PHYSICAL REVIEW B VOLUME 14, NUMBER 9 1 NOVEMBER 1976

Spin dynamics near bicritical points in uniaxial antiferromagnets*

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(Received 12 January 1976)

Spin dynamics near bicritical points in uniaxial antiferromagnets is investigated. When there is rotational symmetry about the zero-field easy axis the transverse staggered susceptibilities and the longitudinal direct susceptibility are characterized by a dynamic exponent $z = \phi/\nu$ (≈ 1.78 in three dimensions), where ϕ is the crossover exponent. The longitudinal staggered susceptibility has a dynamic exponent $z = 2$. These results hold to first order in $4-d$, where d is the dimension, provided the transverse magnetization and the energy density are omitted from the primary set of dynamical variables. The effects of energy conservation and the dynamics of the transverse fluctuations in the magnetization are discussed, and a comparison is made between the dynamics in the bicritical region and the dynamics along the antiferromagnetic-paramagnetic and spin-flop–paramagnetic phase boundaries.

I. INTRODUCTION

Recently, Fisher and Nelson pointed out the unusual thermodynamic properties of antiferromagnets near the point of intersection of the antiferromagnetic, spin flop, and paramagnetic phase boundaries[1] (see Fig. 1). The details of the behavior in the neighborhood of this point, designated the "bicritical point" have been further explored by Nelson, Kosterlitz, and Fisher[2] and Fisher.[3,4] Confirmation of a number of predictions of the theoretical analysis has been reported by Rohrer in experiments on $GdAlO_3$,[5,6] by King and Rohrer in MnF_2,[7] and by Landau and Binder using Monte Carlo techniques.[8]

In an earlier note we have outlined a "first-approximation" analysis of the dynamic behavior near a bicritical point.[9] The purpose of this paper is to develop in detail the arguments leading to the conclusions reported in Ref. 9. Our results follow from the application of dynamic-scaling,[10] mode-coupling,[11,12] and renormalization-group[13-16] concepts and techniques to the bicritical problem. We will show that a qualitative characterization of bicritical dynamics is possible in the paramagnetic phase, at least to first order in $4-d$, d being the dimension. Bicritical dynamics in the spin-flop and antiferromagnetic phases is mentioned but not discussed in detail.

The remainder of the paper is divided into three parts. In the first of these we consider the dynamics near the antiferromagnetic-paramagnetic and spin-flop–paramagentic phase boundaries but away from the bicritical point. In the second we show how the dynamics is modified in the bicritical region. Finally, we discuss our findings with reference to potential experimental tests of the theory. It should be emphasized that unless explicitly stated otherwise our analysis pertains to spin systems with Hamiltonians with rotational symmetry about the zero-field easy axis, which we take to be the z direction.

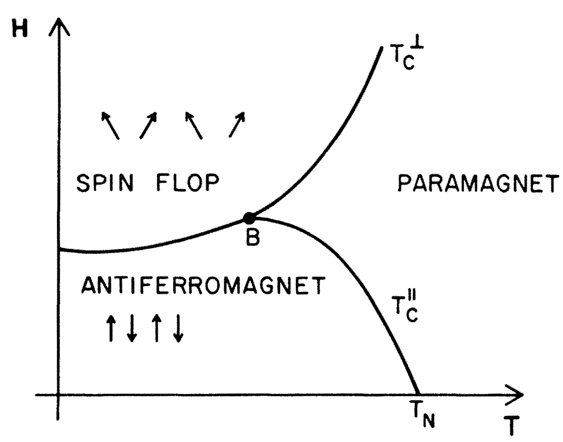

FIG. 1. Schematic phase diagram of a uniaxial antiferromagnet in a uniform field along the easy axis. B denotes the bicritical point. The antiferromagnetic–spin-flop transition is first order; the antiferromagnetic-paramagnetic and spin-flop–paramagnetic transitions are second order. See Ref. 1.

II. PHASE BOUNDARIES

A. Antiferromagnetic-paramagnetic

In a recent paper one of us has outlined a quasihydrodynamic theory for the spin dynamics of easy-axis antiferromagnets in an external field which was based on a linear model with purely relaxational behavior.[17] In the model it was assumed that there was (approximate) rotational symmetry about the zero-field easy axis and that the spin system was weakly coupled to a thermal bath. The basic variables in the theory were the longitudinal component of the magnetization $M_z(\vec{q})$, the longitudinal component of the staggered magnetization $N_z(\vec{q})$, and the energy density $E(\vec{q})$, in each case $\vec{q}$ denoting the wave vector.

