Precision sound-velocity measurements in He II†

J. Heiserman, J. P. Hulin,* J. Maynard, and I. Rudnick
Department of Physics, University of California, Los Angeles, California 90024
(Received 10 May 1976)

Acoustic resonators were used to measure simultaneously the velocities of first, second, and fourth sound in He II to a typical precision of 0.2%. Data were taken at temperatures from 1.2 K to the λ line in 50-mK steps, and at pressures from the saturated vapor pressure to the melting curve in 1-bar steps. Temperature and pressure were determined to better than 0.1%. Fourth sound was measured in a coarse (~ 1-μm) and fine (~ 90-Å) packed powder. Smoothed and raw data values are tabulated for more than 1500 data points.

I. INTRODUCTION

One of the most significant successes of the two-fluid theory of superfluid helium was the accurate prediction of new modes of sound propagation in the bulk liquid.[1-3] In addition to ordinary sound composed of compressional waves (first sound), there appears a thermal or entropy wave (second sound), and a pure superfluid wave which propagates in a superleak (fourth sound). The velocities of propagation of these modes have been thoroughly studied along the saturated vapor pressure (SVP) line, and the velocity expressions of the two-fluid theory have been verified.[4,5]

Prior to the work we report here, the pressure as well as temperature dependence of the first- and second-sound velocities was measured,[6,7] but the full potential of such data had not been fully exploited. The pressure dependence of fourth sound had not until now been measured. There is a very compelling reason for carefully measuring the sound velocities: the velocities can be measured easily to high precision, and such data, when supplemented with a small amount of data at SVP, will completely determine the thermodynamics of He II. It can be shown that the sound velocities determine the most important thermodynamic derivatives directly and that the only derivative which must be taken numerically can be found in two independent ways and hence can be found accurately. The thermodynamic information contained in the sound velocities will be the subject of the following paper.[22]

In this paper we describe an experiment which uses acoustic resonators to measure simultaneously the velocities of first, second, and fourth sound to ~0.2% precision. Data were taken at temperatures from 1.2 K to the λ transition temperature T_λ in 50-mK steps, and at pressures from SVP to the melting curve in 1-bar steps. The temperature was measured to within 1 mK, and the pressure readings were accurate to better than 0.1%. Fourth sound was measured in a coarse packed powder and in a fine packed powder which gave significant size effects. The experiment was intended to be the "last word" in sound velocity measurements in He II above 1.2 K.

II. EXPERIMENT

A. General method

The sound velocities were found by measuring the resonant frequencies of plane-wave modes in cylindrical cavities. Long, narrow cavities were used to produce sequences of plane-wave resonances without the interference of radial modes. By measuring a resonant frequency f belonging to a harmonic sequence, the sound velocity C can be found from

$$C = 2lf/m\,, \tag{1}$$

where l is the length of the cavity and m is the harmonic number. The velocity of fourth sound given by Eq. (1) must be multiplied by a "scattering correction," n, which accounts for changes in the effective compressibility and scattering (arising from Kelvin drag effects) due to the powder particles. The determination of n will be discussed in Sec. III A.

B. Experimental cell

The experimental cell is shown in Fig. 1. It consists of a brass cylinder drilled to make four 10.16 ± 0.01 cm long, 0.6-cm diam resonators. The resonators are interconnected with drilled passageways and are filled and pressurized through a $\frac{1}{8}$-in. copper tube. Drive and pickup sound transducers seal the ends of each resonator. Two of the resonators are empty and used for first and second sound. The other two are packed with powders for fourth sound. One is packed with a 1-μm aluminum oxide powder to an average porosity (open volume divided by total volume) of 57% which gives pores just small enough to lock the normal fluid and give negligible size effects. The

