Flicker ($1/f$) noise in Josephson tunnel junctions*

John Clarke and Gilbert Hawkins†
Department of Physics, University of California, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 19 April 1976)

We have measured the power spectrum of the voltage fluctuations in shunted Josephson junctions biased at a constant current I greater than the critical current I_c. Over the frequency range 5×10^{-2}–50 Hz the power spectra vary approximately as $1/f$, where f is the frequency. At any single frequency, the noise decreases as I is increased. Experimental evidence is presented to show that the voltage noise arises from equilibrium fluctuations in the temperature T of the junction, which in turn modulate I_c and hence the voltage V across the junction. The magnitude of the power spectra is consistently predicted to within a factor of 5 by an extension of the semiempirical formula of Clarke and Voss: $S_V(f) = (dI_c/dT)^2(\partial V/\partial I_c)_I^2 k_B T^2/3C_V f$. In this formula, we postulate that C_V is the heat capacity of an "effective" junction volume given by the product of the junction area and the sum of the coherence lengths of the two superconductors. The dependence of $S_V(f)$ on $(\partial V/\partial I_c)_I^2$ and $(dI_c/dT)^2$ is experimentally established.

I. INTRODUCTION

Apart from a passing reference by Kanter and Vernon,[1] the question of excess low-frequency noise in Josephson junctions[2] appears to have received little attention hitherto. Nevertheless, it is important to determine whether or not excess low-frequency noise exists in Josephson junctions, because such noise many impose a limitation on the low-frequency performance of devices such as superconducting quantum interference devices (SQUIDs). In this paper,[3] we report measurements of voltage noise in resistively shunted Josephson junctions, biased at a current greater than the critical current I_c over a frequency range from 5×10^{-2} to 50 Hz. We find that the power spectrum varies approximately as $1/f$ at the lower frequencies and becomes white at the higher frequencies, the crossover frequency depending on the experimental parameters. Moreover, at any single frequency, the power spectrum decreases with increasing bias current and is proportional to $(dI_c/dT)^2$. These results strongly suggest that the observed $1/f$ noise is generated by equilibrium temperature fluctuations in the junction, which in turn induce fluctuations in the critical current (provided $dI_c/dT \neq 0$). The critical current fluctuations are observed as voltage fluctuations when the junction is current biased at a nonzero voltage. The thermal diffusion theory for $1/f$ noise developed by Clarke and Voss[4,5] for thin metal films can be readily adapted to the case of Josephson junctions. We find good numerical agreement between the predictions of this model and the measured $1/f$ spectra.

In Sec. II, we briefly review and develop the relevant theory, and in Sec. III, we describe the experimental apparatus and techniques. Section IV presents the experimental results and analysis, and Sec. V contains our conclusions.

II. THEORETICAL CONSIDERATIONS

In the thermal fluctuation model[4,5] for $1/f$ noise in thin metal films, one assumes that the film and its substrate are in thermal equilibrium at temperature T. Energy is exchanged between the film and the substrate, producing a mean-square temperature fluctuation in the film $\langle(\Delta T)^2\rangle = k_B T^2/C_V$, where C_V is the heat capacity of the film. If the film has a nonzero temperature coefficient of resistance β, there will be a corresponding resistance fluctuation. A steady current I passed through the film thereby produces a fluctuating mean-square voltage $\langle(\Delta V)^2\rangle = \bar{V}^2\beta^2 k_B T^2/C_V$, where $\bar{V}$ is the mean voltage across the film. The power spectrum $S_V(f)$ of these fluctuations can be calculated using a diffusion model[4-6] in which the temperature fluctuations are uncorrelated in space and time, $\langle \Delta T(\vec{r}+\vec{s},t)\Delta T(\vec{r},t)\rangle \propto \delta(\vec{s})$. This model does not yield a $1/f$ power spectrum. Clarke and Voss[4,5] adopted an empirical approach and introduced a $1/f$ region into the power spectrum over the frequency range $f_1 < f < f_2$, where $f_i = D/\pi l_i^2$, D is the thermal diffusion coefficient, and l_1 and l_2 are the length and width of the film. The spectrum was assumed to be white for $f < f_1$, and to vary as $f^{-3/2}$ for $f > f_2$. The power spectrum was normalized by setting $\int_0^\infty S_V(f)df = \bar{V}^2\beta^2 k_B T^2/C_V$. In the $1/f$ region, this procedure leads to a power spectrum

$$S_V(f) = \frac{\bar{V}^2\beta^2 k_B T^2}{C_V[3 + 2\ln(l_1/l_2)]f} . \tag{1}$$

Equation (1) correctly predicts the magnitude of the $1/f$ noise power spectrum observed in both metal films at room temperature, and superconducting films at the resistive transition.[7]

