Resistance maximum in spin glasses

Ulf Larsen
Physics Laboratory I, H. C. Ørsted Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

(Received 9 February 1976)

The temperature T_m of the resistance maximum in spin-glass magnetic alloys is calculated as a function of the Kondo temperature T_K for the single impurities and the average Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction strength Δ_c between different impurities. In addition T_m is shown to be independent of several other less relevant system parameters. The temperature dependence of the resistivity is obtained in the parquet approximation in the "noise model," where, besides the usual exchange scattering between conduction electrons and impurity spins, there is a transition rate Δ_c for spin-flip processes due to the RKKY interactions. The result is shown to be valid when $\Delta_c \gg T_K$, and applies to the low-T_K spin-glass systems like *Au*Mn, *Ag*Mn, *Cu*Mn, *Au*Cr, and *Au*Fe. For example, it permits the recently observed pressure variations of T_m to be understood in terms of simpler behaviors of the more fundamental system parameters Δ_c and T_K. Asymptotically for $\Delta_c \gg T_K$ one has $T_m \sim \Delta_c \ln(\Delta_c / T_K)$, which is bigger than the spin-glass freezing temperature $T_0 \sim \Delta_c$ identified by the cusp in the magnetic susceptibility.

I. INTRODUCTION

The electrical resistance in dilute magnetic alloys, like *Au*Fe, in the spin-glass[1] range of concentration of magnetic impurity, shows a maximum as a function of the temperature. This and several related effects are thought to be due ultimately to the s-d exchange interaction of the conduction electrons of the host metal with the localized spin of the magnetic impurity.

In the first place, this interaction at each individual impurity gives rise to the Kondo effect[2] with a resistance contribution that rises as the temperature is decreased, eventually reaching the unitarity limit in very dilute systems. Second, by the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction mechanism,[3] the conduction electrons mediate a Heisenberg-type exchange interaction between different impurities in the alloy. This eventually quenches the rising resistance as the temperature decreases into the range of the average RKKY interaction strength between neighboring impurities. The two effects thus result in the formation of a maximum at the temperature T_m.

It is reasonable to view this as the competition between two physical mechanisms, each of which can be characterized by a certain energy scale. For the single-impurity Kondo effect the scale is the Kondo temperature T_K, which depends on the s-d coupling constant in the usual way (defined in Sec. VI). The RKKY interaction depends on the inverse cube of the distance between the impurities and is second order in the same s-d coupling. The interaction energy of two impurities at the average neighbor distance, which is proportional to the inverse cube root of the concentration, will thus be proportional to the concentration and to the square of the s-d coupling constant. The scale of this interaction energy, which is also held responsible for the spin-glass ordering at low temperatures, will be denoted by Δ_c.

It is therefore reasonable to expect the temperature of the resistance maximum to be a function of these two energies

$$T_m = T_m(\Delta_c, T_K) \tag{1}$$

or ultimately of the s-d coupling and the concentration. In principle other mechanisms can influence T_m as well.

A number of calculations[4-7] have been done in the second Born approximation, which is the lowest order in which the Kondo effect is present. The s-d coupling must be extremely small for this approximation to be justified, i.e., T_K must be extremely small compared to the temperatures where the experiment is performed and hence compared to T_m. Even though this *may* be true in systems like *Au*Mn, where T_K's in the range 10^{-12}–10^{-4} K have been considered,[6,8] it is definitely not true in *Au*Fe,[9,10] with $T_K \sim 0.2$ K, as was pointed out by Matho and Béal-Monod.[6]

In order to provide a better calculation regarding the Kondo effect, the problem will be considered in the parquet approximation within the *noise model* as introduced originally by Riess and Ron.[7,11] This seems to be the simplest nontrivial way to represent the RKKY interactions, and in turn allows a more sophisticated treatment of the Kondo effect. The parquet approximation[12] is the simplest *nonperturbative* approximation and valid for temperatures much larger than T_K, but lower by several orders of magnitude than in the perturbation treatment. From the analysis of very-low-concentration alloys it is known that this is essen-

tially adequate for *Au*Fe for temperatures in the liquid-helium range,[9] although deviations become increasingly marked in the mK range as the temperature becomes of the order of T_K.[10]

Experimentally the concentration dependence of T_m has been established in many systems.[8,10,13-17] Recently a second dimension has been added by the demonstration of a rich variety of pressure dependences of T_m in different spin-glass systems at different concentrations.[18,19] The importance of this can be appreciated by noting that in (1) the concentration *only* influences Δ_c, whereas pressure *mainly* influences T_K.[20]

In order to provide a theoretical framework for the analysis of these experiments, in the present paper the mathematical details of the calculation of the resistance in the noise model will be given.[21] Second, relation (1) for T_m will be derived and expressed in a form convenient for numerical calculation. The actual analysis of the experiments will be given elsewhere.[19] Here it shall just be mentioned that given T_K for a spin-glass system (1) allows the compound quantity T_m to be "decoded" into the more basic Δ_c. In this way it is possible to understand the complicated behavior of T_m quantitatively in terms of much simpler behaviors of Δ_c and T_K, for example as the pressure is changed. The determining factor for the behavior of T_m in a particular alloy turns out to be the relative magnitude of Δ_c and T_K, i.e., the relative strengths of the RKKY interaction and Kondo effects.

II. NOISE MODEL

The Hamiltonian of the local interaction between the conduction electrons and a single impurity spin at the origin $\vec{R}=\vec{0}$ is given by

$$\mathcal{H}=-J\vec{\sigma}(\vec{0})\cdot\vec{S}. \tag{2}$$

Here $\vec{\sigma}(\vec{0})$ is the Pauli spin operator of the electron spin density at the impurity site. J is the s-d exchange coupling constant and $\vec{S}$ is the spin of the magnetic impurity. The electrons form a symmetrical Lorentzian conduction band of width D and density of states ρ_0 (per electron of each spin projection), centered at the Fermi energy (zero of energy).

The individual impurity spins of magnitude S have $2S+1$ projections $|S_z\rangle$ along the common axis of quantization. In the present calculation there is no external magnetic field. Hence, in the pseudofermion formalism each of these states is represented by a free pseudofermion of zero energy.[12,22] This corresponds to the spectral function of zero linewidth

$$\mathcal{A}_0(\omega)=\delta(\omega), \tag{3}$$

where ω is the pseudofermion energy. In this picture the lifetime of the pseudofermion is infinite, which means that left to itself the spin stays in a definite state $|S_z\rangle$ infinitely long.

In the noise model a transition rate Δ_c (i.e., transition probability per unit time, $\hbar=k_B=1$) is introduced into the pseudofermion spectral function. Now when a pseudofermion is scattered out of its state and into another, a flip of the spin takes place $|S_z\rangle\rightarrow|S_z'\rangle$. The presence of the *average* transition rate Δ_c for such processes means that the pseudofermions now have only a lifetime Δ_c^{-1}. Alternatively one may say that spin memory is lost by the impurities on the average after a time Δ_c^{-1} due to a source of *noise* located in the surroundings.[11] The spectral function now becomes a Lorentzian

$$\mathcal{A}(\omega)=\frac{\Delta_c/\pi}{\omega^2+\Delta_c^2}. \tag{4}$$

The combination of (4) with (2) forms the mathematical representation of the noise model.[7] It should be noted that in this model the impurities scatter incoherently as individual scattering centers.

When the model is stated in this way one does not have to commit oneself to any particular physical noise source in order to calculate the properties of the model. It is thus possible to test such physical models of the noise Δ_c against values of Δ_c obtained from experiments. In particular this holds for the concentration and pressure dependence of Δ_c.[19]

When introducing the noise model Riess and Ron assumed that RKKY interactions were the source of the noise and later calculated Δ_c in terms of the concentration c and J.[23] In this calculation Δ_c is thus identified with the average RKKY interaction energy.

