

Intraband absorption in a low-density Hubbard chain*

S. K. Lyo and T. Holstein

Department of Physics, University of California, Los Angeles, California 90024

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We have investigated the intraband absorption in a single-band Hubbard model at a low concentration c (c is unity for a filled band) of electrons with a particular emphasis on one-dimensional nearest-neighbor electron transfer. Using a two-particle approximation, we find, for the latter in the frequency range $c \ll \hbar\omega$ (the bandwidth is unity) that the absorption is independent of frequency and vanishes as U^{-2} for a large on-site Coulomb repulsion U . The absorption occurs when an electron is accelerated by another electron in the presence of an oscillating field (i.e., inverse bremsstrahlung). The transport relaxation rate is given by $\tau^{-1} \propto c\omega^2 U^{-2}$. In the limit $U = \infty$ the present result supports the spinless fermion model, according to which the intraband absorption vanishes for $\omega \neq 0$. In the presence of second-nearest-neighbor electron transfer (t_2) such that $c \ll |t_2|$, the ω^{-2} -frequency dependence of the absorption obtains as in two or three dimensions.

I. INTRODUCTION

Recently, one-dimensional phenomena have received a considerable amount of theoretical and experimental attention because of their many interesting properties.¹ In one-dimensional molecular crystals, Coulomb correlation between electrons is particularly important, and the subsequent many-body effect complicates the problem. In this connection the Hubbard Hamiltonian² has been studied extensively.

The dc transport properties of one-dimensional systems have been studied by various authors. Brinkman and Rice³ and Beni, Holstein, and Pincus⁴ found that the mobility of a hole or an extra electron in an otherwise half-filled nearest-neighbor hopping Hubbard chain is infinite when the intrasite Coulomb repulsion (U) is infinite; for an arbitrary density, barring a half-filled band, the electrons seem to behave as spinless fermions, giving rise to an absorption rate proportional to a delta function $\delta(\omega)$, where ω is an external frequency. However, there has been no rigorous proof of the latter assertion. Recently, Lyo⁵ found for a low-density nearest-neighbor hopping Hubbard chain that the dc conductivity based on a two-particle approximation is infinite. He also found that the conventional dc Boltzmann equation based on a two-particle T -matrix approximation is insufficient in a nearest-neighbor hopping Hubbard chain and that many-particle scattering is important.

In this paper, we study the effect of strong Coulomb correlation between electrons on intraband absorption with a particular emphasis in a one-dimensional lattice, using a Hubbard model. The absorption occurs when an electron is accelerated by another electron of opposite spin in the presence of an oscillating field (i.e., inverse brems-

strahlung). A particularly interesting question is the frequency dependence of the absorption in the limit $U = \infty$ in a low-density one-dimensional system with nearest-neighbor electron transfer. The approach employed in the present paper improves the result obtained in Ref. 5 for this specific case. We find that the absorption is independent of the frequency and vanishes as U^{-2} for a large on-site Coulomb repulsion U within a two-particle approximation. In the limit $U = \infty$, the present result supports the spinless fermion model. By introducing a sufficiently large second-nearest-neighbor electron transfer, one obtains the ω^{-2} frequency dependence of the absorption as in two or three dimensions.

In Sec. II, we develop a theory of intraband absorption in a Hubbard band for a sufficiently large frequency (i.e., $\omega\tau \gg 1$), and at a low electron density, using a two-particle approximation. In Sec. III, we study how the absorption rate depends on dimensionality, frequency, and U . The effect of second-neighbor electron transfer to a nearest-neighbor hopping Hubbard chain is also investigated. In Sec. IV, a brief discussion is given.

II. FORMULATION

At a low electron concentration, it is sufficient to use a two-particle approximation. We investigate the mutual interaction of a pair of electrons of opposite spins in the presence of an oscillating field, which is described by a vector potential

$$\vec{A} = (\vec{E}_0 c_0 / i\omega)(e^{-i\omega t} - e^{i\omega t}), \quad (2.1)$$

where $2\vec{E}_0$ and c_0 are amplitude of the electric field and speed of light, respectively. The interaction of the pair with the external field is given by

