Field dependence of the superconductive penetration depth in In and dilute In Bi alloys

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We have studied the superconductive penetration depth $\lambda(T, H)$ by mutual inductance at 75 kHz in small, single spheres of In and InBi (0.19, 0.395, and 0.60 at.%). The corresponding Ginzburg-Landau parameters $\kappa(t = 1)$ are 0.061, 0.155, 0.240, and 0.349, respectively. The temperature dependence of the penetration depth is equally well described by $\lambda(t) = \lambda_0/(1-t^4)^{1/2}$ and $\lambda(t) = \lambda_0' \mathbb{Z}_{BCS}(t)$. Respective values of λ_0 are 395, 510, 630, and 685 ± 30 Å, while λ'_0 is 275, 355, 440, and 480 ± 20 Å. The field dependence of λ was obtained at $t = 0.983$ for fields up to the ideal superheating field $H_{\rm sh}$. At $H_{\rm sh}$, we find $\lambda(H_{\rm sh})/\lambda(H = 0) = 1.46$, 1.44, 1.46, and 1.53 \pm 0.05, respectively, confirming the previous measurements on Sn. In "weak" fields $H \le H_{\rm sh}$, we find $\lambda(H)/\lambda(0) \approx 1+\alpha(H/H_{\rm th})^2$, with $\alpha = 0.08$, 0.16, 0.18, and 0.19 \pm 0.08. We discuss in detail the agreement with theory, which is quite good.

I. INTRODUCTION

We recently carried out the first measurements¹ of the field dependence of the superconduetive penetration depth $\lambda(H)$ in "strong" fields approaching the bulk superheating field $H_{\rm sh}$. These measurements were performed on single, flawless tin spheres of diameter $15-30 \mu m$. Because of the demagnetizing field, the total field varies over the surface of a supercondueting sphere, and the raw data therefore contain information about an *averaged* penetration depth $\bar{\lambda}(H)$. In Ref. 1 we developed, and successfully carried out, a procedure for extracting the real field dependence $\lambda(H)$ from the averaged data. We found that $\lambda(H)/\lambda(0)$ increased very strongly close to the superheating limit, reaching a value of 1.51 ± 0.04 at $H_{\rm sh}$, with a divergent derivative. We present in this paper an investigation of the field dependence $\lambda(H)$ in a series of alloys with different values of the Ginzburg-Landau (GL) parameter κ . We present separately (following paper²) the measurements of the superheating and supereooling fields, the GL paxameter, the properties of the intermediate state, and the bulk resistivity.

II. THEORY

A. Signal

With minor modifications, we shall follow our previous treatment' of the variation of the transition signal with temperature and field. We give here only some of the main points. We assume a penetration depth of the form

$$
\lambda(T, H) = (\lambda_0 y) f(H/H_{\rm sh}). \tag{1}
$$

The proportionality to $y = 1/(1-t^4)^{1/2}$ is justified by the experimental results. [See Eq. (10)]. The temperature dependence of the signal is then, for a sphere of radius $R,$ ¹

$$
\frac{S(T, 0)}{S(0, 0)} = \frac{1 - 3\lambda_0 y/R + 3(\lambda_0 y/R)^2}{1 - 3\lambda_0/R + 3(\lambda_0/R)^2},
$$
\n(2)

which determines λ_0 from the experimental data $S(T)$. In order to analyze the field dependence of the signal, we normalize away the temperature dependence by defining a reduced signal

$$
\xi(h) = \frac{S(T, h) - S^0}{S(T, 0) - S^0},
$$
\n(3)

where h is the reduced equatorial field

$$
h \equiv \frac{H_{\text{eq}}}{H_{\text{sh}}} \cong \frac{\frac{3}{2}H[1 - \lambda_0 y/R + (\lambda_0 y/R)^2]}{H_{\text{sh}}},
$$
(4)

and $S⁰$ is the signal for zero penetration depth, i.e., Eq. (2) with $y = 0$. Let the "average" penetration depth $\overline{\lambda}$ be given by

$$
\overline{\lambda}(h)/\lambda(0) \equiv \overline{f}(h) = \int_0^{\pi/2} d\theta \sin\theta f(h\sin\theta). \tag{5}
$$

