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Flicker (1/*f*) noise: Equilibrium temperature and resistance fluctuations*

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We have measured the $1/f$ voltage noise in continuous metal films. At room temperature, samples of pure metals and bismuth (with a carrier density smaller by 10^5) of similar volume had comparable noise. The power spectrum $S_V(f)$ was proportional to $\bar{V}^2/\Omega f^\gamma$, where $\bar{V}$ is the mean voltage across the sample, Ω is the sample volume, and $1.0 \lesssim \gamma \lesssim 1.4$. $S_V(f)/\bar{V}^2$ was reduced as the temperature was lowered. Manganin, with a temperature coefficient of resistance (β) close to zero, had no discernible noise. These results suggest that the noise arises from equilibrium temperature fluctuations modulating the resistance to give $S_V(f) \propto \bar{V}^2\beta^2 k_B T^2/C_V$, where C_V is the total heat capacity of the sample. The noise was spatially correlated over a length $\lambda(f) \approx (D/f)^{1/2}$, where D is the thermal diffusivity, implying that the fluctuations obey a diffusion equation. The usual theoretical treatment of spatially uncorrelated temperature fluctuations gives a spectrum that flattens at low frequencies in contradiction to the observed spectrum. However, the empirical inclusion of an explicit $1/f$ region and appropriate normalization lead to $S_V(f)/\bar{V}^2 \propto \beta^2 k_B T^2/C_V[3 + 2\ln(l/w)]f$, where l is the length and w is the width of the film, in excellent agreement with the measured noise. If the fluctuations are assumed to be spatially correlated, the diffusion equation can yield an extended $1/f$ region in the power spectrum. We show that the temperature response of a sample to δ- and step-function power inputs has the same shape as the autocorrelation function for uncorrelated and correlated temperature fluctuations, respectively. The spectrum obtained from the cosine transform of the measured step-function response is in excellent agreement with the measured $1/f$ voltage noise spectrum. Spatially correlated equilibrium temperature fluctuations are not the dominant source of $1/f$ noise in semiconductors and discontinuous metal films. However, the agreement between the low-frequency spectrum of fluctuations in the mean-square Johnson-noise voltage and the resistance fluctuation spectrum measured in the presence of a current demonstrates that in these systems the $1/f$ noise is also due to equilibrium resistance fluctuations.

I. INTRODUCTION

Hooge and Hoppenbrouwers[1] have measured the $1/f$ noise voltage generated in continuous gold films (with physical properties close to bulk values) in the presence of a steady current. They found that the noise power spectrum $S_V(f)$ for samples at room temperature could be expressed by the empirical formula

$$S_V(f)/\bar{V}^2 \approx 2\times 10^{-3}/N_c f. \tag{1.1}$$

In Eq. (1.1), N_c is the number of charge carriers in the sample, f is the frequency, and $\bar{V}$ is the average voltage across the sample. This dependence on $\bar{V}$ is universally found in resistive systems. The $1/f$ noise is, consequently, often considered as arising from resistance fluctuations that generate a fluctuating voltage in the presence of a constant current.[2] Hooge and Hoppenbrouwers pointed out that the inverse volume dependence for samples of the same material implied by Eq. (1.1) was strong evidence for believing that the $1/f$ noise is a bulk effect rather than a surface effect in metal films. They found that the noise was still present when the samples were immersed directly in liquid nitrogen or liquid helium, and concluded that $S_V(f)$ had a temperature dependence no stronger than $T^{1/2}$.

Williams and co-workers[3,4] studied very-thin metal films which no longer have bulk properties, and in which electrical conduction is probably partially via a hopping process. Such films exhibit much more noise than is predicted by Eq. (1.1).

Hooge[5] has examined measurements of $1/f$ noise in semiconductors, and has found that, with a few notable exceptions, $S_V(f)$ was again quite well expressed by Eq. (1.1). Agreement with this formula was also found in single-crystal III-V compounds by Vandamme.[6] Both results imply that $1/f$ noise in semiconductors is a bulk effect. Hooge, who studied noise in ionic cells,[7] and Kleinpenning,[8] who studied noise in the thermoelectric emf of intrinsic and extrinsic semiconductors, both concluded that the noise arises from fluctuations in carrier mobility. However, the view that $1/f$ noise in semiconductors is a bulk effect arising from mobility fluctuations is not universally held. McWhorter's[9] theory suggests that $1/f$ noise arises from surface traps with an appropriate distribution of trapping times that generate noise by inducing fluctuations in the number of carriers. This theory has considerable experimental support.[10-12] However, it is possible that, in general, the $1/f$ noise in semiconductors arises from both bulk and surface effects.

In this paper we report results of our work on $1/f$ noise primarily in continuous metal films. Initial results were reported earlier.[13] The paper follows the chronological progression of the work, and oscillates between experiment and theory. In Sec. II, we report measurements on $1/f$ noise in thin films made of a variety of materials. We found general agreement with Eq. (1.1), with two important exceptions. First, Bi, a semimetal with a carrier density about 10^5 smaller than gold, exhibited about the same $1/f$ noise for similar sized samples. Second, manganin, an alloy with a temperature coefficient of resistance close to zero, showed no observable $1/f$ noise. The absence of $1/f$ noise in manganin indicated that the $1/f$ noise in metal films could be caused by temperature fluctuations that modulate the sample resistance R and generate voltage fluctuations in the presence of a steady current I. Thus, we expect

$$S_V(f) \propto I^2 \left(\frac{\partial R}{\partial T}\right)^2 \langle(\Delta T)^2\rangle = \bar{V}^2 \beta^2 \langle(\Delta T)^2\rangle ,$$

where

$$\beta = \frac{1}{R}\frac{dR}{dT} ,$$

and $\langle(\Delta T)^2\rangle$ is the mean-square temperature fluctuation. The similarity of the $1/f$ noise in Bi and other metals suggests that $S_V(f)/\bar{V}^2 \propto \Omega^{-1}$ (where Ω is the sample volume), not N_c^{-1}; and, consequently, that the temperature fluctuations may be those of an equilibrium system. In thermal equilibrium, $\langle(\Delta T)^2\rangle = k_B T^2/C_V$, where C_V is the heat capacity of the sample. At room temperature, $C_V \approx 3Nk_B$, where N is the number of *atoms* in the sample, and $S_V(f) \propto \bar{V}^2\beta^2T^2/3N$.

Energy fluctuations ($\Delta E = C_V \Delta T$) are expected to obey a diffusion equation, and in Sec. III we describe the spectrum of such fluctuations in a small subvolume of a uniform medium, assuming the fluctuations to be uncorrelated in space. This system has been extensively studied in the past.[14] The diffusion model was rejected as an explanation for $1/f$ noise because, in this system, it fails to predict a $1/f$ power spectrum over many decades of frequency,[15] and because it seriously underestimates the noise in semiconductors. However, the experimental configuration involving a metal film on a glass substrate is a poor approximation of the uniform medium for which the spectra are calculated. If an explicit $1/f$ region is empirically included in the spectrum, and the spectrum normalized by setting

$$\langle(\Delta T)^2\rangle = k_B T^2/C_V = \int_0^\infty S_T(f)\,df ,$$

the calculated noise is in excellent agreement with the data.

The diffusion theory introduces a frequency-dependent correlation length $\lambda(f) \approx (D/f)^{1/2}$, where D is the thermal diffusivity. $\lambda(f)$ is roughly the length over which a fluctuation at frequency f is correlated. Frequency-dependent correlation is thus an identifying characteristic of fluctuations in a diffusive medium. In Sec. IV, we describe an experiment on Bi samples in which the noise across two sections becomes more correlated in the predicted manner as the frequency is lowered.

The absence of $1/f$ noise in manganin, the dependence of $S_V(f)$ on $\bar{V}^2\beta^2/N$, and the observation of frequency-dependent spatial correlation for the $1/f$ noise provide overwhelming experimental evidence that equilibrium temperature fluctuations are the physical origin of $1/f$ noise in metal films. Moreover, the introduction of an explicit $1/f$ region in the spectrum enables us to make quantitative predictions of the $1/f$ noise in excellent agreement with experiment. The manner in which the temperature fluctuations produce the $1/f$ spectrum is, however, an open question. One possibility is that the nonuniform nature of the experimental system modifies the simple diffusion theory to produce a $1/f$ spectrum. Indeed, experiments[16] on the $1/f$ noise in Sn films at the superconducting transition have shown that a change in the thermal coupling between the film and the substrate can dramatically affect the spectrum. Another possibility is that the temperature fluctuations have some spatial correlation. In Sec. V, we show that spatial correlation of the temperature fluctuations can produce a spectrum with an extended $1/f$ region.

Fluctuation spectra are calculated on the assumption that the fluctuations are on the average governed by the same decay laws (in this case, the diffusion equation) as macroscopic perturbations.[17] In Sec. VI, we show that the temperature response of a subvolume of a diffusive system to a δ-function power input uniform over the subvolume has the same shape as the autocorrelation function for uncorrelated temperature fluctuations. On the other hand, the response to a step function in power corresponds to the autocorrelation function for correlated temperature fluctuations. We then describe an experiment in which we measure the temperature response of a small Au film to δ-function and step-function power inputs. The cosine transforms of the responses yield power spectra that are compared with the measured noise power spectrum. The spectrum obtained from the δ function is similar to that calculated for uncorrelated fluctuations, flattening at low frequencies, and is unlike the measured noise spectrum. The spectrum obtained from the step function, however, is not only $1/f$ over many decades, but, when appropriately normalized, has a magnitude and shape in excellent agreement with the measured noise power spectrum.

In Sec. VII, we briefly summarize measurements on superconducting films at the transition[16] and Josephson junctions[18] that strongly suggest that the $1/f$ noise in these systems is also due to equilibrium temperature fluctuations. However, although equilibrium temperature fluctuations should generate noise in all systems (except those for which $\beta=0$), they may not be the dominant noise source. For example, the $1/f$ noise in semiconductors and discontinuous metal films is too large to be explained by temperature fluctuations, and lacks the spatial correlation characteristic of a diffusive process. However, we show in Sec. VIII that the $1/f$ noise in these systems is due to *equilibrium* resistance fluctuations. The measured low-frequency spectrum (appropriately normalized) of fluctuations in the mean-square Johnson noise voltage across these samples is $1/f$, and is in excellent agreement with the resistance fluctuation spectrum obtained in the presence of a current. Section IX contains our concluding discussion.

II. MEASUREMENTS OF $1/f$ NOISE IN METAL FILMS

We have measured the spectrum of current-induced $1/f$ voltage noise in small samples of evaporated or sputtered metal films on glass substrates. Our films were 250 to 2000 Å thick, and had resistivities close to bulk values. Each film was cut with a diamond knife in a micromanipulator to produce a small bridge or necked down region of typical dimensions 10×150 μm with large areas of metal at either end suitable for contacts. Two variations of the sample geometry are shown in Figs. 1(a) and 2(a). Four pressed indium contacts were placed on each sample and the contacts were checked for excessive resistance. A current source, consisting of a bank of batteries and a large wire-wound resistor (which exhibited no $1/f$ noise) of resistance $R_0 \gg R$ (R is the sample resistance), was connected to two of the contacts. The other two contacts were used as voltage leads. The average voltage across the sample $\bar V$ ranged from 0.2 to 2 V. The high-resistance current source and the four-terminal configuration were necessary to eliminate contact $1/f$ noise at the current carrying contacts. The current and voltage leads were often reversed to further assure the absence of contact noise. Moreover, the sample was replaced by a wire-wound resistor of the same resistance to insure that no significant noise arose from the current source.