Subsequently, Voss and Clarke[5] proposed a model in which the temperature fluctuations were spatially correlated, with $\langle \Delta T(\vec{r}+\vec{s},t)\Delta T(\vec{r},t)\rangle \propto 1/|\vec{s}|$. This model yielded an explicit $1/f$ region in the power spectrum for $f_1<f<f_2$ that differed from Eq. (1) only by a numerical factor close to unity. It should be noted, however, that the measured $1/f$ region sometimes extended to frequencies below f_1. The theoretical difficulty of determining the range of frequencies over which the $1/f$ power spectrum extends has been discussed elsewhere,[5] and is not likely to be resolved until a more microscopic model of the noise becomes available that takes into account the thermal inhomogeneities of the metal-on-glass system. From an empirical point of view, this problem is not very serious, since the ratio f_1/f_2 enters logarithmically. A reduction in f_1 by many orders-of-magnitude makes a relatively insignificant change in the value predicted for $S_V(f)$.

The thermal fluctuation model is readily adapted to noise in a Josephson tunnel junction. We shall consider junctions having a nonhysteretic current-voltage (I-V) characteristic. This requirement implies that the hysteresis parameter[8] $\beta_c = 2\pi I_c R^2 C/\phi_0 \lesssim 1$, where R is the resistance shunting the junction, C is the junction capacitance, and ϕ_0 is the flux quantum. In the limit $\beta_c \ll 1$, the I-V characteristic is then described[8] by

$$V = R(I^2 - I_c^2)^{1/2}. \tag{2}$$

The temperature fluctuations are assumed to generate fluctuations in $I_c(T)$, provided that $dI_c/dT \neq 0$. If the junction is biased with a current $I > I_c$, the critical current fluctuations will induce voltage fluctuations whose power spectrum is given by replacing the coupling terms $\bar{V}\beta$ in Eq. (1) by $(dI_c/dT)(\partial V/\partial I_c)_I$,

$$S_V(f) = \frac{(dI_c/dT)^2(\partial V/\partial I_c)_I^2 k_B T^2}{C_V[3+2\ln(l_1/l_2)]f}. \tag{3}$$

From Eq. (2), $(\partial V/\partial I_c)_I$ is given by

$$\left(\frac{\partial V}{\partial I_c}\right)_I = \frac{-R}{[(I/I_c)^2-1]^{1/2}}. \tag{4}$$

The theory assumes that the instantaneous Josephson critical current is a function of the spatially averaged temperature of the junction. There is some uncertainty whether or not the entire volume of the tunnel junction contributes to C_V. Temperature fluctuations at a given point in the superconducting film generate fluctuations in the energy gap Δ, over distances up to the Ginzburg-Landau coherence length ξ. The critical current is related to the values of the gap Δ_s at the surfaces of the films adjacent to the tunnel barrier. As a result, fluctuations in Δ that occur at distances greater than ξ from the barrier are not expected to affect Δ_s or I_c. We shall assume, therefore, that if ξ is less than the thickness of the superconducting film, the "effective" fluctuating volume is given by the product of the junction area with the sum of the coherence lengths, rather than with the total thickness of the junction. Accordingly, we shall use for C_V the expression

$$C_V = l_1 l_2 (c_1\xi_1 + c_2\xi_2), \tag{5}$$

where c_1 and c_2 and ξ_1 and ξ_2 are the specific heats and coherence lengths of the two superconductors. We have neglected the contribution of the barrier to the heat capacity.