Starting from a different point of view one obtains essentially the same result. The impurities are located at random sites in the host metal, interacting through the long-range oscillatory RKKY interaction. The individual spins will experience quite different surroundings and there will be no possibility of long-range order in the conventional sense. It has been shown[11,24] that for Heisenberg-type interactions—and in the absence of short-range order too—the distribution of internal magnetic fields H_z experienced by the spins is the Lorentzian

$$P(H_z)=\frac{\Delta_c/\pi}{H_z^2+\Delta_c^2}. \tag{5}$$

Averaging the free pseudofermion Green's function (retarded $\delta\rightarrow 0^+$, cf. Sec. III)

$$\mathcal{G}_0^R(\omega) = -(\omega + i\delta)^{-1} \tag{6}$$

in the presence of the field H_z (z being the axis of quantization), one gets

$$\mathcal{G}^R(\omega) = -\int_{-\infty}^{\infty} dH_z \frac{P(H_z)}{\omega - H_z + i\delta} = -\frac{1}{\omega + i\Delta_c}\,, \tag{7}$$

which is exactly equivalent to the spectral function (4) through the relation

$$\mathcal{A}(\omega) = (1/\pi)\,\mathrm{Im}\mathcal{G}^R(\omega)\,. \tag{8}$$

In fact, $P(H_z)$ may be directly interpreted as the spectral function, as can be seen from (7). In this approach Δ_c is again essentially equal to the average RKKY interaction energy

$$\Delta_c \propto cJ^2/D\,. \tag{9}$$

The different calculations do not agree entirely about the spin and concentration dependence, so a spin-dependent prefactor was left out and a linear concentration dependence chosen in (9). This is only needed as a physical guideline as the following calculations in the noise model do not require the specification of Δ_c. It turns out that using the results of this theory (9) is corroborated by the experiments.[19]

Most of the recent theories of the spin-glass freezing, *not* considering the Kondo effect, have been able to produce the key experimental feature, namely the cusp in the magnetic susceptibility at the freezing temperature T_0.[25] These theories rely on mean-field-type calculations in models both with and without short-range order below the freezing point.[26] Generally, as T comes below T_0 a local order parameter is induced, growing up in the usual way for a mean-field theory as $(T_0 - T)^{1/2}$. Above T_0 this order parameter vanishes. The existence of the nonvanishing order parameter below T_0 reflects the sudden freezing of the spin system into a *disordered* state, where each spin becomes locked into a fixed but random direction. The freezing is a macroscopic cooperative phenomenon, which accounts for the sharpness of the susceptibility cusp.

A distribution of fields $P(H_z)$ or a corresponding noise spectrum of width Δ_c at temperatures *above* T_0 is in accordance with these spin glass theories, even though they may disagree at temperatures *below* T_0, where a simple noise model is hardly applicable anyway. In general, when the Kondo effect is included in the theory of the spin-glass freezing one would expect a relation similar to (1) for the freezing temperature

$$T_0 = T_0(\Delta_c, T_K)\,. \tag{10}$$

Although so far no such calculation has been performed it is conceivable that for T_0 the influence of the Kondo effect will be restricted to the transition region where $\Delta_c \sim T_K$. Here one expects that both T_m and T_0 vanish (Fig. 1), such as to reflect the impossibility of forming the spin-glass condensate in the presence of a strong dominant Kondo effect. Conceivably, when $\Delta_c \gg T_K$ one has to a good approximation[26]

$$T_0 \simeq \Delta_c\,. \tag{11}$$

In contrast to this it will be shown in the following that

$$T_m \gg \Delta_c \tag{12}$$

in the same limit. It is thus a fortunate circumstance that around the maximum at T_m the noise model can be expected to be valid, undisturbed by the freezing that happens in the real spin glass at $T_0 \ll T_m$. *Formally* the model is valid at any temperature.

III. FORMALISM

The interaction (2) is a contact interaction and is therefore only felt by the conduction electrons when they occupy the Wannier state at the impurity site. The problem of calculating the scattering amplitude for electrons by the impurity at the origin can therefore be formulated exclusively in terms of local quantities. This is perhaps

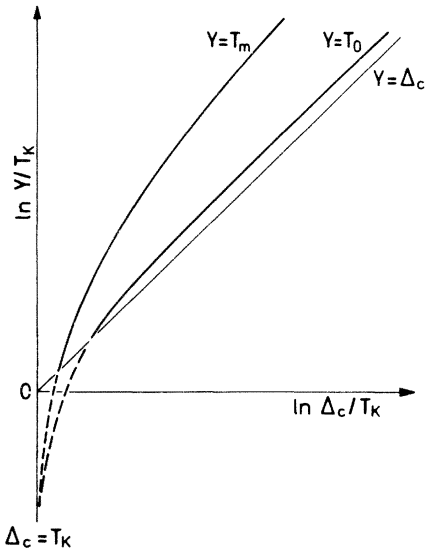

FIG. 1. Schematic representation of T_m and T_0 as functions of Δ_c and T_K based on experiments and the result of the present calculation. Δ_c is assumed roughly proportional to the concentration. Both T_m and T_0 are expected to vanish as Δ_c becomes of the same magnitude as T_K. In the opposite limit of low T_K and high Δ_c, T_0 is expected to be proportional to Δ_c, while in this work it is shown that $T_m \gg \Delta_c \simeq T_0$.

easiest to see by including the electron momenta and observing that since momentum is not conserved by (2) each momentum becomes independently summed over, leaving exclusively local Green's functions for the free-electron intermediate states in the scattering processes

$$G_0(n) = \frac{1}{N}\sum_{\vec{k}} \frac{1}{\mathcal{E}_{\vec{k}} - i\omega_n} = \int_{-\infty}^{\infty} dx \frac{\rho_0(x)}{x - i\omega_n} . \quad (13)$$

Here $\mathcal{E}_{\vec{k}}$ is the energy of a free electron of momentum $\vec{k}$, N is the total number of conduction electrons in the solid, $\rho_0(x)$ is the density of states, and $i\omega_n$ is the fermion Matsubara frequency of finite-temperature field theory[27,28]

$$i\omega_n = i\pi T(2n+1), \quad n = 0, \pm 1, \pm 2, \ldots . \quad (14)$$

The last expression of (13) is the spectral representation of the local free-electron Green's function, where $\rho_0(x)$ is the spectral function analogous to (4) for the similarly local pseudofermions. In terms of the Matsubara frequencies one has for the pseudofermions

$$\mathcal{G}(n) = \int_{-\infty}^{\infty} dx \frac{\mathcal{A}(x)}{x - i\omega_n} . \quad (15)$$

The spectral functions $\rho_0(x)$ and $\mathcal{A}(x)$ are normalized to unity, and they are even functions of x, due to particle-hole symmetry.

The spectral representations (13) and (15) exemplify the analytic structure of the Green's functions. They are defined at the points $i\omega_n$ along the imaginary axis. When analytically continued into the complex plane they define analytic functions in the z plane

$$\mathcal{G}(z) = \int_{-\infty}^{\infty} dx \frac{\mathcal{A}(x)}{x - z} , \quad (16)$$

which have a branch cut along the real axis. Similarly to this all the Green's functions used in the present analysis are defined on the double Riemann sheet structure shown in Fig. 2. The retarded function $\mathcal{G}^R(z)$, defined by analytical continuation from $i\omega_n$ in the upper half plane (UHP), where it is analytic, may be continued through the cut into the second sheet of the lower half plane (LHP), where it may have singularities. Similarly, the advanced function $\mathcal{G}^A(z)$, defined by analytical continuation from $i\omega_n$ in the LHP, where it is analytic, may be continued through the cut into the second sheet of the UHP, where it may have singularities similar to those of $\mathcal{G}^R(z)$ in the LHP. Thus

$$\mathcal{G}(z) = \begin{cases} \mathcal{G}^R(z) & \text{in UHP}, \\ \mathcal{G}^A(z) & \text{in LHP}. \end{cases} \quad (17)$$

The following useful relations hold on the real axis:

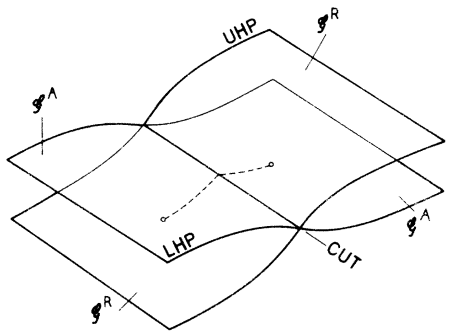

FIG. 2. Double Riemann sheet to define analytic properties of Green's functions. The cut is along the real axis and the upper sheet is the physical where the functions are analytic. Singularities may appear when functions defined on the upper sheet are analytically continued through the cut into the lower sheet. Symbols are defined in the text, cf. (17).

$$[\mathcal{G}^R(x)]^* = \mathcal{G}^A(x), \quad (18)$$

$$\mathrm{disk}\mathcal{G}(x) \equiv (1/2i)[\mathcal{G}^R(x) - \mathcal{G}^A(x)]$$
$$= \mathrm{Im}\mathcal{G}^R(x) = \pi\mathcal{A}(x), \quad (19)$$

where disk$\mathcal{G}$ is the discontinuity across the cut, and the last relation of (19) is identical to (8).