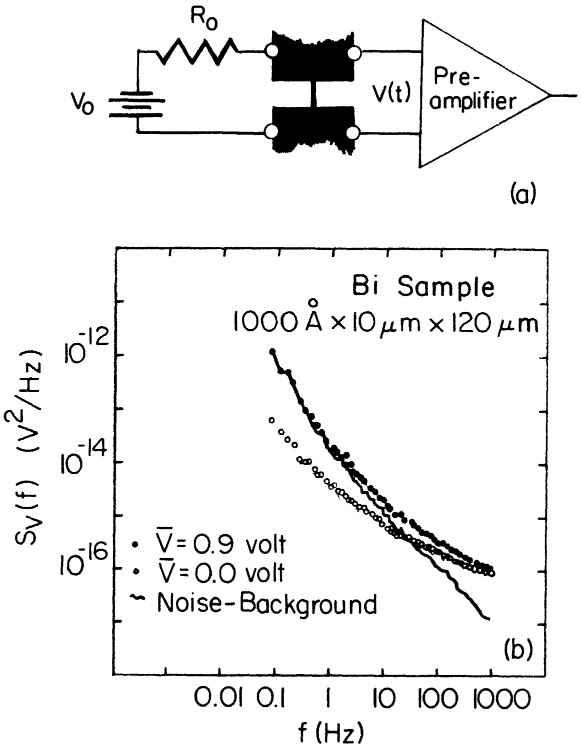

FIG. 1. (a) Sample configuration for Bi film noise measurement. (b) Measured spectrum, $\bar V=0.9V$ (●); background spectrum, $\bar V=0.0V$ (○); and $S_V(f)$, noise background (solid line).

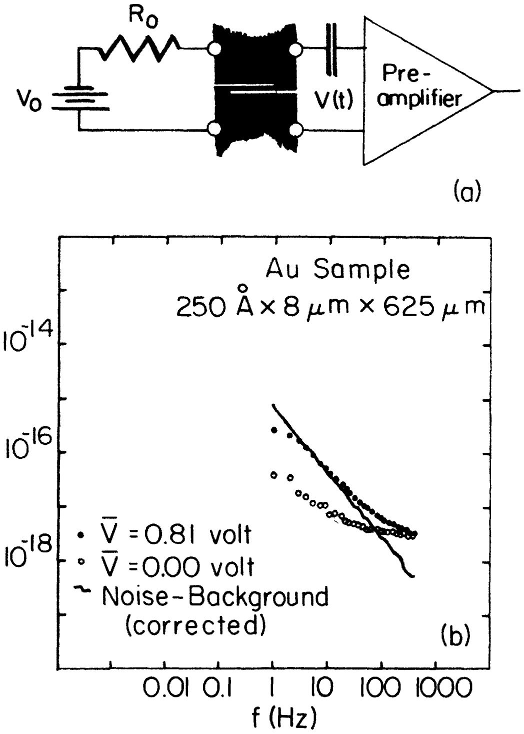

FIG. 2. (a) Sample configuration for Au noise measurement. (b) Measured spectrum, $\bar V=0.81V$ (●); background spectrum $\bar V=0.0V$ (○); and $S_V(f)$ corrected for amplifier and capacitor frequency response.

The sample, current supply, and battery-operated preamplifier were placed in an electrically screened room to reduce pickup of external noise.

In some cases, the samples were also placed in a vacuum with no change of the measured noise spectrum. For the high-resistance Bi samples ($R \sim 1\text{k}\Omega$), the voltage leads were ac coupled directly to a Princeton Applied Research 113 preamplifier. To improve the sample-noise–to-preamplifier-noise ratio for the low-resistance samples ($R \lesssim 100\ \Omega$) it was necessary to provide a better impedance match. These samples were either transformer coupled to the PAR113 or ac coupled through a large capacitor to a Keithley 824 preamplifier. Correction to the spectrum was made for the transformer response or the low-frequency amplifier roll off. In some cases, the sample was used as one arm of a Wheatstone bridge to allow dc coupling to the preamplifier. Although such arrangement is a three-terminal measurement and more susceptible to contact noise, by cutting two symmetric arms from the same continuous film, the bridge arrangement could be made insensitive to contact noise.

The voltage noise spectrum was measured by an interfaced PDP-11 computer. The preamplifier output was filtered to eliminate unwanted high frequencies and was fed to a 1-MHz voltage-to-frequency converter. The converter, in turn, drove an internal counter in the computer. An external oscillator of frequency f_0 generated an interrupt in the computer every $\tau_0 = 1/f_0$. On the first interrupt, the counter was cleared and started. On successive interrupts, the counter was read, cleared, and restarted. This arrangement provided a highly accurate analog-to-digital converter (up to 24 bits) with automatic averaging over τ_0. Successive counts stored in the computer thus provided a digital record of the noise. Once 1024 points had been accumulated, a fast Fourier transform was used to calculate the 512 sine and 512 cosine transforms of the data. These values were squared and added to an accumulating array of 512 frequency points. The entire process was repeated at least 40 times to provide an average measure of the noise spectrum in the frequency range $\frac{1}{1024} f_0$ to $\frac{1}{2} f_0$. By changing f_0, the spectrum could be measured over any desired range, although the digitizing electronics and interrupt delays gave an upper frequency limit of about 10 kHz.

The spectrum $S(f)$ was measured with an average voltage $\overline{V}$ across the sample. The background spectrum $S_0(f)$ was then measured with zero average voltage across the sample and included contributions from both external pickup and amplifier noise. The difference $S_V(f)$ between $S(f)$ and $S_0(f)$ was thus the current-induced voltage noise in the sample. These measured spectra for a typical Bi sample ($R \sim 400\ \Omega$) coupled directly to a PAR113 preamplifier are shown in Fig. 1. The increasing steepness of the spectrum below 1 Hz was found in most samples and was probably due to a gradual deterioration of the sample caused by the high current densities ($\sim 10^6\ \text{A cm}^{-2}$). With the FFT method of measuring the spectrum a slow monotonic drift generates a $1/f^2$ spectrum. This effect can be eliminated if the cosine transforms alone are used.

Figure 2 shows the measured spectra for a Au sample coupled through a large capacitor to a Keithley 824 preamplifier. In this case, it was necessary to correct for the low-frequency roll off of the amplifier and capacitor as well as to subtract out the background to obtain $S_V(f)$. In Figs. 1 and 2, the corrected spectra show a behavior close to $1/f$.

Similar measurements were made on a wide variety of samples of different materials. We found $S_V(f) \propto \overline{V}^2/f^\gamma$, where $1.0 \lesssim \gamma \lesssim 1.4$. By varying sample size, it was possible to show that $S_V(f)$ was roughly proportional to Ω, the sample volume. Otherwise identical samples often showed noise spectra whose magnitude differed by up to a factor of 3. This irreproducibility between different samples and our inability to change Ω over a wide range while still observing the $1/f$ noise made a more accurate determination of the Ω dependence impossible. A summary of the measured noise spectra for various samples (typically, $10 \times 120\ \mu\text{m} \times 1000\ \text{Å}$) of different materials, including metals and a semimetal (Bi), is shown in Table I. The measured temperature coefficient of resistivity β is also shown for each of the metal films.

Hooge and Hoppenbrouwers[1] reported no consistent variation of the $1/f$ noise in their Au films when immersed directly in liquid N_2 or liquid He. These measurements, however, may not be indicative of the temperature dependence of $S_V(f)/\overline{V}^2$. We found that placing the samples directly in the liquids caused the spectrum to become steeper than $1/f$ and to be dominated by bubbling in the liquid. Moreover, at all temperatures, the high current densities (up to $10^6\ \text{A cm}^{-2}$) and high levels of power dissipation (up to $1\ \text{kW cm}^{-2}$) necessary to observe the $1/f$ noise caused the film to operate much above ambient temperature. In the case of some "room-temperature" metal films, the departure of the I-V characteristics from linearity together with the known value of β showed that the sample was as much as 40 °C above room temperature. The nonlinearity of the I-V characteristic at high currents due to heating is shown in Fig. 3 for the Au film whose noise spectrum is shown in Fig. 2. The heating causes an increase in resistance at high currents. The somewhat amorphous nature of our Bi films caused a negative temperature coefficient of resistivity. The Bi films, consequently, exhibited I-V character-

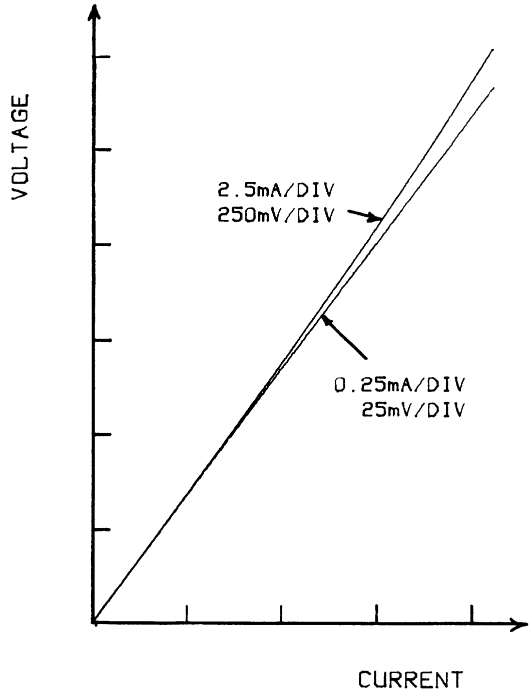

FIG. 3. Nonlinearity of I-V characteristic caused by heating of Au sample of Fig. 2.

istics which curved toward lower resistance at high currents.

In order to get some indication of the temperature dependence of $S_V(f)/\overline{V}^2$ it was necessary to place the samples in a vacuum can and to isolate them from temperature fluctuations in the liquid bath by a long thermal time constant. We found that $S_V(f)/\overline{V}^2$ for both Au and Bi samples decreased by about an order of magnitude in going from room temperature to a liquid-N_2 bath, but that $S_V(f)/\overline{V}^2$ did not change further in going to a liquid-He bath. In all cases, however, the presence of heating nonlinearities indicated that the samples were much above the bath temperature. In liquid-N_2 and liquid-He baths, we were unable to make an accurate measurement of β and determine the actual temperature of the samples. Although we can say that with careful measurement $S_V(f)/\overline{V}^2$ is found to decrease as the temperature is lowered, we can make no quantitative statement about the temperature dependence.

