Generation and detection of high-energy phonons by superconducting junctions*

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It is demonstrated that phonons, with energy greater than the superconducting gap, escape Sn junction generators in sufficient number to be detected by Pb superconducting junctions. The phonon transconductance signal, observed with high generator bias, can be largely understood in terms of the relaxation of injected quasiparticles which emit high-energy phonons. The high-energy phonons are in turn reabsorbed to form secondary high-energy quasiparticles and phonons. These processes are repeated until some form of quasiparticle-phonon equilibrium is reached. Certain features of the transconductance signal indicate that phonons of frequency at least as high as 1.31 THz escape the Sn junction. Sharp onsets observed in the signal require that the high-energy phonons are produced by relaxation of the quasiparticle distribution injected into the Sn generator junction, and not by a thermalized distribution of quasiparticles. Apparently, the presence of quasiparticles produced thermally plays a major role in the escape of phonons from the generator. An observed dependence of the transconductance signal on the total thickness of the generator suggests that the injected quasiparticle distribution, not the high-energy phonons, transport the excitation energy across the generator junction.

I. INTRODUCTION

The generation and detection of phonons by superconducting junctions have, since the original paper by Eisenmenger and Dayem,[1] received considerable additional attention.[2-4] It has generally been assumed that phonons of energy greater than the superconducting gap, i.e., $\hbar\omega > 2\Delta$, generated in a junction made of the more "strongly coupled" superconductors as, e.g., Sn or Pb, do not escape the junction if the thickness of the junction exceeds of the order of a thousand angstroms. In fact, in two experimental attempts to verify this point, no phonons with energy $\hbar\omega > 2\Delta$ were observed to escape a Sn-junction generator.[5,6] We report here experiments, on the transport of phonons from Sn-junction generators to Pb-junction detectors, in which signals resulting from phonons with energy $\hbar\omega > 2\Delta(\mathrm{Sn})$ are readily observed. In fact, signals from phonons with $\hbar\omega \geq 4.6\times 2\Delta(\mathrm{Sn})$ (i.e., with frequency $\nu \geq 1.31$ THz) can be distinguished.

In Sec. III, the experimental results are presented on the dependence of the observed phonon transconductance signal on the bias voltage applied to the generator junction, together with the dependence on temperature and on the thickness of the generator junction. Section IV contains an analysis of the experimental results in terms of, mainly phenomenological, extensions of models introduced earlier by others.[7-10]

II. EXPERIMENTAL TECHNIQUES

Sn-I-Sn junctions are used as phonon generators and Pb-I-Pb junctions as phonon detectors. The insulating barrier in Sn junctions is formed by plasma oxidation, and is formed in Pb junctions by exposing Pb films to an oxygen flow for approximately six hours followed by plasma oxidation. The Pb detector junctions are placed either directly behind the Sn junctions (overlay configuration) or after varying thicknesses (from 0.025 to 0.375 in.) of single crystals of sapphire (standard configuration). In the overlay configuration the two junctions are separated by a thin (approximately 100 Å), electrically isolating Pb-oxide layer[11] with resistance greater than $10^5\ \Omega\ \mathrm{mm}^2$.

Experiments have been carried out using both configurations. The standard configuration has several advantages, although it leads to a weaker flux of phonons transported between the junctions. On the other hand, it results in higher quality junctions which are more reliable. It determines a fixed subtended detector angle, and it provides the same phonon transport medium for each experiment and therefore is more dependable than the plasma-formed, electrically isolating layer of the overlay configuration.

The quality of the junctions is judged primarily on current voltage (I-V) characteristics. The temperature dependence of the junction characteristics is monitored as the ambient temperature is lowered from 4.2 °K. Samples are rejected if little or no temperature dependence is observed. (See Secs. III and IV B for the criteria used for an acceptable temperature response.) Since the interest here is in phonon generation and detection by single particle tunneling, Josephson currents and associated resonances are suppressed through a small external magnetic field (from 0 to 30 G) applied so as to avoid trapped flux.

The experiment is designed to measure the phonon induced tunneling current in the detector junction. The current I^D was monitored by biasing the detector with an electronically regulated voltage source.[12] Actually, only the derivative of the detector current is normally obtained as a function of the generator bias. The derivative stems from a small current modulation at 100 Hz superimposed on the generator bias current I^G and by lock-in detection of the detector response. The resultant signal, which is proportional to the incremental detector current δI^D per incremental change in the generator current δI^G, is hereafter referred to as the phonon transconductance signal. All measurements are made with the samples immersed in pumped liquid helium. In order to extend the life of the junctions, a very thin layer of G.E. varnish was applied immediately after deposition. We shall return to this point in Sec. III.

Phonon transconductance measurements were carried out over a temperature range of 1.3 to 2.2 °K. The temperature is stabilized during each run to variations of no more than 5 m deg °K. The range of experimental parameters is as follows. The total thickness of the generator junction t varied from 2270 to 7500 Å, and the junction resistance R^G_∞ varied from 20 to 800 mΩ. The total thickness of the detector junction is in all cases about 3000 Å, and the junction resistance R^D_∞ varied from 7 to 400 mΩ. The junction area is 1 mm^2 throughout.

III. EXPERIMENTAL RESULTS

Phonon transconductance measurements are obtained as described in Sec. II. The Pb detector junction is typically biased at $V^D = 0.4$ mV in order to obtain optimal response. The sweep rate of the generator bias V^G is adjusted to the time constant of the lock-in amplifier. Some noise is observed in all measurements, especially with samples containing a high-resistance Sn generator junction or a high-resistance Pb detector junction.

