar\omega - i\hbar s)\langle\alpha|\rho'|\alpha'\rangle = f_{\alpha\alpha'}\langle\alpha|V|\alpha'\rangle e^{-(s+i\omega)t} + \langle\alpha|[\rho_0, F]|\alpha'\rangle + \sum_{\alpha''}\frac{f_{\alpha\alpha''}\langle\alpha|V|\alpha''\rangle e^{-(s+i\omega)t} + \langle\alpha|[\rho', V]|\alpha''\rangle + \langle\alpha|[\rho_0, F]|\alpha''\rangle}{\epsilon_{\alpha\alpha''} + \hbar\omega - i\hbar s}\langle\alpha''|V|\alpha'\rangle - \sum_{\alpha''}\langle\alpha|V|\alpha''\rangle\frac{f_{\alpha''\alpha}\langle\alpha''|V|\alpha'\rangle e^{-(s+i\omega)t} + \langle\alpha''|[\rho', V]|\alpha'\rangle + \langle\alpha''|[\rho_0, F]|\alpha'\rangle}{\epsilon_{\alpha''\alpha'} + \hbar\omega - i\hbar s}. \quad (18)$$

This is an exact equation in the Ohmic limit. The iteration could be continued further, but we stop here to make some approximations. At this point, we will assume that V describes an electron-phonon interaction in the nonpolar model.[9,15] All the first-order terms in V will then drop out in the ensemble average.[9] Moreover, all the second-order terms will also drop out unless they can be reduced[9] to the form $|\langle\alpha|V|\alpha''\rangle|^2$. Now, we take the limit $s \to 0$ and use the identity

$$\lim_{s\to 0}\frac{1}{x - is} = P\left(\frac{1}{x}\right) + i\pi\delta(x), \quad (19)$$

to get the steady-state equation for $\langle\alpha|\rho'|\alpha'\rangle$

$$(\epsilon_{\alpha\alpha'} + \hbar\omega)\langle\alpha|\rho'|\alpha'\rangle = \langle\alpha|[\rho_0, F]|\alpha'\rangle + \langle\alpha|\rho'|\alpha'\rangle(\hbar\nu_{\alpha\alpha'} + i\hbar/\tau_{\alpha\alpha'}), \quad (20)$$

which can be easily solved to give

$$\langle\alpha|\rho|\alpha'\rangle = \frac{\langle\alpha|[\rho_0, F]|\alpha'\rangle}{\epsilon_{\alpha\alpha'} + \hbar\nu_{\alpha\alpha'} - i\hbar(1/\tau_{\alpha\alpha'} + i\omega)}, \quad (21)$$

with

$$1/\tau_{\alpha\alpha'} = 1/2\tau_\alpha + 1/2\tau_{\alpha'}, \quad (22)$$

$$\frac{1}{\tau_\alpha} = \frac{2\pi}{\hbar}\sum_{\alpha''_\pm}|\langle\alpha'|V|\alpha''\rangle|^2\delta(\epsilon_{\alpha\alpha''} + \hbar\omega \pm \hbar\omega_q), \quad (23)$$

$$\frac{1}{\tau_{\alpha'}} = \frac{2\pi}{\hbar}\sum_{\alpha''_\pm}|\langle\alpha|V|\alpha''\rangle|^2\delta(\epsilon_{\alpha'\alpha''} - \hbar\omega \pm \hbar\omega_q), \quad (24)$$

$$\nu_{\alpha\alpha'} = \frac{1}{\hbar}P\sum_{\alpha''_\pm}\left(\frac{|\langle\alpha|V|\alpha''\rangle|^2}{\epsilon_{\alpha'} - \epsilon_{\alpha''} - \hbar\omega \pm \hbar\omega_q} - \frac{|\langle\alpha'|V|\alpha''\rangle|^2}{\epsilon_\alpha - \epsilon_{\alpha''} + \hbar\omega \pm \hbar\omega_q}\right). \quad (25)$$

$\pm$ on the summation stands for phonon absorption and emission, respectively. Equation (21) has built in it a Breit-Wigner type of collision broadening which could not be obtained if the iteration procedure of other workers[17] were followed. The presence of the last term in Eq. (20) is thus quite important. This has been described by Arora and Peterson[9] as the scattering dynamics which goes beyond the strict Born approximation and is equivalent to Van Hove's "V^2t limit" technique.[18] Moreover, the energy levels are shifted by an amount proportional to the square of the scattering interaction.