The dependence of $S_V(f)$ on $\overline{V}^2$ suggests that the $1/f$ noise may be caused by resistance fluctuations. The measurements summarized in Table I provide important clues as to the nature of $1/f$ noise in continuous metal films. The absence of detectable $1/f$ noise in manganin with $\beta \approx 0$ indicates that temperature fluctuations generate the $1/f$ noise. The similarity of the spectra from Bi ($\beta < 0$) and the metals ($\beta > 0$) indicates that thermal feedback, in which a resistance fluctuation changes the power dissipated in its neighborhood thereby raising or lowering the local temperature, do not play a role. The observation that Bi, with a carrier density 10^5 smaller than metals, has roughly the same relative noise spectrum suggests that the size effect is not N_c^{-1}, as suggested by Hooge and Hoppenbrouwers.[1] However, both our measurements and those of Hooge and Hoppenbrouwers[1] on Au are consistent with $S_V(f)/\overline{V}^2 \propto \Omega^{-1}$. In thermal equilibrium a body of total heat capacity $C_V = c_V\Omega$ has a mean-square temperature fluctuation $\langle(\Delta T)^2\rangle = k_B T^2/C_V$. Thus, the absence of the $1/f$ noise in manganin, the scaling of $S_V(f)/\overline{V}^2$ as $1/\Omega$, and the fact that $S_V(f)/\overline{V}^2$ decreases with decreasing temperature are all consistent with the idea that the $1/f$ noise voltage in continuous metal films supplied with a steady current is due to equilibrium temperature fluctuations modulating the resistance.

III. FLUCTUATION SPECTRA FOR DIFFUSIVE SYSTEMS

A temperature fluctuation ΔT in a resistor of resistance R and temperature coefficient of resistivity $\beta = (1/R)\partial R/\partial T$ will be observed as a voltage fluctuation $\Delta V = IR\beta\Delta T$ in the presence of a constant current I. The voltage-fluctuation spectrum $S_V(f)$ is then related to the temperature-fluctuation spectrum $S_T(f)$ by

$$S_V(f) = \overline{V}^2\beta^2 S_T(f), \tag{3.1}$$

where $\overline{V} = IR$ is the average voltage across the resistor. If the temperature fluctuations are due to equilibrium exchange of energy between the resistor and its environment, $S_T(f) \propto k_B T^2/C_V$, where C_V is the total heat capacity of the resistor. In this case $S_V(f) \propto \overline{V}^2\beta^2 k_B T^2/C_V$, which predicts the observed behavior of the $1/f$ noise in metal films. It is necessary, however, to determine whether or not the idea of equilibrium temperature fluctuations can account for both the observed magnitude of the $1/f$ noise and the $1/f$ spectrum. In this section,

TABLE I. The measured temperature coefficient of resistance for several materials and the measured and calculated noise power at 10 Hz (measured $S_V(f)/\overline{V}^2$ differs by $2/\pi$ from previous table in Ref. 13 because of recalibration).

Material	Measured β (K^{-1})	$S_V)f)/\overline{V}^2$ measured at 10 Hz (10^{-16} Hz^{-1})	$S_V(f)/\overline{V}^2$ calculated at 10 Hz (10^{-16} Hz^{-1})
Cu	0.0038	6.4	16
Ag	0.0035	6.4	2
Au	0.0012	0.6	0.76
Sn	0.0036	7.7	7.7
Bi	−0.0029	13	9.3
Manganin	$\lvert\beta\rvert < 10^{-4}$	$< 7\times10^{-3}$	$< 3.5\times10^{-3}$

we shall use a Langevin-type approach to calculate $S_T(f)$ for a system characterized by a single correlation time, and for uniform diffusive systems. Although many of the results have been previously derived,[2,14,15,19–24] neither the generalized three-dimensional spectra nor the frequency-dependent spatial correlation length $\lambda(\omega)\propto(D/\omega)^{1/2}$ have been emphasized. Our simple physical derivation, which stresses the importance of $\lambda(\omega)$ in determining the shape of the spectra, not only provides the basis for our later experiments, but also introduces methods that can easily be extended to the case of correlated fluctuations discussed in Sec. V.

We begin by considering the system shown in Fig. 4(a). A mass of total heat capacity C is coupled via a thermal conductance G to a heat reservior at temperature T_0. Macroscopic deviations in T will obey the decay equation

$$C\frac{dT}{dt}=-G(T-T_0). \tag{3.2}$$

In the Langevin approach[25] equilibrium fluctuations are also assumed to obey Eq. (3.2). The stochastic nature of the fluctuations is introduced by adding a "random driving term" $F(t)$ to the right-hand side of Eq. (3.2). $F(t)$ is assumed to have zero average and to be uncorrelated in time $[\langle F(t)\times F(t+\tau)\rangle=F_0^2\delta(\tau)]$ for the time scales in which we are interested. Physically, $F(t)$ represents the random exchange of energy between the mass and the reservior through the thermal conductance. The equilibrium temperature fluctuations thus obey the equation

$$\frac{dT}{dt}=\frac{-(T-T_0)}{\tau_0}+\frac{F(t)}{C}, \tag{3.3}$$

where $\tau_0=C/G$ is the time constant for decay of a given fluctuation. If we assume that $T(t)$ and $F(t)$ fluctuate only for $|t|\leq\theta$, the frequency spectrum is defined[25] by $S_T(\omega)=\lim_{\theta\to\infty}\langle|T(\omega)|^2\rangle/2\theta$, where

$$T(\omega)=(2\pi)^{-1/2}\int(T(t)-T_0)e^{i\omega t}\,dt.$$

From Eq. (3.3), $T(\omega)=F(\omega)/C[\tau_0^{-1}+i\omega]$ and $S_T(\omega)=S_F(\omega)/C^2[\tau_0^{-2}+\omega^2]$. The Wiener-Khintchine relations

$$c_x(\tau)=\langle x(t)x(t+\tau)\rangle=\int_{-\infty}^{\infty}S_x(\omega)\cos\omega\tau\,d\omega \tag{3.4a}$$

and

$$S_x(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}c_x(\tau)\cos\omega\tau\,d\tau \tag{3.4b}$$

that connect the autocorrelation function $c_x(\tau)$ of a fluctuating quantity x with its spectrum $S_x(f)$ may be used to calculate $S_F(\omega)$ from $\langle F(t)F(t+\tau)\rangle=F_0^2\delta(\tau)$. Since $F(t)$ is uncorrelated in time, $S_F(\omega)=F_0^2/2\pi$ is "white" (independent of frequency). Thus, $S_T(\omega)=F_0^2/2\pi C^2(\tau_0^{-2}+\omega^2)$. F_0^2 may be determined from the normalization condition [Eq. (3.4a)] that

$$c_T(0)=\langle(\Delta T)^2\rangle=k_BT^2/C=\int_{-\infty}^{\infty}S_T(\omega)\,d\omega.$$

We find that $F_0^2=2k_BT^2G$ and

$$S_T(\omega)=k_BT^2/\pi G(1+\omega^2\tau_0^2). \tag{3.5}$$

$S_T(\omega)$ is the usual Lorentzian spectrum characteristic of processes with a single correlation time τ_0.[25] This spectrum is obviously not $1/f$. In fact, the $1/f$ spectrum can only arise from physical processes characterized by the appropriate distribution of correlation times.[26] One process with a distribution of correlation times is diffusion, which, moreover, represents a better approximation to the heat flow in the metal samples.

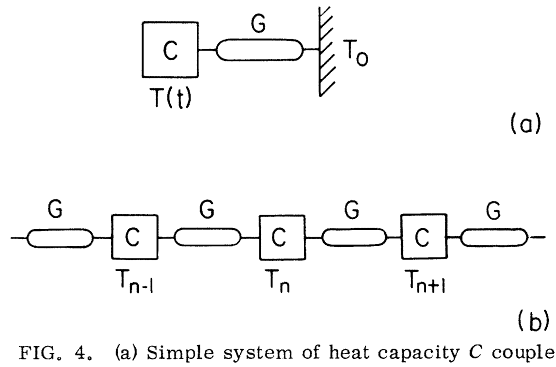

FIG. 4. (a) Simple system of heat capacity C coupled to reservoir at temperature T_0 by thermal conductance G. (b) String of these simple systems which approximate a one-dimensional diffusive system.

With a simple extension of this single correlation time system, one may approach a one-dimensional diffusion system. Figure 4(b) shows a string of equal masses of heat capacity C connected by thermal conductances G. The temperature of the nth mass obeys the Langevin equation

$$\frac{C\,dT_n}{dt}=G(T_{n+1}+T_{n-1}-2T_n)+F_{n+1/2}-F_{n-1/2}. \tag{3.6}$$

Each of the random driving terms $F_{n+1/2}$ is independent of the others. If we assume that each of the masses is separated by a distance l_0, we may define $c=C/l_0$ as the heat capacity per unit length and $g=Gl_0$ as the thermal conductivity. In the limit $l_0\to 0$, T becomes a continuous function of position and time, $T=T(x,t)$, and obeys the diffusion equation

$$\frac{\partial T}{\partial t}=D\frac{\partial^2 T}{\partial x^2}+c^{-1}\frac{\partial F}{\partial x}, \tag{3.7}$$

where $D=g/c$ is the thermal diffusivity and $F(x,t)$ obeys the relation

$$\langle F(x,t)F(x+s,t+\tau)\rangle = 2\pi F_0^2\delta(s)\delta(\tau).$$

The quantity of interest is now the spatial average of the temperature $\overline{T}(t)$ over the length $2l$ from $x=-l$ to $x=l$:

$$\overline{T}(t)=\frac{1}{2l}\int_{-l}^{l} T(x,t)\,dx. \tag{3.8}$$

$T(x,t)$ may be defined in terms of its space and time Fourier transform:

$$T(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} dk\int_{-\infty}^{\infty} d\omega\, e^{ikx}e^{-i\omega t}T(k,\omega). \tag{3.9}$$

From Eq. (3.7) we find

$$T(k,\omega)=ikF(k,\omega)/c[Dk^2-i\omega]. \tag{3.10}$$

Since from Eq. (3.8),

$$\overline{T}(\omega)=\frac{1}{2l}\int_{-l}^{l} T(x,\omega)\,dx,$$

using Eq. (3.9) we have

$$\overline{T}(\omega)=(2\pi)^{-1/2}\int_{-\infty}^{\infty}\frac{\sin kl}{kl}\,T(k,\omega)\,dk. \tag{3.11}$$

The frequency spectrum is defined by $S_T(\omega)=\lim_{\theta\to\infty}\langle|T(\omega)|^2\rangle/2\theta$. The uncorrelated nature of F in space and time implies that it has a white spectrum in ω space and k space. We thus set

$$\langle F(k,\omega)F^*(k',\omega)\rangle/2\theta = F_0^2\delta(k-k')/2\pi,$$

so that $S_T(\omega)$ reduces to

$$S_T(\omega)=\frac{F_0^2}{(2\pi)^2c^2}\int_{-\infty}^{\infty}\frac{\sin^2 kl}{k^2l^2}\,\frac{k^2\,dk}{D^2k^4+\omega^2}\;. \tag{3.12}$$