A typical phonon transconductance signal, obtained with the sample at 1.4 °K, is shown in Fig. 1. The magnitude of the noise is indicated only at the low and high V^G parts of the curve. Several features can be readily distinguished which can be related to the superconducting gap of the generator and detector junction (see Sec. IV). Phonon transconductance is first observed as the generator bias voltage reaches $V^G_0 = 2.8$ mV. At $V^G_1 \simeq 3.9$ mV a strong, sharp onset is observed, followed by a fairly broad peak and a decrease in the signal. (Note that the width of the peak exceeds the resolution $\Delta V \approx 200\ \mu$V.) At $V^G_2 \simeq 5$ mV a further, less sharp, onset of additional phonon transconductance

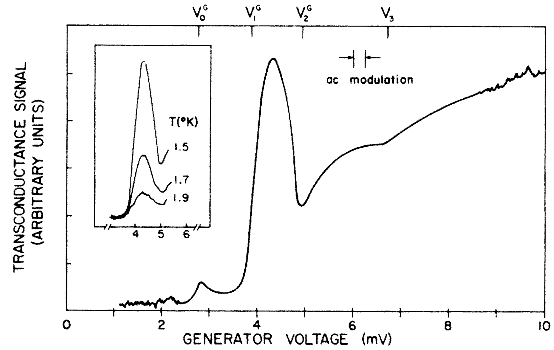

FIG. 1. Experimentally obtained transconductance signal as a function of generator bias voltage. The sample is held at 1.4 °K. The magnitude of the noise is indicated only at low and high values of the bias voltage. The magnitude of the ac modulation is indicated in an insert, as is the variation in the peak near V^G_1 as a function of temperature (left-hand insert).

is observed, followed by another onset at $V^G_3 \simeq 6.6$ mV.

Similar features are observed in the phonon transconductance measurements over the wide range of experimental parameters mentioned in Sec. II, but with two exceptions. In the overlay configuration, the phonon flux from a low-resistance Sn generator can cause overinjection in the Pb detector.[13] Although features in an overinjected detector resemble those in a linear detector, the signal is larger and more sharply peaked at V^G_1 and does not rise as fast beyond V^G_2. In what follows only data taken from samples in the standard configuration will be used. The second exception concerns the magnitude of the peak near the onset at V^G_0, which was observed to vary from one sample to another at the same ambient temperature.

Recordings of the phonon transconductance as a function of V^G were made with the sample held at selected temperatures in the range from 1.3 to 2.2 °K. Examples of portions of such curves near V^G_1 for three temperatures are indicated in the insert of Fig. 1. A measure of the temperature dependence of the transconductance near V^G_1 is determined either by graphical integration in the range $3.5 < V^G < 5.0$ mV or by measuring the height of the peak near V^G_1 relative to a baseline established at $V^G = 3.5$ mV. There are difficulties in obtaining accurate data with either method. The baseline and the peak heights are difficult to determine, especially for weak signals at higher temperatures. The graphical integration includes uncertainties resulting from fluctuations due to noise. Moreover, examples of data inconsistency are particularly observed in runs which require scale changes. Values of $\delta I^D/\delta I^G$ at a given temperature may differ by up to 8% for two different current increments δI^G. Most of the error is attributable to temperature drifts, which can be minimized by carefully reheating and then recooling the liquid-helium bath after making a scale change. Overall, both methods give the same temperature dependence, except at the lowest temperatures and for the lowest modulation voltage. For this case, the peak narrows slightly, and its height increases slightly faster than the integrated area. However, when the experimental uncertainty is included, no significant differences between the two methods can be found. Hence, only peak height data will be quoted hereafter.

To display the temperature dependence of the detected signal, the peak height is plotted on a semilog graph versus $1/T$. Two such plots for high-quality samples are shown in Fig. 2. The data points can be fitted at higher temperatures ($2.2 > T > 1.6$ °K) to a straight line, which gradually levels off as the temperature decreases below $T \simeq 1.6$ °K. A least-squares fit to $e^{b/kT}$ is obtained for the straight-line portion of the curve. The fits for the data shown in Fig. 2 are $b = 1.0$ and 0.74 meV.

An attempt has been made to determine a thickness dependence of the magnitude of the phonon transconductance signal. A series of nine samples have been used with Sn-junction generators of varying total thickness t each with a Pb-junction detector of 3000 Å thickness biased at $V^D = 0.4$ mV. The temperature dependence of the peak for each case is plotted as discussed above. The transconductance at a fixed temperature of 1.6 °K is determined for each sample.[14] This quantity is calculated in two ways. The first is to take the actual value of the transconductance as measured at 1.6 °K. The second is obtained by extrapolating the high-temperature, exponentially varying, values to the temperature of 1.6 °K. These two values are taken as the lower and upper limits of the transconductance in each case. (For three cases, the two values coincide.) The resulting values are then "normalized" by dividing by the tunneling probability of the detector junction.[15] (The tunneling probability is proportional to the inverse of the normal resistance of the detector R_∞^D.) The values of $\delta I^D R_\infty^D / \delta I^G$ are plotted, as a function of thickness, in Fig. 3. Two least-square fits to the function e^{-t/t_0} are performed on the data; the first uses only the high-temperature extrapolated values, and the second, the average values, i.e., the average of the measured plus extrapolated values. The resulting fits, indicated by the solid lines in Fig. 3, are t_0 (extrapolated) = 5700 Å, and t_0 (average) = 6000 Å.