In the antiferromagnetic phase the dynamics reflects a linear coupling among all three variables, while in the paramagnetic phase N_z is no longer linearly coupled to M_z and E. The extent of the linear coupling between M_z and E is determined by the effective dimensionless coupling constant $1-\chi_S/\chi_T$, where χ_S and χ_T are the zz adiabatic and isothermal uniform-field susceptibilities, respectively. It was suggested that the various decay rates appearing in the relaxation equations, as first approximation, should have a temperature dependence which is in accord with the conventional picture of thermodynamic slowing down.[18]

In Ref. 13 Halperin, Hohenberg, and Ma studied the critical dynamics of a class of models based on the time-dependent Ginzburg-Landau equation. Using renormalization-group techniques, they showed that the conventional theory holds only in the linear approximation. Nonlinear interactions among the fluctuations lead to departures from the conventional theory. The details of the departures depend on the model considered. In particular, their model C with $n=1$, $d=3$ (energy conserved, one-component order parameter not conserved, three dimensions) is the model corresponding most closely to the zero-field limit of the system studied in Ref. 17. When $n=1$ and $d=3$ it was found that the relaxation of the order parameter was characterized by a dynamic exponent $z^{\text{AFM-P}}=2+\alpha/\nu$, where α is the specific-heat exponent and ν is the exponent associated with the correlation length.[19]

In contrast, the thermal conductivity for fixed $T>T_c$, T_c being the critical temperature, was equal to the bare value determined by the short-wavelength fluctuations. The value of $z^{\text{AFM-P}}$ is thus slightly larger than the prediction of the conventional theory $z^{\text{AFM-P}}=2-\eta$, whereas the relaxation of the long-wavelength (hydrodynamic) fluctuations in the energy density is in accord with the conventional theory. In the absence of an external field the nonlinear coupling of M_z to N_z and E goes to zero upon iteration of the renormalization-group procedure so that M_z also follows the conventional theory.[13]

The behavior we have outlined is expected to change to some extent when there is a uniform field present. In the field there is a linear coupling between M_z and E since $1-\chi_S/\chi_T\neq 0$. Furthermore, the direct susceptibility shows singular behavior analogous to that of the specific heat.[20] Thus in the case where both $M_z(\vec{q}=0)$ and $E(\vec{q}=0)$ are constants of the motion we have a situation similar to model C except that the order parameter now has nonlinear interactions with the coupled M_z-E modes. Accordingly we do not anticipate changes in $z^{\text{AFM-P}}$ relative to the zero-field value. Likewise in the hydrodynamic region the decay of the magnetization-energy fluctuations is predicted to be characterized by the bare transport coefficients with thermodynamic slowing down characteristic of the specific heat.

It must be emphasized that these predictions apply to the paramagnetic phase. The behavior in the antiferromagnetic phase may be complicated by the linear coupling between the order parameter and the other variables.[17] Since the nonlinear dynamics of model C below T_c has not been worked out we refrain from making predictions about antiferromagnetic dynamics in a finite field.

B. Spin-flop–paramagnetic

For an antiferromagnet with rotational symmetry about the easy axis the dynamic behavior along the spin-flop–paramagnetic line is distinctly different from the behavior near the antiferromagnetic-paramagnetic phase boundary. The difference reflects the fact that the ordering in the flop phase is characterized by a broken continuous symmetry. As a result there are propagating hydrodynamic modes whose frequency is given by the equation[21]

$$\omega_q = g\mu_B\langle N_x(\vec{q}=0)\rangle/[\chi^*_{xx}(\vec{q})\chi_S]^{1/2}\,, \qquad (1)$$

where $\langle N_x(\vec{q}=0\rangle$ is the thermal average of the staggered magnetization, taken to be along the x axis, g denotes the electronic g factor, and μ_B is the Bohr magneton. Also, $\chi^*_{yy}(\vec{q})$ is the transverse staggered susceptibility and χ_S, as before, is the zz adiabatic direct susceptibility.