III. EXPERIMENTAL DETAILS

Our Josephson tunnel junctions are resistively shunted by means of either a disk of copper under the intersection of the two superconducting strips, or by a diagonal strip of copper connecting the two strips near their intersection. After the shunts had been evaporated onto a glass substrate, niobium strips 150 μm wide and approximately 0.2 μm thick were sputtered onto the substrate using a Sloan S-300 Sputtergun. The niobium strips were thermally oxidized, and lead or tin cross strips of similar dimensions were then evaporated. In the case of junctions with diagonal shunts, the area near the junction was coated with an insulating layer and a lead disk evaporated over the junction and the shunt. This superconducting ground plane reduced the inductance of the superconducting strips and the shunt to a negligible level. The shunt resistance was typically 10 mΩ, and the critical currents at 4.2 K were in the range 200 μA–5 mA. Assuming a junction capacitance of 500 pF, we find the corresponding range of β_c was 0.03–0.75. Each slide was mounted in thermal contact with a copper block suspended in a vacuum can. A carbon resistance thermometer was attached to the reverse side of the block. At liquid-helium temperatures, the thermal time constant of the block was about 15 min, so that effects due to fluctuations in the temperature of the helium bath were minimized. For measurements at temperatures below 4.2 K, the temperature of the helium bath was regulated with a manostat to within ± 100 μK.

The measurement configuration is shown in Fig. 1. Two tunnel junctions of comparable critical current and resistance were connected in series with a superconducting coil mounted inside a dc SQUID,[9] and a standard resistor R_v ($\gtrsim R$) of resistance 0.01–0.1 Ω. Niobium leads 50 μm in

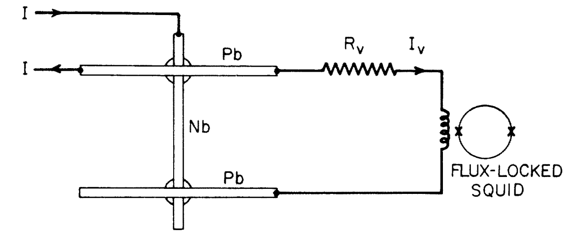

FIG. 1. Configuration used for measuring $1/f$ noise in a shunted Josephson tunnel junction.

diameter were spot welded to 2×2-mm brass tabs that were then soldered to the lead or tin films with indium-bismuth solder. The entire measuring circuit, including the SQUID, was surrounded by a long lead tube that greatly attenuated fluctuations in the external magnetic field. The ambient magnetic field was reduced to below 10^{-6} T by a double μ-metal can surrounding the cryostat. We obtained fluctuation spectra in the following manner. The temperature of the copper block was allowed to stabilize at the desired value. With the feedback loop of the dc SQUID unlocked, the bias current I (see Fig. 1) was slowly increased through one of the junctions. (In a separate experiment, the current was found to have negligible noise and drift.) When I exceeded the junction critical current, a current I_v was induced in the measurement circuit; the output of the SQUID oscillated as I was further increased. When the bias current had been set at the desired value ($>I_c$), the SQUID was switched into its flux-locked mode in which small changes of current in the coil could be detected. Thus, the zero of the SQUID voltmeter was effectively dc offset, so that the voltmeter measured only fluctuations in the voltage across the tunnel junction. This offset was achieved with no loss in the stability of the voltmeter. The bias current was always kept below the value at which I_v exceeded the critical current of the second junction. Thus, the only resistances in the circuit were R_v and the resistance of the tunnel junction. At the frequencies of interest (<100 Hz), the reactances of the superconducting coil (<1 μH), and of the stray inductances were negligible, compared with R_v.

Since the junction resistance and R_v were often comparable, the voltage fluctuations developed across the junction were less than the value that would be measured by a voltmeter with an infinite input resistance. A straightforward calculation shows that the effect of R_v reduces $(\partial V/\partial I_c)_I$ to a shunted value

$$\left(\frac{\partial V}{\partial I_c}\right)_I^{(s)} = R_v\left(\frac{\partial I_v}{\partial I_c}\right)_I = \frac{-R}{[(I/I_c)^2 - 1 + (R/R_v)^2]^{1/2}}\,. \quad (6)$$

Equation (6) is used in Eq. (3) in subsequent calculations of $S_v(f)$.

