Method for determining the electron-phonon and electron-impurity collision frequencies in metals*

P. B. Johnson, J. C. Kimball, and R. G. Goodrich
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70893
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A method for determining accurate values of the average electron-phonon and the electron-impurity collision frequencies (ν_p and ν_I) around orbits on the Fermi surface of metals using the radio-frequency size effect is discussed. The effects of an electron making multiple orbits in the sample before being scattered are considered in the derived theory. Measurements of signal amplitude versus temperature were made for one particular orbit on the Fermi surface of copper, and the results were fit to the theory with good agreement. Results for ν_p/T^3 and ν_I were obtained for this orbit, and a method for accurately determining the anisotropy of the electron-phonon and electron-impurity collision frequencies over the Fermi surface is proposed.

INTRODUCTION

Over the past few years several attempts have been made to determine the anisotropy of the electron-phonon (e-p) collision frequencies in metals. One of the more promising techniques for this purpose was developed and first applied to copper by Gantmakher and Gasparov.[1] The technique is to measure the average e-p collision frequency around different closed orbits on the Fermi surface. An inversion process is then used to obtain a parametrized function for the anisotropy. On each orbit the electrons experience collisions with both phonons and imperfections in the lattice. The measured scattering rate includes both of these effects, and the e-p part of the scattering is normally extracted from the temperature dependence of the size-effect amplitude. It is the purpose of this paper to describe the effect on the observed scattering rates of an electron completing several cyclotron orbits during the measurement.

Experimentally it is observed that the scattering rate is proportional to T^3. This is in accordance with simple theory.[2] Also the logarithm of the amplitude of the radio-frequency size effect (RFSE) is found to vary linearly with T^3 when an electron completes only one traversal of the sample prior to being scattered out of the orbit, that is for $\pi\nu_{\text{eff}}/\Omega \gg 1$, where ν_{eff} is the effective scattering rate and Ω is the angular cyclotron frequency. For sufficiently pure samples at very low temperatures the above condition may not hold and the effects of multiple cyclotron orbits on the signal may be observable.

Since the value of the magnetic field at which a size-effect signal is observed is inversely proprotional to the sample thickness, d, the cyclotron frequency at resonance can be varied by making measurements on samples of different thicknesses. We have measured the change in the derived e-p scattering rate for an orbit in copper as a function of sample thickness and show how these data should be corrected to give the true value of ν_p, the average e-p scattering frequency for the orbit. An approximate expression for the total line amplitude as a function of temperature is derived so that the extrapolation to the true value of ν_p can be done from plots of the log of the amplitude vs T^3. One factor which enters this analysis is the impurity scattering frequency, ν_I, and a consequence of the analysis is that values of ν_I can be obtained.

THEORY

In the presence of a magnetic field, $\vec{H}$, the electrons in a metal move in reciprocal space on the Fermi surface of a metal in a plane perpendicular to the direction of $\vec{H}$. Such motion about extremal dimension trajectories on the Fermi surface results in closed orbits in real space. These orbits will just fit inside of a sample consisting of a thick, flat, metal crystal of thickness d when H is parallel to the sample surface and when $k_c = (e/\hbar c)Hd$, where k_c is an extremal diameter of the Fermi surface, e is the electron charge, $\hbar$ is Planck's constant divided by 2π, and c is the speed of light. Such orbits can be detected using the RFSE.

The amplitude of the resulting RFSE signals is governed by the probability that an electron can traverse the sample along an orbit without being scattered. The probability of an electron being scattered along an orbit in traversing the sample thickness once is proportional to $e^{-\pi\nu_{\text{eff}}/\Omega}$. If the sample is sufficiently pure and the temperature sufficiently low, an electron could possibly make many such traverses without being scattered. In this case the amplitude of an RFSE signal would be given by[2]:

$$A = A_0 \sum_{n=1}^{\infty} \exp\left(\frac{-n\pi\nu_{\text{eff}}}{\Omega}\right) = A_0\left(\frac{\exp(-\pi\nu_{\text{eff}}/\Omega)}{1-\exp(-\pi\nu_{\text{eff}}/\Omega)}\right), \quad (1)$$

where ν_{eff} is the average collision frequency for all types of scattering on an orbit, $\Omega = eH/m^*c$ is the cyclotron frequency, and m^* is the cyclotron mass. The total scattering frequency can be written as $\nu_{eff} = \nu_I + \nu_p(T)$ where ν_I is the temperature-independent collision frequency of electrons with static impurities and $\nu_p(T)$ is the temperature-dependent frequency of collision between electrons and phonons. For the orbits we are considering it is assumed that $\nu_p = CT^3$ where C is a constant for a particular orbit and a particular sample thickness. Now substituting for ν_{eff} in Eq. (1) we obtain

$$\ln A = \ln A_0 - (\pi/\Omega)CT^3 - (\pi/\Omega)\nu_I - \ln(1 - e^{-\pi\nu_{eff}/\Omega}). \tag{2}$$