The spectral representations are examples of the dispersion relation for functions $F(z)$ with this analytic structure

$$F(z) - F_\infty = \frac{1}{\pi}\int_{-\infty}^{\infty} dx \frac{\mathrm{disk}F(x)}{x - z} , \quad (20)$$

where F_∞ is the value of $F(z)$ at $|z| \to \infty$. For the single-particle Green's functions $G_\infty = \mathcal{G}_\infty = 0$.

For the Lorentzian conduction band

$$\rho_0(x) = \frac{D/\pi}{x^2 + D^2} , \quad \rho_0 \equiv \rho_0(0) = (\pi D)^{-1} , \quad (21)$$

one has similarly to (7)

$$G_0^R(\omega) = -(\omega + iD)^{-1} , \quad (22)$$

and (18)–(20) apply.

To illustrate approximations the inventory of Feynman diagrams shown in Fig. 3 is used. Employing the reduced graph expansion[27] of the discontinuities "disk" in conjunction with the dispersion relation (20) is a most efficient way of summing the relevant contributions from diagrams in the various approximations. Here it will be used to produce the parquet approximation.

IV. SCATTERING AMPLITUDE

The total scattering amplitude $t(z)$ is isotropic s wave and independent of the momenta of the conduction electrons in the initial and final states. This follows from the reasoning in Sec. III. It is obtained by summing up all the repeated scattering

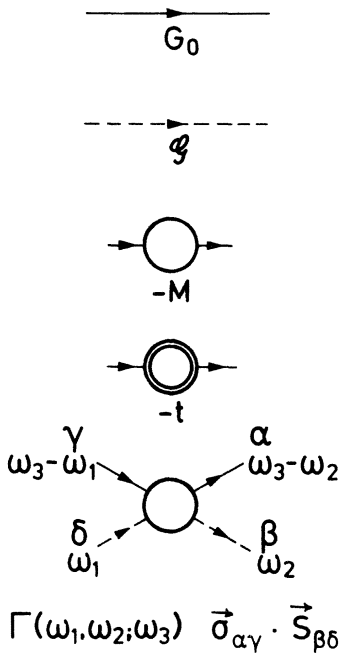

FIG. 3. Feynman diagrams symbolizing quantities defined in the text. α, β, γ, and δ are indices for the spin matrices and ω_1, ω_2, and ω_3 are single-particle energies. Γ is the renormalized exchange interaction.

processes due to (2) that take place while the electron is at the impurity site. Let $M(z)$ denote the irreducible part which does not include any single-electron intermediate states. Summing the terms of Fig. 4 gives

$$t^R(\omega)=M^R(\omega)/[1+G_0^R(\omega)M^R(\omega)]. \tag{23}$$

To obtain the resistivity only the value at $\omega=0$ is needed. Owing to the particle-hole symmetry of the model one can easily show that the real and imaginary parts of all the Green's functions are either even or odd functions of ω, respectively. For example, $\mathrm{Re}M^R(\omega)$ is odd and thus $M^R(0)=i\,\mathrm{disk}M(0)$. Using the auxiliary real function

$$h\equiv-\pi\rho_0\,\mathrm{disk}M(0), \tag{24}$$

one obtains for the imaginary part of (23)

$$-\pi\rho_0\,\mathrm{Im}t^R(0)=H(h)=h/(1+h). \tag{25}$$

For a random distribution of impurities, at low concentration the leading contribution to the resistivity is proportional to the concentration times the contribution from a single impurity. Introducing the correction factor $C(S)$ for pseudofermion statistics[22] the single-impurity contribution in units of the unitarity limit for s-wave scattering is

$$R=-\pi\rho_0 C(S)\,\mathrm{Im}t^R(0)=C(S)H(h). \tag{26}$$

Including then a source of temperature-independent

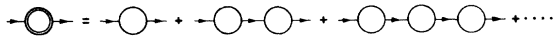

FIG. 4. Sum of irreducible parts making up the scattering amplitude for conduction electrons on the impurity site.

defect scattering A, the total "measurable" resistivity can be written in the form

$$\mathfrak{R}=A+BR, \tag{27}$$

where A depends on the concentration of nonmagnetic impurities, pressure, etc., and B depends on the magnitude of the unitarity limit and the concentration of magnetic impurities (leading term $\propto c$). In writing this expression the Fermi functions in the integrand of the transport coefficient for the resistivity were approximated by the zero-temperature δ function in the usual way so that only the $\omega=0$ scattering amplitude is needed. It was also assumed that contributions from coherent scattering of electrons simultaneously at two or more impurities is negligible (nonleading in the concentration) compared to the incoherent part kept in the noise model here. This was shown to be justified in second Born approximation by Matho and Béal-Monod,[6] and is assumed to be also valid in the present calculation.

All the temperature dependence of $\mathfrak{R}$ comes from h, which is to be calculated in Sec. V. The result of this is that h can be expressed in the form

$$h=f_S h_1(T/T_K;\Delta_c/T_K), \tag{28}$$

where f_S is a function of the impurity spin.

The equation for T_m is obtained by selecting the appropriate solution to

$$\left.\frac{\partial\mathfrak{R}}{\partial T}\right|_{T=T_m}=0. \tag{29}$$

As $H(h)$ is well behaved this ultimately reduces to

$$\frac{\partial}{\partial\tau}h_1\left(\tau;\frac{\Delta_c}{T_K}\right)_{\tau=T_m/T_K}=h_1'\left(\frac{T_m}{T_K};\frac{\Delta_c}{T_K}\right)=0. \tag{30}$$

It follows that among the ingredients of the model giving the $\mathfrak{R}$ given by (27), *T_m only depends on the two "relevant" model parameters T_K and Δ_c.* Apart from these *T_m is independent of* A, B, f_S, and the form of $H(h)$, i.e., of: (i) Size of sample (pressure dependent); (ii) effects of nonmagnetic impurities and lattice defects (pressure induced) other than those which influence Δ_c; (iii) magnitude of the unitarity limit, metal host parameters (except through the influence on Δ_c and T_K); (iv) concentration of magnetic impurities (except through Δ_c); (v) inclusion of multiple scattering, other effects on the form of $H(h)$; (vi) magnitude of the impurity spin (except through Δ_c). It must be em-

phasized that it is only T_m that is not dependent on (i)–(vi). The resistivity $\mathfrak{R}$ depends on all of them.

It also follows from (30) that the expression for T_m as a function of T_K and Δ_c, i.e., (1), is reduced from a function of *two* variables to a function of *one* variable, that may be written in the form

$$x_0 = x_0(d), \tag{31}$$

where

$$x_0 \equiv \ln(T_m/T_K) \quad \text{and} \quad d \equiv \ln(\Delta_c/T_K). \tag{32}$$

This is due to the fact that for each value of $\Delta_c/T_K > 1$ there is *one* resistivity curve with a maximum, hence *one* T_m/T_K.

The establishment of the relation (31) constitutes the principal result of this paper. In the Secs. V–VIII its actual form will be derived in the region of parameters where $\Delta_c > T_K$, together with conditions for the validity of the result.