For the two cases, respectively, of the static field and tickling field being perpendicular and parallel to each other, we then have, to first order in λ/R :

$$
\xi_{\perp}(h) \cong \overline{f}(h) = \overline{\lambda}/\lambda(0), \tag{6}
$$

$$
\zeta_{\text{II}}(h) \cong \frac{d}{dh} \big[h \overline{f}(h) \big] = \frac{d}{dh} \frac{(h \overline{\lambda})}{\lambda(0)} \,. \tag{7}
$$

Because the derivative of $f(h)$ diverges¹ at H_{sh} , there is a striking difference in the field effect as observed in perpendicular and parallel fields, respectively. In Fig. 1, we show two experimental hysteresis loops that bring this out very clearly. The two field sweeps are taken on the same sample at the same temperature. The upper sweep is carried out in parallel fields; it clearly shows the depression of the signal close to the super-

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FIG. 1. Field sweeps taken at the same temperature, and for the same sample, show the difference between having the tickling field parallel (upper half) or perpendicular (lower half) to the static field. Upper sweep gives a marked depression of the signal close to H_{sh} , due to the field dependence of the penetration depth, while the effect is almost undetectable in the lower sweep.

heating field. This depression is always reproducible, it is due to the field dependence $\lambda(H)$, and is proportional to $\xi_{\mu}(h)$. The lower sweep is carried out in perpendicular fields, here the field effect is almost undetectable. Careful measurements, however, show a definite signal depression in this case too, it is proportional to $\xi_1(h)$ and is down by roughly a factor of 10 at $H_{\rm sh}$ compared with the parallel-field case. Experiment thus nicely confirms the theoretical expectation.

Summing up, the field-effect data are analyzed as follows. $\zeta_{\parallel}(h)$ is measured, then integrated over field h [Eq. (7)] to give $\bar{\lambda}(h)$. Equation (5) is then inverted empirically to give $\bar{\lambda}(h)$. [We in fact use the second-order expression for $\bar{\lambda}(h)$, given in Eq. (32) of Ref. 1.]

B. Theory for $\lambda(T, H)$

The temperature dependence $\lambda(T)$ of the penetration depth has given rise to considerable confusion in the literature.³ The reason is that up till now, no method has been devised which permits very precise measurements of the $absolute$ value of λ , while the *variation* of λ with T can be measured very accurately. In the BCS theory, two parameters are needed to describe the field penetration in a pure metal. They are the coherence length ξ_0 and the London penetration depth $\lambda_L(T)$ whose temperature dependence is given by

$$
\frac{\lambda_L(t)}{\lambda_L(0)} \equiv Z_{\rm BCS}(t) = \left(\frac{1}{\Delta(t)} \frac{d[\Delta(t)/t]}{d(1/t)}\right)^{-1/2}.
$$
 (8)

Values of $Z_{\text{BCS}}(t)$ have been tabulated by Mühlschlegel.⁴ The actual penetration depth $\lambda(t)$ is. equal to $\lambda_L(t)$ if $\lambda_L(t) \gg \xi_0$. For local superconductors, this inequality is satisfied even at $t=0$, but, in general, it will only be true close to T_c . In type-I superconductors, $\lambda(0) > \lambda_L(0)$ because of the nonlocal effects.

For alloys, the mean-free path l enters as a. third parameter affecting the actual $\lambda(t)$. In this case, the temperature dependence will still be given roughly by Eq. (8) , but with a rapidly increasing zero-temperature penetration depth λ_0' which in the extreme case $l \ll \lambda_L(0)$ is given by⁵

$$
\lambda'_0 \cong \lambda_L(0) \left(\xi_0/l\right)^{1/2}.\tag{9}
$$

In our case, however we shall find that $l \leq \xi_0$, in which case we must rely on the numerical calculations of Miller⁵ to relate the different lengths involved. Experimentally, $\lambda(t)$ in pure and impure superconductors alike roughly fits the empirical relation

$$
\lambda(t) = \lambda_0 / (1 - t^4)^{1/2} = \lambda_0 y.
$$
 (10)

Close to T_c , $Z_{BCS} \approx \sqrt{2} y$, which implies $\lambda_0 \approx \sqrt{2} \lambda'_0$. Assuming $\lambda \sim Z_{BCS}$ or $\lambda \sim y$ makes very little difference close to T_c : the *absolute* value of λ at $t=0.984$ (y=4) differs only by 7% in the two cases. In the analysis of the field effect, we can therefore safely assume $\lambda \sim \gamma$.