When the SQUID was in a flux-locked loop, the current resolution in the superconducting coil was about 10^{-11} A Hz$^{-1/2}$ at 1 Hz. Over the frequency range 10^{-2}–10^{2} Hz, the resolution of the experiment was limited by the white current noise generated by R_v and the tunnel junction, of order 10^{-10} A Hz$^{-1/2}$ at 4.2 K. The current noise corresponds to a voltage resolution of about 10^{-12} V Hz$^{-1/2}$ for a circuit resistance of order 10^{-2} Ω. Below 10^{-2} Hz, $1/f$ noise from the SQUID became significant. The power spectrum $S_V(f)$ of the voltage fluctuations was obtained by digitizing the signal from the output of the flux-locked SQUID, and taking a fast Fourier transform of the digitized signal with a PDP-11/20 computer. The transform was squared and stored, and the process repeated, typically 30 times, to obtain an averaged power spectrum.[5]

IV. EXPERIMENTAL RESULTS

In Fig. 2 (Ref. 10) we plot the power spectra of the measured low-frequency noise in a Nb–Nb-oxide–Pb tunnel junction at 4.2 K with $I_c = 3.4$ mA, $R = 5\times 10^{-3}$ Ω, $R_v = 1.5\times 10^{-2}$ Ω, and $dI_c/dT = 0.8$ mA K^{-1}. The spectra are shown for three values of bias current: $I = 3.5$, 4.0, and 7.0 mA. The power spectra were reproducible to within about a factor of two under a given set of experimental conditions. The power spectra at low frequencies vary as $f^{-\alpha}$, where $0.9<\alpha<1.15$. At the higher frequencies (>1 Hz for $I=7.0$ mA, and >5 Hz for $I=4.0$ mA), the spectra flatten with increasing frequency as the contribution of the white noise becomes significant. Note that the magnitude of the power spectra *decreases* as the bias current *increases*. This

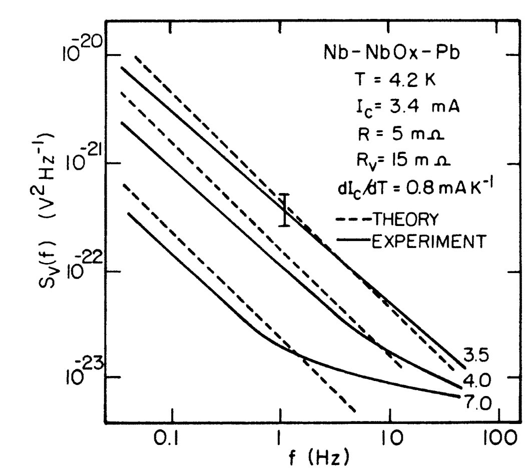

FIG. 2. Power spectra of the voltage fluctuations of a Nb–Nb-oxide–Pb junction for three values of bias current I. The error bar indicates the reproducibility of the experimental data.

current dependence is in marked contrast to that observed for thin metal films, where $S_v(f)$ is proportional to I^2. The dashed lines in Fig. 2 are calculated from Eqs. (3), (5), and (6), using $\xi_{Pb} = 80$ nm, $\xi_{Nb} = 50$ nm, $c_{Pb} = 8 \times 10^{-3}$ J K^{-1} cm^{-3}, and $c_{Nb} = 2.5 \times 10^{-3}$ J K^{-1} cm^{-3}.[11] In each case, the theoretical line is within a factor of 2 of the corresponding experimental curve in the frequency range where the measured curves vary approximately as $1/f$. The ratios of the magnitudes of the three theoretical curves for $I = 7.0$, 4.0, and 3.5 mA, 1:6.8:19.6 are in excellent agreement with the ratios of the magnitudes of the experimental curves, 1:6.8:20 at 0.1 Hz.