Once again F_0^2 may be determined from the normalization condition

$$\langle(\Delta T)^2\rangle=\frac{k_BT^2}{2lc}=\int_{-\infty}^{\infty}S_T(\omega)\,d\omega.$$

We find $F_0^2=2k_BT^2g$. $S_T(\omega)$ may now be explicitly integrated to give

$$S_T(\omega)=\frac{k_BT^2D^{1/2}}{4\sqrt{2}\,l^2c\pi\omega^{3/2}}\,[1-e^{-\theta}(\sin\theta+\cos\theta)], \tag{3.13}$$

where $\theta\equiv(\omega/\omega_0)^{1/2}$, and $\omega_0=D/2l^2$ is the natural frequency defined by the problem. $S_T(\omega)\to k_BT^2/2\sqrt{2}\pi D^{1/2}\omega^{1/2}c$ for $\omega\ll\omega_0$ and $S_T(\omega)\to k_BT^2D^{1/2}/4\sqrt{2}\pi l^2c\omega^{3/2}$ for $\omega\gg\omega_0$.[20,21]

As a check on the formalism one may obtain from Eq. (3.10) the space-time correlation function, $c_T(s,\tau)\equiv\langle\Delta T(x+s,t+\tau)\Delta T(x,t)\rangle$,

$$c_T(s,\tau)=[k_BT^2/c(4\pi D\tau)^{1/2}]\exp(-s^2/4D\tau), \tag{3.14}$$

which is the familiar result for one-dimensional diffusion processes.[27] The physical insight into the connection between diffusion and the $1/f$-like spectrum, however, comes from a calculation of the frequency-dependent correlation function, $c_T(s,\omega)=\langle T(x+s,\omega)T^*(x,\omega)\rangle/2\theta$. For the one-dimensional case, we obtain from Eq. (3.10)

$$c_T(s,\omega)=\frac{k_BT^2\cos(\frac{1}{4}\pi+|s|/\lambda)}{2\pi cD^{1/2}\omega^{1/2}}\;e^{-|s|/\lambda}, \tag{3.15}$$

where $\lambda(\omega)\equiv(2D/\omega)^{1/2}$ is the ω-dependent correlation length and is a measure of the average spatial extent of a fluctuation at frequency ω. A low-ω fluctuation effectively samples $F(x,t)$ over a large coherent volume giving a large amplitude.

When $\omega\ll\omega_0$, $\lambda(\omega)\gg 2l$ and the fluctuations become correlated across the entire length. In this case $S_T(\omega)$ can also be expressed

$$S_T(\omega)=\int_{-l}^{l}\frac{dx_1}{2l}\int_{-l}^{l}\frac{dx_2}{2l}\,c_T(x_1-x_2,\omega). \tag{3.16}$$

Since $c_T(s,\omega)$ is independent of s as $\omega\to 0$, $S_T(\omega)\to c_T(0,\omega)$ as $\omega\to 0$, leading to the same low-ω limit as that obtained from Eq. (3.13).

In the high-ω region $(\omega\gg\omega_0)$ $\lambda(\omega)\ll 2l$. Although $2l$ may be divided into many correlated regions each of length λ only the two end regions can fluctuate independently of the others. Energy exchange between any of the internal lengths cannot change $\overline{T}(t)$. The behavior is then best understood in terms of one-dimensional energy flow across the boundaries. The energy flow $j(x,t)$ obeys the equation

$$j=-g\,\frac{\partial T}{\partial x}-F(x,t).$$

From Eq. (3.10),

$$j(k,\omega)=i\omega F(k,\omega)/[Dk^2-i\omega].$$

If $E(t)$ represents the total energy on one side of the boundary at $x=l$ and we consider only flow across this single boundary, then

$$\frac{dE(t)}{dt}=j(l,t)$$

and

$$E(\omega)=-i(2\pi)^{-1/2}\omega^{-1}\int_{-\infty}^{\infty}\exp(ikl)\,j(k,\omega).$$

Thus,

$$S_E(\omega)=\frac{F_0^2}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{dk}{D^2k^4+\omega^2}=\frac{k_BT^2g}{2^{3/2}D^{1/2}\pi\omega^{3/2}} \tag{3.17}$$

for energy fluctuations due to flow across a single boundary. For $\omega\gg\omega_0$ the flows across the two ends are independent; and, since $\Delta T=\Delta E/2lc$,

$$S_T(\omega)=2S_E(\omega)/4l^2c^2=k_BT^2/4\sqrt{2}\pi l^2c\omega^{3/2}$$

as before.

This formalism may readily be extended to more than one dimension. $T(\vec{\mathrm{x}},t)$ obeys the Langevin diffusion equation

$$\frac{\partial T}{\partial t} = D\nabla^2 T + \frac{\vec{\nabla}\cdot\vec{\mathrm{F}}}{c}\ , \qquad (3.18)$$

where

$$\langle \vec{\mathrm{F}}(\vec{\mathrm{x}}+\vec{\mathrm{s}}, t+\tau)\cdot\vec{\mathrm{F}}(\vec{\mathrm{x}}, t)\rangle = (2\pi)^m F_0^2 \delta(\vec{\mathrm{s}})\delta(\tau).$$

If $\overline{T}(t)$ is the spatially averaged temperature of a box of volume $\Omega = 2^m l_1 \ldots l_m$ in m-dimensions, then

$$S_T(\omega) = \frac{F_0^2}{(2\pi)^{m+1}c^2}\int \frac{d^m k k^2}{D^2k^4+\omega^2}\prod_{i=1}^{m}\frac{\sin^2 k_i l_i}{k_i^2 l_i^2}\ . \qquad (3.19)$$

The requirement that

$$\langle(\Delta T)^2\rangle = \int S_T(\omega)\, d\omega$$

gives $F_0^2 = 2k_B T^2 g$. Although we have been unable to determine a general analytic expression for $S_T(\omega)$, we can determine its limiting forms from the behavior of the appropriate ω-dependent correlation function, which retains its dependence on $\exp(-|s|/\lambda)$ in all dimensions. Thus, in two dimensions

$$c_T(\vec{\mathrm{s}}, \omega) = (k_B T^2/2\pi Dc)\, ker(\sqrt{2}|\vec{\mathrm{s}}|/\lambda), \qquad (3.20)$$

where

$$c_T(\vec{\mathrm{s}}, \omega) \to (k_B T^2/2\pi^2 Dc)\ln(\sqrt{2}\lambda/|\vec{\mathrm{s}}|) \text{ for } |\vec{\mathrm{s}}| \ll \lambda$$

and

$$c_T(\vec{\mathrm{s}}, \omega) \to |\vec{\mathrm{s}}|^{-1/2}\exp(-\sqrt{2}|\vec{\mathrm{s}}|/\lambda) \text{ for } |\vec{\mathrm{s}}| \gg \lambda.$$

In three dimensions,

$$c_T(\vec{\mathrm{s}}, \omega) = \frac{k_B T^2}{4\pi^2 Dc|\vec{\mathrm{s}}|}\cos(|\vec{\mathrm{s}}|/\lambda)\, e^{-|\vec{\mathrm{s}}|/\lambda}\ . \qquad (3.21)$$

For a regular three-dimensional volume of lengths $l_1 \gg l_2 \gg l_3$, the three natural frequencies, $\omega_i = D/2l_i^2$, separate the spectrum into four regions. In the frequency region $\omega \gg \omega_2$ the lengths l_1 and l_2 may be considered infinite and the spectrum becomes one-dimensional with temperature fluctuations only due to energy flow in the x_3 direction. Thus as calculated above $S_T(\omega) \propto \omega^{-3/2}$ for $\omega \gg \omega_3$ and $S_T(\omega) \propto \omega^{-1/2}$ for $\omega_2 \ll \omega \ll \omega_3$. If $\omega \gg \omega_1$, the spectrum looks two-dimensional with temperature fluctuations only due to energy flow in the x_2 and x_3 directions. The low-frequency limit of the two-dimensional spectrum may be calculated from Eq. (3.20) and the observation that

$$S_T(\omega) = \Omega^{-2}\int_\Omega d\vec{\mathrm{x}}\int_\Omega d\vec{\mathrm{x}}\, c_T(\vec{\mathrm{x}} - \vec{\mathrm{x}}', \omega). \qquad (3.22)$$

From the limiting form of Eq. (3.20) as $\omega \to 0$ it can be seen that[24]

$$S_T(\omega) \propto [\text{const} + \ln(1/\omega)] \text{ for } \omega_1 \ll \omega \ll \omega_2.$$

For $\omega \ll \omega_1$, $S_T(\omega)$ is determined from the low-frequency limit of the three-dimensional $c_T(\vec{\mathrm{s}}, \omega)$ [Eq. (3.21)] and Eq. (3.22). Thus, for $\omega \ll \omega_1$,[14] $S_T(\omega) \propto (\text{const})$. The behavior of $S_T(\omega)$ for the four regions of the spectrum of a regular three-dimensional volume are shown in Fig. 5(a). The $\omega^{-3/2}$ behavior at high ω for all dimensionalities is characteristic of diffusive flow across a sharp boundary.[23] When λ is $\ll$ any length $2l$, only the outer shell of an arbitrary volume Ω can fluctuate independently of the remainder and then only by local one-dimensional flow across the boundary. A generalization of Eq. (3.17) gives

$$S_T(\omega) \to k_B T^2 A/2^{3/2}\pi\Omega^2 c\,\omega^{3/2} \text{ as } \omega \to \infty, \qquad (3.23)$$

where A is the total surface area of Ω. If, on the other hand, the boundary is not sharp but has a finite width w, $S_T(\omega)$ varies as ω^{-2} for $\omega \gg D/w^2$.

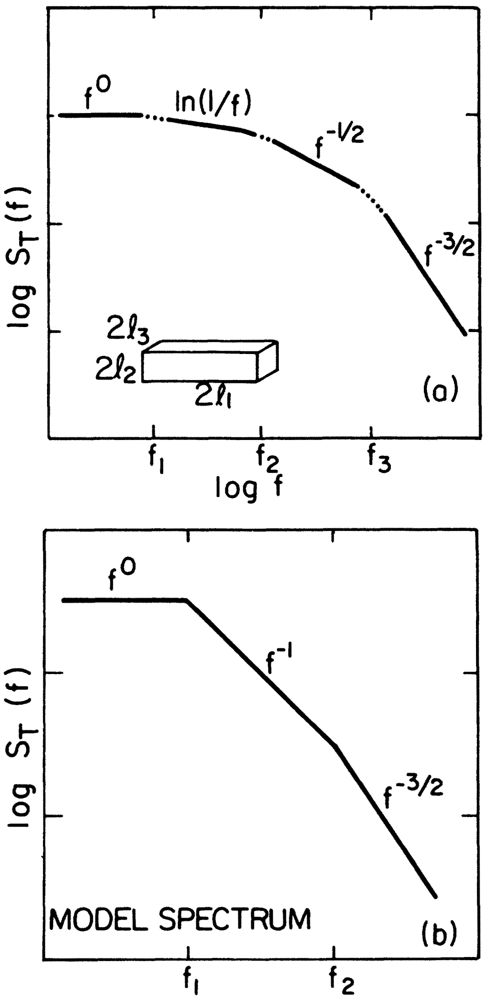

FIG. 5. (a) $S_T(f)$ for spatially uncorrelated temperature fluctuations of a box $2l_1 \times 2l_2 \times 2l_3$: $f_i = \omega_i/2\pi = D/4\pi l_i^2$. (b) Model $S_T(f)$ for a metal film on glass substrate: $f_1 = D/\pi l^2$, where l is the length of the film, and $f_2 = D/\pi w^2$, where w is the width of the film.