To simplify the above expression we assume that $\nu_I \gg CT^3$ for all T under consideration. This approximation is easily met under usual experimental conditions as will be demonstrated later. Let us now consider two cases: $\pi\nu_I/\Omega \gg 1$ and $\pi\nu_I/\Omega \sim 1$. First when

$$\pi\nu_I/\Omega \gg 1, \quad \ln(1 - e^{-\pi\nu_{eff}/\Omega}) \to 0$$

and we obtain

$$\ln A = \ln A_0 - (\pi/\Omega)\nu_I - (\pi/\Omega)CT^3.$$

We will call this expression the thick sample limit. It is in this limit where one would like to perform experiments since the relation between amplitude and temperature becomes most simple. Second, where $\pi\nu_I/\Omega$ is on the order of one then $\pi CT^3/\Omega \ll 1$ and

$$\ln(1 - e^{-\pi\nu_{eff}/\Omega}) = \ln(1 - e^{-\pi\nu_I/\Omega}e^{-\pi CT^3/\Omega}) \simeq \ln[1 - e^{-\pi\nu_I/\Omega}(1 - \pi CT^3/\Omega)]$$

$$\simeq \ln(1 - e^{-\pi\nu_I/\Omega}) + \ln\left(1 + \frac{e^{-\pi\nu_I/\Omega}}{1 - e^{-\pi\nu_I/\Omega}}\frac{\pi CT^3}{\Omega}\right) \simeq \ln(1 - e^{-\pi\nu_I/\Omega}) + \left(\frac{e^{-\pi\nu_I/\Omega}}{1 - e^{-\pi\nu_I/\Omega}}\right)\frac{\pi CT^3}{\Omega}.$$

Therefore

$$\ln A = \ln A_0 - \frac{\pi\nu_I}{\Omega} - \ln(1 - e^{-\pi\nu_I/\Omega}) - \frac{\pi}{\Omega}CT^3(1 - e^{-\pi\nu_I/\Omega})^{-1}. \tag{3}$$

It should be noted that when $\pi\nu_I/\Omega \gg 1$ this expression reduces to the thick sample limit and so is valid everywhere provided $\nu_I \gg CT^3$.

This expression is convenient to use in analyzing experimental data because it shows that the observed linear dependence of $\ln A$ on T^3 is the expected result. However, the slopes and intercepts of these curves are dependent on ν_I. Furthermore, by measuring the Ω dependence of the amplitude of the signal for a particular orbit the value of ν_I can be obtained for that orbit.

EXPERIMENT

Our measurements were performed on thin single-crystal samples of copper which were spark cut from a large oxygen annealed single crystal having a resistance ratio of about 16 000. After spark cutting, the samples were acid polished to remove the spark damage and produce smooth, strain free surfaces. Typically about 1 mm of material had to be removed from each surface in order to eliminate the spark damage.

Measurements of the temperature dependence of the amplitude of parallel field RFSE signals were performed using a 6-MHz limiting oscillator and a field modulation RFSE detection system.[3] The sample coil and Dewar system were located in the gap of a 12'' diameter pole face iron core magnet. The temperatures of the sample was controlled from 2 to 11 K by a double-wall vacuum-can system which isolated the sample from the liquid-helium bath. The inner can, which contained an exchange gas was heated above the temperature of the 4.2 K bath and the temperature was monitored using a carbon resistor. This resistor was calibrated versus helium vapor pressure from 1.5 to 4.2 K and extrapolated above 4.2 K to fit the measured resistance at 77 and 300 K.

RESULTS

We have investigated the extremal central orbit about the Fermi surface obtained when the magnetic field is oriented 15° from the [100] direction in an [001] normal sample. This particular orbit was selected since it did not overlap the noncentral belly orbits and the open orbits obtained when the field is near the [100] direction and since the signal is much larger than the signals near the [100] directions. The orientation of the field with respect to the sample axis was determined to within 0.5° from the symmetry of the RFSE signals.

Measurements of signal amplitude versus temperature were made over the temperature range 2 to 11 K at each of the four different thicknesses of a copper sample. Between each set of measurements, the sample was thinned by the acid

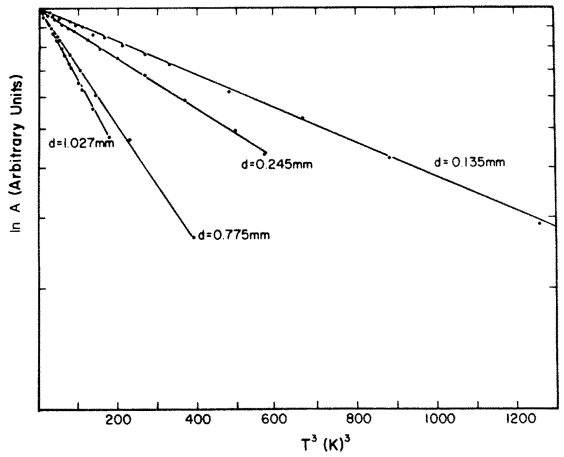

FIG. 1. $\ln A$ vs T^3 for the measured orbit about the Fermi surface at four different sample thicknesses.