The error bars noted with the data points of Fig. 3 reflect the uncertainties in the data reduction. The analysis, however, ignores other major un-

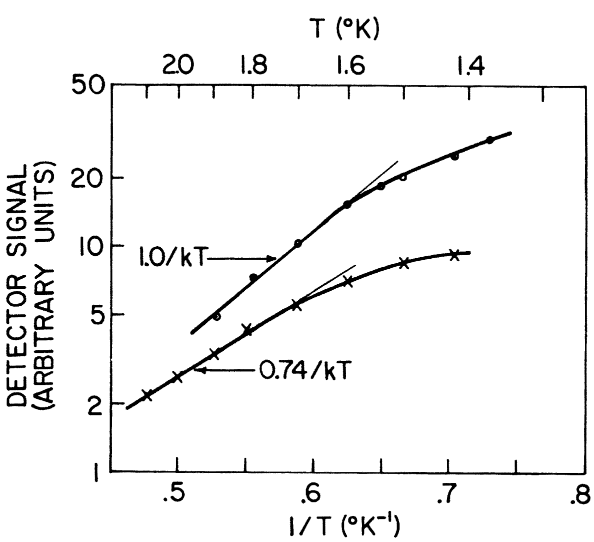

FIG. 2. Detector signal as a function of the temperature as determined by the height of the peak near V_1^G. The respective exponent associated with the straight-line portions is as indicated with each curve.

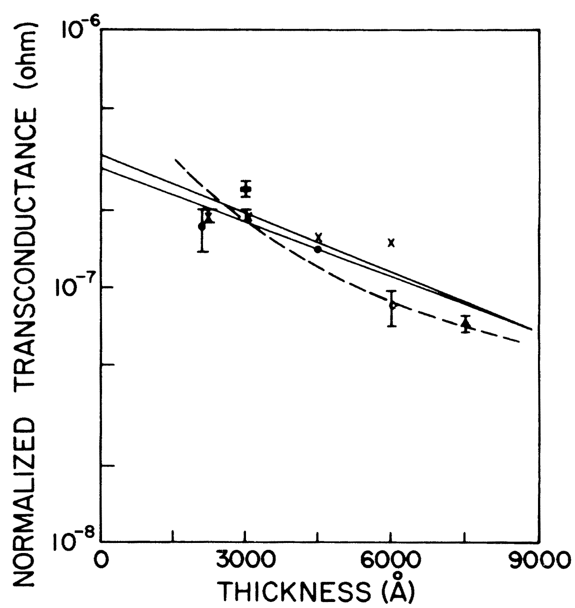

FIG. 3. Normalized transconductance signal as a function of the total thickness of the generator junction. The data are referenced to a sample temperature of 1.6 °K, and the solid lines are least-square fits to e^{-t/t_0} as described in the text. The dashed line is the thickness dependence predicted on the basis of Eq. (12a). The crosses and circles refer to data from the phonon transport through Czochralski-grown sapphire samples.' The triangles refer to Verneuil-grown samples, and the rectangle to an unidentified sample.

certainties in the experiment. Although certain parameters (e.g., geometry, thickness, temperature, detector response) have been accounted for, others, specifically related to the transport properties of high-energy phonons, have not. Among these is the question of phonon transport through different samples of sapphire. Four sapphire single crystals have been used in the thickness experiments: two Czochralski grown and two Verneuil grown, with surface dislocation densities of $10^3/\mathrm{cm}^2$ and $10^5/\mathrm{cm}^2$, respectively.[16] The results from seven Czochralski samples, one Verneuil sample, and one unidentifiable sample are indicated in Fig. 3. (The identification of the crystal in one of the runs was lost.) Since no particular pattern stands out for any given crystal, nor is the phonon transconductance of the Verneuil sample substantially smaller than the others each datum is treated equally. A second point is the question of how critically the attenuation of high-energy phonons depends on the junction-substrate interface.[17] The interface transmission and the recombination lifetimes of quasiparticles may differ from sample to sample since the Verneuil and Czochralski substrates received different surface polishes.[16] Moreover, as noted in Sec. II, the junctions were coated with a thin layer of G.E. varnish. Since it has been demonstrated experimentally[18] that normally a significant fraction of high-energy phonons escape into the helium bath, some variation in the fraction of phonons reaching the detector, as well as variations in the detector detectivity, can be expected.

Finally, it is important to note that, for equivalent Pb-junction detectors, the magnitude of the normalized phonon transconductance signal observed from Sn generator junctions biased at V_1^G is roughly 1% of that obtained from a Pb generator junction biased at $V^G = 2\Delta(\mathrm{Pb})/e$. This result implies that the flux of high-energy phonons escaping Sn junctions is roughly two orders of magnitude smaller than those escaping Pb junctions at the corresponding gap energy.

IV. THEORETICAL BACKGROUND AND ANALYSIS OF EXPERIMENTAL RESULTS

A. Dependence on generator bias

Existing models[1,4,7-10] for the generation and detection of phonons by superconducting junctions lead to the following picture of the mechanisms involved. Biasing a superconducting junction, such that $V^G > 2\Delta^G/e$, results in an "injected" quasiparticle population. These quasiparticles relax and recombine through phonon emission. Phonons with energy $\hbar\omega \geq 2\Delta^G$ are strongly reabsorbed (in strongly coupled superconductors) to yield a secondary quasiparticle population which, in turn, decays yielding a secondary phonon distribution. This process repeats itself until some quasiparticle phonon equilibrium is established. If some of the phonons, so generated, escape from the junction and reach a superconducting junction detector, a quasiparticle tunneling current is caused to flow provided $\hbar\omega \geq 2\Delta^D$ and a small biasing voltage is applied to the detector.

The formalism required to describe in detail the quasiparticle and phonon populations appears in the literature as noted above. It will not be repeated here except those parts necessary to define relevant terms and to indicate areas which have been modified by us in an attempt to account for the present experimental results.

In a junction biased at $V > 2\Delta^G/e$ quasiparticles, which were previously bound in Cooper pairs, are injected into each film of the junction at a rate given by[19]

$$\dot{D}_{1Q}(\epsilon, V)\, d\epsilon = (d\epsilon/e^2 R_\infty)\, \rho(\epsilon)\, \rho(eV - \epsilon)\,, \tag{1a}$$

in which ρ is the normalized BCS density of quasiparticle states at the specified energy. The injected quasiparticle population is distributed over an energy range $\Delta^G \leq \epsilon \leq eV^G - \Delta^G$. (Parameters which refer to quasiparticles will carry the subscript Q; those which refer to phonons the subscript P. The subscripts 1 and 2 refer to primary and secondary processes, and T, to thermal processes.)