Only very recently have these calculated spectra for diffusive systems been verified experimentally by a direct measurement of the spectra.[28] Light scattered from independent particles undergoing Brownian motion was used to measure the number fluctuation spectrum for a small volume. Both the predicted low-frequency roll off and the high-frequency $f^{-3/2}$ behavior of the correct magnitude were observed.

Unlike the measured $S_V(f)$ for metal films shown in Figs. 1 and 2, Fig. 5(a) does not show an extended region of $1/f$ behavior. Figure 5(a), however, was calculated for temperature fluctuations in a regular subvolume of a uniform medium. On the other hand, the experimental system of metal film on glass substrate does not present a uniform medium for heat conduction. We expect diffusive flow along the film to dominate the heat conduction creating primarily a two-dimensional system; but with some effects due to coupling to the substrate. The importance of coupling to the substrate on the spectrum has, in fact, been demonstrated for superconducting films at T_c.[16] It is also possible that path switching effects, in which a temperature configuration that does not change $\overline{T}(t)$ does change R, $(\Delta V \neq \overline{V}\beta\Delta\overline{T})$ may play a role. However, at the frequencies measured, λ is large enough that temperature fluctuations are expected to be correlated across the cross section of the strip, hence $\Delta V = \overline{V}\beta\Delta\overline{T}$.

If we assume that the temperature fluctuations in the metal films obey a diffusion equation, but that the complex nature of the system introduces an explicit $1/f$ region into the spectrum at intermediate frequencies, we may form a model spectrum that will allow quantitative comparison of the measured noise with that predicted from temperature fluctuations. Since the thermal conductivity of the film is so much higher than the substrate, we expect the high-frequency behavior to be two-dimensional while at low enough frequencies the spectrum must become three-dimensional and independent of frequency. This simple model spectrum is illustrated in Fig. 5(b). The limits of the $1/f$ region are defined by the natural frequencies of the film, $D/\pi l^2$ and $D/\pi w^2$, where l and w are the length and width of the film. The high- and low-frequency limits are taken to be diffusionlike: $S_T(f) \propto f^{-3/2}$ for $f > D/\pi w^2$ and $S_T(f) \propto$ const for $f < D/\pi l^2$.

The normalization condition

$$\langle(\Delta T)^2\rangle = k_B T^2/C_V = \int_0^\infty S_T(f)\,df$$

then determines the magnitude of the spectrum. In the $1/f$ region

$$S_V(f)/\overline{V}^2 = \beta^2 k_B T^2/C_V[3+2\ln(l/w)]f, \qquad (3.24)$$

independent of D. The term $\ln(l/w)$ makes Eq. (3.24) extremely insensitive to changes in the limits of the $1/f$ region. For metals at room temperature $C_V \approx 3Nk_B$, where N is the total number of atoms in the sample, and

$$S_V(f) = \overline{V}^2\beta^2T^2/3N[3+2\ln(l/w)]f. \qquad (3.25)$$

For the samples of Hooge and Hoppenbrouwers,[1] Eq. (3.25) predicts $S_V(f)/\overline{V} \approx 3.6\times10^{-3}/Nf$, which is within a factor of two of their experimental results [Eq. (1.1)] if we replace N_c by N. The last column of Table I shows the calculated values of $S_T(f)$ from Eq. (3.25) for our samples. The agreement is excellent.

Although the calculated $S_T(\omega)$ for simple uniform diffusive media do not have an explicit $1/f$ region, the assumption of such a $1/f$ region in $S_T(f)$ for the complex experimental systems allows a quantitave prediction of the $1/f$ noise in excellent agreement with experiment.

IV. MEASUREMENT OF SPATIAL CORRELATION OF 1/f NOISE

The usual diffusion theory does not provide an explanation for the $1/f$ spectrum. It does, however, suggest an important experimental test of the correctness of a diffusion mechanism. Fluctuations in a diffusive medium are characterized by the frequency-dependent correlation length, $\lambda(f) = (D/\pi f)^{1/2}$. Thus, the temperature fluctuations of two regions separated by a length l should be independent if $l \gg \lambda(f)$, and correlated if $l \ll \lambda(f)$. The extent of the correlation depends on the dimensionality of the diffusion process and the exact geometry of the two regions.

Figure 6(a) shows the experimental configuration for an experiment designed to measure the frequency dependence of the correlation of the $1/f$ noise from two regions of a single Bi film. A Bi film of thickness 1000 Å was cut to form two strips each of length l and width 12 μm. Separate batteries and large resistances R_0 were used to supply a constant current to each strip and prevent any correlation via a common power supply. The two noise voltages $V_1(t)$ and $V_2(t)$ were separately amplified with PAR113 preamplifiers and the spectrum of their sum or difference measured with the PDP-11 as described in Sec. II. If $S_+(f)$ is the spectrum of $[V_1(t)+V_2(t)]$ and $S_-(f)$ is the spectrum of $[V_1(t)-V_2(t)]$, the fractional correlation between the strips $C(f)$ is given by

$$C(f) = [S_+(f) - S_-(f)]/[S_+(f) + S_-(f)]. \qquad (4.1)$$

When $V_1(t)$ and $V_2(t)$ are independent, $S_+(f) = S_-(f)$ and $C(f) = 0$. When the two strips are completely correlated, $V_1(t) = V_2(t)$, $S_-(f) = 0$, and $C(f) = 1$. For temperature fluctuations at high f, $\lambda(f) \ll l$ and $C(f) \to 0$, while at low f, $\lambda(f) \gg l$ and $C(f)$

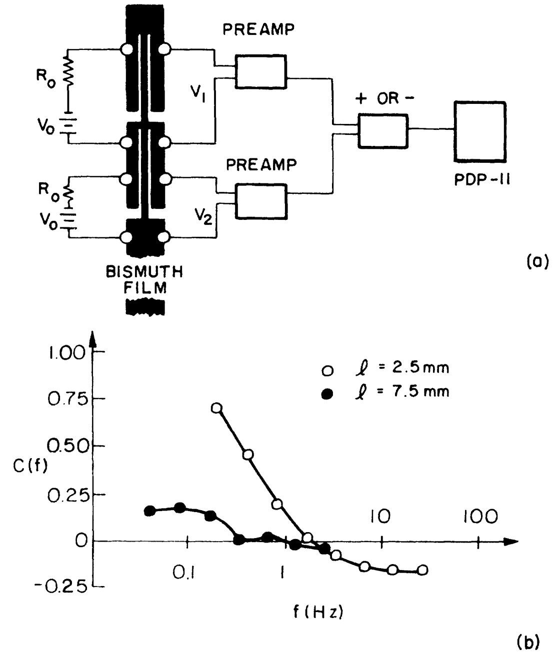

FIG. 6. (a) Experimental configuration for correlation measurement. (b) Fractional correlation for two samples.

→ const. The change from correlated to uncorrelated behavior occurs when $\lambda(f) \sim l$. Experimental results for two different values of l are shown in Fig. 6(b). The condition $\lambda(f) = l$ corresponds to $f = 0.13$ Hz for $l = 7.5$ mm, and $f = 1.2$ Hz for $l = 2.5$ mm (with $D \approx 0.2$ cm^{2}sec^{-1}), in good agreement with the frequencies at which $C(f)$ changes rapidly. As l is increased, the low-frequency limit of $C(f)$ decreases because a fluctuation in one strip has an increasing probability of decaying without influencing the other strip. For l much greater than 7.5 mm, it became increasingly difficult to observe any correlation. Since we could not measure $S_+(f)$ and $S_-(f)$ simultaneously, we often observed errors due to slow changes with time of $S(f)$. Depending upon whether we measured $S_+(f)$ or $S_-(f)$ first, a slow change would appear either as a positive or negative offset to $C(f)$, as in the $l = 2.5$-mm case in Fig. 6(b).

These measurements of the frequency-dependent spatial correlation of the $1/f$ noise in metal films and the observation that the change from uncorrelated to correlated behavior occurs at a frequency predicted by the thermal diffusivity provide strong experimental evidence that the $1/f$ noise arises from a thermal diffusion mechanism.

V. SPATIALLY CORRELATED FLUCTUATIONS

The absence of $1/f$ noise in manganin, the scaling of $S_V(f)/\overline{V}^2$ as $1/\Omega$, the general decrease of $S_V(f)$ with temperature, the observed frequency-dependent spatial correlation, and the ability of temperature fluctuations to correctly predict the absolute magnitude of $S_V(f)$ [with an assumed $1/f$ spectrum for $S_T(f)$] provide overwhelming evidence that the $1/f$ noise in metal films is due to equilibrium temperature fluctuations modulating the film resistance. Yet, the inability of the usual diffusion theory as outlined above to yield a $1/f$ spectrum suggests a reexamination of the theory.

In Sec. III we presented a physically simple derivation of the spectrum of temperature fluctuations in infinite, uniform, diffusive media. The results have been verified experimentally for independent particles undergoing Brownian motion.[28] As mentioned in Sec. III, it is possible that the nonuniform nature of a metal film on a glass substrate is responsible for the $1/f$ spectrum. However, more sophisticated models of the experimental configuration (for example, a diffusive medium coupled to a constant-temperature substrate, or two coupled diffusive media) were unsuccessful in generating a $1/f$ spectrum. In most cases the increasing complexity of the models only brought in more low-frequency flattening. The measured spectra correspond to the frequency range $f < f_3$ in Fig. 5(a) and, consequently, would be expected to vary as $\lesssim f^{-1/2}$. Any coupling to a substrate could only be expected to cause a temperature fluctuation to decay more rapidly and further flatten the spectrum. The measured spectra, on the other hand, have a $1/f$ behavior down to frequencies as low as $\frac{1}{1000} f_1$.