$$H^F(t) = (e/mc_0)\vec{A} \cdot (\vec{p}_1 + \vec{p}_2), \quad (2.2)$$

where $e (= |e|)$, m , and \vec{p}_i are, respectively, charge, mass, and momentum operator of an electron. One studies the scattering of the pair from an initial state $|\phi_s\rangle = |\vec{k}_1 \vec{k}_2\rangle$, into a final state $|\phi_f\rangle = |\vec{k}_3 \vec{k}_4\rangle$ after going through intermediate states $|\phi_n\rangle = |\vec{k}_5 \vec{k}_6\rangle$. Here \vec{k}_i is the crystal momentum. The normalized orbital wave functions $|\phi_r\rangle$ are symmetrized. The matrix elements of the electron-field, and electron-electron interaction are given, respectively, by

$$\langle f | H^F(t) | n \rangle = [(e\vec{E}_0/i\omega) \cdot (\vec{v}_5 + \vec{v}_6) \delta_{f,n}] \times (e^{-i\omega t} - e^{i\omega t}) \quad (2.3)$$

and

$$V_{fn} = (1/N)U\Delta_{f,n}, \quad (2.4)$$

where $\vec{v}_i [\equiv (1/\hbar)(\partial\epsilon_{\vec{k}}/\partial\vec{k}_i)]$, $\epsilon_{\vec{k}}$ is a Bloch energy] is a Bloch velocity, and $\delta_{f,n}$ and N are the Kronecker delta and the number of sites, respectively. The quantity $\Delta_{f,n}$ is unity, when $\vec{k}_3 + \vec{k}_4 = \vec{k}_5 + \vec{k}_6 + \vec{g}$ (\vec{g} is a reciprocal-lattice vector), and zero otherwise.

The Schrödinger equation is given by

$$i\hbar \frac{d}{dt} c_f = \sum_n V_{fn} c_n e^{i\omega_{fn}t} + \sum_n \sum_{\pm} (\pm) H_{fn}^F c_n e^{i(\omega_{fn} \mp \omega)t}, \quad (2.5)$$

where H_{fn}^F is the quantity inside the square brackets of (2.3), $\omega_{fn} = (\epsilon_f - \epsilon_n)/\hbar$, and $\epsilon_f = \epsilon_{\vec{k}_3} + \epsilon_{\vec{k}_4}$, etc. The initial condition is given by $c_r(-\infty) = \delta_{r,s}$. The Schrödinger equation (2.5) is solved by introducing an ansatz⁶

$$c_f(t) = \frac{T_{fs} e^{i\omega_{fs}t + \alpha t}}{-\epsilon_{fs} + i\hbar\alpha} + \delta_{f,s} + \sum_{\pm} (\pm) \frac{\tau_{fs}^{(\pm)} e^{i(\omega_{fs} \mp \omega)t + \alpha t}}{-\epsilon_{fs} \pm \hbar\omega + i\hbar\alpha}, \quad (2.6)$$

where $\epsilon_{fs} = \hbar\omega_{fs}$ and α is a positive infinitesimal. Inserting (2.6) in (2.5) and dropping the quadratic terms in the external field, one finds

$$T_{fs} = \sum_n \frac{V_{fn} T_{ns}}{-\epsilon_{ns} + i\hbar\alpha} + V_{fs} \quad (2.7)$$

and

$$\tau_{fs}^{(\pm)} = \sum_n \frac{V_{fn} \tau_{ns}^{(\pm)}}{-\epsilon_{ns} \pm \hbar\omega + i\hbar\alpha} + \sum_n \frac{H_{fn}^F T_{ns}}{-\epsilon_{ns} + i\hbar\alpha} + H_{fs}^F. \quad (2.8)$$

One solves (2.7) and (2.8), using (2.3) and (2.4), and obtains

$$T_{fs} = T_s \Delta_{f,s}; \quad (2.9)$$

$$T_r = \frac{U}{N} \left(1 - \frac{U}{N} \sum_n \frac{\Delta_{n,s}}{\epsilon_r - \epsilon_n + i\hbar\alpha} \right)^{-1},$$

and ($H_n^F \equiv H_{nn}^F$),

$$\tau_{fs} = \frac{\Delta_{f,s}}{\hbar\omega} \left[T_f H_s^F - T_s H_f^F + T_f T_s \sum_n \Delta_{n,s} H_n^F \left(\frac{1}{\epsilon_s - \epsilon_n + i\hbar\alpha} - \frac{1}{\epsilon_f - \epsilon_n + i\hbar\alpha} \right) \right] + H_{fs}^F. \quad (2.10)$$

In (2.10), use is made of the energy conservation $\epsilon_f = \epsilon_s \pm \hbar\omega$ for $\tau_{fs} = \tau_s^{(\pm)}$. Here the upper (lower) sign denotes a photon-absorption (emission) process.