Quasiparticles present in the junction as the result of thermal excitation, or other pair breaking processes, also tunnel across the barrier. The production of quasiparticles generated thermally is proportional to

$$\dot{D}_{TQ}(\epsilon, V)\, d\epsilon \propto f(\epsilon - eV)\, \rho(\epsilon - eV)\, d\epsilon\,, \tag{1b}$$

and in which the energy range is $eV + \Delta \leq \epsilon \leq \infty$, and f is the Fermi-Dirac occupation probability.

The excited quasiparticles of both types can decay by phonon emission. Quasiparticles are said to relax, and relaxation phonons[7] are generated, when a quasiparticle of energy $\epsilon > \Delta$ decays to an energy state ϵ' such that $\Delta \leq \epsilon' \leq \epsilon$. Two quasiparticles are said to recombine, and a recombination phonon is generated, when the pair of quasiparticles at the gap energy decay to a Cooper pair state. In what follows, primary interest is in the relaxation processes. The decay probability of a quasiparticle of energy ϵ through emission of a phonon of energy $\hbar\omega$, at $T = 0$ °K, is proportional to

$$\Gamma(\epsilon, \omega) = \omega^2 \rho(\epsilon - \hbar\omega)\, K(\epsilon, \omega)\,. \tag{2}$$

The decay probability exists over an energy range $0 \leq \hbar\omega \leq \epsilon - \Delta$ and is zero otherwise. In Eq. (2), ω^2

follows from the phonon density of states, and $K(\epsilon, \omega)$ is the BCS coherence factor.[19] A normalized decay probability is defined as $\Gamma(\epsilon, \omega)/\Gamma(\epsilon)$, in which $\Gamma(\epsilon)$ is obtained by integrating Eq. (2) over energies $0 \leq \hbar\omega \leq \epsilon - \Delta$.[7] The spectrum of phonons produced by the decay of the injected quasiparticles is here referred to as the "primary" relaxation phonon spectrum. The spectrum is obtained[4,10] by convolution of $\dot{D}_{1Q}$ with $\Gamma(\epsilon, \omega)/\Gamma(\epsilon)$. In a junction biased at $V \geq 2\Delta^G/e$, the rate at which primary relaxation phonons of energy $\hbar\omega$ are generated in each film is

$$\dot{N}_{1P}(\omega, V)\, d\omega = d\omega \int d\epsilon\, \dot{D}_{1Q}(\epsilon, V)\, \frac{\Gamma(\epsilon, \omega)}{\Gamma(\epsilon)}, \quad (3a)$$

in which the integration is from $\epsilon = \hbar\omega + \Delta$ to $\epsilon = eV - \Delta$. The spectral range of the phonons is $0 \leq \hbar\omega \leq eV - 2\Delta$.

Phonons are also emitted through the decay of thermalized quasiparticles. The rate at which phonons are emitted in this process is

$$\dot{N}_{TP}(\omega, V)\, d\omega = d\omega \int d\epsilon\, \dot{D}_{TQ}(\epsilon, V)\, \frac{\Gamma(\epsilon, \omega)}{\Gamma(\epsilon)}. \quad (3b)$$

Phonons of energy $0 \leq \hbar\omega \leq \infty$ are generated at all bias voltages. The dominant feature in N_{TP} is the "infinity" in the phonon spectrum as $\hbar\omega$ approaches eV. This infinity has been observed by Welte *et al.*,[5] in phonon transconductance measurements using a Al generator junction and a Sn detector junction. The relevant structure appears as a small peak at a bias voltage $V^G = 2\Delta^D/e$. An analogous peak is observed in the present results as indicated in Fig. 1 at V_0^G. Although, in principle, the relaxation phonons from thermalized quasiparticle decay can be treated in the same manner as those arising from the decay of injected quasiparticles, their presence does not significantly alter the remaining features of the transconductance curve. The remaining analysis is, accordingly, restricted to phonons arising from the decay of injected quasiparticles.

The primary phonon distribution in the generator may contain phonons of energy $\hbar\omega \geq 2\Delta^G$ provided that the junction is biased at voltages $V^G \geq 4\Delta^G/e$. In strongly coupled superconductors such phonons are rapidly reabsorbed by pair breaking, creating thereby secondary quasiparticles. The probability of reabsorption is proportional to

$$A(\omega, \epsilon) = \rho(\epsilon)\rho(\omega - \epsilon)K(\epsilon, \omega) \quad (4)$$

for $\Delta \leq \epsilon \leq \hbar\omega - \Delta$, and is zero otherwise. The rate at which the secondary quasiparticles are created is

$$\dot{D}_{2Q}(\epsilon, V)\, d\epsilon = 2\, d\epsilon \int d\omega\, \dot{N}_{1P}(\omega, V)\, \frac{A(\omega, \epsilon)}{A(\omega)} \quad (5)$$

for $\Delta \leq \epsilon \leq eV - 3\Delta$. In Eq. (5) the factor of 2 reflects the production of two quasiparticles for each absorbed phonon. $A(\omega, \epsilon)/A(\omega)$ is the normalized pair breaking probability, obtained in analogy to the normalized quasiparticle decay probability. The integration is from $\hbar\omega = \epsilon + \Delta$ to $\hbar\omega = eV - 2\Delta$. Equation (5) is, clearly, obtained by convoluting the primary phonon relaxation spectrum with the pair breaking probability.

