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Order in metallic chains. II. Coupled chains*

R. A. Klemm†
Department of Applied Physics, Stanford University, Stanford, California 94305

H. Gutfreund‡
Department of Physics, Stanford University, Stanford, California 94305
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The possibility of long-range order in a quasi-one-dimensional metal is studied from the standpoint of the two existing exactly soluble models of the interacting electron gas in one dimension. The presence of interchain low-momentum-transfer electron-electron interactions does not give rise to a phase transition of any type but serves only to renormalize the effective intrachain interactions. In the region of the space of intrachain interactions for which the purely one-dimensional charge-density wave response function is divergent as $T \to 0$, the presence of nearest-neighbor interchain large-momentum-transfer electron-electron scattering gives rise to a phase transition only of the charge-density-wave type. When the electronic motion is not restricted to one spatial dimension, phase transitions of either the singlet superconducting or charge-density-wave type may occur, depending upon the effective intrachain interactions. These phase transitions are investigated in the "mean-field" and "self-avoiding random-walk" approximations, and the effects of fluctuations in the interchain couplings are found to be small.

I. INTRODUCTION

In the preceding paper[1] (referred to hereafter as I) we have discussed the onset of long-range order in a single metallic chain as $T \to 0$. The electrons were described by the Luther-Emery[2] (LE) Hamiltonian and the four response functions, corresponding to singlet and triplet superconducting (SS and TS) and to charge- and spin-density wave ordering (CDW and SDW) were calculated on the LE line of solutions (the backward scattering interaction strength $\tilde{g}_1 = g_1/\pi v_F = -\frac{6}{5}$).

In the present paper we discuss the quasi-one-dimensional system of coupled metallic chains. Many of the previous treatments which attempted to describe the experimental results in real materials like TTF-TCNQ (tetrathiafulvalene tetracyanoquinodimethane) and KCP (potassium cyanoplatinide) applied mean-field theory in one dimension. In such an approach, one assumes implicitly that some interchain coupling of an unspecified nature exists, which is sufficiently strong to suppress the one-dimensional thermodynamic fluctuations, and that the actual results depend only weakly on the strength of this coupling. As these assumptions are not obvious, a microscopic treatment of the coupled-chain problem must include the interchain coupling explicitly.

There are several approaches to this problem. One approach is to represent the coupled-chains system by a three-dimensional but very anisotropic system in which the single-particle tunneling between the chains is described by the tight-binding model. The advantage of this approach is that for sufficiently strong interchain coupling the mean-field theory is valid. One can therefore compute the transition temperatures and discuss the competition between the various types of order in the mean-field approximation. It is then possible to check *a posteriori*, by calculating the fluctuations, when this approximation breaks down. This line of approach was adopted by Horovitz *et al.*[3] A different treatment of the coupled-chain problem was recently presented by Gorkov and Dzyaloshinskii.[4] They have extended the summation of the parquet diagrams performed by Bychkov *et al.*,[5] for the single metallic chain, to the system of coupled chains. In this approach both intrachain and interchain correlations are treated by mean-field theory.

The treatment presented in this paper has an important advantage over the other two methods, at least for weakly coupled chains, in that it treats the one-dimensional interactions exactly. The interchain interactions are treated perturbatively, and thus all properties of the system reduce to exact results for a one-dimensional system in the absence of interchain coupling. We consider a lattice of coupled one-dimensional electron-gas chains, each described by the values of g_1 and g_2 for which there is an exact solution (LE and Luttinger[7] models). Thus, in the limit of zero coupling, we may solve the problem exactly, and use that solution as a starting point for a perturbation expansion.

There are two types of interchain interaction we shall consider. One may be referred to, in a broad sense, as an interchain Coulomb interaction in which the interacting electrons are confined to their chains and the other is the interchain single-particle tunneling interaction. In the Coulomb interaction, one may distinguish between two processes: interchain forward scattering and interchain backward scattering. The first process is discussed in Sec. II. The addition of the term describing this process to the Hamiltonian gives an exactly solvable extension of the LE model. However, this type of interchain coupling does not order the chains, and hence does not give rise to a phase transition at a finite temperature, but merely modifies the exponents of the response functions for the single chain.

In Sec. III we discuss the effect of interchain backscattering. In this case the problem cannot be solved exactly, and we treat the interchain interaction in mean-field theory. The whole approach is very similar to that of Scalapino *et al.*,[6] for the system of coupled one-dimensional Ising and Ginzburg-Landau chains. We find that of the four response functions, only the one corresponding to CDW is correlated from chain to chain, and thus that only this type of long-range order may occur at a finite temperature. This occurs for those values of g_2 on the LE line for which the one-dimensional CDW response function diverges. The validity of the mean-field approximation for the interchain coupling is investigated by explicitly calculating the fluctuations, and it is found that for a large region of g_2 values, the fluctuations do not greatly suppress T_P from its mean-field value.

In Sec. IV we discuss the effect of interchain tunneling, which again, has to be treated in the mean-field approximation. The semi-invariant expansion in the interchain tunneling results (to lowest order) in complicated integral equations for the response functions. These equations simplify greatly at the point $g_1 = 2g_2$ on either the Luttinger or the LE line. At this point, the transition temperatures to the SS state (T_c) and the CDW state (T_P) are equal. In the vicinity of this point we linearize the kernel of the integral equation in the quantity $g_1 - 2g_2$, and we find that for $g_1 - 2g_2 < 0$, $T_P > T_c$, and for $g_1 - 2g_2 > 0$, $T_c > T_P$.

Finally, in Sec. V we summarize our results for the phase transitions brought about by interchain Coulomb and tunneling interactions. We find that for $g_1 - 2g_2 < 0$, the system will only undergo CDW phase transition. In the region where the one-dimensional SS response function is divergent but the CDW response function is not divergent, the tunneling interaction insures that the system will be superconducting. In the intermediate region where both one-dimensional response functions are divergent but the SS function predominates, there are competing effects, as the Coulomb interactions tend to enhance the CDW instability, but the effect of the tunneling is to favor the SS transition. Thus, in this region, both types of phase transitions may occur. We then attempt to correlate this picture with real materials.

II. THE HAMILTONIAN FOR UNCOUPLED CHAINS

Prior to the discussion of the various interchain coupling mechanisms, let us define the Hamiltonian for a square lattice of parallel uncoupled electron-gas chains. The latter is the LE Hamiltonian [Eqs. (10) and (11) of I]

$$\mathcal{H}_0 = \mathcal{H}_\rho + \mathcal{H}_\sigma ,$$
$$\mathcal{H}_\rho = \frac{2\pi v_F}{L} \sum_{\substack{k>0 \\ n}} [\rho_1^n(k)\rho_1^n(-k) + \rho_2^n(-k)\rho_2^n(k)] + \frac{1}{L}\sum_{k,n} [2V(k) - U_\parallel]\rho_1^n(k)\rho_2^n(-k) ,$$
$$\mathcal{H}_\sigma = \frac{2\pi v_F}{L} \sum_{\substack{k>0 \\ n}} [\sigma_1^n(k)\sigma_1^n(-k) + \sigma_2^n(-k)\sigma_2^n(k)] \tag{1}$$
$$- \frac{U_\parallel}{L}\sum_{k,n} \sigma_1^n(k)\sigma_2^n(-k) + \frac{U_\perp}{(2\pi\alpha)^2}\sum_n \int dx \left[\exp\left(\sqrt{2}\sum_k A_k(x)[\sigma_1^n(k) + \sigma_2^n(k)]\right) + \text{H.c.}\right] ,$$

where

$$A_k(x) = 2\pi \exp(-\alpha|k|/2 - ikx)/Lk .$$

Each of the density and spin-density operators carries the two-component chain index n. In Eq. (1) we have used the Bose representation for $\psi_{is}^n(x)$, analogous to Eq. (9) of I. We transform Eq. (1) to momentum representation in the transverse direction by defining

$$\rho_j(k,\vec{q}) = \frac{1}{N^{1/2}}\sum_n e^{i(\operatorname{sgn} k)\vec{q}\cdot\vec{r}_n}\rho_j^n(k), \quad \sigma_j(k,\vec{q}) = \frac{1}{N^{1/2}}\sum_n e^{i(\operatorname{sgn} k)\vec{q}\cdot\vec{r}_n}\sigma_j^n(k), \tag{2}$$

where $j=1, 2$, $\vec{r}_n$ is the coordinate of the nth chain and N is the total number of chains. In view of the boson commutation relations satisfied by the σ_j^n and ρ_j^n [Eq. (4) of I], we obtain

$$[\rho_i(-k,\vec{q}), \rho_j(k',\vec{q}\,')] = \pm\,\delta_{ij}\,\delta_{\vec{q}\vec{q}\,'}\delta_{qq'}\,(Lk/2\pi)\,, \quad [\sigma_i(-k,\vec{q}), \sigma_j(k',\vec{q}\,')] = \pm\,\delta_{ij}\,\delta_{kk'}\,\delta_{\vec{q}\vec{q}\,'}(Lk/2\pi)\,,$$
$$[\rho_i(k,\vec{q}), \sigma_j(k',\vec{q}\,')] = 0\,. \tag{3}$$

The transformed Hamiltonian reads

$$\mathcal{H}_\rho = \frac{2\pi v_F}{L}\sum_{k,\vec{q}}\,[\rho_1(k,\vec{q})\rho_1(-k,\vec{q}) + \rho_2(-k,\vec{q})\rho_2(k,\vec{q})] + \frac{1}{L}\sum_{k,\vec{q}}\,[2V(k) - U_\parallel]\,\rho_1(k,\vec{q})\rho_2(-k,\vec{q}) \tag{4}$$

$$\mathcal{H}_\sigma = \frac{2\pi v_F}{L}\sum_{k,\vec{q}}[\sigma_1(k,\vec{q})\,\sigma_1(-k,\vec{q}) + \sigma_2(-k,\vec{q})\,\sigma_2(k,\vec{q})]$$
$$- \frac{U_\parallel}{L}\sum_{k,\vec{q}}\sigma_1(k,\vec{q})\,\sigma_2(-k,\vec{q}) + \frac{U_\perp}{(2\pi\alpha)^2}\sum_n \int dx\left[\exp\left(\sqrt{2}\sum_k A_k(x)[\sigma_1^n(k) + \sigma_2^n(k)]\right) + \text{H.c.}\right]. \tag{5}$$

It is convenient to leave the last term in $\mathcal{H}_\sigma$ in the chain-site representation. In complete analogy to the single-chain case, $\mathcal{H}_\rho$ is diagonalized by the canonical transformation $e^{iG_\rho}\mathcal{H}_\rho e^{-iG_\rho}$, with

$$G_\rho = \frac{2\pi i}{L}\sum_{k,\vec{q}}\frac{\varphi(k)}{k}\,\rho_1(k,\vec{q})\rho_2(-k,\vec{q})\,, \tag{6}$$

where

$$\tanh 2\varphi(k) = -(2V(k) - U_\parallel)/2\pi v_F\,. \tag{7}$$

Similarly, the bilinear part of $\mathcal{H}_\sigma$ is diagonalized by a canonical transformation generated by

$$G_\sigma = \frac{2\pi_i}{L}\sum_{k\vec{q}}\frac{\psi}{k}\,\sigma_1(k,\vec{q})\,\sigma_2(-k,\vec{q})\,, \tag{8}$$

where

$$\tanh 2\psi = U_\parallel/2\pi v_F\,. \tag{9}$$

The last transformation introduces the factor e^ψ into the exponent in the last term of $\mathcal{H}_\sigma$. For the particular case $\sqrt{2}\,e^\psi = 1$, this term may be represented in terms of fictitious spinless fermion fields on the individual chains, exactly as in I.