polishing technique used in the initial preparation of the sample. Measurements were made on the same sample to insure that ν_I was the same for each set of measurements. The thickness of the samples was determined using the known caliper values of the orbit and the measured magnetic field value at which the signal occurred. In all four cases plots of $\ln A$ vs T^3 gave the expected linear results. These results are displayed in Fig. 1. The slopes of the plots were determined using the method of least squares, and the coefficient of determination (r^2) for each plot was at least 0.998. Values of the thicknesses and corresponding slopes of the $\ln A$ vs T^3 plots are listed in Table I.

In addition the results obtained from a 0.158-mm thick sample are presented in Table I. The 0.158 mm sample was cut from a different portion of the crystal and was noticeably strained while being handled.

TABLE I. Sample thickness, d, and corresponding values of the slope of $\ln A$ vs T^3 plots for the measured orbit. The first four values were obtained from the same sample as it was thinned to different thicknesses. The fifth measurement was made on a separate slightly strained sample. Both samples had $\vec{n}$||[001] and $\vec{H}$ 15° from [100].

d (mm)	slope $(°K)^{-3}$
1.027	4.188×10^{-3}
0.775	3.340×10^{-3}
0.245	1.479×10^{-3}
0.135	1.016×10^{-3}
0.158	0.914×10^{-3}

DISCUSSION

Equation (3) can be cast into the form

$$\ln(A/A_0) = C_1 + (\pi/\Omega)C_2T^3,$$

where

$$C_1 = -\frac{\pi\nu_I}{\Omega} - \ln(1 - e^{-\pi\nu_I/\Omega}) \quad \text{and} \quad C_2 = \frac{C}{1 - e^{-\pi\nu_I/\Omega}}.$$

Therefore the slope, s, of a $\ln A$ vs T^3 plot is $s = (\pi/\Omega)C_2$ yielding

$$\pi/\Omega s = (1/C)(1 - e^{-\pi\nu_I/\Omega}).$$

Substituting $\Omega = eH/m^*c$ where $m^* = 1.351m_e$, and $H = k_c\hbar c/ed$ where $k_c = 2.736\,90A^{-1}$ for our observed orbit,[1,4] we obtain

$$d/s = (k/C)(1 - e^{-d\nu_I/k}), \tag{4}$$

where $k = 7.4593\times10^8$ mm/sec. By picking appropriate values for C and ν_I, the experimental values of d/s can be fit with good accuracy (Fig. 2). The best fit is obtained with $C = 3.04\times10^6$ sec^{-1} K^{-3} and $\nu_I = 3.7\times10^9$ sec^{-1} where C is the e-p scattering rate and ν_I is the impurity scattering rate. The value of C corresponds to the e-p scattering rate in a sample in which multiple electron orbits can be completely neglected (i.e., $\pi\nu_{\text{eff}}/\Omega \gg 1$). Therefore C is the true average value of the collision frequency around the orbit on the Fermi

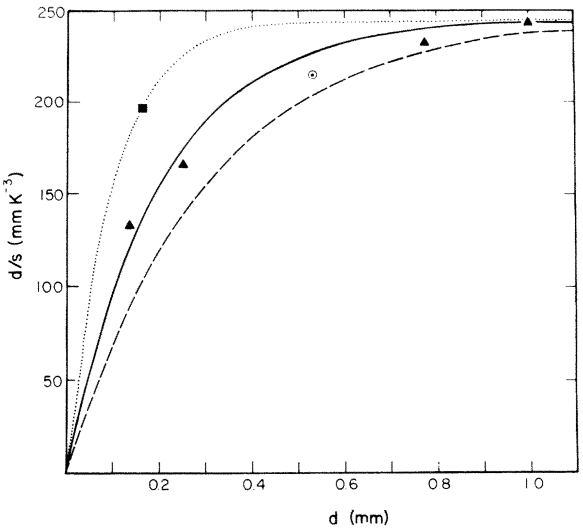

FIG. 2. Plots of d/s vs d obtained from Eq. (4). $C = 3.04\times10^6$ sec^{-1}K^{-3} for each curve. $\nu_I = 2.49\times10^9$ sec^{-1} for the dashed curve, 3.73×10^9 sec^{-1} for the solid curve, and 7.46×10^9 sec^{-1} for the dotted curve. Triangles represent the four data points obtained from the same sample as it was thinned to different thicknesses, the square corresponds to the values from the strained sample, and the circle corresponds to Gantmakher and Gasparov's value.