By making use of dynamic[10] and static[22] scaling arguments we find a dynamic exponent $z^{\text{SF-P}}$ given by

$$z^{\text{SF-P}}=\tfrac{1}{2}d, \quad d\leq 4\,. \qquad (2)$$

In obtaining this equation we have utilized the Wheeler-Griffiths proof that the adiabatic direct susceptibility is finite along a line of antiferromagnetic critical points.[23]

It should be noted that there is an apparent inconsistency in our value of $z^{\text{SF-P}}$. An analysis of the mode-coupling equations or the dynamic renormalization-group equations to first order in $\epsilon\equiv 4-d$, without including the energy modes, leads to the result[11,16]

$$z^{\text{SF-P}}=\tfrac{1}{2}d+\tilde{\alpha}/2\nu\,, \qquad (3)$$

where $\tilde{\alpha}=\alpha$ if $\alpha>0$ and 0 otherwise. Since current estimates of α for planar systems range from[4] -0.02 to[24] 0.0 calculating z^{SF-P} with or without the energy modes leads to identical results.

On the other hand were $\alpha > 0$ we would associate Eq. (2) with systems where energy was conserved and Eq. (3) with systems without energy conservation. Such a conclusion is consistent with the expression obtained for the spin-wave frequency in the absence of energy conservation

$$\omega_q = g\mu_B \langle N_x(0)\rangle / [\chi^*_{yy}(\vec{q})\chi_T]^{1/2} . \quad (4)$$

Because of the specific-heat-like singularity in χ_T Eq. (4) leads directly to the result given in Eq. (3).

In the absence of energy conservation it is expected that Eq. (3) will characterize $\chi^*_{xx}(\vec{q},\omega)$, $\chi^*_{yy}(\vec{q},\omega)$, and $\chi_{zz}(\vec{q},\omega)$. When energy is conserved Eq. (2) is appropriate for $\chi^*_{xx}(\vec{q},\omega)$ and $\chi^*_{yy}(\vec{q},\omega)$. Its applicability to $\chi_{zz}(\vec{q},\omega)$ is less certain because of the coupling between M_z and E; however, heuristic arguments suggest that it will apply for $\alpha < 0$.

III. BICRITICAL REGION

The static susceptibilities near the bicritical point of a system with rotational symmetry about the zero-field easy axis have the same set of critical indices as the isotropic Heisenberg antiferromagnet in zero field with one important exception.[1-3] There is a divergence in the direct susceptibilities $\chi_{\alpha\alpha}(\vec{q}=0)$, which is characterized by an exponent $\tilde{\gamma} \simeq 0.40$. Since the specific heat at constant field C_H is finite and $\chi_S = \chi_T C_M / C_H$, where C_M is the specific heat at constant magnetization, there is also a $\tilde{\gamma}$ singularity in χ_S.[25]

Our approach to bicritical dynamics is based on the dynamical equations of Refs. 12, 14, and 15 which are of the time-dependent Ginzburg-Landau type augmented by appropriate streaming terms. Initially we will omit the energy modes from our analysis as is appropriate when $E(\vec{q}=0)$ is not a constant of the motion.

The change in the dynamics in the bicritical region relative to the behavior along the two second-order lines reflects both the change in the thermodynamic properties and the enlargement of the set of appropriate dynamical variables. In connection with the latter we note that since both the direct and staggered susceptibilities are divergent it would appear at first sight that the primary set of dynamical variables should be N_x, N_y, N_z, M_x, M_y, and M_z as in the case of the isotropic antiferromagnetic.[15]

However, although all three direct susceptibilities are characterized by the exponent $\tilde{\gamma}$ the long-wavelength fluctuations in M_z will decay more slowly than the fluctuations in M_x and M_y since $M_x(\vec{q}=0)$ and $M_y(\vec{q}=0)$ are not constants of the motion. In order to see the implications of the lack of rotational symmetry about the x and y axes we will use the formalism of Refs. 14 and 15 to generate equations of motion for M_x, M_y, and M_z.