The first term in the square brackets of (2.10) describes a two-stage process, whereby the pair absorbs or emits a photon at a state ϕ_s and is, then, scattered into a final state ϕ_f . The second term represents a two-stage process, whereby the pair is scattered from an initial state ϕ_s into a final state ϕ_f and then absorbs or emits a photon. Finally the third term corresponds to a three-stage process, whereby the pair is scattered from an initial state ϕ_s into an intermediate state ϕ_n , absorbs or emits a photon, and is then scattered into a final state ϕ_f .

Finally the net power absorption per volume is given by

$$W = \frac{\hbar\omega}{\Omega} \{ 1 - e^{-\beta\hbar\omega} \} \frac{2\pi}{\hbar} \times \sum_{fs} f_1 f_2 f_3^{(+)} f_4^{(+)} |\tau_{fs}|^2 \delta(\epsilon_f - \epsilon_s - \hbar\omega), \quad (2.11)$$

where Ω , $f_i [\equiv f(\epsilon_{\vec{k}_i})]$ are, respectively, volume of the sample, the Fermi function, $f_i^{(+)} = 1 - f_i$, and $\beta = (k_B T)^{-1}$. Here k_B , T are Boltzmann's constant and temperature. The first and second terms in the curly brackets of (2.11) stand for absorption and emission processes, respectively. Equation (2.11) is valid in the limit $\omega\tau \gg 1$. In this limit, one can rewrite (2.11) in a Drude form

$$W = 2\sigma_R(\omega) E_0^2 = (2ne^2/m^* \omega^2 \tau) E_0^2 \quad (2.12)$$

where $\sigma_R(\omega)$, n , and τ are, respectively, real part of the conductivity, density of electrons, and transport relaxation time. Assuming that the field

is in the x direction, the effective mass m^* is given by⁵ $1/m^* = -(1/cN)\sum_k f'_k(v_k^x)^2$, where c is the electron concentration (c is unity for a filled band) and prime means a derivative with respect to the argument. As will be seen in the next section, (2.12) coincides with the result obtained by Lyo,⁵ using the Boltzmann equation in three dimensions in the limit $\beta\hbar\omega \ll 1$. In a simple cubic lattice, or a one-dimensional lattice, m^* is given at a high-temperature limit, $\beta B \ll 1$ (B is the bandwidth), by

$$1/m^* = \frac{1}{2}\beta(2ta/\hbar)^2 \quad (2.13)$$

where a, t are lattice constant and nearest-neighbor electron transfer, respectively. In the remainder of the paper we calculate the relaxation rate τ^{-1} defined by (2.11) and (2.12) for several situations.

III. INTRABAND ABSORPTION

A. Three-dimensional lattice

In two and three dimensions, the last term in the square brackets of (2.10) is negligibly small in the limit $\hbar\omega \ll B$. One then obtains, approximating $T_f \approx T_s$,

$$\tau_{fs} = (T_s \Delta_{f,s}/\hbar\omega)(H_s^F - H_f^F), \quad f \neq s. \quad (3.1)$$

This result combined with (2.11) and (2.12) is equivalent to that obtained earlier by Lyo⁵ using the Boltzmann equation. For a large Coulomb repulsion (i.e., $U \gg B$), one finds at high temperatures ($\beta B \ll 1$), and in a simple cubic crystal⁵

$$\tau_{3D}^{-1} \approx 3.1tc/\hbar. \quad (3.2)$$

As is expected, the relaxation rate is proportional

$$W = \frac{\beta(\hbar\omega)^2}{2\Omega} \frac{2\pi c^2}{\hbar} N^3 \int \int_{A_0} \frac{2d\theta_0 d\phi_0}{(2\pi)^2} \int \int_{A_0} \frac{2d\theta d\phi}{2\pi} \sum_g \{ |\tau_{fs}(g)|^2 \delta(\Delta\epsilon(g)) \} \delta(2\theta_0 - 2\theta - g), \quad (3.8)$$

where A corresponds to A_0 in $\theta - \phi$ plane, $f_1 = f_2 = c$, $f_3^{(*)} = f_4^{(*)} = 1$ in the present nondegenerate limit, and

$$\begin{aligned} \Delta\epsilon(g) &\equiv \epsilon_f - \epsilon_s - \hbar\omega \\ &= 4 \sin \frac{1}{2}(\phi_0 + \phi + \frac{1}{2}g) \\ &\quad \times \sin \frac{1}{2}(\phi_0 - \phi - \frac{1}{2}g) \\ &\quad \times \{ \cos \theta_0 + 2\epsilon \cos \theta_0 [\cos \phi_0 + \cos(\phi + \frac{1}{2}g)] \} - \hbar\omega. \end{aligned} \quad (3.9)$$

The integrand on the right-hand side of (3.8) with respect to the θ_0 integration is an even function of

to the bandwidth and the number of electrons of opposite spin.