The fact that $S_V(f)$ at a given frequency decreases with increasing bias current is very strong evidence that the measured $1/f$ voltage noise arises from fluctuations in the critical current. This current dependence arises from the denominator of the term $(\partial V/\partial I_c)_I$ in Eq. (3). As the bias current is increased, the junction resistance becomes progressively more ohmic and less sensitive to fluctuations in critical current. If the noise were generated by a fluctuating resistance, $S_V(f)$ would *increase* with increasing bias current. Alternatively, if the noise were inherent in the bias current (a possibility excluded in any case by separate measurements), $S_V(f)$ would also increase as the current was increased.

Approximately fifteen Nb–Nb-oxide–Pb or Nb–Nb-oxide–Sn junctions were studied, in one case down to a frequency of 5×10^{-3} Hz. The spectra shown in Fig. 2 are representative of the spectra observed. In all cases, the low-frequency power spectrum scaled approximately as $1/f$ with a magnitude that was within a factor of 5 of the prediction of Eq. (3). In particular, the decrease of $S_V(f)$ with increasing bias current was always observed.

It should be noted that in calculating the theoretical power spectra of Fig. 2, the logarithmic term in the denominator of Eq. (3) was zero, because $l_1 = l_2$ for our junctions. The vanishing of the logarithmic term indicates a deficiency of the semiempirical model. Since f_1 and f_2 are the lower and upper limits of the $1/f$ region, f_1 and f_2 clearly cannot be equal, and therefore cannot be given by $D/\pi l_1^2$ and $D/\pi l_2^2$. Because the *observed* ratio f_2/f_1 is at least 10^3, the normalization constant $[3 + \ln(f_2/f_1)]$ must be at least 10. If one uses this value in Eq. (3), the calculated spectra in Fig. 2 will be reduced by about a factor of 3. However, since the theoretical predictions in Fig. 2 tend to lie somewhat above the data, this reduction would still maintain a reasonable agreement between theory and experiment. This deficiency can only be removed by a more microscopic understanding of the noise, and for the moment we shall use Eq. (3) in calculating the power spectra.

We have investigated the temperature dependence of $S_V(f)$ by lowering the sample temperature. In the case of Nb–Nb-oxide–Pb junctions, the noise decreased by a factor of between 2 and 5 when the temperature was lowered from 4.2 to 1.5 K. This rather weak temperature dependence can be readily understood from Eq. (3). As the temperature was decreased from 4.2 to 1.5 K, the terms $T^2(dI_c/dT)^2$ in the numerator decreased by about two orders-of-magnitude (using measured values of dI_c/dT), while the term $C_V(\propto T^3)$ in the denominator decreased by a factor of about 20. Thus, $S_V(f)$ is expected to decrease by roughly a factor of 5. The rather wide variation in the observed temperature dependence from junction to junction was due to variations in the value of dI_c/dT at a given temperature.

Because the temperature variation of dI_c/dT and C_V tended to cancel in Eq. (3), it was difficult to convincingly establish that $S_V(f)$ scaled as $(dI_c/dT)^2$ from the temperature dependence of the power spectra for the Nb–Nb-oxide–Pb junctions. Fortunately, the critical current of one Nb–Nb-oxide–Sn junction had a highly anomalous temperature dependence that enabled us to clearly show that $S_V(f)$ varied as $(dI_c/dT)^2$. The variation of I_c with temperature for this junction is shown in Fig. 3. As the temperature was lowered from 2.6 K, the critical current exhibited a local maximum near 2.0 K. This behavior was reproducible after the junction had been warmed to room temperature and recooled three times. We have no detailed explanation for the effect, but can speculate on a possible origin. After the niobium was sputtered onto the substrate, the sputtering current was not switched off abruptly, but was decreased to zero over a period of several seconds. As a result, the last few atomic layers of niobium were deposited slowly, and may not have been superconducting above 1 K. Thus, the junction contained a normal layer (N) between the superconducting niobium film (S) and the insulating

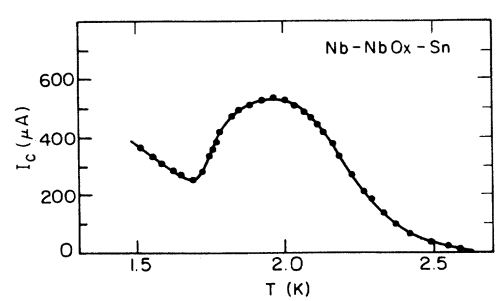

FIG. 3. Anomalous temperature dependence of the critical current of a Nb–Nb-oxide–Sn junction.

barrier (I). *SNIS* junctions with this configuration have been analyzed theoretically by McMillan.[12] He finds that the amplitude of the order parameter at the outer normal surface of a *SN* layer oscillates with temperature, implying that the Josephson critical current of a *SNIS* junction also oscillates with temperature. It is therefore possible that we have observed the effect predicted by McMillan. Dynes[13] has observed a similar effect in a Pb–Pb-oxide–Ag–Pb junction.