The field dependence $\lambda(H)$ is best described in terms of Ginzburg-Landau (GL) theory. In GL theory, an external field decreases the surface order parameter Ψ_{s} below the value Ψ_{0} in the screened interior, leading to an increased penetration depth. For general κ , only numerical computations of $\lambda(H)$ exist.⁶ In the limit $\kappa \ll 1$, the surface order parameter is given by'

$$
[\Psi_s(H)/\Psi_0]^2 = \frac{1}{2} \big\{ 1 + \big[1 - (H/H_{\rm sh})^2 \big]^{1/2} \big\}.
$$
 (11)

In the local case, $\lambda(H)$ is simply proportional to Ψ_s^{-1} , and in the extreme nonlocal limit to $\Psi_s^{-2/3}$.

Expansion of Eq. (11) in the local case gives for weak fields $H \ll H_{\rm sh}$:

$$
\frac{\lambda(H)}{\lambda(0)} = f\left(\frac{H}{H_{\rm sh}}\right) \approx 1 + \frac{1}{8} \left(\frac{H}{H_{\rm sh}}\right)^2 + \frac{7}{128} \left(\frac{H}{H_{\rm sh}}\right)^4 \cdots
$$
 (12)

Substituting $H_{\rm sh} = H_c/(\kappa \sqrt{2})^{1/2}$, the quadratic term in Eq. (12) gives back the familiar quadratic field dependence derived by Ginzburg and Landau' for weak fields. Close to $H_{\rm sh}$, Eq. (11) leads to a rapid increase in λ . For the local case, $\lambda(H_{sh})$ $=\sqrt{2}\lambda(0)$, while $d\lambda/dH$ diverges at $H_{\rm sh}$. These features were both verified in our experiments on $Sn.^1$ We have recently shown analytically⁹ to the next order in κ that

$$
\lambda(H_{\rm sh})/\lambda(0) = \sqrt{2} \left(1 + \frac{3}{32} \sqrt{2} \kappa\right). \tag{13}
$$

This result apparently contradicts a recent proof by Esfandiari¹⁰ stating that $\lambda(H_{sh})=\sqrt{2}\lambda(0)$ for all κ .

III. EXPERIMENTAL

The cryostat and experimental technique have been described before.¹ Sample production and characterization is described in the following paper² dealing with superheating and supercooling; some relevant results are given in Table I.

Determination of $\lambda(T, H)$ requires an accurate measurement of the variation with T and H of the transition signal S. 8 can be measured either directly from a hysteresis loop such as the one shown in the upper Fig. 1, or by stopping the field sweep, and inducing a transition with a transient field. The latter method requires averaging five or ten transitions to get enough accuracy. Figure 1 is deceptive in that no drift with time is present. Usually, there is a time drift which makes direct measurement of $S(H)$ very difficult. We have therefore used the second method almost exclusively, except for the $InBi~0.6$ -at.% concentration, where we measured the total time drift for each sweep, assumed it to be linear in time, and found $S(H)$ by correcting accordingly. This latter procedure gives less random uncertainty

(as can be seen from Fig. 4), but may entail an increased systematic error. Both methods permit measurement of S to about 1% accuracy for 20 μ m spheres.

IV. RESULTS AND DISCUSSION

A. Temperature dependence of λ

Figure 2 shows how the normalized signal varies with $y = (1 - t^4)^{-1/2}$. By fitting the data to Eq. (2), λ_0 can be determined to an accuracy of about ± 30 Å. We thus obtain λ_0 =395, 510, 630, and 685 Å for the four samples, respectively. These fits are shown as solid curves, which follow the data very nicely for all four concentrations. Note that T_c has not been used as a variable parameter, it is independently determined to ± 0.2 mK relative accuracy for each run by a zero-field sweep similar to that shown in Fig. 2 of Ref. 11. The fact that $S(y)$ does not deviate from the fit even very close to T_c ($y \approx 12$, or $t \approx 0.9983$), attests to the correctness of this determination of T_c .