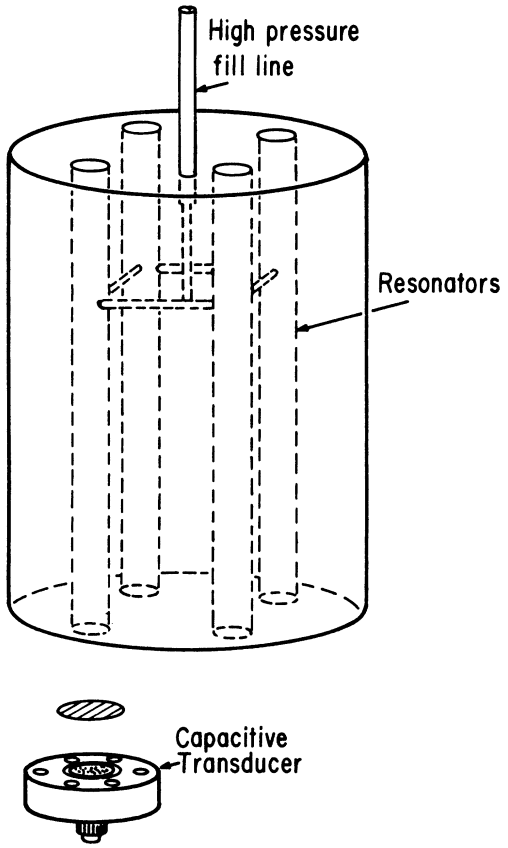

FIG. 1. Experimental cell.

other is packed with ~90-Å carbon black to a porosity of ~60% and gives large size effects. A thorough discussion of fourth-sound resonators and size effects is given in Ref. 5.

The sound transducers were capacitive devices; details are shown in Fig. 2. The first- and fourth-sound transducers used electreted (charge implanted) teflon sheet (1.3×10^{-3} cm thick) for the oscillating element. The stationary element was a conducting button epoxied into a mounting flange. The epoxied surfaces were grooved to add strength and to help ensure a leak-tight seal at high pressures. Pressures in excess of 60 bar could be maintained. The exposed surface of the button was roughened by sandblasting to improve the transducer response. The surface of the teflon sheet facing the resonator had a conducting layer of evaporated aluminum which was grounded through contact with the brass cylinder. The transducer was driven by applying a sinusoidally varying voltage of amplitude ~80 V to the stationary plate.

The second-sound transducers were identical to the first and fourth except that the oscillating membranes were porous superleaks (1-μm pore

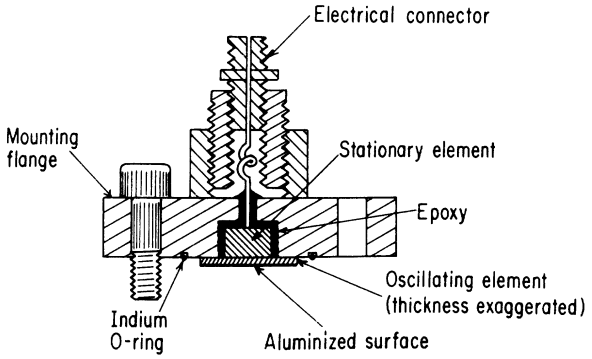

FIG. 2. Details of the sound transducers.

diameter Nuclepore[8]). Operation of these transducers is described in detail elsewhere.[9]

Each transducer was sealed to the brass cylinder with an indium O ring and ten 6-32 machine screws.

C. Resonator performance

Typical spectra (pickup amplitude versus drive frequency) of the resonators are shown in Fig. 3. In the first-sound spectrum [Fig. 3(a)] the seventh, eighth, and ninth harmonics were particularly strong and gave velocities which agreed to within 0.2%. The seventh harmonic consistently gave the average of the three and was used to give the tabulated values of the first-sound velocity C_1. The quality factor, given by $Q=f/\Delta f$, where Δf is the full width at half maximum, was at least 500 at all temperatures and pressures and hence the resonant frequency could easily be determined to 0.2%.