The relaxation of the secondary quasiparticles leads to the generation of secondary relaxation phonons at a rate

$$\dot{N}_{2P}(\omega, V)\, d\omega = d\omega \int d\epsilon\, \dot{D}_{2Q}(\epsilon, V)\, \frac{\Gamma(\epsilon, \omega)}{\Gamma(\epsilon)} \quad (6)$$

for $0 \leq \hbar\omega \leq eV - 4\Delta$, and in which the integration is from $\epsilon = \hbar\omega + \Delta$ to $\epsilon = eV - 3\Delta$. The processes described by Eqs. (5) and (6) repeat themselves to yield yet further quasiparticle and phonon distributions. However, we will not follow these additional processes analytically.

The quasiparticle and phonon rates given by Eqs. (1a), (3a), (5), and (6) have been evaluated numerically by a weighted Gaussian quadrature scheme. The results, calculated for $V^G \gg 2\Delta^G/e$, are illustrated in Fig. 4. The significant feature in each curve is the structure at the maximum energy. $\dot{D}_{1Q}$ has a singularity of the form $1/(\epsilon_{max} - \epsilon)^{1/2}$ at $\epsilon = eV - \Delta$; $\dot{N}_{1P}$ has a finite-value discontinuity at $\hbar\omega = eV - 2\Delta$; $\dot{D}_{2Q}$ varies as $(\epsilon_{max} - \epsilon)^{1/2}$ at $\epsilon = eV - 3\Delta$; whereas $\dot{N}_{2P}$ decreases smoothly to zero as $\hbar\omega$ approaches $eV - 4\Delta$. It is of particular importance to recognize that the above formulation predicts an abrupt (finite-value) discontinuity in the phonon distribution N_{1P}

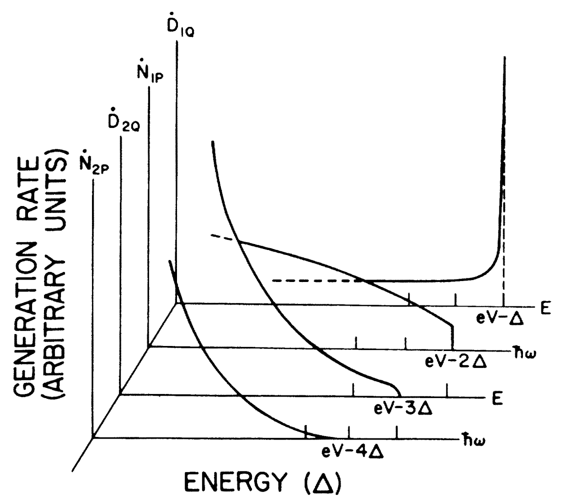

FIG. 4. Calculated rates of generation of primary and of secondary quasiparticles and phonons as a function of particle energy (in units of the gap energy) for a junction biased to a potential $V \gg 2\Delta/e$.

at $\hbar\omega = eV - 2\Delta$. This discontinuity stems from the singularity in the injected quasiparticle distribution D_{1Q} at $\epsilon = eV - \Delta$. It will be shown below that the discontinuity in N_{1P} gives rise to a sharp onset in the phonon transconductance signal, and that N_{2P} gives rise to a further onset.

The response of the detector depends on the energy distribution of the incoming phonons in the following way. Only phonons with energy $\hbar\omega \geq 2\Delta^D$ can generate quasiparticles by pair breaking in the detector. The onset of a phonon transconductance signal can, accordingly, occur only when $eV^G = 2\Delta^D + 2\Delta^G$ if, as in the present case, $2\Delta^G < 2\Delta^D$. If phonons $\hbar\omega > 4\Delta^D$ reach the detector, they give rise to a secondary quasiparticle population in a manner analogous to that discussed above for the generator, and therefore lead to an additional onset of tunneling current in the detector at $eV^G = 4\Delta^D + 2\Delta^G$.

The combined properties of the generator and detector result in the transconductance signal. The dependence of the transconductance on V^G has been calculated on the basis described above and is displayed in Fig. 5, curve a. For comparison an experimental signal is displayed as curve b. First, and foremost, it should be noted that since the Pb detector junction can respond only to phonons with $\hbar\omega \geq 2\Delta(\text{Pb})$, the observation of any transconductance signal implies that phonons of at least that energy escape from the generator junction. Moreover, we can show that a number (though not all) of the features of the observed signal correspond directly to features in the predicted transconductance signal. For example, the sharp onset at V_1^G corresponds to the onset of response by the detector to a flux of primary phonons of energy $\hbar\omega = 2\Delta(\text{Pb})$ generated in and escaping from the Sn junction. The bias voltage for this to happen is, obviously, $V^G = 2\Delta(\text{Pb})/e + 2\Delta(\text{Sn})/e$, which indeed corresponds to V_1^G.

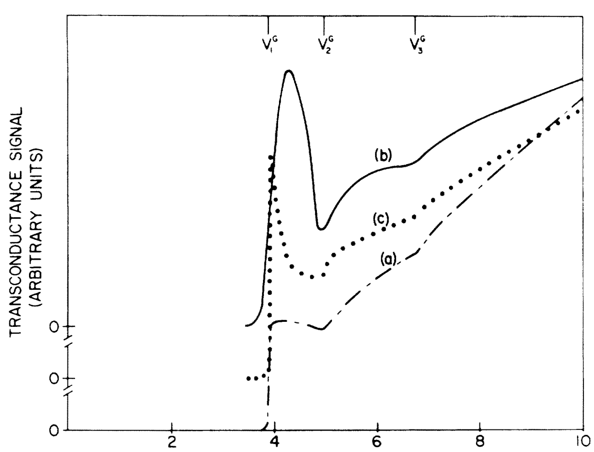

FIG. 5. Transconductance signal as a function of generator voltage as calculated on the basis of the simple model described in the text (curve a), and with the inclusion of an *ad hoc* resonance term at V_1^G (curve c). An experimentally obtained signal is shown for comparison as curve b. Note that the three curves were first scaled to coincide at V_2^G and then displaced as indicated on the ordinate.

