Comments and Addenda

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Decay threshold in the phonon spectra of liquid helium

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The thresholds for phonon decay in two, three, etc., phonons in liquid helium are calculated, supposing anomalous dispersion.

In 1970 Maris and Massey proposed[1] that the dispersion curve for phonons in liquid helium is anomalous, since for small momenta q the group velocity $v(q)=\partial\omega/\partial q$ increases with increasing q. This means that the expansion

$$\omega(q)=sq(1-\gamma q^2-\delta q^4-\cdots) \tag{1}$$

has $\gamma<0$. In the case of anomalous dispersion contrary to the normal case, the phonons can decay, which changes some properties of liquid helium. Recently experiments were performed[2,3] to determine the γ and δ dependence on pressure.

While interpreting these experiments the phonons with dispersion relation (1) were assumed not to decay, provided $q>q_0$, where $q_0^2=\frac{3}{5}(|\gamma|/\delta)$. The argument was that $v(q)<s$ for $q>q_0$, and the emission of long-wave acoustical phonons is, thus, impossible. The purpose of this note is to show that $q=q_0$ is not the true decay threshold. The decay into two phonons is also possible for $q>q_0$ up to $q=q^*\equiv(\frac{4}{3})^{1/2}q_0\approx1.16q_0$. (Very recently two papers were published in which this decay threshold is also found.[4,5]) In this case the phonon decays into two phonons with finite momenta q_1 and q_2. The threshold q^* may be compared with numerical calculations of the decay width $\Gamma(q)$[6]; the calculated $\Gamma(q)$ shows a maximum at $q\approx1.1q_0$ followed by a rapid decrease at $q\approx1.15q_0$.

The decay thresholds are determined by conservation laws. For the decay $\vec{q}\to\vec{q}_1+\vec{q}_2$ it follows from energy-momentum conservation that

$$\omega(q_1)+\omega(q_2)=\omega(q), \tag{2}$$

$$q_1+q_2>q>|q_1-q_2|. \tag{3}$$

Equation (2) defines in the plane (q_1,q_2) a curve C, symmetrical to the line $q_1=q_2$. Inequality (3) defines a domain $\mathfrak{D}$ in the same plane. The decay is possible if at least a part of the curve C is inside the domain $\mathfrak{D}$.

It is evident that the ends of C are in the corners of $\mathfrak{D}$—in the points $(q,0)$ and $(0,q)$ (see Fig. 1). The parts of C near the ends correspond, therefore, to the long-wave acoustical-phonon emission. It is easy to verify that along C in the point $(q,0)$

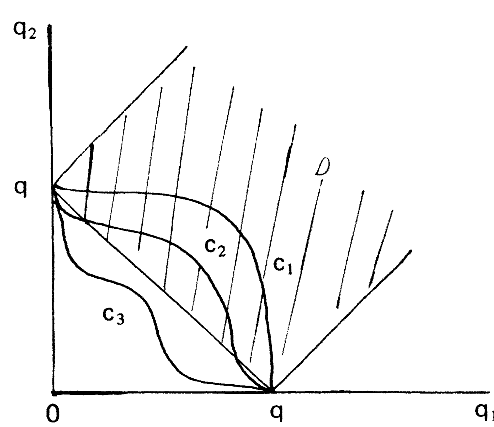

FIG. 1. Decay threshold determination from conservation laws (see text). Curves C_1, C_2, and C_3 correspond to various q: C_1—$q<q_0$; C_2—$q_0<q<q^*$; C_3—$q>q^*$.

$$\frac{\partial q_1}{\partial q_2} = -\frac{v(q)}{s} .$$

If $q > q_0$, then $v(q) < s$, and therefore the end parts of C are outside $\mathfrak{D}$, which means that long-wave acoustical-phonon emission is not possible. But the central part of C is inside $\mathfrak{D}$ if $q < q^*$, and this means that decays with finite q_1 and q_2 are possible. To demonstrate this let us consider decays to identical phonons ($q_1 = q_2 \equiv u$) and collinear decays ($q_1 + q_2 = q$). In the first case Eq. (2) reduces to $2\omega(u) = \omega(q)$; this equation has a root inside $\mathfrak{D}$ ($u > \frac{1}{2}q$), when $q < q^*$. In the second case which corresponds to the boundary of $\mathfrak{D}$, Eq. (2) has roots when $q_0 < q < q^*$. It can be seen now that at the threshold q^* a phonon decays into two identical collinear phonons: $2\omega(\frac{1}{2}q^*) = \omega(q^*)$.

Curves C for various q are displayed in Fig. 1. When $q < q_0$, the whole curve C is inside $\mathfrak{D}$, and the decay generates phonons with momenta from zero up to q. At $q = q_0$ the end parts of C pass the boundary of $\mathfrak{D}$, and if $q > q_0$ the decays with long-wave acoustical-phonon emission are forbidden. But only when $q > q^*$, the whole curve C is outside $\mathfrak{D}$, and the phonons become absolutely stable with respect to the three-phonon processes.

If we consider the possibility of higher-order processes, when the phonon q decays into three and more phonons, the decay is also allowed for $q > q^* \equiv q_2^*$. The thresholds for these higher-order decays can be analyzed in a similar way. Using the dispersion law (1), it can be shown that at the threshold the decay into collinear phonons with equal momenta occurs, and hence, the equation for the threshold momentum is $n\omega(q/n) = \omega(q)$. Solving this equation the threshold momentum is

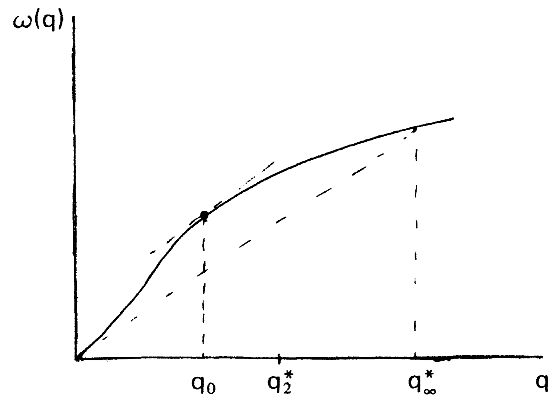

FIG. 2. Thresholds in the dispersion relation for liquid helium.

found to be

$$q_n^* = q_\infty^* (1 + 1/n^2)^{-1/2} , \quad q_\infty^* = (\tfrac{5}{3})^{1/2} q_0 . \tag{4}$$

For example $q_3^* = (\frac{3}{2})^{1/2} q_0 \approx 1.22\, q_0$. The thresholds q_n^* converge to the limit q_∞^*. For $n \to \infty$ the above equation for the threshold momentum reduces to $sq = \omega(q)$. Hence q_∞^* corresponds to the point where the phase velocity is equal to the sound velocity (see Fig. 2).

It must be noted that only q_∞^* is a true threshold. Since the phonon damping $\Gamma(q)$ is nonzero at every $q < q_\infty^*$, the uncertainties in energy conservation lead to the smoothing of all the thresholds q_n^* with finite n. The damping $\Gamma(q)$ is going to zero as $q \to q_\infty^*$. It is a hard task to find the exact behavior of $\Gamma(q)$ near the limiting point q_∞^*, but it is natural to believe that $\Gamma(q)$ goes to zero exponentially.

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