The Q of the second-sound resonances [shown in Fig. 3(b)] was lower than that of the first-sound resonances, but by using the twentieth harmonic the sound velocity could be determined to within 0.04 m sec^{-1}. Although some radial modes appeared between the tenth and twentieth harmonics, the twentieth harmonic agreed with the harmonic sequence below the tenth to within 0.2%.

The 1-μm fourth-sound spectrum [Fig. 3(c)] showed structure due to the acoustic interaction of the packed powder and the fluid. However, if a portion of the spectrum is expanded as shown in Fig. 3(d), then the powder structure becomes a slowly rolling background and the fourth-sound resonance appears as a high Q peak. The second harmonic could be consistently located to within a few Hz for a precision of 0.4 m sec^{-1}.

The 90-Å fourth-sound spectrum was similar to the 1-μm fourth-sound spectrum. However, at some pressures, a few of the resonance peaks would disappear into the background. Hence it

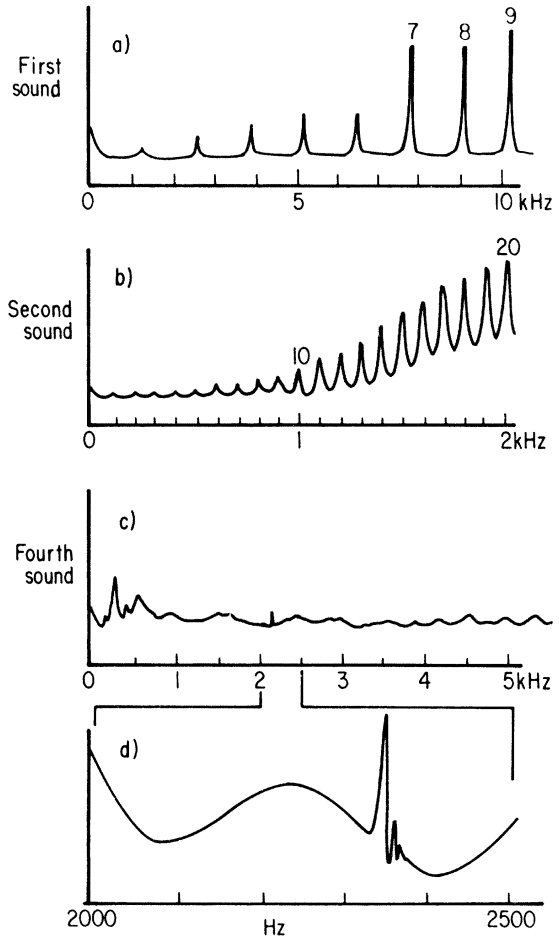

FIG. 3. Typical resonator spectra. (a) First sound, (b) second sound, (c) fourth sound, (d) fourth sound, expanded scale. Harmonic numbers are indicated.

was necessary to follow several peaks in order to obtain data over the complete pressure range. There was also a problem in the shifting of the resonances due to the interaction with the powder, so that some of the resonances did not fall in a harmonic sequence. However, there was always a set of resonances which showed the same temperature and pressure dependence, and hence could be normalized to give a sound velocity accurate to an estimated (1–2)%.

D. Cryostat

The cryostat system is shown in Fig. 4. The temperature was regulated with a carbon resistor thermometer and heater in a feedback circuit. The temperature was found by measuring the vapor pressure of the helium bath and using the T_{58} scale.[10] The vapor pressure was measured with a quartz Bourdon gauge[11] calibrated above 3 Torr