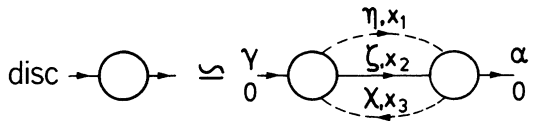

FIG. 5. Reduced graph expressing the discontinuity of the irreducible part. Only the contributions from intermediate states with the smallest number of particles are included. η, ζ, χ are indices for spin matrices; x_1, x_2, and x_3 are single-particle energies in the intermediate state.

V. CALCULATION OF h

The calculation of h according to (24) is done with the first term in the expansion of disk$M$, shown in Fig. 5. This includes the three-particle intermediate state, which is the smallest number in the irreducible part, together with the renormalized exchange interaction Γ to be calculated below. From the diagram in Fig. 5 one gets

$$h = \pi\rho_0\phi_S\pi(-1)^2\times 2\int_{-\infty}^{\infty} dx_1\, \mathfrak{A}(x_1)f(x_1)\int_{-\infty}^{\infty} dx_2\,\rho_0(x_2)f(x_2)\int_{-\infty}^{\infty} dx_3\,\mathfrak{A}(x_3)f(-x_3)$$
$$\times\delta(x_1+x_2-x_3)\Gamma^R(x_3,x_1;x_3)\Gamma^A(x_1,x_3;x_3), \tag{33}$$

where $f(x) = (1+e^{x/T})^{-1}$ is the Fermi function and the spin matrix for the diagram is (summing over repeated indices)

$$\phi_S\delta_{\alpha\gamma} = (\vec{\sigma}_{\zeta\gamma}\cdot\vec{S}_{\eta\chi})(\vec{\sigma}_{\alpha\zeta}\cdot\vec{S}_{\chi\eta}) = (2S+1)S(S+1)\delta_{\alpha\gamma}. \tag{34}$$

In the present calculation, which is standard parquet theory except for the inclusion of the noise width Δ_c in $\mathfrak{A}$, summing the leading logarithmic terms one takes Γ outside the integrals in (33) with all arguments equal to zero. When solving the equation for Γ below, it is sufficient to consider

$$\Gamma(z) \equiv \Gamma(z,z;z). \tag{35}$$

The $\rho_0(x)$ is also slowly varying and can be taken outside as ρ_0, and using that $\mathfrak{A}(x)$ is even in x one gets

$$h = \tfrac{2}{3}\phi_S|-2\rho_0\Gamma^R(0)|^2P(T/\Delta_c), \tag{36}$$

where

$$P\left(\frac{T}{\Delta_c}\right) \equiv \tfrac{3}{4}\pi^2\int_{-\infty}^{\infty} dx_1\int_{-\infty}^{\infty} dx_3\,\mathfrak{A}(x_1)\mathfrak{A}(x_3)f(x_1)f(x_3)f(-x_1-x_3), \tag{37}$$

which may be written in the form suitable for numerical integration

$$P(x) \equiv \tfrac{3}{4}\int_0^{\infty} dy\int_0^{\infty} dz\left\{(1+y^2)(1+z^2)\left[\cosh\left(\frac{y}{x}\right)+\cosh\left(\frac{z}{x}\right)\right]\right\}^{-1}. \tag{38}$$

To calculate the renormalized interaction Γ consider the expansion shown in Fig. 6. The spin matrices of the two terms are

$$\kappa_1 = (\vec{\sigma}_{\eta\gamma}\cdot\vec{S}_{\zeta\delta})(\vec{\sigma}_{\alpha\eta}\cdot\vec{S}_{\beta\xi}) = S(S+1)\delta_{\alpha\gamma}\delta_{\beta\delta} - \vec{\sigma}_{\alpha\gamma}\cdot\vec{S}_{\beta\delta},$$
$$\kappa_2 = (\vec{\sigma}_{\alpha\eta}\cdot\vec{S}_{\zeta\delta})(\vec{\sigma}_{\eta\gamma}\cdot\vec{S}_{\beta\xi}) = S(S+1)\delta_{\alpha\gamma}\delta_{\beta\delta} + \vec{\sigma}_{\alpha\gamma}\cdot\vec{S}_{\beta\delta}.$$

The full right-hand side then becomes

$$\kappa_1\pi(-1)^0(e^{\omega/T}-1)\int_{-\infty}^{\infty} dx_1\,\rho_0(x_1)f(x_1)\int_{-\infty}^{\infty} dx_2\,\mathfrak{A}(x_2)f(x_2)\delta(x_1+x_2-\omega)\Gamma^R(\omega,x_2;\omega)\Gamma^A(x_2,\omega;\omega)$$
$$+\kappa_2\pi(-1)^1(e^{\omega/T}-1)\int_{-\infty}^{\infty} dx_1\,\rho_0(x_1)f(-x_1)\int_{-\infty}^{\infty} dx_2\,\mathfrak{A}(x_2)f(x_2)\delta(-x_1+x_2-\omega)\Gamma^R(\omega,x_2;x_2)\Gamma^A(x_2,\omega;x_2).$$

Using (35), the $\Gamma^R\Gamma^A$ goes outside, and since $\rho_0(x)$ is even, the two terms are equal except for the opposite signs. Thus the $\delta\delta$ terms in the κ_1 and κ_2 cancel (and one easily shows that if a $\delta\delta$ term was included in Γ when calculating κ_1 and κ_2 these extra terms also cancel) and Γ multiplies the $\vec{\sigma}\cdot\vec{S}$ spin matrix. Then

$$\mathrm{disk}\Gamma(\omega) = -2\pi(e^{\omega/T}-1)\Gamma^R(\omega)\Gamma^A(\omega)\int_{-\infty}^{\infty} dx_1\,\rho_0(x_1)\mathcal{A}(\omega-x_1)f(x_1)f(\omega-x_1)\,. \tag{39}$$

Using (19) and

$$(e^{\omega/T}-1)f(x_1)f(\omega-x_1) = f(-x_1)-f(\omega-x_1)\,, \tag{40}$$

one has

$$\begin{aligned}\mathrm{disk}\frac{1}{\Gamma(\omega)} &= 2\pi\int_{-\infty}^{\infty} dx\,\rho_0(x)\mathcal{A}(\omega-x)[f(-x)-f(\omega-x)]\\ &= 2\pi\int_{-\infty}^{\infty} dx\,\rho_0(x)f(x)\mathcal{A}(\omega+x) - 2\pi\int_{-\infty}^{\infty} dx\,\mathcal{A}(x)f(x)\rho_0(\omega-x)\,.\end{aligned} \tag{41}$$

Then the dispersion relation (20) with $\Gamma_\infty = J$ gives

$$\begin{aligned}\frac{1}{\Gamma^R(\omega)}-\frac{1}{J} &= 2\int_{-\infty}^{\infty} dx\,\rho_0(x)f(x)\int_{-\infty}^{\infty} dy\,\frac{\mathcal{A}(y+x)}{y-\omega-i\delta} - 2\int_{-\infty}^{\infty} dx\,\mathcal{A}(x)f(x)\int_{-\infty}^{\infty} dy\,\frac{\rho_0(y-x)}{y-\omega-i\delta}\\ &= 2\int_{-\infty}^{\infty} dx\,\rho_0(x)f(x)\mathcal{G}^R(x+\omega) - 2\int_{-\infty}^{\infty} dx\,\mathcal{A}(x)f(x)G^R(\omega-x)\,.\end{aligned} \tag{42}$$

To obtain h by (36) Γ is only required at $\omega=0$. Then using (4), (7), (21), and (22) the integrals are easily written

$$\frac{1}{\Gamma^R(0)}-\frac{1}{J} = \frac{2}{\pi}\,\mathrm{Im}\int_{-\infty}^{\infty} dx\,f(x)\frac{1}{x+iD}\,\frac{1}{x+i\Delta_c}\,. \tag{43}$$