The value λ_0 = 395 ± 25 Å in pure In is in agreement with Dheer's result¹² of 430 ± 20 Å. The values for the dilute $InBi$ alloys are in qualitative agreement with Connell's results¹³ on dilute $InSn$ alloys. If the data are plotted versus Z_{RCS} instead of y, equally good fits are obtained. This is shown for two of the concentrations in Fig. 3. The values of $\lambda'_{0} = d\lambda/dZ_{\text{BCS}}$ are given in Table I. As expected, λ_0/λ_0' roughly, but not exactly, equals $\sqrt{2}$. For pure In, our value of $\lambda'_{0} = 275 \pm 15 \text{ Å}$ should equal approximately $\lambda_L(0)$, since most of the data are taken close to T_c . This is in fair agreement with the value of $\lambda_L(0) = 250 \text{ Å}$ obtained from ultrasonic attenuation.¹⁴

Finally, the increase in $\lambda'(0)$ with impurity concentration can be compared to the numerical calculations of Miller,⁵ which are based on the BCS theory. We use our value of $\lambda_L(0)$, and $\xi_0 = 0.96$ $\times\lambda_L(0)/\kappa$ = 4300 Å for κ = 0.061.² With the mean-free path obtained from resistance measurements,² we have given in the last columns of Table I experi-

TABLE I. Temperature dependence of λ in In and In Bi dilute alloys.

	T_c (⁴ He scale) ^a	Sphere diameter	$\lambda_0 = d\lambda/dy$	$\lambda_0' = d\lambda/dZ_{BCS}$	Mean-free path l^b	$\left[\lambda^{\text{impure}}(t=1)\right]/\left[\lambda^{\text{pure}}(t=1)\right]$	
	(K)	(μm)	(A)	(Ă	(μm)	Experiment	Theory ^c
Pure In	3.4089	18.6 ± 0.5	395 ± 25	275 ± 15	>300		
<i>In</i> Bi $(0.19 \text{ at.})\%$	3.4051	18.1	510 ± 35	355 ± 25	0.438	1.29	1.28
<i>In</i> Bi $(0.395$ at $\%)$	3.4396	18.8	630 ± 30	440 ± 20	0.220	1.60	1.62
<i>In</i> Bi (0.60 at.%)	3.4949	17.8	685 ± 30	480 ± 20	0.144	1.75	1.88

^a See following paper (Ref. 2).

 b See following paper (Ref. 2).</sup>

^c Reference 5.

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FIG. 2. Decrease of signal with temperature in In and $In \mathbb{B}$ i spheres, due to temperature dependence of penetration depth. Solid curves are fits assuming $\lambda(T) = \lambda_0 y$, determining λ_0 to \pm 30 Å accuracy.

mental and theoretical values of $\lambda^{impure}(t=1)/$ $\lambda^{\text{pure}}(t=1)$ ($\approx \lambda_0'/\lambda_0^{\text{pure}}$). The agreement is surprisingly good, much better than one would expect considering the uncertainties involved.

B. Field dependence of λ

The main purpose of this work is to determine $\lambda(H)/\lambda(0) = f(H/H_{sh})$, for different values of κ . The limiting behavior close to H_{sh} is of particular interest. We shall use the method of analysis developed earlier' and summarized in Sec. II. To maximize the sensitivity, most field measurements were carried out at $t \approx 0.983$ ($y \approx 3.9$), which is just outside the size effect region where H_{sh} starts decreasing from its bulk value. A few measurements were carried out at lower temperatures. Note that all fields are normalized to $H_{\rm sh}$, which decreases drastically as κ increases with the im-

FIG. 3. Decrease of signal plotted vs Z_{BCS} , Eq. (8), instead of y. Fits are seen to be equally good.

purity concentration. At $t = 1$, H_{sh}/H_c equals 3.39, 2.44, 2.08, and 1.85 for pure In and the respective alloys.² The values of $\kappa(t = 1)$, determined from the supercooling results,² are 0.061, 0.155, 0.240, and supercooling results,² are 0.061, 0.155, 0.240, and 0.349, respectively.