A marked feature not predicted by the analytical model is the broad peak in the region between V_1^G and V_2^G. Consider first the analytically predicted contribution of the generator to the transconductance signal in this bias region. No discontinuity is predicted in the generation of phonons in the Sn junction until V^G is such as to permit the production of secondary phonons of $\hbar\omega = 2\Delta(\text{Pb})$. The latter discontinuity is, in fact, observed as the onset at $V_2^G \simeq 5$ mV $= 2\Delta(\text{Pb})/e + 4\Delta(\text{Sn})/e$. In regard to the detector, no discontinuity is predicted in its response until V^G is such as to permit the production of secondary quasiparticles by absorption of phonons of $\hbar\omega = 4\Delta(\text{Pb})$. The latter discontinuity is also, in fact, observed as the onset at $V_3^G \simeq 6.6$ mV $= 4\Delta(\text{Pb})/e + 2\Delta(\text{Sn})/e$. In the range $V_1^G < V^G < V_3^G$, however, in the simplest model, each incoming phonon with $2\Delta(\text{Pb}) < \hbar\omega < 4\Delta(\text{Pb})$ is expected to generate in the detector one pair of quasiparticles which make the same contribution to the observed tunneling current I^G regardless of the energy of the phonon. Deviations from this model have been discussed by Dayem and Wiegand,[20] who show that the inclusion of the details of the recombination and relaxation processes in the detector will give rise to a slightly nonconstant signal in this bias region.

The onset at $V_3^G = 6.6$ mV fixes at 1.31 THz the value of the frequency of the highest-energy escaping phonon which can be identified directly from the data. The continued increase of the transconductance signal beyond this bias voltage implies, however, that phonons of even higher energy are generated and escape the generator junction.

The observed peak at V_1^G can be partially reproduced through an addition to the detector response of an *ad hoc* resonant absorption term for phonons with $\hbar\omega = 2\Delta(\text{Pb})$. A transconductance signal, computed on this basis, is indicated as curve c in Fig. 5. We defer discussion of the possible origins of such a resonance until Sec. V.

B. Dependence on temperature

The experimental results displayed in Figs. 1 and 2 indicate a strong temperature dependence of the phonon transconductance signal. This dependence may stem either from the temperature dependence of the production of high-energy phonons in the generator, from the temperature dependence of the response of the detector, or from a combination of these effects. The expected temperature dependence of the detector is the easier to document. The detector response depends on the number of quasiparticles N_Q and their lifetime τ. These quantities depend, in turn, on the phonon population in the junction N_P. The nonequilibrium concentration of quasiparticles and phonons of $\hbar\omega \geq 2\Delta$ has been formulated by Rothwarf and Taylor.[21] An application of their formulation to the present case[22] leads to a steady-state quasiparticle concentration

$$N_Q^2 = N_{Q0}^2 + (\beta/\gamma R) I_P \,, \tag{7}$$

in which I_P is the rate of injection of phonons into the detector; $\frac{1}{2}\beta$ is the rate of pair breaking by phonons of $\hbar\omega \geq 2\Delta$; R is the recombination coefficient for quasiparticles of $\epsilon = \Delta$; γ is the escape rate of phonons of $\hbar\omega \geq 2\Delta$; and the subscript "0" (as in N_{Q0}) refers to the equilibrium thermal concentration. If the superconductor is strongly coupled, i.e., $\beta/\gamma \gg 1$, then Eq. (7) may be rewritten

$$N_Q^2 = N_{Q0}^2 + 2 N_{Q0} I_P \tau \,. \tag{8}$$

τ can be identified as the effective recombination lifetime and is given by[23]

$$\tau = N_{Q0} / 2\gamma N_{P0} \,. \tag{9}$$

τ corresponds to the time it takes quasiparticles of $\epsilon = \Delta$ to recombine to form Cooper pairs, provided that the quasiparticles are produced thermally. The recombination lifetime in a strongly coupled superconductor is enhanced over the intrinsic recombination lifetime through the presence of phonons of energy $\hbar\omega = 2\Delta$. The lifetime enhancement was first pointed out in Ref. 21 and later verified experimentally by Sai-Halasz *et al.*[24]

In the so-called linear limit, i.e., for $(N_Q - N_{Q0})/N_{Q0} \ll 1$,

$$N_Q - N_{Q0} = I_P \tau \,, \tag{10}$$

which can readily be verified from Eq. (8). The change in the quasiparticle population is seen to depend on the product of the rate of injection of phonons into the junction times the lifetime of the quasiparticles. Accordingly, the temperature dependence of the detected signal should follow from the temperature dependence of I_P and τ.

The expected temperature dependence[25] of N_{Q0} is

$$N_{Q0} \propto (kT)^{1/2} e^{-\Delta/kT} \tag{11a}$$

and of N_{P0} is

$$N_{P0} \propto kT\, e^{-2\Delta/kT} \,. \tag{11b}$$

Therefore, according to Eq. (9), the temperature dependence of τ is

$$\tau \propto [(kT)^{1/2}/\gamma]\, e^{\Delta/kT} \,. \tag{11c}$$

Equation (11c) yields that part of the temperature dependence which arises from the response of the detector.[26] The result is valid provided that the quasiparticle concentration is primarily thermal in origin. If the above mechanism is the sole contributor to the observed temperature dependence, then for $kT \ll \Delta$ the observed exponent in Eq. (11c) ought to be $\Delta(\text{Pb})/kT = 1.35\ \text{mV}/kT$, which is to be compared with the observed range of $0.74\ \text{mV}/kT$ to $1.05\ \text{mV}/kT$.