The Hamiltonian $\mathcal{H}_0 = \mathcal{H}_\rho + \mathcal{H}_\sigma$ with $\mathcal{H}_\rho$ defined in Eq. (4) and $\mathcal{H}_\sigma$ in Eq. (5) is the unperturbed Hamiltonian of coupled-chain system to be discussed in the following sections.

III. INTERCHAIN FORWARD SCATTERING

As the simplest natural extension of the Luttinger and Luther-Emery models of the one-dimensional electron gas to include interchain coupling, we shall first investigate the role of forward scattering between electrons on nearest-neighbor chains. To this end we add to the Hamiltonian in Eq. (1) the term

$$\mathcal{H}_{fs} = \frac{1}{2}\sum_{\langle nn'\rangle}\sum_{ss'}\int dx\,dy\,\psi_{1s}^{\dagger n}(x)\,\psi_{2s'}^{\dagger n'}(y)$$
$$\times V_2(x-y)\,\psi_{2s'}^{n'}(y)\,\psi_{1s}^{n}(x)\,, \tag{10}$$

where $V_2(x-y)$ is assumed independent of chain indices and $\langle nn'\rangle$ indicates that n and n' are nearest-neighbor chains. This form of the interchain coupling is particularly simple, as the Fourier transformation in the continuous intrachain and the discreet interchain coordinates gives

$$\mathcal{H}_{fs} = \frac{2}{L}\sum_{k,\vec{q}} V_2(k,\vec{q})\,\rho_1(k,\vec{q})\,\rho_2(-k,\vec{q})$$
$$+ \frac{2}{L}\sum_{k>0,\vec{q}} V_2(k,\vec{q})\,[\rho_1(k,\vec{q})\,\rho_1(-k,\vec{q})$$
$$+ \rho_2(-k,\vec{q})\,\rho_2(k,\vec{q})]\,, \tag{11}$$

where

$$V_2(k,\vec{q}) = V_2(k)\,(\cos q_x d + \cos q_y d) \tag{12}$$

and d is the chain-lattice constant. The first term corresponds to the g_2 and the second to the g_4 process in Fig. 1 of I, but now the electron lines represent electrons on nearest-neighbor chains. The interaction in Eq. (11) may be added to $\mathcal{H}_\rho$ in Eq. (4) and the whole Hamiltonian is diagonalized as before, the only difference being that Eq. (7) now reads

$$\tanh 2\varphi(k,\vec{q}) = \frac{-[2V(k) + 2V_2(k,\vec{q}) - U_\parallel]}{2[\pi v_F + V(k) + V_2(k,\vec{q})]}\,. \tag{13}$$

We may now calculate the appropriate response functions in order to see the effect of this interchain interaction. First of all we note that $\mathcal{H}_\sigma$ remains unchanged and therefore gives the same contribution to the response functions as for the uncoupled-chains problem. For the ρ-dependent factors [see Eq. (22) of I] we obtain

$$S_\rho^\pm(x,t;nn')=\frac{e^{2ik_Fx}}{(2\pi\alpha)^2}\exp\left(\frac{2\pi}{L\sqrt{N}}\sum_{k\vec{q}}\frac{e^{\pm\varphi(k,\vec{q})}}{\sqrt{2k}}e^{-id\,\mathrm{sgn}\,k(n_1q_x+n_2q_y)}\times[e^{iks}\rho_1(k,\vec{q})+e^{-iks}\rho_2(k,\vec{q})]\right)$$

$$\times\exp\left(-\frac{2\pi}{L\sqrt{N}}\sum_{k\vec{q}}\frac{e^{\pm\varphi(k,\vec{q})}}{\sqrt{2k}}e^{-id\,\mathrm{sgn}\,k(n_1'q_x+n_2'q_y)}[\rho_1(k,\vec{q})+\rho_2(k,\vec{q})]\right), \tag{14}$$

where $s=v_F't-x$, $s'=v_F't+x$, $v_F'=v_F\,\mathrm{sech}\,2\varphi(k,\vec{q})\simeq v_F\,\mathrm{sech}\,2\varphi(0,0)$, and n_1,n_2 and n_1',n_2' are the x,y coordinates of the chain index vectors $\vec{n},\vec{n}'$. Using the result

$$\langle\rho_1(k,\vec{q})\rho_1(k,\vec{q})\rangle=k(L/2\pi)\,\delta_{k',-k}\,\delta_{q_xq_x'}\,\delta_{q_yq_y'},$$

and performing the $\vec{q}$ integrations in the exponent, we find

$$S_\rho^\pm(xt,\vec{n},\vec{n}')=\tilde{S}_\rho^\pm(x,t)\,\delta_{\vec{n},\vec{n}'}, \tag{15}$$

where $\tilde{S}^\pm(x,t)$ is calculated as in Luther and Peschel[7] with the result quoted in Eq. (26) of I, but with $\delta_\pm$ replaced by

$$\delta_\pm'=\int_0^\pi\frac{dx}{\pi}\int_0^\pi\frac{dy}{\pi}\left(\frac{1\mp K(x,y)}{1\pm K(x,y)}\right)^{1/2}, \tag{16}$$

where

$$K(x,y)=\frac{\tilde{g}_2+\frac{3}{5}+\tilde{V}_2(\cos x+\cos y)}{1+\tilde{g}_2+\tilde{V}_2(\cos x+\cos y)}, \tag{17}$$

where $g_2=\tilde{V}(0)/\pi v_F$ and $\tilde{V}_2=V_2(0)/\pi v_F$.

Thus, the presence of the interchain forward scattering does not change the qualitative behavior of any of the four response functions, as the chains act as if they were completely decoupled, the response functions for particles on different chains vanishing identically. The interaction with particles on other chains serves only as a correction to the effective intrachain interaction and thus renormalizes the effective plasmon velocity v_F' and the exponents $\delta_\pm$.

By applying the scaling arguments of Chui and Lee,[8] we find, as in I, that the $\frac{3}{5}$ in Eq. (17) should be replaced by $-\frac{1}{2}\tilde{g}_1$. Expanding $\delta_\pm'$ for small $\tilde{V}_2$, we find $\delta_+'>\delta_+$ and $\delta_-'<\delta_-$. Since we have $\mathcal{Im}\,\chi\sim\omega^{-2+\delta_\pm'}$, where the upper (lower) sign corresponds to a CDW (SS) response, we conclude that the interchain forward scattering tends to suppress the tendency towards CDW order, and to enhance the tendency towards SS order as $T\to 0$. This is shown explicitly in Figs. 1 and 2, in which we have plotted the regions in g_1/g_2 space of different physical behavior at $T=0$ for two characteristic values of the ratio $|\tilde{V}_2/\tilde{g}_2|$. We observe that the curves represented by $\delta_\pm'=2$ and $\delta_+'=\delta_-'$, which describe the boundaries of the regions in which the response functions for SS and CDW order at $T=0$ are divergent, and for which one of them diverges more rapidly than the other, respectively, are all distorted to more positive values of $\tilde{g}_2$ than in Fig. 3 of I. However, the boundary of the region in which the model can be solved is dramatically shifted to more positive $\tilde{g}_2$ values, so that for $|\tilde{V}_2/\tilde{g}_2|=\frac{1}{2}$, the region of purely SS-ordering behavior as $T\to 0$ is completely outside of the region in which the model can be solved. In order to interpret the behavior of the system in the region $0\leqslant 1+2\tilde{g}_2-\tilde{g}_1/2<4|\tilde{V}_2|$, which is in the region of g_1/g_2 space for which the model can be solved in the absence of V_2, but outside the soluble region for finite V_2, let us consider expanding the zero-temperature response functions in powers of V_2 within the $\mathcal{S}$ matrix formalism. If we do so, we pick up logarithmic corrections to the response functions, which when summed give exactly the corrections to the exponents $\delta_\pm$ that would arise from a power series expansion of δ_+' in powers

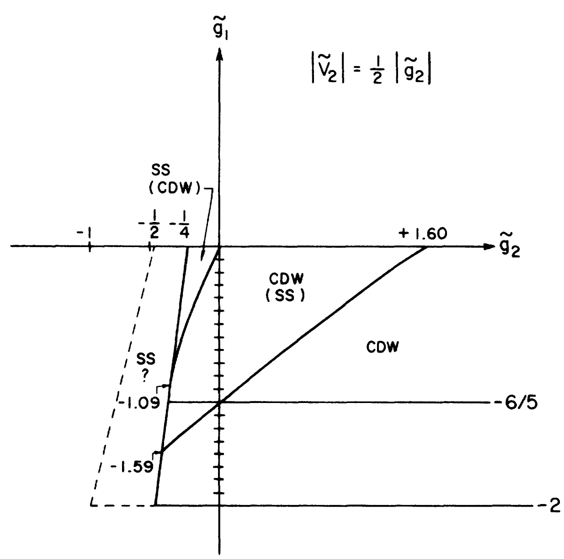

FIG. 1. Plot in g_1/g_2 space for $\tilde{g}_1<0$, $\tilde{g}_4=\tilde{g}_2$, and $|\tilde{V}_2|=\frac{1}{2}|\tilde{g}_2|$. The model can be solved for $\tilde{g}_1>-2$ and $4\tilde{g}_2-\frac{1}{2}\tilde{g}_1>-1$. In the dashed region, it appears that superconductivity is predominant over CDW behavior at $T=0$. The line $\delta_+'=\delta_-'$ that separates the regions in which SS and CDW dominate, and the line $\delta_-'=2$ that defines the region of a divergent superconducting response at $T=0$, are both distorted to more positive $\tilde{g}_2$ values than in Fig. 3 of I. There is no region for which the model can be solved for which only a divergent SS response at $T=0$ exists.

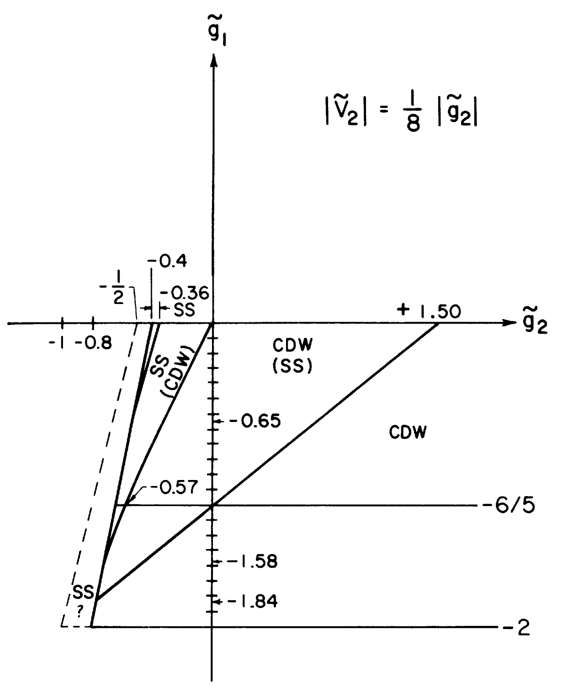

FIG. 2. Plot in g_1/g_2 space for $\tilde{g}_1 < 0$, $\tilde{g}_4 = \tilde{g}_2$, and $|\tilde{V}_2| = \frac{1}{8}|\tilde{g}_2|$. The model can be solved for $\tilde{g}_1 > -2$ and $5\tilde{g}_2 - \tilde{g}_1 > -2$. The curves are all distorted to more positive $\tilde{g}_2$ values than in Fig. 3 of I, but more negative $\tilde{g}_2$ values than in Fig. 1. All four regions that were pres- in Fig. 3 of I are also present in this figure.

of $x = 2|\tilde{V}_2|/(1 + 2\tilde{g}_2 - \frac{1}{2}\tilde{g}_1)$. Since for $x > \frac{1}{2}$, the series for δ_+ diverges to $+\infty$ and that for δ_- diverges to $-\infty$, we thus interpret the region $0 \leq 1 + 2\tilde{g}_2 - \frac{1}{2}\tilde{g}_1 < 4|\tilde{V}_2|$ as perhaps exhibiting only a tendency towards SS ordering as $T \to 0$, although a calculation of the precise form of the response functions in that region is not readily apparent.