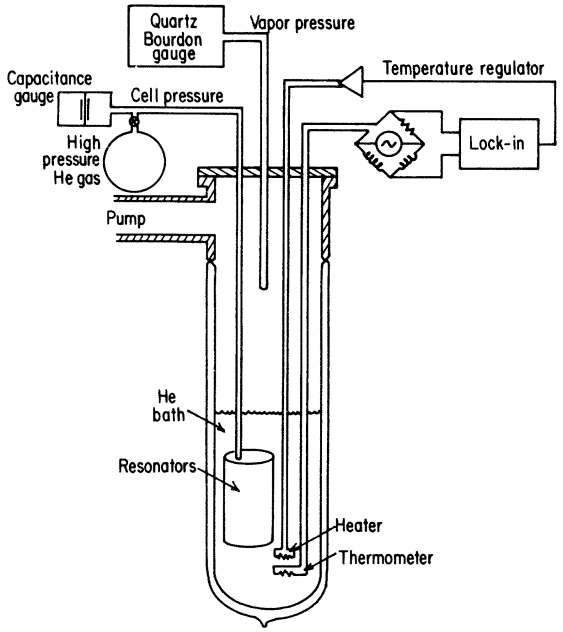

FIG. 4. Cryostat system.

with a mercury manometer and cathetometer and below 3 Torr with a calibrated capacitance gauge.[12] The temperature could be determined to within 1 mK over the entire temperature range.

The pressure in the resonators was measured with a capacitance gauge[13] factory calibrated to better than 0.1% accuracy in a 0–10-bar range. In order to realize the full 0–25-bar range in the experiment, the reference pressure of the gauge was changed at 10 and 20 bar. When the gauge read 10.000 bar, the reference was adjusted until the gauge read 0.000 bar. A second precision high-pressure gauge ensured that the resonator pressure was constant to less than 0.1% during the reference change. Hence the accuracy of 0.1% was maintained over the full 0–25-bar range.

Both the vapor pressure and resonator pressure gauges were temperature regulated.

E. Electronics

The electronics setup shown in Fig. 5 was used to measure the first-, second-, and one of the fourth-sound velocities simultaneously. The first- and second-sound resonances had sufficiently high Q's and low backgrounds so that their resonance frequencies could be tracked automatically using wave analyzers[14] in a phase-locked loop. The fourth-sound resonances had high Q's but the background prevented automatic tracking. The fourth-sound resonances were each found in turn using a spectrum analyzer[15] with a cathode-ray tube dis-

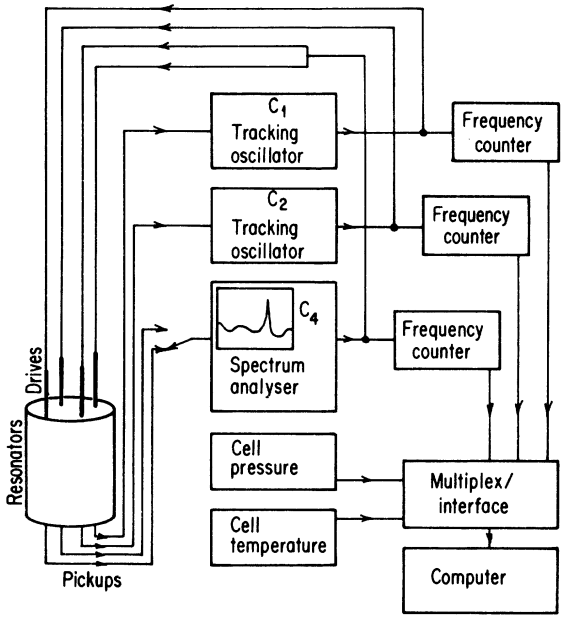

FIG. 5. Electronics.

play. The analyzer could sweep a frequency range and locate the resonance in a matter of seconds. Once the fourth-sound resonance was found a computer was signaled, and all the resonant frequencies together with the cell temperature and pressure were simultaneously stored and tabulated. Then the spectrum analyzer was switched to the other fourth-sound resonator and the process was repeated.

F. Procedure

At the beginning of a run, the temperature was regulated at a value close to an integral multiple of 50 mK. The resonator fill line was interconnected with the bath vapor and the cell was given time to come to thermal equilibrium. When it was observed that the sound velocities were stable, the SVP data were taken. The resonator pressure was then incremented by 1 bar and the data taking procedure was repeated. After each pressure increment, the sound velocities were found to become stable after only a few minutes.