Adding a vanishing integral along an infinite semicircle in the upper half-plane this becomes by the residue theorem

$$\begin{aligned}\frac{1}{\Gamma^R(0)}-\frac{1}{J} &= \frac{1}{\pi^2}\,\mathrm{Im}\left(\frac{i}{T}\sum_{n=1}^{\infty}\frac{1}{n-\frac{1}{2}+D/2\pi T}\,\frac{1}{n-\frac{1}{2}+\Delta_c/2\pi T}\right) = \frac{2\pi T}{\pi^2 T(D-\Delta_c)}\sum_{n=1}^{\infty}\left(\frac{1}{n-\frac{1}{2}+\Delta_c/2\pi T}-\frac{1}{n-\frac{1}{2}+D/2\pi T}\right)\\ &= 2\rho_0[\psi(\tfrac{1}{2}+D/2\pi T)-\psi(\tfrac{1}{2}+\Delta_c/2\pi T)] = 2\rho_0[\ln(D/2\pi T)-\psi(\tfrac{1}{2}+\Delta_c/2\pi T)]\,,\end{aligned} \tag{44}$$

where terms $O(\Delta_c/D, T/D)$ have been neglected. Defining the dimensionless exchange coupling constant

$$g \equiv 2\rho_0|J| \tag{45}$$

and the Kondo temperature

$$T_K \equiv D e^{-1/g}\,, \tag{46}$$

then for $J<0$ one gets

$$-2\rho_0\Gamma^R(0) = [\ln(2\pi T/T_K)+\psi(\tfrac{1}{2}+\Delta_c/2\pi T)]^{-1}\,. \tag{47}$$

The vanishing of the [ ] as T is lowered signals the familiar divergence in Γ indicating the breakdown of the parquet approximation, when a pole of $\Gamma^R(z)$ from the second sheet of the LHP crosses the real axis and enters the first sheet in the UHP. This spurious pole in the UHP violates the analytical properties of $\Gamma^R(z)$ once $T \lesssim T_K$.

The key feature of the noise model is the presence of Δ_c, depressing for $0<\Delta_c<T_K$ the temperature where the pole crosses the real axis. When $\Delta_c = T_K$ this happens only just at $T=0$. If $\Delta_c > T_K$ the function has the correct analytic structure at any T. Inserting (47) in (36)

$$\begin{aligned}h &= \tfrac{2}{3}(2S+1)S(S+1)P(T/\Delta_c)\\ &\quad\times[\ln(2\pi T/T_K)+\psi(\tfrac{1}{2}+\Delta_c/2\pi T)]^{-2}\,,\end{aligned} \tag{48}$$

which is the desired result having the form (28), with

$$\begin{aligned}f_S &= \tfrac{2}{3}(2S+1)S(S+1)\,,\\ h_1(T/T_K;\Delta_c/T_K) &= P(T/\Delta_c)[\ln(2\pi T/T_K)\\ &\qquad+\psi(\tfrac{1}{2}+\Delta_c/2\pi T)]^{-2}\,.\end{aligned} \tag{49}$$

VI. RESISTIVITY

The resistivity is given by (27) in terms of (26), (25), and (48)

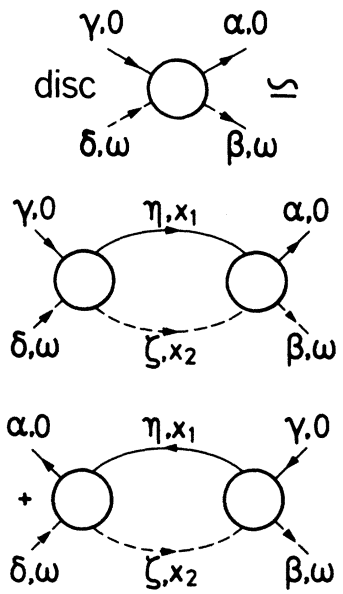

FIG. 6. Reduced graphs expressing the discontinuity of the renormalized exchange interaction. Only the contributions from intermediate states with the smallest number of particles are included in the parquet approximation. η and ζ are indices for the spin matrices; x_1 and x_2 are single-particle energies in the intermediate states.

$$R\left(\frac{T}{T_K};\frac{\Delta_c}{T_K}\right) = \frac{C(S) f_S P(T/\Delta_c)}{[\ln(2\pi T/T_K)+\psi(\frac{1}{2}+\Delta_c/2\pi T)]^2 + f_S P(T/\Delta_c)}\,. \tag{50}$$

In order to infer $C(S)$ expand this in leading order as $T\to\infty$, where $P(x)\to\frac{3}{32}\pi^2$

$$R\sim 3\pi^2 C(S) f_S/32\ln^2(T/T_K)\,. \tag{51}$$

Noting that this must agree with Abrikosov's[12] and Hamann's[29] result in the same limit where the noise plays no role

$$R\sim \pi^2 S(S+1)/4\ln^2(T/T_K)+O(\ln^{-3}(T/T_K))\,, \tag{52}$$

it follows that

$$C(S)=4/(2S+1)=\begin{cases}2 & \text{for } S=\frac{1}{2}\,,\\ \frac{4}{3} & \text{for } S=1\,,\end{cases} \tag{53}$$

in agreement with the result of the exact calculation of $C(S)$ in the high-temperature limit.[22] Thus one gets the result

$$R\left(\frac{T}{T_K};\frac{\Delta_c}{T_K}\right) = \frac{\frac{8}{3}S(S+1)P(T/\Delta_c)}{[\ln(2\pi T/T_K)+\psi(\frac{1}{2}+\Delta_c/2\pi T)]^2+\frac{2}{3}(2S+1)S(S+1)P(T/\Delta_c)}\,. \tag{54}$$

Letting $\Delta_c\to 0$ one recovers the noise-free parquet result valid for $T\gg T_K$

$$R\left(\frac{T}{T_K};0\right) = \frac{\frac{1}{4}\pi^2 S(S+1)}{[\ln(2\pi T/T_K)+\psi(\frac{1}{2})]^2+\frac{1}{16}\pi^2(2S+1)S(S+1)}\,. \tag{55}$$

According to Sec. V (47) only has the correct analytical structure at all T when $\Delta_c > T_K$. Considering (54) in this case for $T\ll\Delta_c$ and using that

$$P(x)\simeq\tfrac{3}{16}\pi^2x^2 \quad \text{for } x\to 0\,, \tag{56}$$

one gets

$$R(T/T_K;\Delta_c/T_K)\simeq\tfrac{1}{2}\pi^2 S(S+1) \times[T/\Delta_c\ln(\Delta_c/T_K)]^2+O(T^4)\,. \tag{57}$$

As T is increased R goes through a maximum (at T_m calculated in Sec. VI) and then decreases, eventually making contact with the slowly decreasing loagrithmic parquet resistivity (52) at very high temperatures $T\gg\Delta_c$.[21] The validity of the expression (54) is discussed below and in Sec. VIII.

In the Kondo model even terms of lower order than those included in the parquet approximation continue to have some potential influence. Thus the Kondo contribution to the width of the spectral function is[30]

$$\Delta_K\sim T/\ln^2(T/T_K) \tag{58}$$

and in fact increases with T, though slower than linearly, which is the reason why at high temperatures it does not contribute significantly compared to the thermal fluctuation rate $\sim T$. But as $T\to\infty$ it will eventually exert a stronger influence on the zero linewidth parquet resistivity (55) than does the noise rate Δ_c. In (54) the deviation from (55) as T comes down from ∞ is of the order

$$\sim\Delta_c/T\,,$$

in agreement with the virial expansion,[31] while at the same temperature the leading Kondo deviation would be of the order

$$\Delta_K/T\sim 1/\ln^2(T/T_K)$$

and is clearly the bigger part when T is large enough, though a small correction. Fortunately this happens only at $T\gg T_m$ where such corrections are insignificant anyway, while at $T\simeq T_m$ it turns

out that the noise dominates the Kondo fluctuations as long as $\Delta_c \gg T_K$. The features of the maximum that will be derived in Sec. VII are thus genuine effects of the noise, not strongly influenced by the use of the parquet approximation.

VII. MAXIMUM

The temperature of the maximum is readily extracted from measurements of the resistance. The analysis of T_m is a source of information about the "essential" parameters Δ_c and T_K, that does not rely on detailed fits of the temperature dependence of $\mathfrak{R}$ to the experimental resistance that would involve the consideration of a great many additional parameters, cf. Sec. IV.