B. One-dimensional lattice

In the following we compute the relaxation rate in a high-temperature limit ($\beta \ll 1$). One writes

$$\epsilon_k = -\cos k - \epsilon \cos 2k, \quad (3.3)$$

where $a \equiv 1$, $2t \equiv 1$, and ϵ is the second nearest-neighbor electron transfer. It is convenient to rewrite (2.10) as

$$\begin{aligned} \tau_{fs}(g) &= (\Delta_{f,s}/\hbar\omega) T_s T_f \\ &\quad \times [(N/U)(H_s^F - H_f^F) + X_{fs}(g)], \quad f \neq s \end{aligned} \quad (3.4)$$

where

$$X_{fs}(g) = \sum_n \Delta_{n,s} \left(\frac{H_n^F - H_s^F}{\epsilon_s - \epsilon_n + i\hbar\alpha} - \frac{H_n^F - H_f^F}{\epsilon_f - \epsilon_n + i\hbar\alpha} \right). \quad (3.5)$$

The dependence on the reciprocal lattice vector g is shown explicitly in (3.4). In order to perform the various k integrations, one transforms

$$\begin{aligned} k_1 + k_2 &= 2\theta_0, \quad k_1 - k_2 = 2\phi_0, \\ k_3 + k_4 &= 2\theta + g, \quad k_3 - k_4 = 2\phi. \end{aligned} \quad (3.6)$$

One then has, for example,

$$\int \int_{-\pi}^{\pi} dk_1 dk_2 (\dots) = \int \int_{A_0} 2d\theta_0 d\phi_0 (\dots), \text{ etc.}, \quad (3.7)$$

where A_0 is the region inside the rhomb with vertices at $(\theta_0, \phi_0) = (\pm\pi, 0)$, and $(0, \pm\pi)$ in $\theta_0 - \phi_0$ plane.

The absorption rate is then given from (2.11) by

θ_0 . Therefore, assuming $\theta_0 \geq 0$ and setting the quantity in the curly brackets of (3.8) as $F(\theta, \phi)$, one obtains

$$\begin{aligned} &\int \int_A d\theta d\phi \sum_g F(\theta, \phi) \delta(2\theta_0 - 2\theta - g) \\ &= \int_0^{\pi-\theta_0} d\phi F(\theta_0, \phi) + \int_0^{\theta_0} d\phi F(\theta_0 - \pi, \phi). \end{aligned} \quad (3.10)$$

The first term in (3.10) arises from a normal process $g=0$, and the second term from an umklapp process $g=2\pi$. One now transforms $\phi - \pi - \phi$ for the second term in (3.10). It turns out that $F(\theta, \phi)$ depends on θ and ϕ always as a function of $\epsilon_3 + \epsilon_4$

$= -2(\cos\theta \cos\phi + \epsilon \cos 2\theta \cos 2\phi)$ via energy parameters and of $v_s + v_4 = (2/\hbar)(\sin\theta \cos\phi + 2\epsilon \sin 2\theta \cos 2\theta)$ via velocity parameters. Hence $F(\theta_0 - \pi, \pi - \phi) = F(\theta_0, \phi)$, yielding

$$\int \int_A d\theta d\phi \sum_g F(\theta, \phi) \delta(2\theta_0 - 2\phi - g) = \int_0^\pi F(\theta_0, \phi) d\phi. \quad (3.11)$$