In our analysis we adopt the mode coupling viewpoint.[12] That is, we assume that the static properties of the system are known at the outset. As pointed out in Ref. 12 this approach is equivalent to choosing the fixed point *a priori*. In accord with this point of view we take the free-energy functional to be of the form given in Ref. 2.

$$\bar{H} = \frac{1}{2}\int d^dR\,(r_{\parallel}\sigma_{\parallel}^2 + |\nabla\sigma_{\parallel}|^2 + r_{\perp}\vec{\sigma}_{\perp}^2 + |\nabla\vec{\sigma}_{\perp}|^2 + 2u\sigma_{\parallel}^4 + 2v\vec{\sigma}_{\perp}^4 + 4w\sigma_{\parallel}^2\vec{\sigma}_{\perp}^2 + r_2^{\parallel}m_{\parallel}^2 + r_2^{\perp}\vec{m}_{\perp}^2) . \quad (5)$$

Here $\sigma_{\parallel}$ and $\vec{\sigma}_{\perp}$ are proportional to the longitudinal and transverse components of $\vec{N}$ while $m_{\parallel}$ and $\vec{m}_{\perp}$ are similarly related to the components of $\vec{M}$. At the bicritical point we have $u^* = v^* = w^* = \frac{1}{44}\bar{\epsilon}$ and $r_{\parallel} = r_{\perp} = -\frac{5}{22}\bar{\epsilon}$, where $\bar{\epsilon}$ is proportional to $4-d$.

The equations of motion for $m_{\perp x}$ and $m_{\perp y}$ take the form

$$\frac{\partial m_{\perp x}}{\partial t} = A_1\sigma_{\perp y}\frac{\delta\bar{H}}{\delta\sigma_{\parallel}} - A_2\sigma_{\parallel}\frac{\delta\bar{H}}{\delta\sigma_{\perp y}} + A_3 m_{\perp y}\frac{\delta\bar{H}}{\delta m_{\parallel}} - A_4 m_{\parallel}\frac{\delta\bar{H}}{\delta m_{\perp y}} - L_{\perp}^m\frac{\delta\bar{H}}{\delta m_{\perp x}} + f(t) , \quad (6)$$

$$\frac{\partial m_{\perp y}}{\partial t} = A_2\sigma_{\parallel}\frac{\delta\bar{H}}{\delta\sigma_{\perp x}} - A_1\sigma_{\perp x}\frac{\delta\bar{H}}{\delta\sigma_{\parallel}} + A_4 m_{\parallel}\frac{\delta\bar{H}}{\delta m_{\perp x}} - A_3 m_{\perp x}\frac{\delta\bar{H}}{\delta m_{\parallel}} - L_{\perp}^m\frac{\delta\bar{H}}{\delta m_{\perp y}} + f(t) , \quad (7)$$

where the A_i and $L_{\perp}^m$ are constants and $f(t)$ is a noise term. The first four terms in Eqs. (6) and (7) are identified as streaming terms, whereas the fifth term has the conventional Ginzburg-Landau form.

Following Kawasaki[26] we can determine the characteristic frequencies associated with the various terms in the equations of motion for $\vec{m}_{\perp}$ by evaluating the functional derviatives using the scaling form for the free energy. According to Ref. 1, in the bicritical region we have

$$F(t,g,H_{\perp},H_{\parallel}^{\dagger},H_{\perp}^{\dagger}) = t^{2-\alpha}\Phi(g/t^{\phi}, H_{\perp}/t^{\phi}, H_{\parallel}^{\dagger}/t^{\Delta}, H_{\perp}^{\dagger}/t^{\Delta}) , \quad (8)$$

where H^+ is the ordering field, g and $H_\perp$ are the nonordering fields, ϕ and Δ are the crossover and gap exponents, respectively, and $t = |T - T_b|/T_b$, T_b being the bicritical temperature. The functional derivatives are most simply analyzed using the Legendre transform of Φ with respect to the field variables, which we write as $\hat{\Phi}(\vec{m}/t^{\tilde{\beta}}, \vec{\sigma}/t^{\beta})$, where $\tilde{\beta} = 2 - \alpha - \phi$ and $\beta = 2 - \alpha - \Delta$. We thus have

$$\frac{\delta \overline{H}}{\delta \sigma} = t^{2-\alpha-\beta} \hat{\Phi}(\vec{m}/t^{\tilde{\beta}}, \vec{\sigma}/t^{\beta}), \tag{9}$$

$$\frac{\delta \overline{H}}{\delta m} = t^{2-\alpha-\tilde{\beta}} \hat{\Phi}(\vec{m}/t^{\tilde{\beta}}, \vec{\sigma}/t^{\beta}). \tag{10}$$