According to (30) T_m is determined by the zero of the temperature derivative of (49). In order to facilitate the mathematics of the derivation of the relation $x_0(d)$, introduce the set of dimensionless quantities

$$\begin{aligned} &\tau \equiv T/T_K\,, \quad x \equiv \ln\tau\,, \\ &\tau_m \equiv T_m/T_K\,, \quad x_0 \equiv \ln\tau_m\,, \\ &\alpha \equiv \Delta_c/T_K\,, \quad d \equiv \ln\alpha\,, \end{aligned} \tag{59}$$

and the auxiliary functions

$$E(\tau;\alpha) \equiv \ln(2\pi\tau) + \psi(\tfrac{1}{2} + \alpha/2\pi\tau)\,, \tag{60}$$

$$\equiv \ln\alpha + N(\tau/\alpha)\,, \tag{61}$$

$$\equiv d + \mathfrak{N}(x-d)\,, \tag{62}$$

$$P(\tau/\alpha) \equiv \mathcal{P}(x-d)\,, \tag{63}$$

where script symbols denote the functions of the logarithmic arguments. Equations (30) and (49) then determine τ_m through the implicit equation

$$E(\tau_m;\alpha)P'(\tau_m/\alpha) = 2P(\tau_m/\alpha)E'(\tau_m;\alpha)\,. \tag{64}$$

Letting prime denote the derivative with respect to the argument of the logarithmic functions the equivalent equation for x_0 is

$$[d + \mathfrak{N}(x_0-d)]\mathcal{P}'(x_0-d) = 2\mathcal{P}(x_0-d)\mathfrak{N}'(x_0-d)\,. \tag{65}$$

Given $\mathfrak{N}$, $\mathfrak{N}'$, $\mathcal{P}$, and $\mathcal{P}'$ as functions of their argument y this equation allows a direct parametric evaluation of $x_0(d)$

$$\begin{aligned} x_0 &= y + 2\mathcal{P}(y)\mathfrak{N}'(y)/\mathcal{P}'(y) - \mathfrak{N}(y)\,, \\ d &= 2\mathcal{P}(y)\mathfrak{N}'(y)/\mathcal{P}'(y) - \mathfrak{N}(y)\,. \end{aligned} \tag{66}$$

The relation is shown in Fig. 7.

The asymptotic series for $x_0(d)$ as $d \to \infty$ can be generated as follows. One has for use in (64)

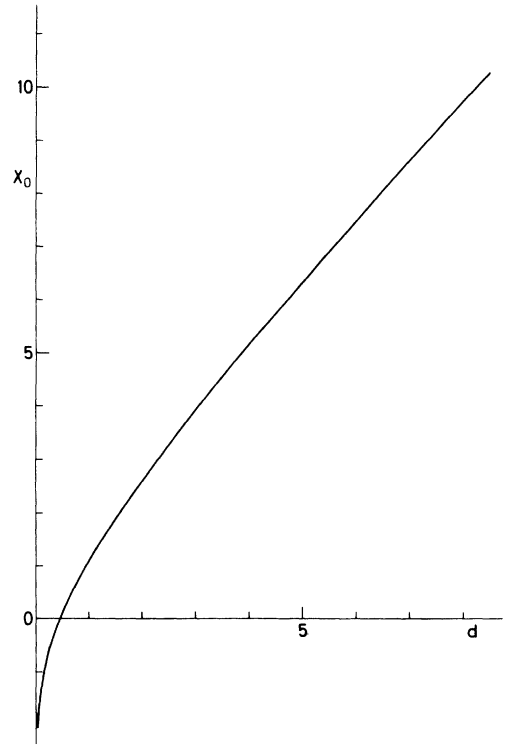

FIG. 7. Relation $x_0 = x_0(d)$ expressing T_m as a function of Δ_c and T_K [cf. (31) and (32)]. The slope $x_0'(d)$ is a monotonically decreasing function of d. In the limit $d \to \infty$ the relation is $x_0 \sim d + \ln d + \text{const}$. Although qualitatively reasonable for all positive d this relation is only strictly valid for large d.

$$E = x_0 + \ln(2\pi) + \psi(\tfrac{1}{2} + \alpha/2\pi\tau_m)\,, \tag{67}$$

$$\tau_m E' = 1 - (\alpha/2\pi\tau_m)\psi'(\tfrac{1}{2} + \alpha/2\pi\tau_m)\,, \tag{68}$$

$$P \simeq \tfrac{3}{32}\pi^2[1 - i_0\alpha/\tau_m - i_0 j(\alpha/\tau_m)^2 + O((\alpha/\tau_m)^3)]\,, \tag{69}$$

$$\tau_m P' \simeq \tfrac{3}{32}\pi^2[i_0\alpha/\tau_m + 2i_0 j(\alpha/\tau_m)^2 + O((\alpha/\tau_m)^3)]\,. \tag{70}$$

The asymptotic expansions of P and P' (69) and (70) are nontrivial, as is the integral (38) itself. The coefficient i_0 has been calculated exactly

$$i_0 = 28\zeta(3)/\pi^3 = 1.085\,509\,028\,, \tag{71}$$

whereas a numerical estimate of j gives

$$j \simeq -0.773\,. \tag{72}$$

It follows that

$$[x_0 + \ln(2\pi) + \psi(\tfrac{1}{2} + \alpha/2\pi\tau_m)][i_0\alpha/\tau_m + 2i_0 j(\alpha/\tau_m)^2 + O((\alpha/\tau_m)^3)]$$
$$\simeq 2[1 - i_0\alpha/\tau_m - i_0 j(\alpha/\tau_m)^2 + O((\alpha/\tau_m)^3)][1 - (\alpha/2\pi\tau_m)\psi'(\tfrac{1}{2} + \alpha/2\pi\tau_m)]\,.$$

Now for $\tau_m \to \infty$ and therefore $x_0 \to \infty$ the leading terms are

$$x_0 i_0 \alpha/\tau_m \simeq 2 ,$$

and it is possible to express α/τ_m as the series

$$\alpha/\tau_m \simeq 2/i_0 x_0 + u/x_0^2 + O(x_0^{-3}) . \tag{73}$$

Using this and expanding the digamma function one gets

$$\{1 + [\ln(2\pi) + \psi(\tfrac{1}{2})]/x_0 + O(x_0^{-2})\}[1 + (i_0 u + 8j/i_0)/2x_0 + O(x_0^{-2})] \simeq [1 - 2/x_0 + O(x_0^{-2})][1 - 2\psi'(\tfrac{1}{2})/2\pi i_0 x_0 + O(x_0^{-2})] ,$$

which is an equation for the constant u giving

$$i_0 u = -2[2 + \psi'(\tfrac{1}{2})/\pi i_0 + \ln(2\pi) + \psi(\tfrac{1}{2}) + 4j/i_0] \simeq -0.946 . \tag{74}$$

Using this result and (59) one may write (73) in the form

$$d \simeq x_0 + \ln\{(2/i_0)[1/x_0 + i_0 u/2x_0^2 + O(x_0^{-3})]\} , \tag{75}$$

valid when $x_0 \to \infty$ and $d \to \infty$. The inverse relation $d = d(x_0)$ is useful, and (75) is a rather good approximation numerically when d is not too small.