The remaining integrations in θ_0 , ϕ_0 , and ϕ in (3.8) can be rearranged, noting that the integrand is invariant (i) under $\theta_0 \rightarrow -\theta_0$, and (ii) under $\theta_0 \rightarrow \pi - \theta_0$, $\phi_0 \rightarrow \pi - \phi_0$, and $\phi \rightarrow \pi - \phi$:

$$L_r = - \int_0^{2\pi} \frac{d\phi'}{2\pi} \frac{1}{\epsilon_r + 2 \cos\theta_r \cos\phi' + 2\epsilon \cos 2\theta_0 \cos 2\phi' + i0}; \quad r = s, f, \quad (3.14)$$

where $\theta_s \equiv \theta_0$, $\theta_f \equiv \theta$, $\epsilon_r = -2 \cos\theta_r \cos\phi_r - 2\epsilon \cos 2\theta_0 \cos 2\phi_r$, $\phi_s \equiv \phi_0$, and $\phi_f \equiv \phi$. One also obtains

$$X_{fs}(g) = \frac{eE_0}{i\hbar\omega} (\sin\theta_0 - 2 \cos\theta_0 \tan 2\theta_0) \times \int_0^{2\pi} \frac{d\phi'}{2\pi} \left[\frac{1}{\cos\theta_0 - 2\epsilon \cos 2\theta_0 (\cos\phi' - \cos\phi_0) - i\delta \operatorname{sgn}(\cos\phi' + \cos\phi_0)} - \frac{1}{\cos\theta_0 - 2\epsilon \cos \frac{1}{2}g \cos 2\theta_0 (\cos\phi' - \cos\phi) - i\delta \cos \frac{1}{2}g \operatorname{sgn}(\cos\phi' + \cos\phi)} \right]. \quad (3.15)$$

Here δ is a positive infinitesimal.

1. Nearest-neighbor hopping Hubbard chain

Setting $\epsilon = 0$ and noting that $\cos\theta_0 \neq 0$ from (3.9), one finds from (3.15),

$$X_{fs}(g) = 0. \quad (3.16)$$

One also finds from (3.14),

$$L_r^{-1} = -2i |\cos\theta_0 \sin\phi_r|. \quad (3.17)$$

Using (3.16) in (3.4), one obtains

$$\tau_{fs}(g) = i \frac{\Delta_{fs}}{\hbar\omega} \frac{T_s T_f N}{U} eE_0 \tan\theta_0 \quad (3.18)$$

where

$$T_r = (1/N) L_r^{-1} / [1 + (L_r U)^{-1}]. \quad (3.19)$$

In (3.18), one power of $\hbar\omega$ has been canceled from the denominator unlike in three dimensions. This arises from the fact that $v_1 + v_2 - v_3 - v_4 \propto \omega$ in one dimension due to the energy conservation. This leads to ω^2 dependence of the relaxation rate unlike in three dimensions. The absorption rate can be calculated in a straightforward way from (3.12) in the region $\hbar\omega \ll 4 \ll k_B T \ll U$. The relaxation rate is given by

$$\tau_{1D}^{-1} = (0.090tc/\hbar)(\hbar\omega/U)^2. \quad (3.20)$$

Again, the relaxation rate is proportional to the bandwidth and number of scatterers. It is also fre-

$$W = \frac{4\beta(\hbar\omega)^2 c^2}{\Omega} \frac{2\pi}{\hbar} N^3 \int \int_0^\pi \frac{d\theta_0 d\phi_0 d\phi}{(2\pi)^3} F(\theta_0, \phi), \quad (3.12a)$$

where

$$F(\theta_0, \phi) = |\tau_{fs}(g)|^2 \delta(\Delta\epsilon(g)) \Big|_{g=0, \text{i.e., } \theta=\theta_0}. \quad (3.12b)$$

To compute the T matrix, one defines

$$L_r = - \frac{1}{N} \sum_n \frac{\Delta_{rn}}{\epsilon_r - \epsilon_n + i0}; \quad r = s, f. \quad (3.13)$$

Using the method described in (3.6)–(3.11), one finds

quency dependent and vanishes as $(\hbar\omega/U)^2$ for large U . The relaxation arises purely from normal processes. The umklapp processes give rise to higher-order contributions in $\hbar\omega$. This is due to the fact that a U process requires that $\cos \frac{1}{2}\theta_0 \sim \frac{1}{4}\hbar\omega$, namely, $k_1 + k_2 \simeq \pm\pi$, and the T matrix becomes small (i.e., of order $\hbar\omega$) in view of (3.17) and (3.19).