The calculated diffusion spectra of Sec. III assume that the fluctuations are spatially uncorrelated:

$$\langle \Delta T(\vec{x}+\vec{s}, t)\, \Delta T(\vec{x}, t)\rangle \equiv c_0(\vec{s}) \propto \delta(\vec{s}).$$

A spatial correlation of the temperature fluctuations, $c_0(\vec{s}) \neq \delta(\vec{s})$, could drastically alter the shape of $S_T(f)$; and, as suggested by Lundström, McQueen, and Klason,[29] in certain cases could give an explicit $1/f$ region. Such spatial correlation would occur if the free energy of a given temperature configuration is nonlocal and contains higher-order terms such as $(\nabla T)^2$. A familiar example of this effect is the large correlation length of density fluctuations at a critical point.[30] The presence of a term such as $(\nabla T)^2$ implies that configurations with slow spatial variations require a smaller free energy and, consequently, have a greater probability of occurring than configurations with rapid spatial variations.[31]

More explicitly, if $\overline{T}(t)$ is the spatially averaged temperature over some arbitrary volume Ω, then

$$\overline{T}(t) = \int_\Omega T(\vec{x}, t)\, d^3x = \int B^*(\vec{k})\, T(\vec{k}, t)\, d^3k, \qquad (5.1)$$

where

$$B(\vec{q}) \equiv (2\pi)^{-1/2} \int_{\Omega} e^{-i\vec{q}\cdot\vec{x}}\, d^3x. \tag{5.2}$$

Thus, the autocorrelation function $c_T(\tau) = \langle T(t) \times \overline{T}(t+\tau)\rangle$ has the form

$$c_T(\tau) = \int |B(\vec{k})|^2 \langle T(\vec{k},t)\, T^*(\vec{k},t+\tau)\rangle\, d^3k. \tag{5.3}$$

Because the temperature fluctuations obey a diffusion equation,

$$\langle T(\vec{k},t)\, T^*(\vec{k},t+\tau)\rangle = \langle |T(\vec{k})|^2\rangle\, e^{-Dk^2\tau},$$

where $\langle |T(\vec{k})|^2\rangle$ is the mean-square amplitude of temperature fluctuations of wave vector $\vec{k}$. By Eq. (3.4b) the spectrum then has the form

$$S_T(\omega) \propto \int \frac{|B(\vec{k})|^2 Dk^2 \langle |T(\vec{k})|^2\rangle\, d^3k}{D^2k^4+\omega^2}. \tag{5.4}$$

$\langle |T(\vec{k})|^2\rangle$ is related to $c_0(\vec{s})$ by the spatial Wiener-Khintchine relation

$$\langle |T(\vec{k})|^2\rangle = \Omega_0\, (2\pi)^{-3} \int c_0(\vec{s})\, e^{i\vec{k}\cdot\vec{s}}\, d^3s,$$

where Ω_0 is the total volume over which T fluctuates.

For uncorrelated temperature fluctuations, $c_0(\vec{s}) \propto \delta(\vec{s})$ and $\langle |T(\vec{k})|^2\rangle = (\mathrm{const})$. Moreover, for a regular volume of sides $2l_i$,

$$B(\vec{k}) \propto \prod_{i=1}^{3} \frac{\sin(kl_i)}{k_i}\quad ;$$

and Eq. (5.4) reproduces Eq. (3.19) for $S_T(\omega)$.

If, however, the $(\nabla T)^2$ term dominates the free energy we find $\langle |T(\vec{k})|^2\rangle \propto 1/k^2$, $c_0(\vec{s}) \propto 1/|\vec{s}|$, and

$$S_T(\omega) \propto \int \frac{|B(\vec{k})|^2 d^3k}{D^2k^4+\omega^2}. \tag{5.5}$$

We now show that it is possible to treat these spatially correlated fluctuations in the Langevin formalism by replacing the $\vec{\nabla}\cdot\vec{F}$ term in Eq. (3.18) by a random source $P(\vec{x},t)$ to give

$$\frac{\partial T}{\partial t} = D\nabla^2 T + \frac{P(\vec{x},t)}{c}. \tag{5.6}$$

$P(\vec{x},t)$ is assumed to be uncorrelated in space and time,

$$\langle P(\vec{x}+\vec{s},t+\tau) P(\vec{x},t)\rangle = (2\pi)^m P_0^2 \delta(\vec{s})\delta(\tau).$$

$P(\vec{x},t)$ is a random source which adds or subtracts energy from the diffusive system. The $\vec{\nabla}\cdot\vec{F}$ term [Eq. (3.18)], on the other hand, represents a random flow of energy *within* the diffusive system.

As with Eq. (3.18) we may determine the spectrum of the spatially averaged temperature of a box of volume $\Omega = 2^m l_1 \ldots l_m$ in m dimensions. Thus,

$$T(\vec{k},\omega) = P(\vec{k},\omega)/c[Dk^2 - i\omega] \tag{5.7}$$

and

$$S_T(\omega) = \frac{P_0^2}{(2\pi)^{m+1}c^2} \int \frac{d^mk}{D^2k^4+\omega^2} \prod_{i=1}^{m} \frac{\sin^2 k_i l_i}{k_i^2 l_i^2}. \tag{5.8}$$

A comparison of Eqs. (5.4) and (5.8) shows that the introduction of $P(\vec{x},t)$ is equivalent to $\langle |T(\vec{k})|^2\rangle \propto k^{-2}$. The normalization condition

$$\langle(\Delta T)^2\rangle = \int_{-\infty}^{\infty} S_T(\omega)\, d\omega$$

cannot be applied unless $m \geq 3$, since for one and two dimensions $\int S_T(\omega)\, d\omega$ diverges.

The general behavior of the spectrum can be determined from the three-dimensional frequency-dependent correlation function

$$c_T(\vec{s},\omega) = \frac{P_0^2 e^{-|\vec{s}|/\lambda}}{8\sqrt{2}\pi^2 D^{3/2}\omega^{1/2}}\, \frac{\sin(|\vec{s}|/\lambda)}{|\vec{s}|/\lambda}, \tag{5.9}$$

where, as before, $\lambda(\omega) = (2D/\omega)^{1/2}$. In the limit $\omega \to 0$, $(\lambda(\omega) \gg |\vec{s}|)$, $c_T(\vec{s},\omega) \propto \omega^{-1/2}$. Because $P(\vec{x},t)$ is an external source, each correlated region may be considered as fluctuating independently of the other correlated volumes with a spectrum $S_c(\omega) \propto \omega^{-1/2}$ from Eq. (5.9).

If Ω, the volume of interest, consists of N independent correlated volumes, $S_T(\omega) \propto S_c(\omega)/N$. Thus, when $\omega \gg \omega_3$, $\lambda(\omega) \ll l_3$, and Ω is composed of $N = \Omega/\lambda^3(\omega)$ independent volumes,

$$S_T(\omega) \propto S_c(\omega)\lambda^3(\omega)/\Omega \propto \omega^{-2} \quad (\omega \gg \omega_3). \tag{5.10a}$$

When $\omega_2 \ll \omega \ll \omega_3$, $l_2 \gg \lambda(\omega) \gg l_3$, and Ω is composed of $N = l_1 l_2/\lambda^2$ independent volumes,

$$S_T(\omega) \propto S_c(\omega)\lambda^2(\omega)/l_1 l_2 \propto \omega^{-3/2} \quad (\omega_2 \ll \omega \ll \omega_3). \tag{5.10b}$$

When $\omega_1 \ll \omega \ll \omega_2$, $l_1 \gg \lambda(\omega) \gg l_2$, and Ω is composed of $N = l_1/\lambda(\omega)$ independent volumes,

$$S_T(\omega) \propto S_c(\omega)\lambda(\omega)/l_1 \propto \omega^{-1} \quad (\omega_1 \ll \omega \ll \omega_2). \tag{5.10c}$$

And, for $\omega \ll \omega_1$, all of Ω is correlated:

$$S_T(\omega) \propto S_c(\omega) \propto \omega^{-1/2} \quad (\omega \ll \omega_1). \tag{5.10d}$$

The shape of $S_T(f)$ for this type of spatially correlated temperature fluctuation is shown in Fig. 7. Not only does $S_T(f)$ contain an explicit $1/f$ region, but this region corresponds to the low-frequency limit of a two-dimensional system and matches the frequency range over which the metal films are observed to have the $1/f$ spectrum. In fact, if we assume that $\langle(\Delta T)^2\rangle = T^2/3N$, we find

$$\frac{S_V(f)}{\overline{V}^2} = \frac{\beta^2 T^2}{3N[4 - d/w + 2\ln(l/w)]f}, \tag{5.11}$$

where d is the film thickness. This result differs from our earlier model spectrum [Eq. (3.25)] only by a factor close to unity.

Although the introduction of spatially correlated fluctuations provides a means of achieving the $1/f$ spectrum for simple diffusive systems, the theo-

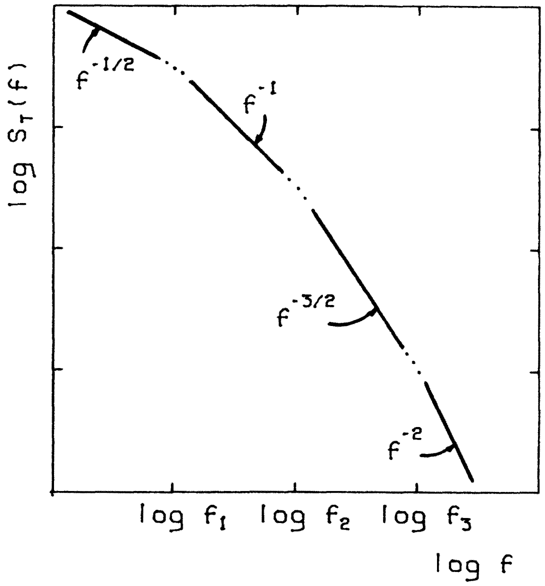

FIG. 7. $S_T(f)$ for spatially correlated temperature fluctuations of a box $2l_1 \times 2l_2 \times 2l_3$: $f_i = D/4\pi l_i^2$.

retical justification of the spatial correlation in the case of equilibrium temperature fluctuations poses new problems: notably, the physical origin of the correlations, and the proper normalization of the spectrum. Moreover, our calculation has been for a homogeneous medium. A metal film on a glass substrate is thermally inhomogeneous and different boundary conditions should be applicable. It is difficult to estimate *a priori* the extent to which these new boundary conditions will change the spectrum.[32] In Sec. VI we demonstrate experimentally that the correlated temperature fluctuations do in fact produce a $1/f$ spectrum for the thermally inhomogeneous experimental systems.

It is interesting to note that, when treated by the Langevin method [the association of an uncorrelated random source $F(t)$ with each thermal conductance], a diffusive system coupled to a substrate at constant temperature T_0 via a thermal conductance G contains an $F_1(\vec{x},t)$ term representing exchange of energy with the substrate. In this case,

$$\frac{c\,\partial T}{dt} = g\nabla^2 T + \vec{\nabla}\cdot\vec{F} - G(T - T_0) + F_1(\vec{x},t). \qquad (5.12)$$

It can be shown, however, that the frequency region in which $F_1(\vec{x},t)$ dominates $\vec{\nabla}\cdot\vec{F}$ corresponds to the region in which the $-GT$ decay term dominates $g\nabla^2 T$ and the spectrum never achieves a $1/f$ behavior.