Although the origin of the anomalous temperature dependence is speculative, the effect is extremely useful for studying the dependence of $S_V(f)$ on dI_c/dT. A relatively small variation in temperature can change dI_c/dT by a large factor, while producing only a small change in T^2/C_V. In Fig. 4, we plot $S_V(f)$ for this junction at 1.8 K, where $dI_c/dT = 2.1$ mA K^{-1}, and at 2.0 K, where $dI_c/dT = 0.6$ mA K^{-1}. In both cases, $I/I_c = 1.2$. The average ratio of the magnitudes of the power spectra at 1.8 and 2.0 K (the slopes differ slightly) is 14.7, while the expected ratio of $(dI_c/dT)/T$ at the two temperatures is about 14. The dashed lines are calculated from Eq. (3), using $\xi_{Nb} = 50$ nm, $t_{Sn} = 150$ nm, $c_{Nb}(2.0\text{ K}) = 1.3\times10^{-4}$ J K^{-1} cm^{-3}, $c_{Nb}(1.8\text{ K}) = 9\times10^{-5}$ J K^{-1} cm^{-3}, $c_{Sn}(2.0\text{ K}) = 3.7\times10^{-4}$ J K^{-1} cm^{-3}, $c_{Sn}(1.8\text{ K}) = 2.6\times10^{-4}$ J K^{-1} cm^{-3}, $R = 0.013\ \Omega$, $R_V = 0.1\ \Omega$, and $I/I_c = 1.2$. Since the coherence length for tin (0.24 μm) is larger than the thickness of the tin film t_{Sn}, we have used t_{Sn} rather than ξ_{Sn} in the calculation of C_V. The magnitudes of the calculated power spectra are in excellent agreement with the measured power spectra. These results clearly demonstrate that $S_V(f)$ is proportional to $(dI_c/dT)^2$.

Finally, we measured the power spectra of junctions biased at currents of 10 mA, or higher. We found that these relatively high levels of power dissipation (> 1 μW) the slope of the power spectra was steeper than that shown in Figs. 2 and 4. The steeper slope indicated that the power dissipation was sufficiently high to cause a significant drift in the temperature of the junction and the copper block. (In our method of data analysis, a function that increases linearly with time has a power spectrum proportional to f^{-2}.) In addition, a few power spectra exhibited structure that we tentatively ascribed to thermal feedback in the junction. We were unable to obtain any useful quantitative results from junctions at high bias currents.

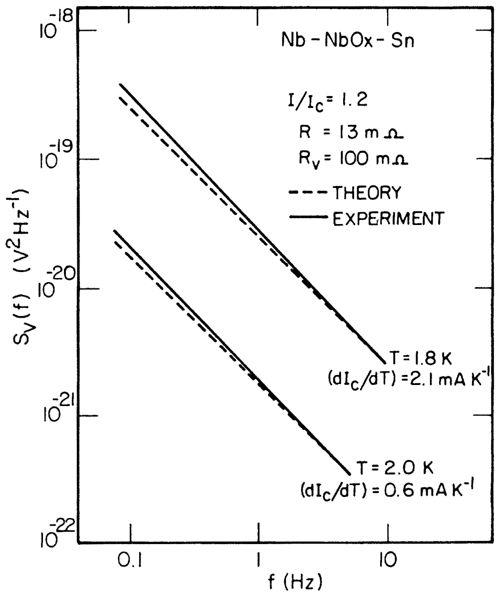

FIG. 4. Power spectra of the voltage fluctuations of the Nb–Nb-oxide–Sn tunnel junction for which I_c vs temperature is plotted in Fig. 3.