We now discuss the validity of (3.20). So far we have restricted ourselves to two-particle scattering. To include the many-body effect, one has to consider a self-energy as well as a scattering correction. As a consequence of the self-energy correction, the second term in the square bracket of (3.4) becomes of order c or smaller. The energy is defined within the accuracy of c , so that one requires $\hbar\omega \gg c$. It is to be remembered that the present approach is valid in the limit $\omega\tau \gg 1$. This condition is satisfied in (3.20) even for an arbitrarily low frequency. This means that there is no intraband absorption for $\hbar\omega > c$ in the limit $U = \infty$ in agreement with the spinless fermion model.

2. Effect of the second-nearest-neighbor hopping

For simplicity we assume $U = \infty$, so that the first term of (3.4) vanishes

$$\tau_{fs}(g) = (\Delta_{fs}/\hbar\omega) T_s T_f X_{fs}(g). \quad (3.21)$$

One distinguishes two situations. For $|\epsilon| \ll \hbar\omega$, one expands $X_{fs}(g)$ in (3.15) to the lowest order in ϵ , obtaining

$$X_{fs}(0) = \frac{4eE_0}{i\hbar\omega} (\sin\theta_0 - 2\cos\theta_0 \tan 2\theta_0) \times \frac{\epsilon \cos 2\theta_0}{\cos^2\theta_0} \left(\sin \frac{\phi_0 + \phi}{2} \sin \frac{\phi_0 - \phi}{2} \right) \quad (3.22)$$

and

$$T_r = N^{-1} L_r^{-1} = -2iN^{-1} |\cos\theta_0 \sin\phi_r|. \quad (3.23)$$

In (3.22) we have used the energy conservation $\Delta\epsilon(0) = 0$. Inserting (3.21)–(3.23) in (3.12), one obtains in the limit $\hbar\omega \ll 1$,

$$\tau_{\text{ID}}^{-1} = 0.62ct\epsilon^2/\hbar, \quad |\epsilon| \ll \hbar\omega. \quad (3.24)$$

For $1 \gg \epsilon \gg \hbar\omega$, one drops $\hbar\omega$ in (3.9)

$$\Delta\epsilon(g) = \left(4 \sin \frac{\phi_0 + \phi}{2} \sin \frac{\phi_0 - \phi}{2} \right) \times [\cos\theta_0 + 2\epsilon \cos 2\theta_0 (\cos\phi_0 + \cos\phi)] = 0. \quad (3.25)$$

In (3.25), the zeroes at $\phi = \pm\phi_0$ do not contribute to the absorption, because $X_{fs}(0)$ vanishes at these points. Therefore the energy conservation requires that $[\] = 0$ in (3.25). Using the energy conservation (3.25), one obtains from (3.15),

$$X_{fs}(0) = \frac{eE_0}{i\hbar\omega} \frac{1}{2|\epsilon|} \frac{|\sin\phi_0| + |\sin\phi|}{|\sin\phi_0 \sin\phi|} \times i \operatorname{sgn}(\cos\phi_0 - \cos\phi). \quad (3.26)$$

One also finds from (3.14), using the energy conservation

$$L_r = \frac{i}{4} \frac{|\sin\phi_0| + |\sin\phi|}{|\epsilon(\cos\phi_0 - \cos\phi) \sin\phi_0 \sin\phi|} \quad \text{for } g=0, r=s, f. \quad (3.27)$$

Using (3.21), (3.26), (3.27), and $T_r = N^{-1} L_r^{-1}$, one finds from (3.12), in the limit $\hbar\omega \ll 1$,

$$\tau_{\text{ID}}^{-1} = 0.47ct\epsilon^2/\hbar, \quad 1 \gg |\epsilon| \gg \hbar\omega. \quad (3.28)$$

One notes that the relaxation rates given in (3.24) and (3.28) are independent of the frequency as in a three-dimensional lattice. The many-body effect can be neglected in the limit $\epsilon \gg c$.

IV. CONCLUSION

We have studied the intraband absorption in a single-band Hubbard model at a low concentration of electrons. In a one-dimensional lattice with nearest-neighbor electron transfer, we find in the frequency range $c \ll \hbar\omega$ that the absorption is independent of frequency and vanishes as U^{-2} . The transport relaxation rate is given in (3.20). In the limit $U = \infty$, the present result supports the spinless fermion model. In the presence of second nearest-neighbor electron transfer (ϵ) such that $c \ll |\epsilon|$, the ω^{-2} frequency dependence of the absorption obtains as in two or three dimensions; the relaxation rate is given in (3.24) and (3.28).

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¹For review articles and references, see *Low-Dimensional Cooperative Phenomena*, edited by H. J. Keller (Plenum, New York, 1975).

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