The characteristic frequencies associated with the first two terms in Eqs. (6) and (7) vary as $m^{-1}\sigma\, \delta\overline{H}/\delta\sigma$ that is to say as $t^{-\tilde{\beta}} t^{\beta} t^{2-\alpha-\beta} \sim t^{\phi}$. The frequencies associated with the third and fourth terms vary as $\delta\overline{H}/\delta m \sim t^{\phi}$. In contrast, the Ginzburg-Landau term has a characteristic frequency $m^{-1}\delta\overline{H}/\delta m \sim t^{-\tilde{\beta}} t^{\phi} \sim t^{\phi-\tilde{\beta}}$. Because $\tilde{\beta} > 0$ the Ginzburg-Landau terms will dominate in the limit $t \to 0$ giving rise to a dynamic exponent $z = (\phi - \tilde{\beta})/\nu = (2\phi - 2 + \alpha)/\nu$. Since $\tilde{\gamma} = 2\phi - 2 + \alpha$[1] the dynamic exponent associated with $\vec{m}_\perp$, $\tilde{\gamma}/\nu$, is the same as that given by conventional theory of critical slowing down where the relaxation rate is inversely proportional to the direct susceptibility. Because $\overline{H}$ is quadratic in $\vec{m}_\perp$ the equation of motion of $\vec{m}_\perp$ without the streaming terms is of the standard linear Langevin form.[14] As a consequence we omit M_x and M_y from the primary set of dynamical variables which we define to be those variables whose dynamical properties are determined by nonlinear kinetic equations. It should be emphasized that these arguments may not be completely rigorous in that a similar analysis applied to model A of Ref. 13 would lead to the conclusion that the corresponding dynamic exponent was that given by the conventional theory, which is only true to order ϵ. However, the approach does give the correct dynamic exponents for the isotropic ferromagnet and the isotropic antiferromagnet.

The behavior of M_x and M_y is distinctly different from the behavior of M_z. Since $M_z(\vec{q} = 0)$ is a constant of the motion, the Ginzburg-Landau term in the equation for $m_\parallel$ is of the form $L_\parallel^m \nabla^2\, \delta\overline{H}/\delta m_\parallel$. As a consequence the characteristic frequency associated with this term varies as $t^{\phi - \tilde{\beta} + 2\nu}$, whereas the frequency associated with the streaming terms varies as t^{ϕ}. Since $2\nu > \tilde{\beta}$ the streaming terms will dominate as $t \to 0$ so that the dynamic exponent is equal to ϕ/ν.

With M_x and M_y omitted from the primary set of dynamical variables we are left with dynamical equations for $\sigma_\parallel$, $\vec{\sigma}_\perp$, and $m_\parallel$. In the dynamical equations for $\sigma_\parallel$ in an isotropic antiferromagnet the streaming terms are of the form $\vec{m}_\perp \times (\delta\overline{H}/\delta\vec{\sigma}_\perp)$ and $\vec{\sigma}_\perp \times (\delta\overline{H}/\delta\vec{m}_\perp)$.[15] Since we are leaving out $\vec{m}_\perp$ the corresponding equation for $\sigma_\parallel$ in the bicritical problem is of the standard Ginzburg-Landau type,

$$\frac{\partial \sigma_\parallel}{\partial t} = -L_\parallel^{\sigma} \frac{\delta \overline{H}}{\delta \sigma_\parallel} + f(t), \tag{11}$$

with no streaming terms.

From Eq. (5) it is evident that $\sigma_\parallel$ couples to $\vec{\sigma}_\perp$ only through the quartic term $4w\sigma_\parallel^2\vec{\sigma}_\perp^2$ in $\overline{H}$. Since the fixed-point value of w is of order ϵ it follows that any contribution to the dynamic exponent associated with $\chi_{zz}^*(q, \omega)$ which comes from this term will be of order ϵ^2. A similar analysis can also be made for $\vec{\sigma}_\perp$ and $m_\parallel$ which shows that the coupling to $\sigma_\parallel$ in the dynamical equations for these variables comes also from the aforementioned quartic term. As a consequence, to order ϵ we can treat N_z and N_x, N_y, M_z as independent sets of dynamical variables.