We notice that the values of $|\tilde{V}_2|$ we have chosen in Figs. 1 and 2 are not unreasonable. Davis[9] has shown by a classical screening calculation that the Coulomb interaction strength between electrons on different chains may be of the same order of magnitude as that between electrons on the same chain. We thus take $\tilde{V}_2$ to be the effective screened Coulomb interaction between nearest-neighbor chains, which accounts for the fact that there are also second and third-nearest-neighbor interactions, and so on.

IV. INTERCHAIN BACKSCATTERING

A. Mean-field theory

Let us now consider an interchain interaction of the form

$$\mathcal{H}_{bs} = V_1 \sum_{\langle nn'\rangle} \sum_{ss'} \int dx\, \psi_{1s}^{\dagger n}(x)\, \psi_{2s'}^{\dagger n'}(x)\, \psi_{1s'}^{n'}(x)\, \psi_{2s}^{n}(x)\,, \tag{18}$$

where again $\langle nn'\rangle$ denotes nearest-neighbor chains. This is the analog of the intrachain backscattering term introduced by LE [Eq. (6) of I]. In the present case, however, there is no reason to distinguish between the processes with $s = s'$ and $s = -s'$, because now the parallel spin process cannot be represented as a bilinear form in the density and spin-density operators. We use the boson representation of the Fermi fields [Eq. (9) of I] to express $\mathcal{H}_{bs}$ in terms of the ρ and σ operators and obtain (after a Fourier transformation)

$$\mathcal{H}_{bs} = \frac{V_1}{(2\pi\alpha)^2} \sum_{n_1, n_2} \sum_{\epsilon_1, \epsilon_2, \epsilon_3 = \pm 1} \int dx \Bigg(\exp \frac{1}{\sqrt{N}} \sum_{k \vec{q}} \frac{A_k(x)}{\sqrt{2}}\, e^{-id\,\mathrm{sgn}\,k(n_1 q_x + n_2 q_y)} \times \{ (1 - e^{-id\,\mathrm{sgn}\,k(\epsilon_1 q_x + \epsilon_2 q_y)})[\rho_1(k,\vec{q}) + \rho_2(k,\vec{q})] + (1 - \epsilon_3 e^{-id\,\mathrm{sgn}\,k(\epsilon_1 q_x + \epsilon_2 q_y)})[\sigma_1(k,\vec{q}) + \sigma_2(k,\vec{q})] \} \Bigg) + \mathrm{H.c.}, \tag{19}$$

where the sums over ϵ_1, ϵ_2 arise from the nearest neighbors of the chains with coordinates n_1, n_2, and the sum over ϵ_3 is due to the possibilities of parallel and antiparallel spins. Unlike the case of intrachain backscattering, the boson representation of $\mathcal{H}_{bs}$ involves the ρ operators as well as the σ operators. It is now impossible to diagonalize by a single transformation the last term of Eq. (5) and Eq. (10), since the coefficients of the σ operators in Eq. (5) and of the σ and ρ operators in Eq. (19) are rotated out of phase with respect to each other. Thus, an exact solution of the Hamiltonian with $\mathcal{H}_{bs}$ is not readily apparent and we must resort to perturbation theory. But first, we note that $\mathcal{H}_{bs}$ may be written in the form

$$\mathcal{H}_{bs} = V_1 \sum_{\langle nn'\rangle} \sum_{ss'} \int dx\, \varphi_s^{\dagger n}(x)\, \varphi_{s'}^{n'}(x)\,, \tag{20}$$

where $\varphi_s^{\dagger n}(x) \equiv \psi_{1s}^{\dagger n}(x)\, \psi_{2s}^{n}(x)$. In this form $\mathcal{H}_{bs}$ is equivalent to a Hamiltonian for tunneling of particle-hole pairs. The response function corresponding to CDW involves such particle hole pairs. Since we are interested in calculating the temperature of a possible phase transition, we shall compute the "imaginary" time-response functions. For the case of CDW we can write

$$\chi_{\rm CDW}^{nn'}(x,\tau)=\langle T_\tau[\varphi_s^n(x,\tau)\varphi_s^{\dagger n'}(0,0)\mathcal{S}]\rangle/\langle\mathcal{S}\rangle\ , \qquad (21)$$

where

$$\mathcal{S}=T_\tau\exp\int_0^{1/T}\mathcal{H}_{bs}(\tau)\,d\tau\ ,$$

T_τ is the τ-ordering operator, the expectation value is taken with respect to the uncoupled Hamiltonian, and the operators are in the Heisenberg representation. Similar expressions may also be written for the other three response functions. For these we find to every order in perturbation theory that (see Appendix A)

$$\chi_M^{nn'}(x,\tau)=\delta_{nn'}\,\tilde{\chi}_M(x,\tau)\ , \qquad (22)$$

where M stands for SDW, SS, or TS. Thus, the chains remain completely uncorrelated for these three response functions and the effect of $\mathcal{H}_{bs}$ is only to renormalize the effective intrachain interaction. Therefore there cannot be a phase transition to any of these types of order because of $\mathcal{H}_{bs}$.

We shall now restrict our consideration to the CDW case. Expanding Eq. (21) in V_1 we obtain in mean-field theory (after Fourier transformation)

$$\chi_{\rm CDW}^{-1}(k\omega)_{\vec{q}}=\chi_{\rm CDW}^{0^{-1}}(k\omega)-\Sigma(k\omega)_{\vec{q}}\ , \qquad (23)$$

where $\Sigma(k\omega)_{\vec{q}}=V_1(\cos q_x d+\cos q_y d)$ and $\chi^0_{\rm CDW}(k,\omega)$ is the Fourier transform of $\chi^{nn}_{\rm CDW}(x\tau)$ in the absence of V_1. The system becomes unstable at the temperature T_P, for which

$$2|V_1|\chi^0_{\rm CDW}(2k_F,0)_{T_P}=1\ . \qquad (24)$$

Equation (23) is the same equation as that used by Scalapino *et al.*[6] in their treatment of coupled Ising chains. Equation (24) gives (see Appendix B)

$$T_P^{\rm MF}=W(2A|V_1|)^{1/\mu}\ , \qquad (25)$$

where W is the bandwidth, $\mu=2-\delta_+'$ for the LE model and $\mu=1-\delta_+'$ for the Luttinger model, δ_+' is given by Eq. (16) with the $\frac{3}{5}$ replaced by $\frac{1}{2}\tilde{g}_1$, and

$$A=\frac{1}{4\pi^2 v_F'}\left(\frac{v_F'}{\pi v_F}\right)^{1-\delta_+'}\times\int_0^\pi d\alpha\int_{-\infty}^{\infty}d\beta\,|\sin(\alpha-i\beta)|^{-\delta_+'}\times\sin(\alpha-(v_F'/v_F)\beta)|^{-1}\ , \qquad (26)$$

$$A=\frac{\alpha\Delta K_0(\alpha'\Delta)v_F}{4\pi^3(v_F''v_F')}\left(\frac{v_F'}{\pi v_F}\right)^{1-\delta_+'}\times\int_0^\pi d\alpha\int_{-\infty}^{\infty}d\beta\,|\sin(\alpha-i\beta)|^{-\delta_+'}$$

for the Luttinger and LE models, respectively. We note that A is finite for $\mu>0$, the region for which the CDW response function for the single chain can diverge as $\omega\to 0$. Finally, we note that since $\Sigma(k\omega)_{\vec{q}}$ depends on the sign of V_1, we find that the unit vector in the ordered phase is $(0,0,2k_F)$ for $V_1<0$ and $(\pi/d,\pi/d,2k_F)$ for $V_1>0$. That is, if the backscattering interaction is attractive the crests of the charge-density waves on adjacent chains line up in phase with each other, and if it is repulsive they line up out of phase so as to minimize the electrostatic energy in each case.

B. Effect of fluctuations

In deriving Eq. (25) for the mean-field Peierls transition temperature owing to the interchain backscattering interaction, we have of course assumed that the particles on a given chain interact with the "mean field" of the particles on adjacent chains. This is not necessarily a bad approximation, as Scalapino *et al.*[6] have shown that for a system of one-dimensional Ising chains with Ising interchain coupling on a one-dimensional lattice of chains, the mean-field transition temperature owing to the interchain coupling differs from the exact two-dimensional Ising transition temperature by much less than a factor of 1/ln2. Thus, in that case, the fluctuations of the spin correlations owing to the interchain coupling do not appear to be important. However, it is not obvious that such remarkably accurate results as those should always arise from a mean-field treatment of the interchain coupling between other types of chains, as most interacting systems do not exhibit a phase transition in two dimensions. For the case we are considering, the interchain backscattering interaction bears a formal similarity with the Ising interaction, if instead of the spin S_n on the nth site, we consider the field $\varphi^n(x)=\sum_s\varphi_s^n(x)$. Thus, we might naively expect that in the absence of intrachain interactions, the problem we are considering may behave similarly to an Ising system with dimensionality equal to that of the lattice of the chains. Thus, for a one-dimensional lattice of chains, it is questionable that a phase transition would exist, but for a two-dimensional lattice of chains, the possibility of such a phase transition does not appear at first sight unreasonable. We shall present here some qualitative arguments, however, that the fluctuation effects are small, although the details of the cal-

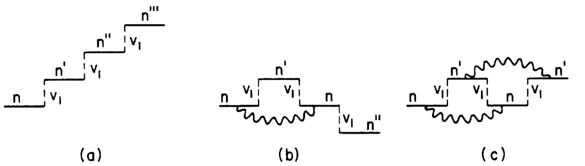

FIG. 3. Shown are typical diagrams of third order in the interchain backscattering interaction. The straight line with index n implies the propagation of a particle-hole pair on the chain with the two-component index $\boldsymbol{n}$. The wavy lines in Figs. 3(b) and 3(c) imply that the particle-hole pair interferes with itself at a later time, and therefore the terms corresponding to these diagrams do not factor as in mean-field theory.

culation of the effect of fluctuations on the system are of sufficient generality and complexity as to merit a separate discussion, and will thus be given elsewhere.[10]

If we expand Eq. (21) for the finite temperature CDW response function $\chi_{\rm CDW}^{nn'}(x,\tau)$ in powers of the interchain backscattering interaction V_1, and Fourier transform with respect to x and τ, we find that there are several distinct types of contributions to $\chi_{\rm CDW}^{nn'}(q=2k_F,\ \omega_n=0)$. In Fig. 3, we have shown diagrammatically the three types of contributions of order V_1^3 that arise. We shall denote the diagram in Fig. 3(a) a "mean-field" diagram, as it has the value V_1 which is the same as in mean-field theory. That is, $\chi_1=V_1^3\chi_0^4(2k_F,0)$ where χ_0 is the single-chain CDW response function, and therefore $\chi_0(2k_F,0)=A(\alpha T)^{-\mu}$, where $\mu=2-\delta_+'$ for the LE model and $\mu=1-\delta_+'$ for the Luttinger model for the respective values of $\tilde{g}_1$, and A depends only upon δ_+', v_F'/v_F'', and $\alpha\Delta$, and is given in Appendix B. Since the interchain backscattering interaction is equivalent to the tunneling of a particle-hole pair from chain to chain, we may describe this type of diagram as arising from a particle-hole pair that "tunnels" from chain to chain three times in a self-avoiding fashion.