TABLE I. Velocity of fourth sound in He II in a 90-Å packed powder ($m\,sec^{-1}$).

T (K) / P (bar)	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.80
SVP	158	156	153	151	148	144	140	135	130	124	118	103
1	161	159	156	154	151	147	142	137	132	126	119	104
2	165	162	159	157	154	149	145	139	134	127	120	105
3	168	166	162	160	157	152	147	141	135	127	120	106
4	171	168	165	162	159	153	149	143	136	128	120	106
5	173	171	166	165	161	155	150	144	138	129	120	106
6	173	173	168	166	163	157	152	145	139	129	120	105
7	178	175	170	168	165	159	153	147	140	128	119	104
8	181	177	171	170	166	160	154	147	141	127	118	102
9	182	178	173	172	167	161	155	148	141	127	117	101
10	184	181	174	173	168	161	155	149	141	126	115	99
11	186	183	175	174	169	162	156	149	141	124	113	97
12	187	184	176	174	169	163	156	149	140	122	111	94
13	189	185	177	175	170	163	156	149	140	120	109	92
14	190	186	178	176	170	163	156	148	139	118	108	89
15	191	187	179	176	170	163	156	147	138	115	103	86
16	192	187	179	176	170	163	156	147	136	112	100	
17	193	187	179	176	170	163	154	145	134	109		
18	194	187	179	176	170	162	153	144	132	106		
19	194	187	179	176	170	161	152	142	130	102		
20	194	187	179	176	169	160	150	141	128	98		
21	194	187	179	175	168	159	149	139	126	93		
22	193	187	178	174	167	158	146	136	123	87		
23	193	187	178	174	165	156	144	133	119	82		
24	192	186	177	172	164	154	142	130	116			
25	191	185	176	170	162	152	140	127	112			

On some runs which immediately followed liquid-helium transfers, the carbon resistor thermometer was found to drift a few millidegrees. Since the actual temperature was measured with the vapor pressure, this caused no problem except for some inconvenience in tabulating the data.

III. ANALYSIS AND RESULTS

A. Analysis; determination of n

The first- and second-sound velocities (C_1 and C_2, respectively) were determined from the resonant frequencies using Eq. (1) with $l = 10.16$ cm, $m(C_1) = 7$, and $m(C_2) = 20$. The 1-μm fourth-sound velocity uncorrected for the powder scattering (C_4^{uncorr}) was found using $m = 2$. The scattering correction n was found by first calculating a low-temperature (1.187 K) SVP value of the fourth-sound velocity from the thermo-hydrodynamic equations

$$C_4^2 = C_1^2\left[1 - \frac{\rho_n}{\rho} + \frac{\rho_n}{\rho}\left(\frac{C_2}{C_1}\right)^2 - \frac{\rho_n}{\rho}\,\frac{2\beta_p C_2^2}{\gamma S}\right] \qquad (2)$$

and

$$\rho_n/\rho = (1 + C_p C_2^2/TS^2)^{-1}\,. \qquad (3)$$

At SVP and 1.187 K, $\beta_p \approx 0$, and the last term in Eq. (2) can be ignored. The only data needed here other than our own C_1 and C_2 are S and C_p which were interpolated from Ref. 16. Note that since ρ_n/ρ is small (≈ 0.02), an error of 5% in C_p/S^2 will produce an error of only 0.05% in C_4. The scattering correction is taken to be

$$n = C_4/C_4^{\text{uncorr}}$$

evaluated at 1.187 K and SVP. The calculated value $n = 1.228$ is in reasonable agreement with the approximate expression[5] $n = (2 - p)^{1/2}$, where p is the porosity of the packed powder.