In order to deal with alloys like AuFe a better numerical approximation is required, though. Rather than working directly with a numerical solution like (66), in the practical analysis of data the following Padé approximation is convenient. The argument of the logarithm in (75) is replaced by the ratio of two polynomials having the same $x_0 \to \infty$ expansion as (75). The degrees of the two polynomials are constrained by the necessity of avoiding the spurious divergence of (75) when $x_0 \to 0$. The actual choice is made by considerations of numerical accuracy compared to (66). The approximation is given by

$$d = x_0 + \ln Q(x_0) , \tag{76}$$

with

$$Q(x_0) \simeq \frac{2(R_2 + R_1 x_0 + x_0^2)}{i_0(R_5 + R_4 x_0 + R_3 x_0^2 + x_0^3)} , \tag{77}$$

$$R_1 = 9.527 , \quad R_2 = 20.792 , \quad R_3 = 10.000 ,$$

$$R_4 = 25.375 , \quad R_5 = 24.610 ,$$

and is accurate to three decimal places in $d > 0.75$ or $x_0 > 0.65$. The expression (76) is easy to invert numerically by iteration to produce the corresponding approximation for $x_0 = x_0(d)$. Also, another quantity of interest is easily obtained from (76) and (77)

$$\frac{\partial x_0}{\partial d} \equiv x_0' = [1 + Q'(x_0)/Q(x_0)]^{-1} , \tag{78}$$

where $Q'(x_0) = \partial Q/\partial x_0$. A useful expression equivalent to (73) is

$$\Delta_c = T_m Q(\ln(T_m/T_K)) , \tag{79}$$

from which Δ_c can be calculated given T_m and T_K. Conversely, from $x_0(d)$ T_m can be calculated given Δ_c and T_K

$$T_m = T_K \exp[x_0(\ln(\Delta_c/T_K))] . \tag{80}$$

Finally, from the inverse function of (77), $x_0 = q(Q)$, and (79) T_K can be obtained from Δ_c and T_m

$$T_K = T_m \exp[-q(\Delta_c/T_m)] . \tag{81}$$

Thus given any *two* of the three quantities Δ_c, T_K, and T_m the third may be calculated. It is noteworthy that these relations, being in general functions of *two* variables, like (1), may in fact be expressed by functions of *one* variable connecting ratios like for example T_m/T_K and Δ_c/T_K. This is as close as one gets to universal scaling behavior in a system with two characteristic energies.

VIII. DISCUSSION

It is of interest to consider the behavior of the leading terms in the asymptotic limit when $d \to \infty$, i.e., when $\Delta_c \gg T_K$. The leading term in Q is for $x_0 \to \infty$

$$Q \sim 2/i_0 x_0 ,$$

so it follows that

$$d \sim x_0 + \ln(2/i_0 x_0) , \tag{82}$$

$$x_0 \sim d + \ln(i_0 d/2) , \tag{83}$$

$$x_0' \sim 1 + 1/x_0 \sim 1 + 1/d . \tag{84}$$

Note that these reflect the general monotonical increase of x_0 with d and decrease of x_0' with d. One further obtains

$$\Delta_c \sim [\pi^3/14\zeta(3)]\, T_m/\ln(T_m/T_K) , \tag{85}$$

$$T_m \sim [14\zeta(3)/\pi^3]\, \Delta_c \ln(\Delta_c/T_K) , \tag{86}$$

$$T_K \sim T_m \exp\{-[\pi^3/14\zeta(3)]\, T_m/\Delta_c\} . \tag{87}$$

Here note that Δ_c and T_m depend weakly on T_K, while in turn T_K depends strongly on their ratio.

The principal result is (86) for T_m which shows that indeed $T_m \gg \Delta_c$ when $\Delta_c \gg T_K$ as was remarked in Sec. II. The logarithm involves T_K in an *essential* way. It seems appropriate that T_K thus appears explicitly in the expression for the temperature of the maximum, which is after all mainly due to the Kondo effect. This has not always been the case in previous theories,[4,5,7] and it is essential to account for the pressure variation of T_m.[19]

One sees that as T_K is decreased, with fixed Δ_c, T_m increases. That is, the weaker the Kondo effect is the sooner it will be effectively quenched by RKKY interactions as the temperature is decreased. Simultaneously the absolute value of the resistivity at T_m decreases, also indicating the weaker Kondo effect.

Another point worthy of comment is the following. As was mentioned before, with two characteristic energies there is no possibility of strict universality of the temperature-dependent resistivity. However, comparing (86) and (57) one notes that for $T_K \ll \Delta_c$ it looks as if the resistivity on the low-temperature side and around the maximum is *approximately* a function only of T/T_m. When T_K is very small there may be a region of $\Delta_c \simeq T_0 < T \ll T_m$, where (57) is a fair approximation. This approximate universality, that will cease to hold when $T \gg T_m$, *may* account for the observations of universal behavior $\mathcal{R} \sim \mathcal{R}(T/T_m)$ at different concentrations in AgMn,[14] which does have a very small $T_K < 1$ mK.

Finally one must check the various conditions for the validity of these results. *Formally,* the relation $x_0(d)$ holds for $d > 0$, i.e., for $\Delta_c > T_K$. Qualitatively the result that $T_m \to 0$ as $\Delta_c \to T_K^+$ seems reasonable. However, the parquet approximation requires $T_m \gg T_K$ for the expression to be strictly valid. This is satisfied when $\Delta_c \gg T_K$. Simultaneously one achieves [cf. (86)] $T_m \gg \Delta_c \simeq T_0$ (cf. Sec. II) getting away from the spin-glass freezing. Using (58), at $T \simeq T_m$ the Kondo fluctuations are of the magnitude

$$\Delta_K \sim \frac{\Delta_c \ln(\Delta_c/T_K)}{[\ln(\Delta_c/T_K) + \ln\ln(\Delta_c/T_K)]^2} < \frac{\Delta_c}{\ln(\Delta_c/T_K)} \ll \Delta_c ,$$

hence insignificant when $\Delta_c \gg T_K$.

When will Δ_K become of the same order of magnitude as Δ_c? According to Sec. VI this will happen for temperatures

$$T \gtrsim T_u \sim \Delta_c \ln^2(\Delta_c/T_K) \sim T_m \ln(\Delta_c/T_K) \gg T_m ,$$

and the relative magnitude of both corrections to noise-free parquet resistivity at these temperatures will then be less than

$$\Delta_c/T_u \sim 1/\ln^2(\Delta_c/T_K) \ll 1 ,$$

and thus insignificant anyway.

Consequently the mathematical criterion for the validity of the $x_0(d)$ relation for T_m is

$$\ln(\Delta_c/T_K) \gg 1 \quad \text{(sharp)},$$

$$\Delta_c/T_K \gg 1 \quad \text{(soft)},$$

where the soft criterion is sufficient to satisfy the parquet approximation, which is perhaps the most important, while the sharp criterion is needed to get clear of T_0 and effectively eliminate corrections to the parquet approximation compared to noise corrections around T_m.

These criteria do not directly point out the *numerical* conditions for the validity of the theory, which can in practice only be assessed by comparison with a more sophisticated theory and by application to the experiments. One can expect all the alloys AuMn, AgMn, CuMn, AuCr, and AuFe for concentrations of the order 0.1 at.% to be at least on the right side of the criteria, all of them satisfying the soft criterion on the 10% level or better, while especially AuFe with the highest T_K may have some difficulty satisfying the sharp criterion fully. Thus it is particularly in AuFe that one may expect to see spin-glass freezing effects rather close below the maximum effects that the present theory may not account for.[32]

ACKNOWLEDGMENTS

I wish to thank Dr. J. S. Schilling, Dr. P. J. Ford, and Dr. J. A. Mydosh for many extremely valuable discussions and for making their experimental results available to me before publication. I have also benefited from discussions with Dr. K. H. Fischer, Dr. H. Højgaard Jensen, Dr. K. Matho, Dr. R. D. Mattuck, and Dr. J. L. Tholence.

[1] J. A. Mydosh, AIP Conf. Proc. 24, 131 (1975).
[2] *Magnetism*, edited by H. Suhl (Academic, New York, 1973), Vol. 5; C. Rizzuto, Rep. Prog. Phys. 37, 147 (1974).
[3] M. A. Ruderman and C. Kittel, Phys. Rev. 96, 99 (1954); K. Yosida, *ibid.* 106, 893 (1957); T. Kasuya, Progr. Theor. Phys. 16, 45 (1956); C. Kittel, Solid State Phys. 22, 1 (1968).
[4] S. D. Silverstein, Phys. Rev. Lett. 16, 466 (1966); H. Suhl, *ibid.* 20, 656 (1968).