VI. AUTOCORRELATION FUNCTIONS FROM DECAY MEASUREMENTS

The theoretical calculations of Sec. III and V, as all such theoretical calculations, are based on the assumption that the spontaneously occurring fluctuations in equilibrium on the average obey the same decay law as small nonequilibrium macroscopic perturbations of the system.[17] The autocorrelation function for temperature fluctuations $c_T(\tau)$ thus reflects the average manner in which a temperature fluctuation decays in time. By perturbing the temperature of the experimental system and measuring its response, we are able to measure $c_T(\tau)$. The cosine transform [Eq. (3.4b)] of $c_T(\tau)$ gives $S_T(\omega)$.

This procedure will be illustrated for the simple system shown in Fig. 4(a), and described by Eq. (3.2). If the temperature at $t = 0$ is raised ΔT above T_0 the decay for $t > 0$ will proceed according to

$$T(t) = T_0 + \Delta T e^{-t/\tau_0} \quad (t > 0). \qquad (6.1)$$

Thus, $c_T(\tau) \propto e^{-\tau/\tau_0}$ for $\tau > 0$. Since $c_T(\tau)$ is symmetric about $\tau = 0$, the normalization condition $c_T(0) = \langle(\Delta T)^2\rangle = k_B T^2/C$ implies

$$c_T(\tau) = (k_B T^2/C)\exp(-|\tau|/\tau_0), \qquad (6.2)$$

which gives the same $S_T(\omega)$ as Eq. (3.5). Thus, the response of the system to a temperature perturbation determines the shape of the spectrum while the normalization condition $c_T(0) = \langle(\Delta T)^2\rangle$ determines the magnitude.

This procedure is not so straightforward for extended media described by a diffusion equation. In this case, we are interested in the spatially averaged temperature of some volume Ω,

$$\overline{T}(t) \equiv \frac{1}{\Omega}\int_{\Omega} T(\vec{x},t)\,d^3x. \qquad (6.3)$$

It is obvious that a given perturbation ΔT in $\overline{T}(t)$ could occur for an infinite variety of perturbation distributions $\Delta T(\vec{x},t)$, each of which might have a different decay in time. We must determine which perturbation distribution corresponds to the desired spectrum. In the simplest distribution, the temperature of Ω is uniformly raised a height ΔT above the surroundings at $t = 0$. This is accomplished by dissipating the power $P(t) = c\Delta T\delta(t)$ uniformly throughout Ω. The decay equation then becomes

$$\frac{\delta T}{\delta t} = D\nabla^2 T + \Delta T\delta(t)B(\vec{x}), \qquad (6.4)$$

where $B(\vec{x}) = 1$ if $\vec{x}$ is in Ω and $B(\vec{x}) = 0$ otherwise. Introducing

$$T(\vec{k},t) \equiv (2\pi)^{-1/2}\int T(\vec{x},t)\,e^{-i\vec{k}\cdot\vec{x}}d^3x\,,$$

we find

$$T(\vec{k},t) = \Delta T B(\vec{k})\,e^{-Dk^2 t} \quad (t > 0), \qquad (6.5)$$

where

$$B(\vec{k}) = (2\pi)^{-1/2}\int B(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, d^3x .$$

Now, since

$$\overline{T}(t) = \int B(\vec{x})\, T(\vec{x},t)\, d^3\vec{x} = \int B^*(\vec{k})\, T(\vec{k},t)\, d^3k , \qquad (6.6)$$

we find from Eq. (6.5) that

$$\overline{T}(t) = \Delta T \int |B(\vec{k})|^2 e^{-Dk^2 t}\, d^3\vec{k} \quad (t > 0). \qquad (6.7)$$

The cosine transform of Eq. (6.7) determines the shape of the spectrum:

$$S_T(\omega) \propto \int_0^\infty \overline{T}(t) \cos\omega t\, dt \propto \Delta T \int \frac{Dk^2 |B(\vec{k})|^2 d^3\vec{k}}{D^2k^4 + \omega^2} . \qquad (6.8)$$

For a regular Ω of sides $2l_1$, $2l_2$, $2l_3$,

$$|B(\vec{k})|^2 \propto \prod_{i=1}^{3} \frac{\sin^2(k_i l_i)}{k_i^2} ,$$

and we see that Eq. (6.8) predicts the same shape for the spectrum as Eq. (3.19) for uncorrelated fluctuations. This is not surprising. For uncorrelated fluctuations the average manner in which a fluctuation ΔT in $\overline{T}(t)$ occurs is by a uniform distribution of temperature over Ω. The importance of this result, however, is that it gives an experimental method of determining the shape of $S_T(\omega)$ for uncorrelated temperature fluctuations in an arbitrary volume with arbitrary coupling.

Another important result comes from a consideration of the temperature response of Ω to a step-function input of dissipated power, $P(t) = p_0\Theta(t)B(\vec{x})$, where $\Theta(t) = 0$ for $t < 0$ and $\Theta(t) = 1$ for $t > 0$. In this case

$$\frac{\partial T}{\partial t} = D\nabla^2 T + p_0\Theta(t)\, \frac{B(\vec{x})}{c} \qquad (6.9)$$

and

$$T(\vec{k},t) = (1 - e^{-Dk^2 t})\, p_0 B(\vec{k})/Dk^2 c . \qquad (6.10)$$

Thus, we find that

$$\overline{T}(t) = \frac{p_0}{c}\int |B(\vec{k})|^2 (1 - e^{-Dk^2 t})\, \frac{d^3\vec{k}}{Dk^2} . \qquad (6.11)$$

The cosine transform of Eq. (6.11) shows that

$$S_T(\omega) \propto \frac{p_0}{c}\int \frac{|B(\vec{k})|^2 d^3k}{D^2k^4 + \omega^2} , \qquad (6.12)$$

which has the same shape as $S_T(\omega)$ for correlated temperature fluctuations given by Eq. (5.5). The response of an arbitrary volume to a step-function input of power thus determines the shape of the spectrum for correlated temperature fluctuations.

Figures 8 and 9 show the response of the same Au sample used in Figs. 2 and 3 to δ-function and step-function power inputs. The sample was one arm of a Wheatstone bridge. The other three arms consisted of wire-wound resistors with a zero temperature coefficient of resistivity. At $t = 0$, a 1-kHz ac current was applied to the bridge. As the Au film became hot, the bridge became unbalanced. The PDP-11 was used as a digital lock-in detector to measure the voltage response of the bridge as a function of time. In this way, the decay of the sample temperature was determined. Each decay was averaged over many repetitions. The ac current provided the heating as well as the bias for measuring the temperature response. This method had the advantage that the necked-down areas of the film, which contributed the most noise, also were weighted the most heavily in the temperature response. The δ-function response was determined from the derivative of the step-function response. Direct δ-function (very narrow pulse) response measurements gave similar results.

Figure 8 shows the temperature response $\overline{T}(t)$ to a δ-function power input on three different time scales. The decay is essentially complete by a few hundredths of a second. Figure 9, on the

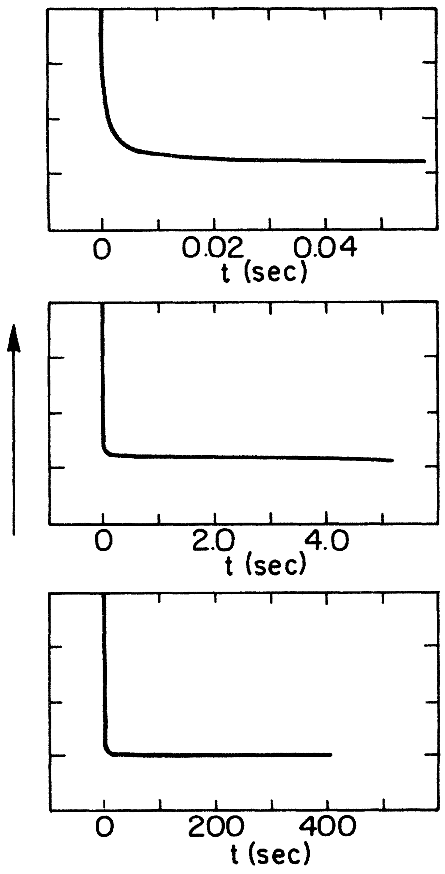

FIG. 8. Temperature response of Au sample of Fig. 2 to δ function of applied power.

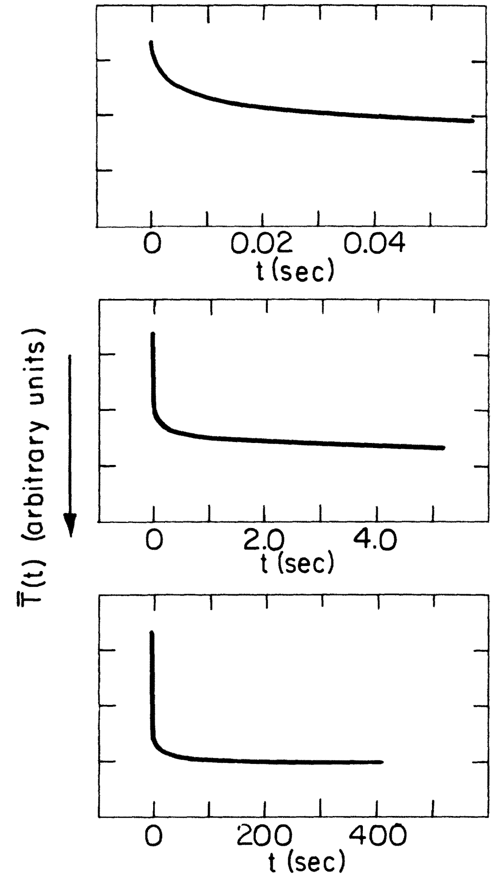

FIG. 9. Temperature response of Au sample of Fig. 2 to step function of applied power.

other hand, shows $\overline{T}(t)$ for a step-function input of power. The decay is much slower and appears to have appreciable contributions on all time scales. Figure 10 shows the cosine transform of these decays over many decades of frequency. The decay was assumed to give the shape of $c_T(\tau)$ and was normalized to $\overline{T}(0) = \langle(\Delta T)^2\rangle = \beta^2 T^2/3N$ to allow comparison with $S(f) = S_V(f)/\overline{V}^2$. The dotted line shows the expected $S(f)$ for uncorrelated temperature fluctuations. As predicted theoretically, $S(f)$ is $1/f$-like for higher frequencies, but flattens rapidly for low frequencies. In this case, the low-frequency cutoff, $f_1 = D/\pi l^2 \approx 80$ Hz, also corresponds to the measured change in behavior. The solid line, however, shows the expected $S(f)$ for correlated fluctuations. This spectrum shows the $1/f$ behavior down to the lowest frequencies measured. The squares in Fig. 10 show the measured relative noise spectrum for the same sample. The normalized cosine transform of the measured step-function response, which contains no fitted parameters, provides an excellent reproduction of the measured noise spectrum both in shape and magnitude. The observation that the cosine transform of the step-function response retains its $1/f$ behavior down to 10^{-2} Hz implies that even on these long time scales the heat conduction is preferentially two dimensional. Similar experiments on Bi films ($\beta < 0$) also show that the cosine transform of the step-function response matches the measured noise spectrum, indicating, in addition, that feedback effects are not important in measuring the decay.