III. MAGNETOCONDUCTIVITY TENSOR

The matrix elements of the one-electron current operator $\vec{J}_{op}$ as obtained from the Heisenberg

equation of motion,

$$\vec{J}_{op} = -e\vec{v}_{op} = -(ie/\hbar)[\mathcal{H}, \vec{r}], \tag{26}$$

are given by

$$\langle \alpha' | J_{op\,x} | \alpha \rangle = -(i\hbar e/\sqrt{2}\,\lambda m^*)[(n+1)^{1/2}\delta_{n',n+1} - n^{1/2}\delta_{n',n-1}]\delta_{k'k}, \tag{27}$$

$$\langle \alpha' | J_{op\,y} | \alpha \rangle = -(\hbar e/\sqrt{2}\,\lambda m^*)[(n+1)^{1/2}\delta_{n',n+1} + n^{1/2}\delta_{n',n-1}]\delta_{k'k}, \tag{28}$$

$$\langle \alpha' | J_{op\,z} | \alpha \rangle = -(e\hbar k_z/m^*)\delta_{n'n}\delta_{k'k}. \tag{29}$$

The ensemble average of the current $\langle \vec{J}_{op} \rangle$ can then be obtained from Eq. (11). The complex magneto-microwave conductivity tensor $\vec{\vec{\sigma}}$ defined by $\langle \vec{J} \rangle = \vec{\vec{\sigma}} \cdot \vec{\mathcal{E}}_0 e^{i\omega t}$ is then obtained as

$$\vec{\vec{\sigma}} = \begin{pmatrix} \sigma_1 & -\sigma_2 & 0 \\ \sigma_2 & \sigma_1 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}, \tag{30}$$

with

$$\sigma_1 = \frac{e^2}{m^*}\sum_{nks} f_{n,n+1}(k)(n+1)\frac{\tau_{n,n+1}(k)[1+i\omega\tau_{n,n+1}(k)]}{[\omega_c + \nu_{n+1,n}(k)]^2\tau^2_{n,n+1}(k) + [1+i\omega\tau_{n,n+1}(k)]^2}, \tag{31}$$

$$\sigma_2 = \frac{e^2}{m^*}\sum_{nks} f_{n,n+1}(k)(n+1)\frac{[\omega_c + \nu_{n+1,n}(k)]\tau^2_{n,n+1}(k)}{[\omega_c + \nu_{n+1,n}(k)]^2\tau^2_{n,n+1}(k) + [1+i\omega\tau_{n,n+1}(k)]^2}, \tag{32}$$

$$\sigma_3 = -e^2\sum_{nks}\frac{\tau_{nn}(k)}{1+i\omega\tau_{nn}(k)}\frac{df}{d\epsilon_{nk}}\left(\frac{\hbar k_z}{m^*}\right)^2, \tag{33}$$

where s stands for two-spin states of the electron. When the zero-frequency limit is taken ($\omega \to 0$), the conductivity tensor of Eq. (28) reduces to that obtained earlier.[9] When zero magnetic field limit is taken the results reduce to those obtained for zero-field microwave conductivity.[15] For the model in which τ is assumed to be constant, independent of quantum numbers n and k, the complex conductivity components of Eqs. (29)–(31) assume a simple form

$$\sigma_1 = \frac{n_e e^2}{m^*}\frac{\tau(1+i\omega\tau)}{\omega_c^2\tau^2 + (1+i\omega\tau)^2}, \tag{34}$$

$$\sigma_2 = \frac{n_e e^2}{m^*}\frac{\omega_c\tau^2}{\omega_c^2\tau^2 + (1+i\omega\tau)^2}, \tag{35}$$

$$\sigma_3 = \frac{n_e e^2}{m^*}\frac{\tau}{1+i\omega\tau}, \tag{36}$$

where n_e is the electronic density per unit volume.

In the works of Srivastava and others[1-5] an averaging of the expressions (34)–(36) was performed, where energy dependence of the relaxation time was included to analyze experimental results. Our results Eqs. (31)–(33) do reduce to their expressions in the limit of low magnetic field, when τ can be taken to be approximately independent of the magnetic field and the microwave frequency. It is interesting to note that in the experiments of Kennedy *et al.*[19] a frequency dependence of the cyclotron resonance in silicon inversion layers has indeed been observed. They also had to assume $B^{1/2}$ temperature dependence of the linewidth to fit their data. Both these features are included in expressions (31)–(33).