The second type of contribution to $\chi^{nn'}$ of order V_1^3, which we shall call χ_2, is shown diagrammatically in Fig. 3(b). This term corresponds to a particle-hole pair that tunnels to an adjacent chain, tunnels back to the original chain, and then tunnels to a third chain. However, when it tunnels back to the original chain, it interferes with itself owing to its previous occupation of that chain. This interference had been neglected in our mean-field treatment. This term may be written formally as

$$\chi_2=V_1^3\int_{-\infty}^{\infty}dx\int_0^{1/T}d\tau\prod_{i=1}^{3}\int_{-\infty}^{\infty}dx_i\int_0^{1/T}d\tau_i\,\langle T_\tau[\varphi(x_3\tau_3)\varphi^\dagger(00)]\rangle\langle T_\tau[\varphi(x_1\tau_1)\varphi^\dagger(x_2\tau_2)]\rangle$$
$$\times\{\langle T_\tau[\varphi(x\tau)\varphi^\dagger(x_1\tau_1)\varphi(x_2\tau_2)\varphi^\dagger(x_3\tau_3)]\rangle$$
$$-\langle T_\tau[\varphi(x\tau)\varphi^\dagger(x_3\tau_3)]\rangle\langle T_\tau[\varphi^\dagger(x_1\tau_1)\varphi(x_2\tau_2)]\rangle\}\,, \qquad (27)$$

where for simplicity of notation, we have dropped the spin subscripts. The term in the brackets with the minus sign is the disconnected part arising from the expansion of $\langle S\rangle^{-1}$. The integrand in Eq. (27) contains the overall factor $[(x_1-x_2)^2+(\tau_1-\tau_2)^2+\alpha^2]^{\mu-2}$ (times terms each of which is of the order unity as $|x_1-x_2|\to 0$ and $|\tau_1-\tau_2|\to 0$),[11] which are not present in the "mean-field" diagrams. This factor arises from the "disconnected" part of the integrand in Eq. (27), and would be absent if the expectation value with four φ fields could be factored as in mean-field theory. If we examine the contribution to χ_2 from the region in the integrand for small $|x_1-x_2|$ and $|\tau_1-\tau_2|$, however, we find that the remaining part of the integrand sums to zero for $x_1=x_2$ and $\tau_1=\tau_2$, and thus that for $\mu>0$, χ_2 does not depend upon α in an essential way. We therefore may write χ_2 as $V_1^3B(\alpha T)^{-4\mu}$, where $B\neq A^4$, and B depends only upon δ_+', v_F'/v_F'', and $\alpha\Delta$. Furthermore, for $\mu>0$, the contribution to χ_2 from the region in the integrand of small $|x_1-x_2|$ and $|\tau_1-\tau_2|$ is less than the contribution of that region χ_1, and it thus appears that $B\lesssim A^4$.

If we now examine the contribution χ_3 to $\chi^{nn'}$ of order V_1^3 shown diagrammatically in Fig. 3(c), we similarly find that for $\mu>0$, χ_3 may be written as $V_1^3C(\alpha T)^{-4\mu}$, where $B\neq C\neq A^4$, $C\lesssim A^4$, and C depends only upon δ_+', v_F'/v_F'', and $\alpha\Delta$. Similarly, to arbitrary order in V_1, the temperature dependence of each type of contribution to $\chi^{nn'}$ is the same as the "mean-field" diagram of that order in V_1. Thus, it is clear that if a phase transition exists, the exact transition temperature T_P will have the same dependence upon V_1 as does $T_P^{\rm MF}$, but is reduced by an overall factor that depends only upon v_F'/v_F'', δ_+', and $\alpha\Delta$. Let us now assume that each of the fluctuation (or "non-mean-field") diagrams can be neglected relative to the "mean-field" diagrams of the same order in V_1. This is the "self-avoiding random walk" approximation, and has been discussed by Domb[12] for the Ising and Heisenberg models. In this approximation, we only consider processes in which a particle-hole pair "tunnels" from chain to chain without ever returning to a chain that it has previously occupied. Within this approximation, we find

$$T_P^{\text{SARW}} = \gamma T_P^{\text{MF}} , \tag{28}$$

where $\gamma = \frac{1}{2}$ for a one-dimensional lattice of chains, and $\gamma \cong 0.6$ for a two-dimensional lattice of chains.

We notice that the region $\mu > 0$ is just the region in which $\chi_0^{\text{CDW}}(2k_F, 0)$ diverges as $T \to 0$. For $\tilde{g}_1 < 0$, the regions in g_1/g_2 space for which $\mu > 0$ are indicated for different values of V_2 in Fig. 3 of I, and in Figs. 1 and 2. On the line $\tilde{g}_1 = 0$, this region is just the region $\tilde{g}_2 > 0$. Examination of the fluctuation terms indicates that for $\delta_+' > 0$ they are of the same sign but smaller in magnitude than the "mean-field" terms of the same order in V_1, and we therefore expect the relationship $T_P^{\text{SARW}} \lesssim T_P < T_P^{\text{MF}}$ to hold. For a large region in g_1/g_2 space, we expect T_P^{SARW} to represent a more reliable estimate of T_P than does T_P^{MF}. A detailed analysis of the fluctuation effects will be presented elsewhere.[10]

V. TUNNELING BETWEEN CHAINS

In the previous two sections, we have investigated the effects of interchain Coulomb interactions, and found that the interchain backscattering interaction could only give rise to a phase transition of the charge-density-wave type. Thus, even in the region in g_1/g_2 space where the one-dimensional superconducting response function is divergent as $T \to 0$, the interchain Coulomb interactions are insufficient to allow for a superconducting phase transition. Apparently, a system in which the motion of the electrons is restricted to one dimension cannot be superconducting, even if the electron-electron interactions are of higher dimensionality. In order to discuss the phenomenon of superconductivity to quasi-one-dimensional metals, we must therefore introduce some additional dimensionality to the motion of the electrons. For a system of coupled chains, the relevant motion perpendicular to the chains is interchain hopping or tunneling. The Hamiltonian for this process is $\mathcal{H}_T$ where

$$\mathcal{H}_T = \frac{J}{2} \sum_{\substack{i=1,2 \\ \langle nn' \rangle \\ s=\pm 1}} \int dx\, \psi_{is}^{\dagger n'}(x)\, \psi_{is}^{n}(x) + \text{H.c.} . \tag{29}$$

In Eq. (29), we have assumed that the interchain transfer strength J is independent of chain index and position along the chains. The Hamiltonian is of the form of the usual transfer Hamiltonian,[13] and is equivalent to the tight-binding model for single-particle motion perpendicular to the chains.

We may calculate the effect of $\mathcal{H}_T$ upon the response functions by the usual $\mathcal{S}$ matrix formulation. For the singlet superconducting pain propagator in the presence of tunneling, we may write

$$\mathcal{P}^{nn'}(1\,2\,3\,4) = \langle T_\tau[\psi_{1+}^{\dagger n}(x_1\tau_1)\,\psi_{2-}^{\dagger n}(x_2\tau_2)\,\psi_{2-}^{n'}(x_3\tau_3)\,\psi_{1+}^{n'}(x_4\tau_4)\,\mathcal{S}]\rangle / \langle \mathcal{S} \rangle , \tag{30}$$

where

$$\mathcal{S} = T_\tau \exp \int_0^{1/T} \mathcal{H}_T(\tau)\, d\tau ,$$

and the operators are all in the Heisenberg representation with respect to the unperturbed Hamiltonian $\mathcal{H}_0 + \mathcal{H}_{fs}$. The inclusion of $\mathcal{H}_{fs}$ in the unperturbed Hamiltonian is not necessary for our discussion of the effects of tunneling, but since it does not present any further complications of significance, we have included it in order to make our investigation of interchain coupling as general as possible.

In order to discuss the long-range order included in the system by the interchain tunneling, we wish to calculate the Fourier transform of $\mathcal{P}^{nn'}(1100)$. Expanding $\mathcal{P}^{nn'}(1100)$ to second order in the tunneling interaction, we have for a square lattice of chains

$$\begin{aligned} \mathcal{P}^{nn'}(1100) = {} & P^0(1100)\delta_{n_1' n_1}\,\delta_{n_2' n_2} + \left(\frac{J}{2}\right)^2 \prod_{i=2}^{3} \int_{-\infty}^{\infty} dx_i \int_0^{1/T} d\tau_i \\ & - [\delta_{n_1' n_1}(\delta_{n_2' n_2+1} + \delta_{n_2' n_2-1}) + \delta_{n_2' n_2}(\delta_{n_1' n_1+1} + \delta_{n_1' n_1-1})]\, P^0(1132)\, P^0(2300) \\ & - 2\delta_{n_1' n_1}\,\delta_{n_2' n_2} \sum_{j=1}^{2} \sum_{s} G_{js}^0(23)\,[Q^0_{1+,2-,js}(112300) - G_{js}^0(32)\,P^0(1100)] + O(J^4) , \end{aligned} \tag{31}$$

where

$$G_{js}^0(12) = -\langle T_\tau[\psi_{js}(x_1\tau_1)\,\psi_{js}^\dagger(x_2\tau_2)]\rangle , \tag{32}$$

$$Q^0_{js\,j's'\,j''s''}(123456) = \langle T_\tau[\psi_{js}^\dagger(x_1\tau_1)\,\psi_{j's'}^\dagger(x_2\tau_2)\,\psi_{j''s''}^\dagger(x_3\tau_3)\,\psi_{j''s''}(x_4\tau_4)\,\psi_{j's'}(x_5\tau_5)\,\psi_{js}(x_6\tau_6)]\rangle \tag{33}$$

and where the fields in P^0, G^0, and Q^0 are all on the same chain. In Eq. (31), the chain indices n_1 and n_2 refer to column and row index, respectively, for the square lattice of chains. We note that for a triangular two-dimensional lattice of chains, the leading corrections (other than with regard to chain index) are of order J^3, whereas for a one-dimensional or two-dimensional square lattice of chains, they are of order J^4. If we examine Eq. (31) in detail, we observe that the term of order J^2 proportional to $\delta_{n_1' n_1}\delta_{n_2' n_2}$ is a single-particle process, as it corresponds to a single-particle tunneling from one chain to an adjacent chain, and then tunneling back to the original chain If we approximate Q^0 by its "mean-field" value, treating the fields at the same position and time as inseparable, then $Q^0_{1+,2-,js}(112300) = G^0_{js}(32)P^0(1100)$, and thus this term of order J^2 proportional to $\delta_{n_1' n_1}\delta_{n_2' n_2}$ vanishes. We note that it is reasonable to treat a product of two fields at the same position and (imaginary) time as inseparable since we calculate all expectation values in the Bose representation, and hence the product $\psi^\dagger_{1+}(x_1\tau_1)\psi^\dagger_{2-}(x_1\tau_1)$ acts like a single Bose field. With this prescription for "mean-field" theory, the only term of order J^2 is shown diagrammatically in Fig. 4(a). If we now consider the terms of order J^4, then there are four possible terms of order J^4 that exist in "mean-field" theory, shown diagrammatically in Figs. 4(b)–4(e). Of these four terms, the first two consist only of P^0 functions but the latter two consist of both P^0 and G^0 functions. These latter two processes, in which a single particle may tunnel to an adjacent chain independently of the other particles, do not allow as much interaction between the particles in question, and thus are less responsible for the pairing processes necessary for superconductivity (or for the particle-hole case, for long-range CDW ordering). We note that a consideration of the fluctuations (see Appendix C) leads us to the conclusion that in fact the most important process is the "pair self-avoiding random-walk" process shown diagrammatically in Fig. 4(b), and we thus expect by analogy with the discussion in Sec. III B that the true superconducting (or CDW) transition temperature T_c (or T_P) can be reasonably estimated by T_c^{PSARW} or (T_P^{PSARW}), its value in the "pair self-avoiding random-walk" approximation. However, since it is easier to handle the equations for the "mean-pair-field" approximation in which only terms consisting exclusively of P^0 functions are included, and since we do not expect T_c^{PSARW} to differ by a large factor from T_c^{MPF}, its value in the "mean-pair-field" approximation,[14] we shall consider here the inclusion of all "mean-pair-field" terms, such as those shown in Figs. 4(a)–4(c).