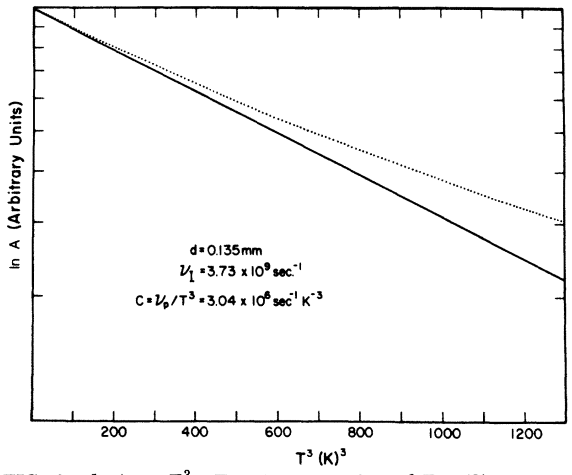

FIG. 3. $\ln A$ vs T^3. Exact expression of Eq. (2) is the dotted curve and the approximate expression of Eq. (3) is the solid curve.

surface. As a check on the order of magnitude of the value of ν_I obtained from this experiment, we have calculated the conductivity using $1/\tau = \nu_I$ and compared it to the measured dc conductivity at 4.2°K. The two numbers agree within a factor of 2.

In a recent determination of the anisotropy of the e-p collision frequency on the Fermi surface of copper, Gantmakher and Gasparov[1] determined a value of $C = 3.46 \times 10^6$ sec^{-1} K^{-3} for the particular orbit we are considering. They determined this value from a 0.53 mm thick sample and the corresponding value of d/s falls slightly below our curve (Fig. 2). This smaller value would result if their sample had a smaller value of ν_I. Figure 2 contains a plot of a curve with a smaller value of ν_I, and as can be seen, a small change in ν_I could easily account for the difference.

As the sample gets thinner, $e^{-\pi\nu_I/\Omega}$ starts to become larger and therefore values of d/s are very dependent on ν_I. Our measured value of d/s for our 0.158 mm thick sample was much larger than our calculated curve (Fig. 2). This sample was, however, badly strained and therefore has a larger value of ν_I. As can be seen (in Fig. 2) by increasing the value of ν_I the theoretical expression can account for the larger value of the impurity scattering rate in the thin sample.

All of our amplitude measurements, except for a few high-temperature points determined on our 0.135 mm sample, were made under the condition that $\nu_I \gg CT^3$. In this regime the exact and approximate expressions [Eqs. (2) and (3)] are nearly identical. Above about 6°K, however, CT^3 starts to become on the order of ν_I and the two results start to diverge (Fig. 3). It should be noted that our data on the $d = 0.135$ mm sample (Fig. 1) remain linear as predicted by the approximate expression, instead of following the slight curve of the exact result. One possible explanation of this behavior is that the e-p scattering rate may not be proportional to T^3 over the entire temperature range for this data. If the temperature law is increased slightly[5] to $T^{3.3}$, the exact curve starts to straighten out and the approximate curve starts to bend.

CONCLUSIONS

We have calculated the effect on the amplitude of RFSE signals that multiple-electron orbits within a sample can have on the resulting values of d/s. The results of the calculation were compared with measurements of d/s determined from an orbit on the Fermi surface of copper at four different sample thicknesses. The same sample was thinned between each set of measurements to ensure that ν_I would be the same in each case. The fit of the theoretical curve to the experimental data was very good and yielded values of ν_I and $C = \nu_p/T^3$. The value of CT^3 is the true average e-p collision frequency around the orbit.

Our data were compared with the data from the same orbit measured by Gantmakher and Gasparov for their determination of the e-p collision frequency over the Fermi surface of copper. The true value of C for this orbit, extrapolated from our data, is about 12% smaller than theirs. If the procedure we have followed on this orbit is extended to many orbits over the Fermi surface, an accurate value of the e-p collision frequency anisotropy can be determined using the technique outlined by Gantmakher and Gasparov. In addition this procecure will allow a determination of the anisotropy of ν_I for electrons on defferent parts of the Fermi surface.

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[1]V. F. Gantmakher and V. A. Gasparov, Zh. Eksp. Teor. Fiz. 64, 1712 (1973) [Sov. Phys.-JETP 37, 864 (1973)].

[2]R. G. Chambers, Phys. Kondens. Mater. 9, 171 (1969).

[3]R. C. Jones, R. G. Goodrich, and L. M. Falicov, Phys. Rev. 174, 672 (1968).

[4]M. R. Halse, Phil. Trans. R. Soc. 265, 507 (1969).

[5]V. F. Gantmakher and Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. 48, 1077 (1965) [Sov. Phys.—JETP 21, 720 (1965)].