In real junctions nonthermal sources such as Josephson currents[27] and trapped flux[28] contribute quasiparticles in addition to those generated thermally and by phonons. Consequently, at very low temperatures N_Q is often dominated by nonthermal sources. Nevertheless, it is still possible to express the phonon-enhanced quasiparticle concentration (in the linear limit) ΔN_Q in terms of an effective lifetime τ' as

$$\Delta N_Q = I_P \tau' \,.$$

However, these processes do not lead to the thermal dependence indicated in Eq. (11c),[29,30] and, accordingly, do not account for the observed results. [It should be noted in passing that the Sn junctions were of such quality that nearly ideal temperature dependence, i.e., $I^G(T) \propto N_{Q0}$, was observed for $V^G < 2\Delta/e$.]

It is, of course, possible that the remaining thermal dependence arises from a temperature dependence of the phonons which escape from the Sn generator. Unfortunately, no generally accepted model exists for the escape of high-energy phonons from strongly coupled superconductors in which the thickness of the superconductor t exceeds the phonon mean-free path Λ.[31] It is possible, however, to rule out one mechanism suggested by Long. In strongly coupled superconductors, Long[32] asserts that both high-energy phonons and quasiparticles are rapidly thermalized, precluding thereby that phonons generated by the primary relaxation process can escape the junction. Only phonons from the thermalized, high-energy tail of the distribution of recombination phonons are thought able to escape. However, such phonons cannot be the source of the observed discontinuity in the transconductance signal at V_1^G since the distribution of phonons in the tail would vary smoothly with phonon energy. This conclusion holds whether the thermalization temperature is that of the ambient or some effective temperature[33] reached through quasiparticle-phonon dynamics.

Although it is clear that thermalized quasiparticles do not directly generate the relevant phonons, there remains the possibility that they influence the mechanism for phonon escape. A mechanism of this type has been proposed by Welte *et al.*,[34] who note that the propagation direction of a tunneling quasiparticle must be perpendicular, or nearly perpendicular, to the barrier. If this is indeed the case, then k-vector conservation limits the relaxation of such quasiparticles to the emission of phonons propagating nearly parallel to the junction-sapphire interface. Accordingly, the propagation direction of the excited quasiparticle must first be randomized prior to the relaxation process in order for phonons to be emitted isotropically and, consequently, be able to escape into the sapphire. Welte *et al.* suggest that the necessary randomization (as distinguished from thermalization) proceeds through electron-electron scattering. In the linear limit, thermalized quasiparticles dominate the quasiparticle concentration and, hence, the electron-electron scattering. Accordingly, the phonon escape rate would depend on the concentration of thermalized quasiparticles, i.e., as $(kT)^{1/2}e^{-\Delta/kT}$. Combining this result with that found above for the detector, leads to a total temperature dependence proportional to

$$\exp[\Delta(\mathrm{Pb})/kT - \Delta(\mathrm{Sn})/kT] = \exp(0.78\ \mathrm{mV}/kT),$$

which compares favorably to the experimentally observed range of $0.74-1.05\ \mathrm{mV}/kT$.

It should also be noted that as the temperature of the sample is lowered, the concentration of thermalized quasiparticles is lowered relative to the concentration of injected quasiparticles. Under these circumstances, the randomization process, though still dependent on the concentration of quasiparticles, may no longer be temperature dependent. That is, a saturation of the temperature dependence of the phonon escape rate will result. Calculations, based on injection rates of quasiparticles and experimentally obtained lifetimes of gap energy quasiparticles,[29] lead to results which are consistent with the observed saturation in Sn near 1.6 °K (see Fig. 2).

It has been shown[35] that an alternative model exists for the escape of high-energy phonons. Calculations involving the ultrasonic attenuation of such phonons[8,9] in a superconductor predict the same temperature dependence for the escape rate as found in the model of Welte *et al.* These calculations also predict an energy dependence of the escape rate, which is, indeed, found to be consistent with the observed transconductance signal.[35] A full discussion of this model is deferred until a later time. At present there is insufficient experimental and theoretical evidence with which to distinguish between the above two mechanisms, or between others.

C. Dependence on junction thickness

It is generally considered that the mean-free path of high-energy phonons in superconducting Sn is of the order of 1000 Å.[22,34,36] This value must be compared to the rather weak thickness dependence obtained experimentally. As shown in Fig. 3, an attempt to characterize the observed dependence by an effective mean length t_0 leads to a value of approximately 6000 Å. In order to understand this result, we investigate somewhat further the model by Welte *et al.* Clearly, restricting the quasiparticle randomization process solely to electron-electron scattering is an idealization of what must actually occur. Inhomogeneities within a real tunneling barrier and at the barrier-superconductor boundary should provide some tunneling quasiparticles with propagation directions other than perpendicular to the boundary. In any event, additional randomization occurs as the quasiparticles are repetitively reflected at the barrier boundary and at the superconductor-sapphire interface. That these reflections must occur can be seen from the following argument. The quasiparticle relaxation time in Sn is of the order of 3×10^{-11} sec at $T=1.6$ °K.[37,38] Since the Fermi velocity is of the order of 10^8 cm/sec, this means that on average the quasiparticles can traverse a 3000-Å film, and undergo scattering at the boundaries of the film of the order of 10^2 times before they decay by phonon emission. If the electron-electron scattering time is less than the relaxation time,[34] the particle will also have undergone a number of further scattering events. The combined scattering should result in a spatial redistribution of quasiparticles, such that they decay by high-energy phonon emission, with equal probability at any distance from the interface. It should be noted that the proposed redistribution of randomized quasiparticles is equivalent to the transport of excitation energy across the superconductor by the excited quasiparticles rather than by the high-energy phonons.