Figure 4 shows the reduced signal $\zeta_{\parallel}(h)$ for the four concentrations. The strong increase in the field effect close to H_{sh} is apparent in all samples. The bulk of the measurements was performed with a peak-to-peak tickling field of about 0.4-0.⁵ G, corresponding to $(3-4)\%$ of H_{sh} at $t = 0.983$. The tickling field superimposes upon the static field at each instant, and thus the superheating transition is induced at a static field of about $0.98H_{\rm sh}$. By halving the tickling field one can get to about $0.99 H_{\rm sh}$, and the signal depression increases, as indicated by the open circles in Fig. 4. Further reduction in the tickling field reduces the signal too much. The tickling field thus limits how close one can get to H_{sh} , where ζ_{H} diverges.

Figure 5 shows the average penetration depth $\overline{\lambda}(h)/\lambda(0)$. It is obtained by graphical integration over field of the data in Fig. 4: the data points for each concentration are connected by a zig-zig curve and integrated over h according to Eq. (7). The final step is to invert Eq. (5) so as to obtain $\lambda(H)$ from $\overline{\lambda}(H)$. We are unable to do this analytically in the general case. We observe from Fig. 4,

FIG. 4. Field dependence of signal, given as $\zeta(h)$, Eq. (3). Note strong effect near the superheating limit. Open circles close to H_{sh} indicate measurements with the tickling field reduced by a factor of 2. 1.00

however, that the experimental data are roughly approximated by the prediction from QL theory, $f(H/H_{sh})=(\Psi_s/\Psi_0)^{-1}$, with Ψ_s given by Eq. (11). We therefore fit the experimental results by assuming an empirical deviation:

$$
f_{\text{expt}}\left(\frac{H}{H_{\text{sh}}}\right) = \left(\frac{\Psi_s}{\Psi_0}\right)^{-1} + A\left(\frac{H}{H_{\text{sh}}}\right)^2 + B\left(\frac{H}{H_{\text{sh}}}\right)^4 + C\left(\frac{H}{H_{\text{sh}}}\right)^6. \tag{14}
$$

Using Eq. (5), this gives

$$
\overline{f}(h) = \langle (\Psi_s/\Psi_0)^{-1} \rangle_{\text{av}} + j_3 A h^2 + j_5 B h^4 + j_7 C h^6, \quad (15)
$$

FIG. 5. Average penetration depth $\bar{\lambda}(h)/\lambda(0)$, obtained by graphical integration of Fig. 4. Solid curve gives GL prediction based on $\lambda \sim \Psi_s^{-1}$.

where¹ $j_3 = \frac{2}{3}$, $j_5 = \frac{8}{15}$, $j_7 = \frac{48}{105}$, and the first term on the right-hand side is found from Eq. (11) by numerical integration. A is determined from the slope of $\bar{\lambda}$ vs h^2 for low h, while B and C are chosen to give the best possible fit close to $h = 1$. In this way, very good fits to the data of Fig. 5

TABLE II. Field dependence of λ in In and In Bi dilute alloys.

	$\kappa_{\rm GI}$ (t = 1) ^a	$\overline{\lambda}(H_{\rm ch})/\lambda(0)$	$\lambda(H_{\rm ch})/\lambda(0)$	$\alpha = [d\lambda/d(h^2)]_0$	$A^{\,b}$	R ^b	c ⁶
Pure In	0.061	1.18 ± 0.02	1.46 ± 0.05	0.08 ± 0.08	-0.048	-0.019	0.114
<i>In</i> Bi $(0.19 \text{ at.})\%$	0.155	1.20 ± 0.02	1.44 ± 0.05	0.15 ± 0.08	0.029	0.173	-0.174
<i>In</i> Bi $(0.395 \text{ at.} \%)$	0.240	1.21 ± 0.02	1.46 ± 0.05	0.18 ± 0.08	0.054	0.158	-0.168
<i>In</i> Bi $(0.60$ at.%)	0.349	1.24 ± 0.02	1.53 ± 0.05	0.19 ± 0.08	0.070	-0.038	0.086

^a Following work (Ref. 2).

 b Parameters of empirical fit, Eq. (14).</sup>

FIG. 6. Final field dependence $\lambda (H/H_{sh})$, obtained from the data of Fig. 5 by empirical inversion of Eq. {5), Differences between concentrations are seen to be slight. Solid curves $\lambda(H) \sim \Psi_s^{-1}$, Eq. (11). Values at $t = 1$ are indicated at upper right.