By omitting the coupling between N_z and the variables N_x, N_y, M_z we are in a situation where to order ϵ the dynamics of N_x, N_y, and M_z is similar to the dynamics of these variables along the spin-flop–paramagnetic line (with allowance made for the $\tilde{\gamma}$ divergence in the direct susceptibility) whereas the dynamics of N_z in the bicritical region is similar to the dynamics of N_z along the antiferromagnetic-paramagnetic line.

An analysis of the equation of motion of $\vec{\sigma}_\perp$ shows that the characteristic frequency associated with the streaming term $\sigma\, \delta\overline{H}/\delta m$ varies as t^{ϕ}, that associated with $m\, \delta\overline{H}/\delta\sigma$ as $t^{\gamma+\tilde{\beta}}$, and the Ginzburg-Landau frequency as t^{γ}. Since[27] $\phi < \gamma$ ($\epsilon > 0$) the $\sigma\, \delta\overline{H}/\delta m$ terms dominate in the $t \to 0$ limit. As a result $\chi_{zz}(\vec{q}, \omega)$, $\chi_{xx}^*(\vec{q}, \omega)$, and $\chi_{yy}^*(\vec{q}, \omega)$ are all characterized by a dynamic exponent z_{I}^B given by

$$z_{\mathrm{I}}^B = \phi/\nu. \tag{12}$$

In view of the scaling relation $d\nu = 2 - \alpha$ we can rewrite (12) as

$$z_{\mathrm{I}}^B = \tfrac{1}{2}(d + \tilde{\gamma}/\nu), \tag{13}$$

which agrees with the value inferred from Eq. (4). In contrast the dynamic exponent characterizing $\chi_{zz}^*(\vec{q}, \omega)$, z_{II}^B, is obtained from the characteristic frequency associated with Eq. (11), which varies as $\sigma^{-1}\, \delta\overline{H}/\delta\sigma \sim t^{-\beta} t^{2-\alpha-\beta} \sim t^{\gamma}$, using the scaling relation $\gamma + 2\beta = 2 - \alpha$. As a consequence we have

$$z_{\mathrm{II}}^B = \begin{cases} \gamma/\nu \\ 2 + O(\epsilon^2). \end{cases} \tag{14}$$

Finally, from our previous analysis we have for z_{III}^B, the dynamic exponent associated with $\chi_{xx}(\vec{q}, \omega)$ and $\chi_{yy}(\vec{q}, \omega)$,

$$z_{\mathrm{III}}^B = (2\phi - 2 + \alpha)/\nu. \tag{15}$$

It should be emphasized that $z_{\rm I}^B$ and $z_{\rm II}^B$ are subject to corrections of order ϵ^2.

Using the values[3] $\phi = 1.25$, $\alpha = -0.10$, $\nu = 0.70$ we obtain the results

$$z_{\rm I}^B = 1.78\,, \tag{16}$$

$$z_{\rm III}^B = 0.57\,, \tag{17}$$

for $d = 3$.

It is of interest to compare Eqs. (16) and (17) with with the corresponding values obtained from the ϵ expansion.[28] We have

$$z_{\rm I}^B = 2 - \tfrac{2}{11}\epsilon + O(\epsilon^2) = 1.82 \quad (\epsilon = 1)\,, \tag{18}$$

$$z_{\rm III}^B = \tfrac{7}{11}\epsilon + O(\epsilon^2) = 0.64 \quad (\epsilon = 1)\,, \tag{19}$$

which are remarkably close to the previous results. From (14), (18), and (19) we see that as $\epsilon \to 0$ $z_{\rm I}^B$ and $z_{\rm II}^B$ approach the value 2, the value obtained in the conventional theory of critical slowing down, whereas $z_{\rm III}^B$ goes to zero. The difference in the $\epsilon \to 0$ limit between $z_{\rm III}^B$ on one hand and $z_{\rm I}^B$ and $z_{\rm II}^B$ on the other is consistent with our having omitted M_x and M_y from the primary set of dynamical variables.