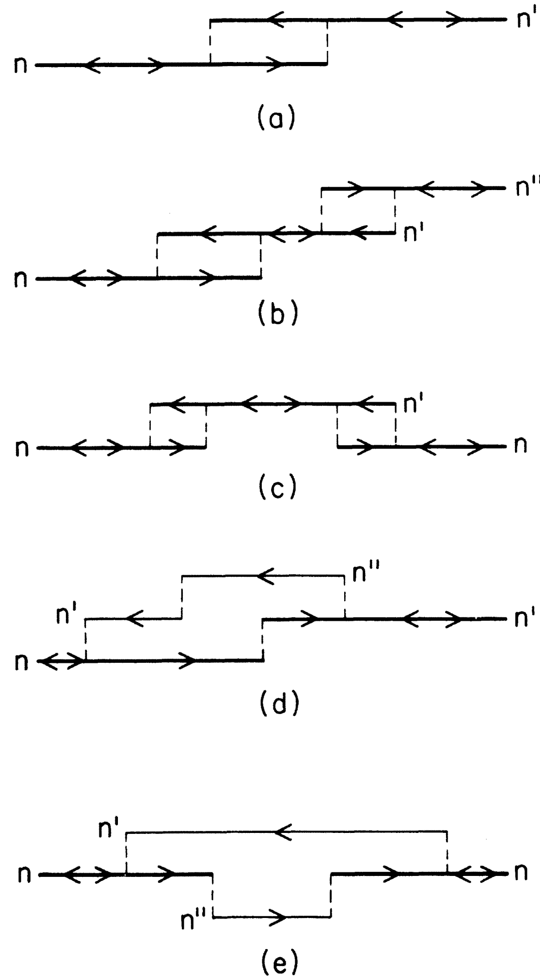

FIG. 4. Shown are the diagrams for the SS pair propagator in mean-field theory through fourth order in the single-particle tunneling interaction. The letters indicate chain indices, the heavy lines represent SS-pair propagators on a single chain, and the thin lines represent single-particle propagators on a single chain. The arrows indicate particles traveling in either the 1 or 2 direction. The diagram shown in Fig. 4(a) is the only one that contributes in mean-field theory to second order in J. In the "pair-mean-field" approximation, the diagrams shown in Figs. 4(d) and 4(e) are neglected.

To all orders in the tunneling interaction, the "mean-pair-field" terms are generated by the integral equation,

$$\mathcal{P}(1100)_{q_\perp} = P^0(1100) - 2J^2(\cos q_x d + \cos q_y d) \times \int_{-\infty}^{\infty} dx_2\, dx_3 \int_0^{T^{-1}} d\tau_2\, d\tau_3 \times P^0(1123)\mathcal{P}(3200)_{q_\perp}\,, \tag{34}$$

where $\mathcal{P}(1200)_{q\perp}$ is the Fourier series transform of $\mathcal{P}^{nn'}$ with respect to the chain indices. From Eq. (34), it appears that in order to calculate $\mathcal{P}(1100)_{q\perp}$, we also need an equation for $\mathcal{P}(1200)_{q\perp}$. Keeping only the "mean-pair field" terms, we have

$$\mathcal{P}(1200)_{q_\perp} = P^0(1200) - 2J^2(\cos q_x d + \cos q_y d)\int_{-\infty}^{\infty} dx_3\, dx_4 \int_0^{1/T} d\tau_3\, d\tau_4\, P^0(1234)\mathcal{P}(4300)_{q_\perp}, \tag{35}$$

which is an integral equation for $\mathcal{P}(1200)_{q_\perp}$ in terms of itself. The kernel $P^0(1234)$ depends upon three relative positions and (imaginary) times, and we shall now focus our attention upon its explicit form.

We may write $P^0(1234)$ in terms of its ρ and σ factors,

$$P^0(1234) = T(1234)(2\pi\alpha)^{-2} e^{ik_F(x_1 - x_2 + x_3 - x_4)} S_\rho^0(1234) S_\sigma^0(1234), \tag{36}$$

where T is the usual Fermion τ-order factor.

We shall first consider the case of $\tilde{g}_1 = 0$ (Luttinger chains), for which both S_ρ^0 and S_σ^0 may be easily calculated. For the particular τ ordering $\tau_1 > \tau_2 > \tau_3 > \tau_4$, we have

$$S_\rho^0(1234) = [f(1,4)f^*(2,3)]^{(\delta'_+ + \delta'_- + 2)/8}[f(2,3)f^*(1,4)]^{(\delta'_+ + \delta'_- - 2)/8}\left(\frac{f(1,2)f^*(1,2)f(3,4)f^*(3,4)}{f(2,4)f^*(2,4)f(1,3)f^*(1,3)}\right)^{(\delta'_+ - \delta'_-)/8} \tag{37}$$

and

$$S_\sigma^0(1234) = [f'(1,4)f'^*(2,3)]^{1/2}, \tag{38}$$

where

$$f(1,2) = \frac{\alpha\pi T[\,|\tau_1 - \tau_2| - i(x_1 - x_2)/v'_F]}{[\alpha + v'_F|\tau_1 - \tau_2| - i(x_1 - x_2)]\sin\{\pi T[\,|\tau_1 - \tau_2| - i(x_1 - x_2)/v'_F]\}}, \tag{39}$$

$f' = f$ with v'_F replaced by v''_F, and $\delta'_\pm$ is given by Eqs. (16) and (17) with the $\frac{3}{5}$ replaced by 0.

We observe that Eq. (37) simplifies greatly at the values of $\tilde{g}_2$ and $\tilde{V}_2$ for which $\delta'_+ = \delta'_- \equiv \delta'$, which is the point on the Luttinger "line" (in g_1/g_2 space) that separates the region of superconductivity from that of Peierls-Fröhlich behavior for the one-dimensional chain at $T = 0$. We note that the analogous kernel for CDW (and SDW, for the case $\tilde{g}_1 = 0$) is identical to that for SS (and TS, for $\tilde{g}_1 = 0$), with the quantities δ'_+ and δ'_- interchanged. Thus, at $\delta'_+ = \delta'_- = \delta'$, we conclude that the transition temperatures for superconductivity and Peierls-Fröhlich behavior are identical, provided that such a transition exists.[15]

For completeness let us assume that $\tilde{V}_2$ is independent of $\tilde{g}_2$, so that δ' may be greater than 1. Since $P^0(1234)$ factors into two parts, one depending upon $x_1 - x_4$ and $\tau_1 - \tau_4$, and the other depending upon $x_2 - x_3$ and $\tau_2 - \tau_3$, we may take advantage of the usual Fourier transform techniques to reduce the integral equations given by Eqs. (34) and (35) to algebraic equations.

We write

$$\mathcal{P}(1200)_{q_\perp} = (2\pi\alpha)^{-2} e^{ik_F(x_1 - x_2)} \mathcal{P}'(1200)_{q_\perp}, \tag{40}$$

and we let $\tilde{\mathcal{P}}'_{q_\perp}(q, \omega_n; q', \omega_m)$, $\tilde{P}^{0\prime}(q, \omega_n; q', \omega_m)$, $\tilde{G}_1(q, \omega_n)$, and $\tilde{G}_2(q, \omega_n)$ be the Fourier transforms of $\mathcal{P}'(1200)_{q_\perp}$, $P^{0\prime}(1200)_{q_\perp}$, $G_1(1,2) = T(1,2) \times f(1,2)^{(\delta'+1)/4} f^*(1,2)^{(\delta'-1)/4} f'(1,2)^{1/2}$ and its complex conjugate $G_2(1,2)$, respectively, where $T(1,2) = \mathrm{sgn}(\tau_1 - \tau_2)$, and $\omega_n = (2n+1)\pi T$ is the fermion Matsubara frequency.

From Eqs. (34) and (35), we find

$$\tilde{\mathcal{P}}'(q' = 0, \omega_m = 0)_{q_\perp} = \tilde{P}^{0\prime}(0,0) + T\sum_{\omega_n}\int \frac{dq}{2\pi} \frac{\tilde{G}_1(q, \omega_n)\tilde{G}_2(-q, -\omega_n)\tilde{P}^{0\prime}(q, \omega_n; -q, -\omega_n)}{1 - [2J^2/(2\pi\alpha)^2](\cos q_x d + \cos q_y d)\tilde{G}_1(q, \omega_n)\tilde{G}_2(-q, -\omega_n)}, \tag{41}$$

where $\tilde{\mathcal{P}}'(q', \omega_m)_{q_\perp}$ is the Fourier transform of $\mathcal{P}'(1100)_{q\perp}$, and we have set $q', \omega_m = 0$ as we are interested in the highest temperature at which long-range order arises. The denominator in the right-hand side of Eq. (41) is the largest for $\omega_n = \pi T$ and $q_x, q_y = 0$, and an expansion for small q reveals that $\tilde{\mathcal{P}}'$ diverges at

$$T_c^{\mathrm{MPF}} = T_p^{\mathrm{MPF}} = \left[J^2\left(\frac{v'_F}{\alpha}\right)^{1-\delta'} g(\delta', v'_F/v''_F)\right]^{1/(3-\delta')}, \tag{42}$$

where

$$g(x,y) = \frac{y}{\pi^{5-x}}\left|\int_{-\infty}^{\infty} d\alpha \int_0^{\pi} d\beta\, e^{i\beta} \sin^{-1/2}(\beta - i\alpha y)\sin^{-(x-1)/4}(\beta + i\alpha)\sin^{-(x+1)/4}(\beta - i\alpha)\right|^2. \tag{43}$$

For $\delta'_+ \neq \delta'_-$, this factorization does not apply. We note that the analogous equation to Eq. (37) for the CDW case is identical to Eq. (37) with δ'_+ and δ'_- interchanged. Thus, in the "mean-pair-field" approximation the line $\delta'_+ = \delta'_-$ separates the regions of superconducting and Peierls-Fröhlich phase transitions. Expanding $S^0_\rho(1234)$ for small $\epsilon = \delta'_+ - \delta'_-$, we have the equation from which T_c^{MPF} and T_P^{MPF} are determined:

$$1 = \frac{J^2}{(\pi\alpha)^2}\bigg(\tilde{G}_2(0,\omega_{-1}) \pm \tfrac{1}{8}(\delta'_+ - \delta'_-)\int \frac{dp}{2\pi}\, T\sum_{\omega_n}\tilde{Z}(p,\omega_n)\{\tilde{G}_1(-p,\omega_{1-n})[\tilde{G}_2(p,\omega_{n-1}) - \tilde{G}_2(0,\omega_{-1})] + \tilde{G}_1(0,\omega_1)[\tilde{G}_2(0,\omega_{-1}) - G_2(-p,\omega_{1-n})]\} + O((\delta'_+ - \delta'_-)^2)\bigg), \quad (44)$$

where the upper (lower) sign refers to superconducting (Peierls-Fröhlich) behavior, respectively, and $\tilde{Z}(p,\omega_n)$ is the Fourier transform of $Z(1,2) = T(1,2)\ln|f(1,2)|^2$. Since $\tilde{Z}(p,\omega_n)$ is sharply peaked for small p and ω_n, and since the leading $\tilde{G}_1\tilde{G}_2\tilde{Z}$ factor in the correction term (for $p\approx 0$, $n=1$) is positive, we see that for $\delta'_+ - \delta'_- > 0$, $T_c^{\text{MPF}} > T_P^{\text{MPF}}$, and for $\delta'_+ - \delta'_- < 0$, we have $T_P^{\text{MPF}} > T_c^{\text{MPF}}$. This is consistent with the behavior of the superconducting and Peierls-Fröhlich response functions at $T=0$ in the absence of tunnelling.