An estimate of the fraction of relaxation phonons that can escape the film of thickness t (at a fixed temperature) under these circumstances can be shown to be

$$F(t) \propto \frac{1}{t}\int_0^t dt'\, e^{-(t-t')/\Lambda} = \frac{\Lambda}{t}(1-e^{-t/\Lambda}). \qquad (12)$$

For $\Lambda \ll t$, Eq. (12) reduces to

$$F(t) \propto \Lambda/t\ , \qquad (12')$$

in agreement with a more detailed calculation of this problem by Gray.[28] A plot of Eq. (12′), scaled to yield the best overall fit to the data, is shown as the dashed line in Fig. 3. In view of the uncertainties in the data (noted in Sec. III), the comparison of $F(t)$ with the data can be said to be qualitatively satisfactory. Since the boundary scattering is not expected to be strongly temperature dependent, the component of the temperature dependence resulting from the generator remains that discussed in Sec. IV B.

V. DISCUSSION

It has definitely been established that high-energy phonons escape Sn superconducting junction generators in sufficient number to be detected by Pb superconducting junction detectors. As noted in Sec. IV A, most of the features of the phonon transconductance signal have been accounted for in terms of a model in which high-energy phonons are repetitively reabsorbed to form quasiparticles which relax to generate more phonons. Nevertheless, a small number of phonons with $\hbar\omega > 2\Delta(\text{Sn})$ escape the generator junction and give rise to easily distinguished features of the transconductance signal. The sharp onset of the transconductance signals is consistent with the generation of the relevant phonons by the relaxation of the injected quasiparticle distribution in the generator junction, and its presence also rules out the possibility that the dominant source of the phonons are thermally excited quasiparticles, or a thermalized redistribution of the injected quasiparticles.

Some features of the transconductance signal are, however, poorly understood and require additional comment. As noted in Sec. IV A, the marked peak in the transconductance signal near V_1^G, which has been observed under all experimental conditions, cannot be explained by the simple model. A similar peak has been reported by Dayem *et al.*[2] in samples containing Pb junctions as both generators and detectors. However, no satisfactory explanation of this peak has as yet been given. The addition, to the simple model, of an *ad hoc* resonance at $V^G = 2\Delta(\text{Pb})/e + 2\Delta(\text{Sn})/e$ leads to a predicted transconductance signal which compares more favorably to the experimental result.[39] However, no immediately obvious physical effect accounts for such a resonance structure. We limit ourselves here to a number of speculative explanations. The possibility exists that the observed peak is an artifact, and that the real effect is an interference in the phonon propagation giving rise to a dip in the 5-mV region; i.e., in the phonon frequency region of about 1 THz. A slight dip in the phonon density of states of Sn does, in fact, occur near 1 THz,[40] and for Pb an even stronger dip is expected, based on the published dispersion curves of Brockhouse *et al.*[41] These dips result in both cases from a sharply decreased contribution, by the lowest transverse-acoustic branch to the total density of states. On the other hand, the peak may be real and reflect an enhanced sensitivity of the Pb detector junction to phonons with energy near the superconducting gap. For example, it has been demonstrated[42] that the onset of photon absorption in Pb exceeds that predicted by BCS theory. Alternatively, it has been suggested[35] that the repetitive absorption of phonons with $\hbar\omega = 2\Delta(\text{Pb})$, followed by the recombination of quasiparticles of $\epsilon = \Delta(\text{Pb})$, followed in turn by the production of phonons of $\hbar\omega = 2\Delta(\text{Pb})$, etc., gives rise to the observed enhanced onset.

In Sec. IV B, it is demonstrated that both the rate of escape of high-energy phonons from the generator junction and the response of the detector junction depends on temperature. The combined temperature dependence can be expressed as an exponential function with an exponent b equal to 0.78 mV, whereas the observed exponent varies as $0.74 < b < 1.05$ mV. In most experimental cases, however, the observed exponent was actually in the high side of the range, i.e., $b \approx 1.0$ meV. Since independent measurements on high-quality Pb-junction detectors indicate nearly ideal behavior at high temperatures, it appears that the temperature dependence of the phonon flux escaping from the Sn generator for $T > 1.6$ °K is somewhat less than the predicted one. We rule out the possibility that this discrepancy is solely ascribable to a higher, than ambient, effective temperature arising from the quasiparticle phonon dynamics. According to the modified heating theory of Parker,[33] in strongly coupled superconductors, an increase in the nonequilibrium concentration of quasiparticles should increase the effective temperature of the quasiparticle-phonon system. Based on the data in Fig. 2, Ref. 33, an estimate of the effective temperature indicates that it is less than that required to bring the exponent to the predicted one. It is possible that the coat of varnish, which was (haphazardly) applied to all junctions, is responsible for temperature variations. A coat of varnish should reduce the Kapitza conductance[17] from the sample into the liquid helium bath possibly increasing, thereby, the effective temperature of the sample. The coat of varnish must also be considered a source of uncertainty in the detected signal. In particular, a variation in the fraction of phonons reaching the detector, as well as a variation in the detector response, can be expected among the different samples.

It is, however, demonstrated that the observed dependence is in rough accord with the combined thermal dependence of the production of phonons which escape the generator and the thermal dependence of the response of the detector junction. If this interpretation is correct, then the escape of the relevant phonons by the relaxation of the injected quasiparticles is seen to depend on the concentration of thermalized quasiparticles. Finally, another feature of the results which is not well understood is the rather large number of phonons which escape Sn junctions of the order of 3000–7000 Å in thickness plus the observed thickness dependence of the transconductance signal. These observations are not in accord with the strong reabsorption expected for high-energy phonons in strongly coupled superconductors. On the other hand, these results can be understood provided some high-energy quasiparticles transport energy to near the generator junction-substrate interface before emitting relaxation phonons.

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