V. CONCLUSIONS

The scaling of $S_V(f)$ with $[(\partial V/\partial I_c)_I]^2$ and $(dI_c/dT)^2$ provides strong evidence that the $1/f$ noise observed in shunted Josephson tunnel junctions arises from equilibrium temperature fluctuations in the junction. The magnitudes of the observed power spectra are generally within a factor of 5 of the predictions of Eq. (3). As pointed out earlier, the geometrical factor $[3+2\ln(l_1/l_2)]$ is unlikely to be correct. In addition, we cannot determine experimentally whether or not the heat capacity C_V used in Eq. (3) corresponds precisely to a thickness given by the sum of the coherence lengths of the two superconductors. A study of the noise in junctions with a wide range of film thickness should resolve this question. Despite these difficulties, we now have an experimentally verified model that enables us to make reasonably accurate predictions of the $1/f$ noise in devices involving Josephson tunnel junctions. One may reduce the $1/f$ noise by minimizing the value of dI_c/dT without affecting the sensitivity of (for example) a SQUID to magnetic fields.

It obviously would be of interest to extend the measurements to other weak links such as the Anderson-Dayem bridge.[14] In this junction, the effective volume is of submicron dimensions, and Eq. (3) predicts that the $1/f$ noise should be much higher than that of a tunnel junction at the same temperature, and with comparable values of dI_c/dT and $(\partial V/\partial I_c)_I$.

*Work supported by the U.S. ERDA and Miller Institute for Basic Research.

†Present address: Eastman Kodak Co., Rochester, N. Y. 14650.

[1]H. Kanter and F. L. Vernon, Jr., Appl. Phys. Lett. 16, 115 (1970).

[2]B. D. Josephson, Phys. Lett. 1, 251 (1962); Adv. Phys. 14, 419 (1965).

[3]Preliminary accounts of this work appeared in IEEE Trans. Magn. MAG-11, 841 (1975); and in *Proceedings of the Fourth International Conference on Physical Aspects of Noise in Solid State Devices*, (Noordwijkerhout, the Netherlands, 1975), p. 48.

[4]John Clarke and Richard F. Voss, Phys. Rev. Lett. 33, 24 (1974).

[5]Richard F. Voss and John Clarke, Phys. Rev. B 13, 556 (1976).

[6]J. M. Richardson, Bell Syst. Tech. J. 24, 117 (1950); G. G. MacFarlane, Proc. Phys. Soc. Lond. B 63, 807 (1950); R. E. Burgess, *ibid.* B 66, 334 (1953); R. L. Petritz, Phys. Rev. 87, 535 (1952); M. Lax and P. P. Mengert, Phys. Chem. Solids 14, 248 (1960); K. M. van Vliet and E. R. Chenette, Physica (Utr.) 31, 985 (1965); K. M. van Vliet and A. van der Ziel, *ibid.* 24, 415 (1958); K. M. van Vliet and J. R. Fassett, in *Fluctuation Phenomena in Solids*, edited by R. E. Burgess (Academic, New York, 1965), pp. 267–354; and Aldert van der Ziel, *Noise* (Prentice-Hall, Englewood Cliffs, N.J., 1954).

[7]John Clarke and Thomas Y. Hsiang, Phys. Rev. Lett. 34, 1217 (1975); Phys. Rev. B 13, 4790 (1976).

[8]W. C. Stewart, Appl. Phys. Lett. 12, 277 (1968); D. E. McCumber, J. Appl. Phys. 39, 3113 (1968).

[9]John Clarke, Wolfgang M. Goubau, and Mark B. Ketchen, Appl. Phys. Lett. 27, 155 (1975).

[10]A minor plotting error exists in earlier versions of Fig. 2 in Ref. 3.

[11]*Properties of Materials at Low Temperature*, edited by V. J. Johnson, (Pergamon, New York, 1961).

[12]W. L. McMillan, Phys. Rev. 175, 559 (1968).

[13]R.C. Dynes (private communication).

[14]P. W. Anderson and A. H. Dayem, Phys. Rev. Lett. 13, 195 (1964).