We now consider the case of $\tilde{g}_1 < 0$. We may still write $P^0(1234)$ as in Eq. (36) where $S^0_\rho(1234)$ is given by Eq. (37) with $\delta'_\pm$ given by Eqs. (16) and (17). However, $S^0_\sigma(1234)$ is no longer given by Eq. (38), as for $\tilde{g}_1 < 0$, there are gaps in the spectra of the magnon modes. In particular, for $\tilde{g}_1 = -\frac{6}{5}$, we have for SS

$$S^0_\sigma(1234) = (2\pi\alpha)^2\langle \mathcal{O}_1^\dagger(x_1\tau_1)\mathcal{O}_2(x_2\tau_2)\mathcal{O}_2^\dagger(x_3\tau_3)\mathcal{O}_1(x_4\tau_4)\rangle, \quad (45)$$

where $\mathcal{O}_1(x\tau) = e^{\tilde{H}_\sigma\tau}\psi_2^{3/4}(x)\psi_1^{1/4}(x)e^{-\tilde{H}_\sigma\tau}$, and $\mathcal{O}_2(x\tau)$ is the same as $\mathcal{O}_1(x\tau)$ with the indices 1 and 2 interchanged, and where $\tilde{H}_\sigma$ is given by Eq. (16) of I. Clearly, the nonintegral powers of the operators greatly complicate S^0_σ. However, we may compare Eq. (45) with the zero temperature S^-_σ for two positions and times given by Eq. (29) in I. We expect that there will be many contributions to S^0_σ that depend upon three relative positions and times, but that there will also be contributions analogous to the constant term in Eq. (33) of I that now depend upon only two relative positions and times. This can more clearly be seen by expanding $S^0_\sigma(1234)$ in powers of $U_\perp$ (as in Chui and Lee[8]), and examining the terms of order $U_\perp^2$. These terms may be approximated by assuming that S^0_σ can be factored as in mean-field theory,

$$S^0_\sigma(1234) \approx (2\pi\alpha)^2[\langle \mathcal{O}_1^\dagger(x_1\tau_1)\mathcal{O}_2(x_2\tau_2)\rangle\langle \mathcal{O}_2^\dagger(x_3\tau_3)\mathcal{O}_1(x_4\tau_4)\rangle + \langle \mathcal{O}_1^\dagger(x_1\tau_1)\mathcal{O}_1(x_4\tau_4)\rangle \times\langle \mathcal{O}_2(x_2\tau_2)\mathcal{O}_2^\dagger(x_3\tau_3)\rangle]. \quad (46)$$

In order to calculate these expectation values, we assume that the dominant contribution arises from a further factorization of fields of like powers, such as

$$\langle \mathcal{O}_1^\dagger(x_1\tau_1)\mathcal{O}_1(x_4\tau_4)\rangle \approx \langle \psi_2^{\dagger 3/4}(x_1\tau_1)\psi_2^{3/4}(x_4\tau_4)\rangle \times\langle \psi_1^{\dagger 1/4}(x_1\tau_1)\psi_1^{1/4}(x_4\tau_4)\rangle. \quad (47)$$

We further assume that the nonintegral powers can be treated as in Paper I, as if they were essentially unimportant. Equation (47) may also contain a phase factor arising from the fermion nature of the factorization procedure, but this can be incorporated into the overall τ-ordering factor $T(1234)$. That this last approximation makes some sense can be seen by noting that $\mathcal{O}_1(x\tau)$ is essentially a single fictitious fermion operator, consisting of both ψ_1 and ψ_2 character. If we let $\mathcal{O}_1(x\tau)$ be either $\psi_1(x\tau)$ or $\psi_2(x\tau)$, we obtain essentially the same result for $\langle \mathcal{O}_1^\dagger(x_1\tau_1)\mathcal{O}_1(x_4\tau_4)\rangle$ as by the procedure we have described. We have (see Appendix A of I)

$$S^0_\sigma(1234) \approx \left(\frac{\alpha\Delta}{v''_F}\right)^2\left[K_0(\Delta S_{12}S^*_{12})K_0(\Delta S_{34}S^*_{34}) + \left(\frac{S_{14}S_{23}}{S^*_{14}S^*_{23}}\right)^{1/2}K_1(\Delta S_{14}S^*_{14})K_1(\Delta S_{23}S^*_{23})\right], \quad (48)$$

where $S^2_{jl} = \alpha + |\tau_j - \tau_l| - i(x_j - x_l)/v''_F$, and K_0 and K_1 are Bessel functions.

We now examine Eq. (35) with $P^0(1234)$ given by Eqs. (36), (37), and (48). Since the integrations over τ_3 and τ_4 can be extended from $-1/T$ to $1/T$, we see that the second term in Eq. (48) gives for $\delta'_+ = \delta'_-$ a vanishingly small contribution to $\mathcal{P}(1200)_{q_\perp}$ as $T\to 0$, because of the presence of the

$$\exp\{-\Delta[(\tau_3-\tau_2)^2 + (x_3-x_2)^2/(v''_F)^2]^{1/2}\}$$

factor in the asymptotic form of the K_1 Bessel function. The first term in Eq. (48) gives a finite contribution, however, which for $\delta'_+ = \delta'_-$ is at first sight considerably more complicated in form than for $\tilde{g}_1 = 0$. However, we note that the K_0 Bessel function is sharply peaked for small argument, owing to the exponential factor in the asymptotic region. If we consider the case of zero temperature, we find that the σ modes give a restriction upon the position and time that the second particle may tunnel to the adjacent chain, relative to the position and time of the tunnelling of the first particle. That is, when the first particle tunnels at (x_1, τ_1), it creates a disturbance in the spin-density modes in the first chain owing to the removal of a fermion from a system consisting entirely of collective modes. This disturbance propagates with velocity v_F'' in both directions, and at some later time τ_2, it causes a second particle to tunnel to the same adjacent chain. Thus, the particles essentially tunnel in pairs, although not necessarily at the same position and time. If two particles are at positions x_1 and x_2, and the first one tunnels at time t_1, the second will tunnel at the time $t_2 = t_1 + |x_2 - x_1|/v_F''$, when it receives the information via the spin-density disturbance that the first particle has tunnelled.

We note that had we included terms in $S_\sigma^0(1234)$ that did not factor as in Eq. (46), we would have picked up factors proportional to

$$\exp\{-\Delta[(\tau_3-\tau_2)^2+(x_3-x_2)^2/(v_F'')^2]^{1/2}\},$$

which when integrated with respect to τ_3, τ_2, x_3, and x_2 in Eq. (35), would give a negligible contribution. A similar result is also obtained if one assumes (as in I) that the factorization does not take place in the manner discussed, but occurs only in that fields of like powers are combined together in separate expectation values. This procedure gives us two quantities, one raised to the $\frac{3}{4}$ power, and one to the $\frac{1}{4}$ power, such as in Eq. (A5) of I. However, expanding these quantities in a power series gives us only one term (identical to the one we have kept by the procedure we have discussed in detail) that does not contain any of the exponential factors in $\tau_3 - \tau_2$ and $x_3 - x_2$, and thus only this term survives the integrations in Eq. (35).

If we now replace the K_0 Bessel functions in Eq. (48) by "delta functions," that is, at $T=0$, $S_\sigma^0 \propto \delta(x_1 - x_2 \pm v_F''(t_1-t_2))\delta(x_3-x_4 \pm v_F''(t_3-t_4))$, which leads to an instability for $\delta_+' = \delta_-' = \delta'$ at

$$\omega = \omega_0 = [\kappa J^2/(\alpha v_F')^{1-\delta'}]^{1/(3-\delta')} \tag{49}$$

in the frequency spectrum of $\mathcal{P}(q=0,\omega)_{T=0}$, where κ depends only upon $\alpha\Delta$, δ', and v_F''/v_F'. At finite temperatures, we consider these Bessel functions to give poles at $x_1 - x_2 = \pm i v_F''(\tau_1-\tau_2)$ and $x_3 - x_4 = \pm i v_F''(\tau_3 - \tau_4)$. Transforming to the variables x_1-x_2, x_1+x_2, x_3-x_4, and x_3+x_4 (and similarly for the τ's, we may solve Eq. (34) directly, and obtain for $\delta_+' = \delta_-' = \delta'$,

$$T_c^{\text{MPF}} = T_P^{\text{MPF}} \cong [J^2 h(\alpha\Delta, \delta', v_F''/v_F')/(\alpha/v_F')^{1-\delta'}]^{1/(3-\delta')}, \tag{50}$$

where

$$h(x,y,z) = \frac{2xK_0(x)}{\pi^{5-y}} \int_{-\infty}^{\infty} d\alpha \int_0^\pi d\beta \int_0^\pi d\gamma\, e^{i(\beta-\gamma)}\{\sin(\beta - i\alpha)\sin[\gamma - z(\beta-\gamma)+i\alpha]\}^{-(y+1)/4} \times\{\sin(\beta+i\alpha)\sin[\gamma - z(\beta-\gamma)-i\alpha]\}^{-(y-1)/4}. \tag{51}$$

We note that this expression is quite similar to that of Eq. (42) in that in both cases, for $\delta_+' = \delta_-' = \delta'$, we have $T_c^{\text{MPF}} = T_P^{\text{MPF}} \sim J^{2/(3-\delta')}$, where δ' is given by Eqs. (16) and (17) with the $\frac{3}{5}$ replaced by $-\frac{1}{2}\tilde{g}_1$. This relation appears to hold along the entire curve $\delta_+' = \delta_-'$ in the g_1/g_2 plane by suitable arguments analogous to those of Chui and Lee.[8] By arguments similar to those discussed previously for $\tilde{g}_1 = 0$, it thus appears that for $\tilde{g}_1 \leq 0$, we have $T_P^{\text{MPF}} > T_c^{\text{MPF}}$ for $\delta_+' < \delta_-'$, and $T_c^{\text{MPF}} > T_P^{\text{MPF}}$ for $\delta_-' < \delta_+'$. Thus, in the entire lower-half g_1/g_2 plane, the line $\delta_+' = \delta_-'$ separates the regions of superconducting and CDW phase transitions.