Although it can be assumed that the scattering correction for the 1 – μm powder is constant, no such assumption can be made for the 90-Å powder. The reason is that as the pressure approaches the melting pressure, solid layers build up on the powder particles. For the 1-μm powder this has negligible effect on the particle and pore size, but in the 90-Å powder the fractional change in particle and pore size is significant and results in a change in the scattering correction. Since there is no clear theory for calculating the correction, we have tabulated the uncorrected 90-Å fourth-sound velocity data.

B. Results

The sound-velocity data have been used to determine continuous thermodynamic functions for He II. As a result, continuous functions fitting the C_1, C_2, and C_4 (1 μm) data have been obtained. Since these functions fit the data to within the experimental precision, we have used them to obtain smoothed values for these sound velocities. In a table[17] we present the smoothed values along with the raw data (in parentheses). The general precisions for the C_1, C_2, and C_4 (1 μm) values are 0.2 m sec^{-1}, 0.04 m sec^{-1}, and 0.4 m sec^{-1}, respectively. For the bulk of the data these correspond to a precision of 0.2%.

The accuracy of the data is limited by the error in the length of the resonators (0.1%) and, for the 1-μm fourth sound, in the scattering correction (0.05%). We assume that these are constant over the temperature and pressure range of the experi-

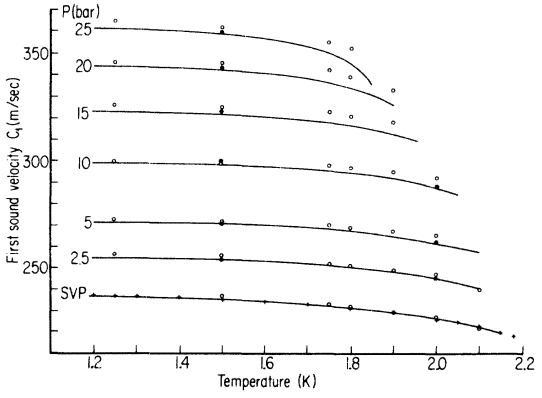

FIG. 6. Velocity of first sound in He II. Solid lines, this experiment; solid circles, Ref. 6; open circles, Ref. 20; crosses at SVP, Ref. 21.

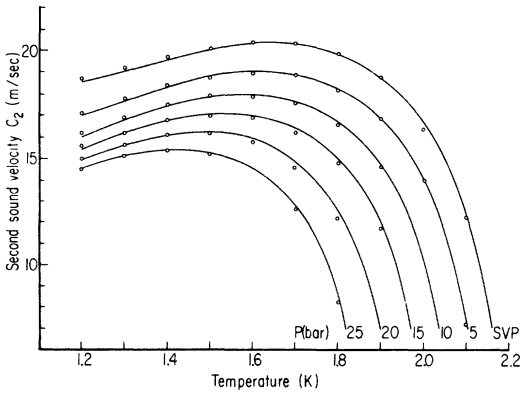

FIG. 7. Velocity of second sound in He II. Solid lines, this experiment; open circles, Ref. 7. Data points of Ref. 7 at elevated pressures have been shifted down by 2%.

ment, and hence the accuracy is within the 0.2% precision.

Very near the λ line the C_4 (1 μm) resonance amplitude decreased and the peak could have been shifted by the background. The accuracy of the temperature measurement also becomes significant near T_λ. Hence the raw data values of C_2 and C_4 for $T/T_\lambda > 0.98$ have precisions of ~1%. However, the functions which generated the smoothed values in the table had the correct asymptotic behavior (as determined from Ahlers[18] and Greywall and Ahlers[19]) built in. For those few points very near T_λ, the smoothed values can be considered as more accurate than the raw data values.

The C_4 (90 Å) data are presented in Table I. The accuracy is estimated at (1–2)%. The significance of these data in regard to size effects will be the subject of a future paper.