It is customary and convenient for the analysis of the microwave experiments to split a linearly polarized wave into two circularly polarized waves.[1] The propagation constants for the two senses of polarization are determined by the conductivity terms

$$\sigma_\pm = \sigma_1 \pm i\sigma_2 = \frac{e^2}{m^*}\sum_{nks} f_{n,n+1}(k)(n+1) \times \frac{\tau_{n,n+1}(k)}{1+i\{\omega \mp [\omega_c + \nu_{n+1,n}(k)]\}\tau_{n,n+1}(k)}, \tag{37}$$

whose real (in-phase component) and imaginary (out-of-phase component) parts are given by

$$\sigma_\pm^R = \frac{e^2}{m^*}\sum_{nks} f_{n,n+1}(k)(n+1) \times \frac{\tau_{n,n+1}(k)}{1+\{\omega \mp [\omega_c + \nu_{n+1,n}(k)]\}^2\tau^2_{n,n+1}(k)}, \tag{38}$$

$$\sigma_{\pm}^{I}=-\frac{e^2}{m^*}\sum_{nks} f_{n,(n+1)}(k)(n+1) \times \frac{\{\omega\mp[\omega_c+\nu_{n+1,n}(k)]\}\tau_{n,n+1}^2(k)}{1+\{\omega\mp[\omega_c+\nu_{n+1,n}(k)]\}^2\tau_{n,n+1}^2(k)}. \quad (39)$$

Investigation of resonance and microwave effects in the transverse configuration involves these terms. The mode with positive sign (σ_+) is characterized as the cyclotron-resonance active mode. At this point, it may be worthwhile to make comparison with the results obtained by Kawabata[12] using the generalized form of the Langevin equation of Brownian motion. The motivation behind Kawabata's work was to explain the line shift and linewidth in cyclotron-resonance absorption, where the power absorbed in the cyclotron-resonance active mode is proportional to σ_+^R [$\mathrm{Re}\sigma_{+-}(\omega)$ in notation of Ref. 12]. When properties of the scattering interaction of electrons via acoustic and optic phonons in the nonpolar model[9] are used, the anisotropic terms involving the ratio of J_+^B's in Eq. (3.6) of Ref. 12 will drop out. Then, the real part of $\Gamma_\alpha(\omega)$ used by Kawabata is equivalent to $1/\tau_{\alpha'\alpha}$ of Eq. (22) above when $\alpha=(n,k_y,k_z)$ and $\alpha'=(n+1,k_y,k_z)$, whereas the imaginary part of $\Gamma_\alpha(\omega)$ is equivalent to $\nu_{\alpha\alpha'}$ of Eq. (25). However, Kawabata approximates these components of $\Gamma_\alpha(\omega)$ by those at $\omega=\omega_c$ near the cyclotron-resonance frequencies.

The in-phase (σ_3^R) and out-of-phase (σ_3^I) components of σ_3 are given by

$$\sigma_3^R=-e^2\sum_{nks}\frac{\tau_{nn}(k)}{1+\omega^2\tau_{nn}^2(k)}\frac{df}{df_{nk}}\left(\frac{\hbar k_z}{m^*}\right)^2, \quad (40)$$

$$\sigma_3^I=e^2\sum_{nks}\frac{\omega\tau_{nn}^2(k)}{1+\omega^2\tau_{nn}^2(k)}\frac{df}{d\epsilon_{nk}}\left(\frac{\hbar k_z}{m^*}\right)^2. \quad (41)$$

The numerical computation of the above expressions can be facilitated by using the transformation and resummation technique described earlier.[7] The effect of multiple valleys and nonparabolicity can be included in a straightforward manner[8,10] if needed. The theory thus can be used to describe microwave experiments at high frequencies in a strong magnetic field. Some applications of this work will be published in future works.

IV. CONCLUSION

A quantum theory, that of using the density matrix, where no *ad hoc* insertion of the relaxation time has been made, is described. The semiclassical Boltzmann transport equation for microwave conductivity has been fairly well successful for problems involving none or at most a low magnetic field, because the de Broglie wavelength (λ_D) of the electron has always been smaller than the mean free path (λ_M) and the radius of the cyclotron orbit (λ), i.e., $\lambda_D<\lambda_M<\lambda$. For such fields, an electron can be treated like a classical particle and the effect of a magnetic field on its motion can be treated as a perturbation. But, for strong magnetic fields, this semiclassical situation may change when $\lambda\sim\lambda_M$ (or equivalently $\omega_c\tau\sim1$). In this case the effect of a magnetic field cannot be treated as a perturbation. The curvature in the mean free path of an electron introduces nondiagonal matrix elements of the velocity operator in any quantum-mechanical representation, and gives zero expectation value for the current if its averaging is attempted with the Boltzmann transport function. It is at this point that the quantum-mechanical treatment has a distinct advantage over Boltzmann-type techniques. This technique has further advantage when $\lambda\sim\lambda_D$ or $\hbar\omega_c\sim\zeta$, the Fermi energy (k_BT, for nondegenerate electrons); then the Landau quantization also plays a prominent role. A review paper by Dresden[20] does an excellent job in explaining why the semiclassical Boltzmann transport equation cannot be used for magnetic field of arbitrary strength.

With the advent of an era of strong magnetic fields available from superconducting magnets, it is hoped that the present work will help others to interpret microwave experiments in terms of more meaningful results.

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