We remark that if we examine Eq. (37) for the ρ part of the kernel for $\delta_+' \neq \delta_-'$, and consider the σ part (for the LE model) to be delta functions, we may make the following observation. If we consider only processes in which pairs tunnel from chain to chain (albeit at different positions and times), but the first particle does not tunnel to a third chain before the second particle tunnels to the second chain, then we have the interesting results that $T_c^{\text{MPF}} \sim W(J/W)^{2/(3-\delta_-')}$ and $T_P^{\text{MPF}} \sim W(J/W)^{2/(3-\delta_+')}$, where W is the bandwidth. These results have the correct property that $T_c^{\text{MPF}} = T_P^{\text{MPF}}$ for $\delta_+' = \delta_-'$, $T_c^{\text{MPF}} > T_P^{\text{MPF}}$ for $\delta_-' < \delta_+'$, and $T_P^{\text{MPF}} > T_c^{\text{MPF}}$ for $\delta_-' > \delta_+'$.

VI. DISCUSSION

We have considered the effects of interchain coupling upon a lattice of interacting electron-gas chains. For the uncoupled lattice of chains, each chain is considered to have two types of interactions between the electrons, low-momentum-transfer, and large-momentum-transfer scattering processes. Of the first type, we consider g_4, the strength of the forward-scattering interaction for particles on the same side of the Fermi "surface," to equal g_2, the strength of the forward-scattering interaction for particles on opposite sides of the Fermi "surface." Of the large-momentum-transfer processes, we consider here only the process (with strength g_1) of the backward scattering between particles on opposite sides of the Fermi "surface," as we assume that the conduction band is not exactly half-filled, and thus that the umklapp processes do not contribute.

Of the possible types of interchain interactions, we consider the three types of nearest-neighbor interchain electron-electron interactions analogous to the three interchain interactions, and we also assume $V_4 = V_2$. The inclusion of the nearest-neighbor interchain forward-scattering processes does not qualitatively change the picture of the low-temperature response behavior of the purely uncoupled lattice of interacting electron-gas chains, as the chains appear to an external perturbation to be uncoupled. Thus, the only changes in the low-temperature response functions owing to the inclusion of interchain forward scattering are the renormalizations of the exponent in the low-frequency behavior from $-2+\delta_\pm$ to $-2+\delta'_\pm$ and of the plasmon Fermi velocity v'_F. The effect in g_1/g_2 space of the inclusion of this interaction is to distort the regions of SS and CDW behavior (for $\tilde{g}_1 \leq 0$) to more positive values of g_2, and thus appears to enhance the tendency towards SS ordering and to suppress the tendency towards CDW ordering for fixed g_1, g_2 as $T \to 0$. However, the region in which the model may be solved (by the Bogoliubov transformation employed by Mattis and Lieb[7]) is more grossly distorted to more positive values of g_2, and thus it is difficult to conclude with certainty that the region in g_1/g_2 space for which the SS response function diverges as $T \to 0$ is larger for $|\tilde{V}_2| > 0$ than for $\tilde{V}_2 = 0$. A power-series expansion in $\tilde{V}_2$ gives support to this conclusion, however.

The presence of interchain backscattering, however, causes a dramatic change in the low-temperature response behavior of the coupled system. This interaction greatly enhances the formation of CDW-type order, and in fact can give rise to a phase transition of the entire lattice. We have shown that the CDW response for the lattice has contributions from particle-hole pairs on different chains, and thus the ordering of the system as T_P is approached from above is not restricted to the single dimension of the electronic motion. This transition temperature was estimated in mean-field theory, and an analysis of the fluctuations leads us to conclude that T_P is not greatly reduced from T_P^{MF}, but may more accurately be predicted from T_P^{SARW}, the transition temperature in the "self-avoiding random-walk" approximation.

With regard to the effect of the interchain backscattering interaction upon the superconducting and SDW responses, however, we have shown that the orderings of these types only grow in the single dimension of the electronic motion. Although we have not attempted to calculate those response functions in detail, this fact alone insures that long-range order of these types could occur only at $T = 0$. Thus, for the SS response, which in the absence of V_1 was characterized by an exponent $-2+\delta'_-$, is now apparently characterized by a new exponent $-2+\delta''_-$, where δ''_- depends upon $\tilde{g}_1$, $\tilde{g}_2$, $\tilde{V}_2$, and $\tilde{V}_1$.

Since the interchain scattering interactions are not sufficient to cause the system to undergo a superconducting phase transition, we have also considered the effect of additional degrees of freedom in the motion of the electrons. Since for a "quasi-one-dimensional" metal, the electrons may propagate more easily in the metallic chains than perpendicular to them, we have assumed the tight-binding model for their motion perpendicular to the chains. This is equivalent to including single-particle tunnelling between the chains.

The interchain tunnelling interaction profoundly alters all of the four response functions of the coupled system, since all of them include correlations between particles (and/or holes) on different chains. However, as the TS and SDW responses in the absence of tunnelling vanish as $T \to 0$ owing to the presence of the gap in the σ modes, we need only consider the effect of tunnelling upon the SS and CDW response functions. Expanding the SS and CDW pair and particle-hole propagators, respectively, we find that the important contributions are from "pair self-avoiding random-walk" tunnellings, in which pairs (or particle-hole pairs, as the case may be) tunnel from chain-to-chain albeit at different positions and (imaginary) times, never returning to a previously occupied chain. For a one-dimensional lattice of chains, this implies that the pair propagates from chain to chain, always tunnelling in the same direction, and the form of the equations for this approximation differs only by a factor of 2 from that in the "pair-mean-field" approximation, in which the particles may tunnel as pairs in either direction (albeit at different positions and imaginary times). For a two-dimensional lattice of chains, however, the combinatorics of the "pair-self-avoiding random-walk" approximation is considerably more complicated than for the "pair-mean-field" approximation, though the power dependences of the transition temperatures upon the tunnelling strength J are identical. For these reasons, we use the "pair-mean-field" approximation to investigate the onset of the phase transitions brought about by tunnelling, recognizing that the proper combinatorics of the "pair-self-avoiding random-walk" approximation will decrease T_c (or T_P) by a factor of 2 for a one-dimensional lattice, and

$\simeq 1.7$ for a square two-dimensional lattice.

Examination of the kernel of the resulting integral equation reveals that at $\delta'_+ = \delta'_-$, the plasmon modes have free-particle behavior (as we might trivially expect), and for $\tilde{g}_1 = 0$, the integral equation may be solved by Fourier transformation. For $\tilde{g}_1 < 0$, the magnon (σ) modes give a restriction upon the relative positions and times at which two particles (or a particle and a hole) may tunnel from one chain to the next. That is, when one particle tunnels to an adjacent chain, it sets up a disturbance in the spin-density modes that eventually causes another particle (or hole) to do likewise. We thus conclude that the quasi-one-dimensional system prefers that the particle (and/or holes) act in some sense as pairs, and that a phase transition of either the SS or CDW type is possible. For $\delta'_+ > \delta'_-$, we have $T_c > T_P$, and for $\delta'_- > \delta'_+$, we have $T_P > T_c$.

Let us now examine the plots in g_1/g_2 space for $\tilde{g}_1 < 0$ and for the values of $\tilde{V}_2$ shown in Figs. 1 and 2 and Fig. 3 of I. In the region $\delta'_- > \delta'_+$, where in the absence of tunnelling and interchain backscattering the low-temperature CDW response is more divergent than the low-temperature SS response, the system may only undergo a CDW phase transition, arising from either tunnelling or interchain backscattering. In the region $\delta'_+ > 2$, the system may only undergo a SS phase transition, brought about by the tunnelling of electrons from chain-to-chain. In the region $2 > \delta'_+ > \delta'_-$, either type of long-range order is possible, as the tunnelling interaction favors the SS type of long-range order over that of the CDW type, but the interchain backscattering interaction also contributes to CDW long-range order. In this region in g_1/g_2 space, these two effects are thus strongly competing, and whichever transition temperature is higher depends upon all the parameters of the system.

We remark that for the interactions we have considered, T_c and T_P depend upon the interchain interactions via power laws, the power depending upon the effective intrachain interactions. If this power is greater than 1, then a strong interaction relative to the respective energy scale is required for the transition temperature to be of the order of the bandwidth. If the power is small (say less than 1), then a weak interaction relative to the respective energy scale will induce a transition at a temperature comparable to the bandwidth. For superconductivity, the energy scale is W, the bandwidth, and the power is less than 1 for $\delta'_- < 1$. However, the smallest possible power is $\frac{2}{3}$, so that it is unlikely that T_c will ever be of the same order of magnitude as the bandwidth (as we always have $J/W \ll 1$). For the charge-density-wave case, however, there are two possible mechanisms that may set up a phase transition. The single-particle tunnelling has the energy scale W, and the power is less than 1 for $\delta'_+ < 1$, and thus may cause an instability at a temperature considerably less than W. On the other hand, the interchain backscattering has the energy scale (πv_F), the inverse density of states, and also has a power <1 for $\delta'_+ < 1$, and so it will dominate over the tunnelling mechanism in that region, and may give a Peierls transition at a temperature comparable to the bandwidth. In the region for $\delta'_+ > \delta'_-$ where both response functions are divergent at $T = 0$ in the absence of interchain couplings, the superconducting transition temperature will be comparable to the Peierls transition temperature when $(J/W)^{2/(3-\delta'_-)} \sim \tilde{V}_1^{1/(2-\delta'_+)}$, where $\tilde{V}_1 = V_1/\pi v_F$. Of course this relation will be modified by considering the effect of V_1 upon the superconducting response, which should change δ'_- to δ''_-, where δ''_- depends upon $\tilde{g}_1$, $\tilde{g}_2$, $\tilde{V}_1$, and $\tilde{V}_2$.

If we attempt to correlate our results with real materials, we may conclude that quasi-one-dimensional materials such as $(SN)_x$ that are superconducting may have $\tilde{g}_1$ and $\tilde{g}_2$ in the region where the one-dimensional superconducting response function is more divergent than the one-dimensional CDW response function, and that interchain tunnelling is important. For materials exhibiting a Peierls instability, such as KCP and TTF-TCNQ, the predominate mechanism is most likely interchain backscattering, although tunnelling may also contribute somewhat to the instability. It is possible that a measure of $\tilde{g}_1$ and $\tilde{g}_2$ may be given by the optical spectrum of the materials.