As noted we have also omitted the energy from the primary set of dynamical variables. The fact that χ_S and χ_T are both divergent with the same exponent makes it plausible that Eq. (13) will apply independent of whether energy is conserved, as indicated by Eqs. (1) and (4). The effect of energy conservation on $z_{\rm II}^B$ and $z_{\rm III}^B$ is less certain. However, since $\alpha < 0$ at the bicritical point we expect on the basis of the analysis of Ref. 13 that the effects of energy conservation will not influence the dynamic exponents to order ϵ.

We can obtain this last result directly in the mode-mode coupling approximation which, as Kawasaki and Gunton[12] have shown, is essentially equivalent to the renormalization group to $O(\epsilon)$. We start directly with the microscopic two-sublattice (A and B) antiferromagnetic Heisenberg Hamiltonian with easy-axis anisotropy (only nearest-neighbor interactions are implied)

$$\mathcal{H} = \sum_{(ij)} J_{ij}^{AB}(\vec{S}_{iA}\cdot\vec{S}_{jB}) - \sum_{ij} D_{ij}^{AB} S_{iA}^z S_{jB}^z - H\sum_i (S_{iA}^z + S_{iB}^z) - H^* \sum_i (S_{iA}^x - S_{iB}^x)\,. \tag{20}$$

From this one can explicitly obtain equations of motion for $\vec{N}(\vec{q})$, $\vec{M}(\vec{q})$, and $\vec{E}(\vec{q})$. However, the equations of motion are insufficient to establish the dynamic exponents and we need expressions for the linewidths of the various fluctuation processes. For these we utilize Mori's equation[29]

$$\Gamma^{AA}(q, iz) = \frac{1}{(A(\vec{q}), A(-\vec{q}))} \times \int_0^\infty e^{-zt}(A(\vec{q}, t), A(-\vec{q}, 0))\,dt\,, \tag{21}$$

where $(A(\vec{q}), A(-\vec{q}))$ is just the static susceptibility $\chi_{AA}(\vec{q})$ for the dynamical variables A (here $\vec{N}$, $\vec{M}$, and $\vec{E}$).

Under the further assumption of decoupling of the four- and six-spin correlations into products of pairings of two-spin correlations a lengthly calculation[30] shows that omitting M_x, M_y from our list of variables, we get the result stated above, namely that when the specific heat does not diverge, the asymptotic time scales for our primary set of dynamical variables are determined by the same dynamic exponents as before.[31] Of course, both from Kawasaki and Gunton's arguments and the above assumptions on the behavior of M_x and M_y, our conclusions may not be valid beyond $O(\epsilon)$.

IV. DISCUSSION

In the preceding section we have analyzed the spin dynamics near a bicritical point in a uniaxial antiferromagnet with a spin Hamiltonian having rotational symmetry about the zero-field easy axis. It should be noted that the analysis applied to the paramagnetic phase. Bicritical dynamics in the spin-flop and antiferromagnetic phases may be complicated by the presence of the first-order antiferromagnetic-spin-flop boundary.

Even in the paramagnetic phase the analysis is incomplete. One would like to have a quantitative estimate of the ϵ^2 corrections to $z_{\rm I}^B$ and $z_{\rm II}^B$. In particular it is important to know whether higher-order terms can establish a common dynamic exponent.[32] A more quantitative assessment of the effects of energy conservation and the role played by the transverse fluctuations in the magnetization is needed as well. It is also important to investigate "crossover" effects which may appear as the system is moved into the bicritical region.

Experimental tests of the theory require systems like MnF_2 where the anisotropy in the XY plane is small. The most useful probe is likely to be inelastic neutron scattering, where it is possible to measure the wave-vector- and frequency-dependent susceptibilities directly. Measurements of $\chi_{xx}(\vec{q}, \omega)$ would be an important test of the overall dynamical picture. The conclusions about the temperature dependence of the characteristic frequency associated with the transverse fluctuations in the magnetization obtained in Sec. III are based on an approximate treatment of a particular dynamical model. Other dynamical models can lead to different results. Thus, for example, were we to omit the Ginzburg-Landau terms in the equations of motion for M_x and M_y we would find that all components of $\vec{M}$ and $\vec{N}$ would be characterized by the dynamic exponent ϕ/ν.

ACKNOWLEDGMENTS

We would like to thank L. W. Bruch, B. I. Halperin, K. K. Murata, and D. R. Nelson for comments on various aspects of this work. We would also like to thank K. Kawasaki and J. Gunton for providing us with preprints of their papers.