C. Comparison with other experiments

Figures 6 and 7 present smooth curves representing a part of our data and points taken from other experiments. It can be seen in Fig. 6 that the C_1 data of Atkins and Stasior[20] deviate by several percent at high pressures whereas the data of Vignos and Fairbank[6] are in good agreement. The data of Chase[21] at SVP agree with our data to ~0.1%.

We found that the C_2 data of Maurer and Herlin[7] agree with our data at SVP but are systematically 2% higher at elevated pressures. However, our C_2 data agree with that of Greywall and Ahlers to ~0.2% at SVP and elevated pressures. In Fig. 7, the open circles for elevated pressures represent the data of Mauer and Herlin after being shifted down by 2%.

---

†Work supported in part by the NSF Grant No. DMR 7500761-000 and the Office of Naval Research, Contract No. N00014-75-C-0246.

*Present address: Ecole Normale Supérieure, Paris V, France.

[1]L. Tisza, J. Phys. Radium 1, 165, 350 (1940).

[2]L. D. Landau, J. Phys. USSR 5, 71 (1941).

[3]J. R. Pellam, Phys. Rev. 73, 608 (1948).

[4]V. P. Peshkov, J. Phys. USSR 10, 389 (1946).

[5]K. Shapiro and I. Rudnick, Phys. Rev. A 137, 1383 (1965); M. Kriss and I. Rudnick, J. Low Temp. Phys. 3, 339 (1970).

[6]J. H. Vignos and H. A. Fairbank, Phys. Rev. 147, 185 (1966).

[7]R. D. Maurer and M. A. Herlin, Phys. Rev. 81, 444 (1951).

[8]Nuclepore Membranes, Nuclepore Corp., Pleasanton, Ca. 94566.

[9]R. Williams, S. E. A. Beaver, J. C. Fraser, R. S. Kagiwada, and I. Rudnick, Phys. Lett. 29, 279 (1969); R. A. Sherlock and D. O. Edwards, Rev. Sci. Instrum. 41, 1603 (1970).

[10]F. G. Brickwedde, H. Van Dijk, M. Durieux, J. R. Clement, and J. K. Logan, Natl. Bur. Std. Mon. 10, 1 (1960).

[11]Precision Pressure Test Set Model 145, Texas Instruments, Inc., Houston, Texas.

[12]Baratron Pressure Meter Type 170 with special 0–3-Torr Head, MKS Instruments, Inc., Burlington, Mass. 01803.

[13]Same as Ref. 12 but with special 0–10-bar head.

[14]Wave analyzer Type 1900A, General Radio Co., West Concord, Mass.

[15]Spectrum Analyzer, Model 3580A, Hewlett-Packard, Loveland, Colorado.

[16]R. G. Hussey, B. J. Good, and J. M. Reynolds, Phys. Fluids 10, 89 (1967).

[17]See AIP document No. PAPS PLRBA-14-3862-9 for nine pages comprising this table. Order by PAPS number and journal reference from American Institute of Physics, Physics Auxiliary Publication Service, 335 East 45th Street, New York, N. Y. 10017. The price is $1.50 for each microfiche (98 pages) or $5 for photocopies of up to 30 pages with $0.15 for each additional page over 30 pages. Airmail additional. Make checks payable to the American Institute of Physics. This material also appears in *Current Physics Microfilm*, the monthly microfilm edition of the complete set of journals published by AIP, on the frames immediately following this journal article. The smoothed sound-velocity data may also be found in the Physical Review article following this one.

[18]G. Ahlers, Phys. Rev. A 8, 530 (1973).

[19]D. S. Greywall and G. Ahlers, Phys. Rev. A 7, 2145 (1973).

[20]K. R. Atkins and R. A. Stasior, Can. J. Phys. 31, 1156 (1953).

[21]C. E. Chase, Phys. Fluids 1, 193 (1958), as tabulated in *Experimental Superfluidity* by R. J. Donnelly (University of Chicago Press, Chicago, 1967).

[22]J. Maynard, following paper, Phys. Rev. B 14, 3868 (1976).