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APPENDIX A

In this Appendix, we wish to elaborate on the assertion that the interchain backscattering only correlates the chains in the charge-density-wave manner, which is the implication of Eq. (22) of the text. For the superconducting responses, we have

$$\chi_S^{nn'}(x,\tau) = \langle T_\tau [\psi_{1+}^{\dagger n}(x\tau)\psi_{2\mp}^{\dagger n}(x\tau)\psi_{2\mp}^{n'}(00)\psi_{1+}^{n'}(00)\,\mathcal{S}]\rangle / \langle \mathcal{S}\rangle , \tag{A1}$$

where the upper (lower) sign refers to SS and TS behavior and

$$\mathcal{S} = T_\tau \exp \int_0^{1/T} \mathcal{H}_{bs}(\tau)\, d\tau ,$$

where $\mathcal{H}_{bs}(\tau)$ is the Heisenberg representation of Eq. (18) in the text, and the unperturbed Hamiltonian $\mathcal{H}_0$ is given by Eq. (1) of the text. For simplicity, we set $\tilde{V}_2 = 0$ and consider only a one-dimensional lattice of chains, although neither of these restrictions is necessary. To order V_1, we have

$$\chi_S^{nn'}(10) = \chi_S^0(10)\delta_{n'n} + V_1 \sum_{ss'} \sum_l \int_{-\infty}^{\infty} dx_2 \int_0^{1/T} d\tau_2 \langle T_\tau \{\psi_{1+}^{\dagger n}(1)\psi_{2\mp}^{\dagger n}(1)\psi_{1s}^{\dagger l}(2)[\psi_{2s'}^{\dagger l+1}(2)\psi_{1s'}^{l+1}(2)$$
$$+\psi_{2s'}^{\dagger l-1}(2)\psi_{1s'}^{l-1}(2)]\psi_{2s}^{l}(2)\psi_{2\mp}^{n'}(0)\psi_{1+}^{n'}(0)\}\rangle + O(V_1^2). \qquad \text{(A2)}$$

Since

$$\langle \psi_{2s'}^{\dagger l}(2)\psi_{1s'}^{l}(2)\rangle = 0,$$

there are no disconnected terms, and thus the only possible terms are for $n' = n$, $n \pm 1$. If $n' = n+1$, we have two terms of order V_1, one of which is proportional to

$$\int_{-\infty}^{\infty} dx_2 \int_0^{1/T} d\tau \langle T_\tau [\psi_{1+}^{\dagger n}(1)\psi_{2\mp}^{\dagger n}(1)\psi_{1s}^{\dagger n}(2)\psi_{2s}^{n}(2)]\rangle\langle T_\tau[\psi_{2s'}^{\dagger n+1}(2)\psi_{1s'}^{n+1}(2)\psi_{2\mp}^{n+1}(0)\psi_{1+}^{n+1}(0)]\rangle \qquad \text{(A3)}$$

and the other is similar. Since these expectation values contain an unequal number of $\psi_{is}^\dagger$ fields, the factors arising from the plasmon modes have exponents which diverge logarithmically to $-\infty$, so that the overall contribution from the plasmon modes is 0. Similarly, for terms of higher order in V_1, for $n' \neq n$ there will always be an unequal number of ψ_{is} and $\psi_{is}^\dagger$ fields, so that to every order in V_1, the plasmon modes give a total contribution of 0. For $n' = n$, however, there will be finite contributions to $\chi_S^{nn}(10)$ of even order in V_1.

For the spin-density-wave-response function, the argument that $n' \neq n$ terms vanish is similar, except that it involves the σ or spin-density parts of the expectation values, rather than the ρ or plasmon modes. This is easiest to see for the Luttinger model, but can also be understood for the LE model by noting that every $n' \neq n$ term contains spin-density expectation values with unequal powers of daggerred and undaggerred fictitious fermion operators. Thus, those expectation values represent "incomplete" processes of creating and destroying fictitious fermions, and must vanish identically. By "incomplete" we mean that an unequal fraction of fictitious fermions is created and destroyed in the process, and thus, the final states of the system are orthogonal.

APPENDIX B

We wish to calculate $\chi_{\text{CDW}}^0(2k_F, 0)$, the Fourier transform of

$$\chi_{\text{CDW}}^0(x,\tau) = \langle T_\tau[\varphi_+^n(x,\tau)\varphi_+^{n\dagger}(0,0)]\rangle \qquad \text{(B1)}$$

at finite temperature. To do so, we calculate $S(x,\tau)$, the finite temperature correlation function, and define

$$S(x,\tau) = (2\pi\alpha)^{-2} e^{2ik_F x} S_\rho(x,\tau) S_\sigma(x,\sigma). \qquad \text{(B2)}$$

We have

$$S(x,\tau) = \left| \frac{\alpha\pi T(\tau - ix/v_F')}{(\alpha + v_F'\tau - ix)\sin[\pi T(\tau - ix/v_F')]} \right|^{\delta_+'}, \qquad \text{(B3)}$$

where δ_+' is given by Eqs. (16) and (17) of the text with the $\frac{3}{5}$ replaced by $-\frac{1}{2}\tilde{g}_1$, and v_F' is the renormalized velocity of the plasmon modes. For the Luttinger model, we find

$$S_\sigma(x,\tau) = \left| \frac{\alpha\pi T(\tau - ix/v_F)}{(\alpha + v_F\tau - ix)\sin[\pi T(\tau - ix/v_F)]} \right|, \qquad \text{(B4)}$$

and for the LE model, we have

$$S_\sigma(x,\tau) = (v_F/v_F'')\alpha\Delta K_0(\alpha'\Delta). \qquad \text{(B5)}$$

Since

$$\chi_{\text{CDW}}^0(2k_F, 0) = (2\pi\alpha)^{-2} \int_0^{1/T} d\tau \int_{-\infty}^{\infty} dx\, S_\rho(x,\tau) S_\sigma(x,\tau), \qquad \text{(B6)}$$

we have

$$\chi_{\text{CDW}}^0(2k_F, 0) = (\alpha T)^{-\mu} A, \qquad \text{(B7)}$$

where A depends upon δ_+', v_F''/v_F', $\alpha\Delta$, and $\alpha'\Delta$, and $\mu = -2 + \delta_+'$ for the LE model and $-1 + \delta_+'$ for the Luttinger model. For $\mu \leq 0$, A also depends upon temperature, as the finiteness of α prevents the integrals from diverging. For the Luttinger model, we have

$$A = \frac{1}{4\pi^2 v_F'} \left(\frac{v_F'}{\pi v_F}\right)^{1-\delta_+'} \int_0^{\pi} d\alpha \int_{-\infty}^{\infty} d\beta\, |\sin(\alpha - i\beta)|^{-\delta_+'}$$
$$\times \sin[\alpha - (v_F'/v_F)\beta]|^{-1}, \qquad \text{(B8)}$$

and for the LE model we have

$$A=\frac{\alpha\Delta K_0(\alpha'\Delta)v_F}{4\pi^3(v_F''v_F')}\left(\frac{v_F'}{\pi v_F}\right)^{1-\delta_+'}\times\int_0^\pi d\alpha\int_{-\infty}^{\infty}d\beta\,|\sin(\alpha-i\beta)|^{-\delta_+'}. \tag{B9}$$

APPENDIX C

Let us consider the terms of order J^4 in the expansion for the finite temperature SS-pair propagator. Let us define

$$\varphi(1)\equiv\psi_{2-}(x_1\tau_1)\psi_{1+}(x_1\tau_1). \tag{C1}$$

For a one-dimensional lattice of chains, we have three separate terms. The first which is the same in mean-field theory, has the value $\chi_1(\delta_{n',n+2}+\delta_{n',n-2})$ where

$$\chi_1=J^4\prod_{i=1}^{5}\int_{-\infty}^{\infty}dx_i\int_0^{1/T}d\tau_i P_0(1123)P_0(3245)P_0(4500), \tag{C2}$$

and corresponds to the Fourier transform (at $q=0,\omega_n=0$) of the diagrams shown in Fig. 4(b). χ_1 may be calculated by Fourier transformation at $\delta_+'=\delta_-'=\delta'$, and has the value $J^4A^3(\alpha T)^{-3\nu}$, where $\nu=3-\delta'$, and A depends upon $\alpha\Delta$, δ', and the renormalized Fermi velocities. The second term has the value $\chi_2(\delta_{n';n+1}+\delta_{n',n-1})$, where

$$\begin{aligned}\chi_2=\frac{J^4}{3!}\sum_{\substack{jj'j''j'''\\ ss's''s'''}}\prod_{i=1}^{5}\int_{-\infty}^{\infty}dx_i\int_0^{1/T}d\tau_i\,[&\langle\varphi^\dagger(1)\psi_{js}^\dagger(2)\psi_{j's'}(3)\psi_{j''s''}(4)\psi_{j'''s'''}(5)\rangle\langle\psi_{js}(2)\psi_{j's'}^\dagger(3)\psi_{j''s''}^\dagger(4)\psi_{j'''s'''}^\dagger(5)\varphi(0)\rangle\\ &+3\delta_{s''s'''}\delta_{j''j'''}\langle\psi_{js}^\dagger(2)\psi_{j's'}^\dagger(3)\varphi(0)\rangle\langle\psi_{j''s''}(4)\psi_{j''s''}^\dagger(5)\rangle\\ &\times\langle\varphi^\dagger(1)\psi_{js}(2)\psi_{j's'}(3)\psi_{j''s''}^\dagger(4)\psi_{j''s''}^\dagger(5)\rangle\\ &+3\delta_{s''s'''}\delta_{j''j'''}\langle\varphi^\dagger(1)\psi_{js}(2)\psi_{j's'}(3)\rangle\langle\psi_{j''s''}(4)\psi_{j''s''}^\dagger(5)\rangle\\ &\times\langle\psi_{js}^\dagger(2)\psi_{j's'}^\dagger(3)\psi_{j''s''}^\dagger(4)\psi_{j''s''}(5)\varphi(0)\rangle\\ &-9\delta_{j''j'''}\delta_{s''s'''}\langle\varphi^\dagger(1)\psi_{js}(2)\psi_{j's'}(3)\rangle\langle\psi_{j''s''}^\dagger(4)\psi_{j''s''}(5)\rangle\langle\psi_{js}^\dagger(2)\psi_{j's'}^\dagger(3)\varphi(0)\rangle\\ &\times\langle\psi_{j''s''}(4)\psi_{j''s''}^\dagger(5)\rangle],\end{aligned} \tag{C3}$$

where all the expectation values are τ ordered. In "mean-field" theory, this term is shown diagrammatically in Fig. 4(d). However, if we examine the term in a manner analogous to the discussion in Sec. III B, we find that this term bears a certain resemblance to the term shown diagrammatically in Fig. 3(b) for the interchain backscattering, that is, $\chi_2\sim BJ^4(\alpha T)^{-3\nu}$, where $B\neq A^3$, but $B\lesssim A^3$. In fact, the ρ parts of the expectation values factor like single-particle Green's functions at $\delta_+'=\delta_-'$, and if we consider the σ modes to factor as in mean-field theory (as we have done in the text), then we have $\chi_2=0$ at $\delta_+'=\delta_-'$. The third term of order J^4 is proportional to $\delta_{n'n}$ and because of the large number of terms, we shall not write it here. However, similar arguments lead to the conclusion that at $\delta_+'=\delta_-'=\delta'$, the assumption of "mean-field" factorization of the σ modes leads to $\chi_3\to 0$ as well.

Thus a consideration of the fluctuations leads us to believe that the "pair self-avoiding random-walk" approximation is a good one for this problem of single-particle tunnelling. Since the resulting equation cannot be written as a single-integral equation, however, it is of some use to calculate T_c in the "pair-mean-field" approximation, which can be shown to have the same dependence upon J as in the "pair-self-avoiding random-walk" approximation.

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†IBM Postdoctoral Fellow.

‡Permanent address: The Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem, Israel.

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[11]This is easy to show for the Luttinger model, for which $\mu = 1 - \delta/4$. For the LE model, however, the σ modes pose a complication similar to that discussed in more detail in Sec. IV. Similar arguments can be invoked to show that the σ modes only contribute a constant to χ_2, as the other possible contribution contains an exponential gap factor, which when integrated, gives a vanishingly small contribution to χ_2 as $T \to 0$.

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[14]It can be shown that T_c^{PSARW} and T_c^{MPF} have the same power dependence upon J.

[15]This can be shown without using the "mean-pair-field" approximation.