[5]R. J. Harrison and M. W. Klein, Phys. Rev. 154, 540 (1967); 167, 878 (1967); Phys. Rev. B 1, 940 (1970).
[6]M. T. Béal-Monod, Phys. Rev. 178, 874 (1969); K. Matho and M. T. Béal-Monod, Phys. Rev. B 5, 1899 (1972); J. Phys. F 3, 136 (1973); K. Sato and Y. Nagaoka, Prog. Theor. Phys. 47, 348 (1972).
[7]I. Riess and A. Ron, Phys. Rev. B 4, 4099 (1971).
[8]J. W. Loram, T. E. Whall, and P. J. Ford, Phys. Rev. B 3, 953 (1971) (*Au* Mn).
[9]J. W. Loram, T. E. Whall, and P. J. Ford, Phys. Rev. B 2, 857 (1970) (*CuAu* Fe).
[10]O. Laborde and P. Radhakrishna, Solid State Commun. 9, 701 (1971) (*Au* Fe).
[11]N. Rivier and K. Adkins, in *Amorphous Magnetism*, edited by H. O. Hooper and A. M. deGraaf (Plenum, New York, 1973), p. 215.
[12]A. A. Abrikosov, Physics 2, 5 (1965).
[13]P. J. Ford, T. E. Whall, and J. W. Loram, Phys. Rev. B 2, 1547 (1970) (*Au* Fe).
[14]H. L. Malm and S. B. Woods, Can. J. Phys. 44, 2293 (1966); D. Jha and M. H. Jericho, Phys. Rev. B 3, 147 (1971); D. A. Tindall, M. H. Jericho, and D. Jha, *ibid.* 9, 3113 (1974); E. D. Ramos, J. Low Temp. Phys. 20, 547 (1975) (*Ag*Mn).
[15]O. Laborde, P. Radhakrishna, J. Phys. F3, 1731 (1973) (*Cu* Mn).
[16]J. A. Mydosh, Phys. Rev. Lett. 33, 1562 (1974) (*PdH* Fe).
[17]A. Nakamura and N. Kinoshita, J. Phys. Soc. Jpn. 27, 382 (1969) (*Cu* Mn); K. Inoue and Y. Nakamura, *ibid.* 32, 441 (1972) (*Cu* Mn, *Ag* Mn); P. J. Ford, E. Babić, and J. A. Mydosh, J. Phys. F 3, L75 (1973) (*Au* Cr); P. J. Ford and J. A. Mydosh, in *Proceedings of the International Conference on Magnetism*, Moscow, 1973 (Nauka, Moscow, 1974), Vol. 2, p. 79; J. A. Mydosh, P. J. Ford, M. P. Kawatra, and T. E. Whall, Phys. Rev. B 10, 2845 (1974) (*Au* Fe); P. J. Ford and J. A. Mydosh, J. Phys. (Paris) 35, 241 (1974); P. J. Ford and J. A. Mydosh, Phys. Rev. (to be published) (*Au* Fe, *Au* Cr, *Au* Mn, *Ag* Mn, *Cu* Mn); J. A. Mydosh and P. J. Ford, Phys. Lett. A 49, 189 (1974) (*Cu* Mn).
[18]J. S. Schilling, J. Crone, P. J. Ford, S. Methfessel, and J. A. Mydosh, J. Phys. F 4, L116 (1974) (*Au* Fe); J. S. Schilling, P. J. Ford, J. Crone, and J. A. Mydosh, Euro-Phys. Conf. Abs. 1, 23 (1975) (*Au* Fe, *Au* Mn); *Cu* Mn).
[19]J. S. Schilling, P. J. Ford, U. Larsen, and J. A. Mydosh, following paper, Phys. Rev. B 14, 4368 (1976); U. Larsen, Solid State Commun. (to be published); U. Larsen, J. S. Schilling, P. J. Ford, and J. A. Mydosh, in Proceedings of the Second International Symposium on Amorphous Magnetism, Troy, 1976, edited by R. A. Levy and R. Hasegawa (Plenum, New York, to be published); J. S. Schilling, P. J. Ford, U. Larsen, and J. A. Mydosh, *ibid.* (to be published); P. J. Ford, J. S. Schilling, U. Larsen, and J. A. Mydosh, Physica B (to be published); U. Larsen, J. S. Schilling, P. J. Ford, and J. A. Mydosh, *ibid.* (to be published).
[20]J. S. Schilling, W. B. Holzapfel, and E. Lüscher, Phys. Lett. A. 38, 129 (1972) (*Cu* Fe); J. S. Schilling, and W. B. Holzapfel, Phys. Rev. B 8, 1216 (1973); G. P. Summers, J. G. Lipham, and L. D. Roberts, *ibid.* 8, 2106 (1973) (*Cu* Fe); J. Crone and J. S. Schilling, Solid State Commun. 17, 791 (1975) (*Au* Fe, *Au* Mn); further references in Ref. 19; J. Crone, thesis (T. U. Münschen, 1975) (unpublished).
[21]A short report of this calculation appeared in U. Larsen, Phys. Lett. A 40, 39 (1972), showing numerically calculated resistivity curves.
[22]U. Larsen, Z. Phys. 256, 65 (1972); only the $T \to \infty$ value $C(S)$ is required in the parquet approximation.
[23]I. Riess and A. Ron, Phys. Rev. B 8, 3467 (1973); 9, 2418 (1973).
[24]K. Adkins, thesis (Imperial College, London, 1974) (unpublished); N. River, Wiss. Z. Tech. Univer. Dres. 23, 1000 (1974); R. E. Walstedt and L. R. Walker, Phys. Rev. B 9, 4857 (1974).
[25]V. Cannella and J. A. Mydosh, Phys. Rev. B 6, 4220 (1972) (*Au* Fe); in *Proceedings of the International Conference on Magnetism, Moscow, 1973* (Nauka, Moscow, 1974), Vol. 2, p. 74 (*Au* Fe, *Au* Mn, *Au* Cr, *Ag* Mn, *Cu* Mn); V. Cannella, in *Amorphous Magnetism*, edited by H. O. Hooper and A. M. deGraaf (Plenum, New York, 1973), p. 195 (*Au* Mn, *Cu* Mn); J. L. Tholence and R. Tournier, J. Phys. (Paris) 35, 229 (1974) (*Au* Fe).
[26]K. Adkins and N. Rivier, J. Phys. (Paris) 35, 237 (1974); D. A. Smith, J. Phys. F 4, L266 (1974); I. Riess, Physica 74, 496 (1974); I. Riess and C. Mavroyannis, *ibid.* 75, 483 (1974); S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975); K. H. Fischer, Phys. Rev. Lett. 34, 1438 (1975); D. Sherrington, J. Phys. C 8, L208 (1975); B. W. Southern, *ibid.* 8, L213 (1975); D. Sherrington and B. W. Southern, J. Phys. F 5, L49 (1975); D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. 35, 1792 (1975); D. A. Smith, J. Phys. F 5, 2148 (1975); I. Riess, *ibid.* 5, 1957 (1975); A. B. Harris, T. C. Lubensky, and Jing-Huei Chen, Phys. Rev. Lett. 36, 415 (1976).
[27]R. D. Mattuck, *A Guide to Feynman Diagrams in the Many-Body Problem*, 2nd ed. (McGraw-Hill, London, 1976), Appendix N for reduced graph expansion.
[28]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971); A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics*, 2nd ed. (Pergamon, New York, 1965).
[29]D. R. Hamann, Phys. Rev. 158, 570 (1967).
[30]A. Zawadowski and P. Fazekas, Z. Phys. 226, 235 (1969).
[31]A. I. Larkin and D. E. Khmel'nitskii, Zh. Eksp. Teor. Fiz. 58, 1789 (1970) [Sov. Phys.-JETP 31, 958 (1970)]; A. I. Larkin, V. I. Mel'nikov, and D. E. Khmel'nitskii, *ibid.* 60, 846 (1971) [*ibid.* 33, 458 (1971)].
[32]N. Rivier and K. Adkins [J. Phys. F 5, 1745 (1975)] discuss the resistivity in the *frozen* state and find the $T^{3/2}$ dependence observed over a temperature range below T_m (Ref. 17).