were constructed. The coefficients are given in Table II. Figure 6 shows the resulting $\lambda \left(H/H_{\rm sh}\right)$ The values of

$$
\lambda(H_{sh})/\lambda(0) = \sqrt{2} + A + B + C, \qquad (16)
$$

are 1.46, 1.44, 1.46, and 1.53 ± 0.05 respectively.
The use of the exact temperature dependence of λ , and an exact field distribution $H(\theta)$ instead of the London solution, $¹$ may lower these numbers by an</sup> amount not exceeding the given uncertainty. Thus, our results confirm $\lambda_{sh}/\lambda(0)$ to be quite close to tell whether there is in fact a slight increase with $\sqrt{2}$, with a very weak κ dependence. In order to κ , as predicted by Eq. (13), or the ratio is exactl as predicted by Eq. (13), or the ratio is ex
 $\overline{2}$ for all κ ,¹⁰ the accuracy of the experiments must be improved, or experiments for high κ must be performed. Figures $4-6$ also confirm the other conclusion drawn from the Sn experiments': that $\sqrt{2}$, with a very weak κ dependence. In order to
tell whether there is in fact a slight increase with
 κ , as predicted by Eq. (13), or the ratio is exactly
 $\sqrt{2}$ for all κ ,¹⁰ the accuracy of the experime

Since the prediction $\lambda_{\rm sh} = \sqrt{2} \lambda(0)$ stems from λ would expect $\lambda(H)/\lambda(0)$ to increase less rapidly in ζ_s^{-1} , which is only valid in the local li

pure In and Sn than in the $InBi$ alloys, which must be considered local. Surprisingly, there is no such tendency is apparent from Figs. 5 and $6,$ which eviation at $H_{\rm sh}$. But in low fields $H \ll H_{\rm sh}$, such a

FIG. 7. Magnitude of field effect at lower tempera tures, shown as reduced signal $\zeta(H_{\rm sh})$. Temperatur de pendence is seen to be slight.

show the initial increase in $\lambda(H)$ to be smaller in pure In than in the alloys. The same is true for pure Sn.¹ Fitting the data to $f(H/H_{sh}) \approx 1 + \alpha (H/H_{sh})^2$ for low H , and correcting for the fourth-order term in Eq. (12), we find $\alpha = 0.08 \pm 0.08$ for In, 0.06 for Sn, and 0.15, 0.18, and 0.19 ± 0.08 for the alloys. This compares to a "local" prediction of 1/8 and a "nonlocal" prediction of $1/12$ in the low- κ limit,¹ assuming $\lambda \sim \Psi_s^{-2/3}$. It would be of interest if the field dependence $\lambda(H)$ were computed in the nonlocal limit.

Finally, we give in Fig. 7 the magnitude of the field effect at lower temperatures. Because of the decreased sensitivity, we have only measured the reduced signal at the superheating transition. ζ_{sh} is seen to decrease slightly at low t , but this may be due to departures from ideality as heterogeneous nucleation of the normal state may appear far away from T_c . We conclude that there is no evidence of a pronounced temperature dependence of the field effect. Table II summarizes our results for the field dependence $\lambda(H)$.

V. CONCLUSION

We have measured $\lambda(T,H)$ in single spheres of In and dilute InBi alloys, with κ ranging from 0.061 in pure In to 0.349 in $InBi$ (0.60-at. %). All samples showed ideal superheating and supercooling (following paper²), and thus permitted measurements in fields up to the ideal superheating field $H_{\rm sh}$. The temperature dependence of λ for all concentrations is equally well described by $\lambda(T) = \lambda_0 y$, or $\lambda(T)$ $=\lambda'_0 Z_{\text{BCS}}$. The increase in λ_0 and λ'_0 with impurity content are in nice agreement with BCS theory. The field dependence of λ comes close to being a universal function of $H/H_{\rm sh}$. At $H_{\rm sh}$, $\lambda(H_{\rm sh})/\lambda(0)$ $=1.46, 1.44, 1.46,$ and 1.53 ± 0.05 , respectively, while $d\lambda/dH$ diverges. This confirms the measurements on Sn.¹ Only in low fields $H \ll H_{\rm sh}$, where the field dependence is quadratic, is there a slight difference between the pure metals (In and Sn) and the alloys. This might be due to nonlocality. Experiments are under way to see if the field dependence can be measured for high κ , i.e., for very dirty alloys

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