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*Research supported by the National Science Foundation.

[1]M. E. Fisher and D. R. Nelson, Phys. Rev. Lett. 32, 1350 (1974).
[2]D. R. Nelson, J. M. Kosterlitz, and M. E. Fisher, Phys. Rev. Lett. 33, 813 (1974); J. M. Kosterlitz, D. R. Nelson, and M. E. Fisher, Phys. Rev. B 13, 412 (1976).
[3]M. E. Fisher, AIP Conf. Proc. 24, 273 (1975).
[4]M. E. Fisher, Phys. Rev. Lett. 34, 1634 (1975).
[5]H. Rohrer, AIP Conf. Proc. 24, 268 (1975).
[6]H. Rohrer, Phys. Rev. Lett. 34, 1638 (1975).
[7]A. R. King and H. Rohrer, AIP Conf. Proc. 29, 420 (1976).
[8]D. P. Landau and K. Binder, AIP Conf. Proc. 29, 461 (1976).
[9]D. L. Huber, Phys. Lett. A 49, 345 (1974).
[10]B. I. Halperin and P. C. Hohenberg, Phys. Rev. 177, 952 (1969).
[11]K. Kawasaki, Ann. Phys. (N.Y.) 61, 1 (1970).
[12]K. Kawasaki and J. Gunton, Phys. Rev. B 13, 4658 (1976).
[13]B. I. Halperin, P. C. Hohenberg, and S. Ma, Phys. Rev. B 10, 139 (1974); see also C. De Dominicis, E. Brézin, and J. Zinn-Justin, *ibid*. 12, 4945 (1975); E. Brézin and J. Zinn-Justin, *ibid*. 12, 4954 (1975).
[14]S. Ma and G. F. Mazenko, Phys. Rev. B 11, 4077 (1975).
[15]R. Freedman and G. F. Mazenko, Phys. Rev. Lett. 34, 1575 (1975); Phys. Rev. B 13, 4967 (1976).
[16]B. I. Halperin, P. C. Hohenberg, and E. D. Siggia, Phys. Rev. Lett. 32, 1289 (1974); Phys. Rev. B 13, 1299 (1976).
[17]D. L. Huber, Phys. Rev. B 10, 3992 (1974).
[18]L. Van Hove, Phys. Rev. 93, 1374 (1954).
[19]The dynamic exponent z describes the wave-vector dependence of the characteristic frequency $\omega_c(\vec{q})$ associated with the corresponding susceptibility through the equation $\omega_c(\vec{q}) = q^z f(q/\kappa)$, where κ is the inverse correlation length. See Ref. 10.
[20]M. E. Fisher, Philos. Mag. 7, 1731 (1962).
[21]B. I. Halperin and P. C. Hohenberg, Phys. Rev. 188, 898 (1969).
[22]L. P. Kadanoff, W. Götze, D. Hamblen, R. Hecht, E. A. S. Lewis, V. V. Palciauskas, M. Rayl, J. Swift, D. Aspnes, and J. Kane, Rev. Mod. Phys. 39, 395 (1971).
[23]J. C. Wheeler and R. B. Griffiths, Phys. Rev. 170, 249 (1968).
[24]D. D. Betts, *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, New York, 1974), Vol. III, p. 569.
[25]Singular behavior in χ_S at an isolated point is allowed by the Wheeler-Griffiths arguments.
[26]K. Kawasaki, in *Proceedings of the International School of Physics "Enrico Fermi" Course LI*, edited by M. S. Green (Academic, New York, 1971).
[27]M. E. Fisher and P. Pfeuty, Phys. Rev. B 6, 1889 (1972).
[28]F. Wegner, Phys. Rev. B 6, 1891 (1972).
[29]H. Mori, Prog. Theor. Phys. 33, 423 (1965).
[30]R. Raghavan, thesis (University of Wisconsin, 1976) (unpublished).
[31]We have also established this result using renormalization-group techniques (see Ref. 30).
[32]For an indication of how this comes about in model C of Ref. 13, see, K. K. Murata, Phys. Rev. B 13, 2028 (1976); B. I. Halperin, P. C. Hohenberg, and S. Ma, *ibid*. 13, 4119 (1976).