We have shown experimentally that correlated temperature fluctuations in the complex experimental systems of metal films on glass substrates can, in fact, produce the measured $1/f$ spectrum. Moreover, we have shown that the usual assumption of uncorrelated fluctuations does not produce the measured $1/f$ spectrum for these samples. The assumption of correlated equilibrium temperature fluctuations in a diffusive medium together with the normalization $\langle(\Delta T)^2\rangle = k_B T^2/C_V$ are, thus, sufficient to predict all the measured characteristics of $1/f$ noise in continuous metal films.

VII. THERMAL FLUCTUATIONS IN OTHER SYSTEMS

It was not possible to test in detail the dependence of $S_V/\overline{V}^2$ [Eq. (3.24)] on Ω, β, and T using the metal films. However, Clarke and Hsiang[16] measured the $1/f$ noise in Sn films at the superconducting transition, where β is larger than at room temperature by a factor of about 10^5. In the first series of experiments, the Sn was evaporated directly onto a glass substrate. The main conclusions were: (i) $S_V/\overline{V}^2 \propto 1/\Omega$ (for a factor of 30 variation in Ω); (ii) $S_V \propto \beta^2$ (for a factor of 30 variation in β^2); (iii) $S_V \propto \overline{V}^2$ (for a factor of 500 var-

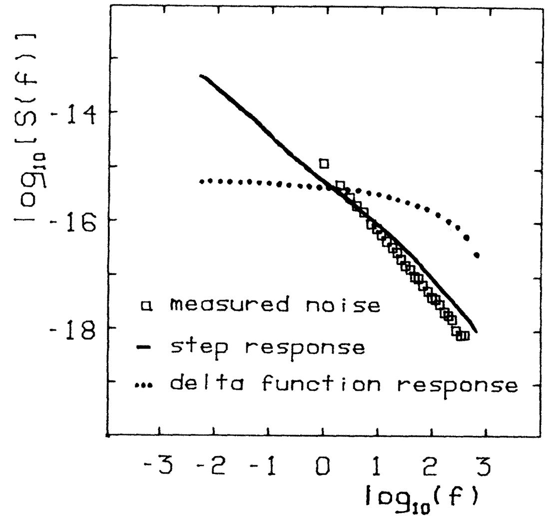

FIG. 10. $S(f)$ from cosine transform of temperature response to δ function (dotted line) and step function (solid line) of applied power from Figs. 7 and 8 normalized to $\overline{T}(0) = \beta^2 T^2/3N$; and measured noise spectrum $S(f) = S_V(f)/\overline{V}^2$ (□).

iation in $\overline{V}^2$); (iv) the noise showed the expected spatial correlation; (v) the magnitude of the noise was well represented by Eq. (3.24), thus verifying the dependence of S_V on T. Equation (3.24) thus correctly predicts the measured $1/f$ noise in Sn both at 4 and 300 K. In the subsequent experiments the Sn evaporation was preceded by a thin underlay of Al, that greatly enhanced the thermal coupling of the film to the substrate. Not only did the observed spectrum flatten at low frequencies to become white, but the degree of spatial correlation of the $1/f$ noise was appreciably reduced. As the coupling to the substrate increased a given temperature fluctuation in one section of the film could delay more rapidly by heat flow into the substrate and, consequently, was less likely to influence the neighboring sections. These results add strong support to the diffusion theory.

Clarke and Hawkins[18] measured the $1/f$ noise in Josephson tunnel junctions that were resistively shunted to eliminate hysteresis in the current-voltage characteristic. The noise was measured by passing a constant current I greater than the critical current I_c, through the junction and measuring the voltage fluctuations with a superconducting voltmeter. If the noise is assumed to be due to equilibrium temperature fluctuations modulating I_c, a suitable modification of Eq. (3.24) leads to the following result for the noise power spectrum:

$$S_V(f) \approx \left(\frac{\partial V}{\partial I_c}\right)_I^2 \left(\frac{\partial I_c}{\partial T}\right)^2 k_B T^2/3C_V f \,. \qquad (7.1)$$

C_V is the heat capacity of a volume given by the product of the junction area and a superconducting coherence length. The dependence of $S_V(f)$ on $(\partial V/\partial I_c)_I$ and $(\partial I_c/\partial T)$ was experimentally verified. Moreover, the magnitude of the noise was accurately predicted by Eq. (7.1).

Weissman and Feher[33] have studied the low-frequency noise in electrolytes in the presence of a current. Their system consisted of a capillary tube connecting two large reservoirs. $S_V/\overline{V}^2$ was proportional to β^2 and was quantitatively predicted by the three-dimensional diffusion model [Eq. (3.19)]. Presumably the thermal conductivities of the solution and the glass capillary were comparable and the boundary resistance between them not too large, so that the system was reasonably thermally homogeneous.

Thus, there are several different systems for which strong evidence exists for a thermal diffusion model of $1/f$ noise. However, in a series of experiments on semiconductors, we found no evidence for this model. In evaporated films of InSb we found that the noise was typically three orders of magnitude larger than that predicted by Eq. (3.25), and that there was no spatial correlation of the noise on a scale of a few mn at frequencies down to 10^{-3} Hz. We also found that the $1/f$ noise in very-thin (~ 100 Å) discontinuous metal films[3,4] was much larger than predicted by Eq. (3.25). In these systems, the noise due to thermal diffusion presumably exists, but is completely dominated by another mechanism. The lack of spatial correlation indicates that, if diffusive in nature, this additional mechanism must be characterized by $D < 10^{-5}$ cm^2 sec^{-1}. However, we were able to show that the $1/f$ noise in semiconductors and very thin metal films is also an equilibrium process.

VIII. THERMAL EQUILIBRIUM MEASUREMENT OF 1/f NOISE

We have observed a $1/f$-like power spectrum for low-frequency fluctuations of the mean-square Johnson noise voltage across a very-small sample of semiconductor or discontinuous metal film in thermal equilibrium. The $1/f$ spectrum is shown to be due to resistance fluctuations in the sample, and closely matches the resistance fluctuation spectrum obtained by passing a current through the sample.

Consider a resistance R of total heat capacity C_V shunted by a capacitance $\mathcal{C}$ and in thermal contact with a reservoir at temperature T_0. The voltage across the capacitor $V(t)$ represents a single degree of freedom that can exchange energy with the resistor via the charge carriers in the resistor. The exchange takes place on time scales of order $\tau = R\mathcal{C}$. In thermal equilibrium the average energy of the capacitor,

$$\langle E_c \rangle = \tfrac{1}{2}\mathcal{C}\langle V^2 \rangle = \tfrac{1}{2} k_B T_0 .$$

These voltage fluctuations (Johnson noise) are limited to a bandwidth of $1/4\tau$, and consequently have a spectrum of the form

$$S_V(f) = 4k_B T_0 R/(1 + 4\pi^2 f^2 \tau^2).$$

If the resistor is assumed to exchange energy with the reservoir on a time scale of order τ_R that is much greater than τ, the capacitor is able to reach equilibrium with the internal degrees of freedom of the resistor before the internal energy of the resistor can change. The temperature of the capacitor is then the same as the temperature of the resistor. $V^2(t)$, like $V(t)$, is a rapidly fluctuating quantity in time due to this exchange of energy between the resistor and capacitor. However, the average of $V^2(t)$ over a time, θ, such that $\tau \ll \theta \ll \tau_R$, $\langle V^2(t)\rangle_\theta = k_B T/\mathcal{C}$ (T is now the instantaneous temperature of the resistor), is sensitive to slow energy or temperature fluctuations in the resistor on time scales τ_B or longer.

Experimentally, the Johnson noise voltage $V(t)$ is passed through a filter with a bandpass from f_0 to f_1, squared, and averaged over a time $\theta > 1/f_0$ to give $P(t)$, a slowly varying signal proportional to the Johnson noise power in the bandwidth f_0 to f_1. Thus,

$$P(t) \approx 4k_B TR \int_{f_0}^{f_1} df/(1+4\pi^2 f^2 \tau^2) + P_0(t), \qquad (8.1)$$

where if R and T are fixed the first term on the right-hand side represents the average of $P(t)$, and $P_0(t)$ represents fluctuations in $P(t)$ about the average due to the rapid exchange of energy between capacitor and resistor. Because this exchange is so rapid, $P_0(t)$ has a spectrum $S_{P_0}(f)$ that is independent of f for the low frequencies in which we are interested. S_{P_0} may be reduced by increasing the bandwidth or by moving the bandwidth to higher frequencies, but in practice $P_0(t)$ severely limits the accuracy of measurements of $P(t)$.

If the bandwidth in Eq. (8.1) is either totally above or totally below the knee at $1/2\pi\tau$, $P(t)$ is sensitive to slow resistance as well as temperature fluctuations. These resistance fluctuations ΔR may be driven by temperature fluctuations with a spectrum $S_T(f)$ so that $\Delta R = \overline{R}\beta\Delta T$; or be temperature-independent fluctuations ΔR_0 with a spectrum $S_{R_0}(f)$ (such as number or mobility fluctuations of the charge carriers). Thus, from Eq. (8.1)

$$\Delta P(t)/\overline{P} = (1 \pm \beta T_0)\Delta T/T_0 + \Delta R_0/\overline{R} + P_0(t)/\overline{P},$$

and the relative power spectrum for fluctuations in $P(t)$ is of the form

$$\frac{S_P(f)}{\overline{P}^2} = (1 \pm \beta T_0)^2 \frac{S_T(f)}{T_0^2} + \frac{S_{R_0}(f)}{\overline{R}^2} + \frac{S_{P_0}}{\overline{P}^2}, \qquad (8.2)$$

where the plus sign corresponds to $f_0 < f_1 \ll 1/2\pi\tau$, and the minus sign corresponds to $f_1 > f_0 \gg 1/2\pi\tau$. If however, most of the noise power and the knee frequency $1/2\pi\tau$ are included in the bandwidth (i.e. (i.e., $f_0 \ll 1/2\pi\tau \ll f_1$), from Eq. (8.1) we find $P(t) \approx k_B T/C + P_0(t)$ and $S_P(f)/\overline{P}^2 = S_T(f)/T_0^2 + S_{P_0}/\overline{P}^2$. In this limit, $P(t)$ is not sensitive to resistance fluctuations. Thus, with an appropriate choice of bandwidth, the low-frequency spectrum of $P(t)$ is an equilibrium measurement of $S_T(f)$ or $S_{R_0}(f)$ provided the temperature or resistance fluctuations are large enough to dominate S_{P_0}.

Our initial measurements were on evaporated InSb films with a thickness of 1000 Å and a resistivity of about 1 Ω cm. As indicated in Sec. VII, we expected to observe only the resistance fluctuations $\Delta R_0(t)$. In order to make the relative resistance fluctuation spectrum $S_{R_0}(f)/\overline{R}^2$ large enough to dominate $S_{P_0}/\overline{P}^2$, the samples were made as small as possible. The resistance of a strip of InSb was monitored while the strip was cut transversely with a diamond knife until only a small bridge containing typically about 10^6 atoms remained. In the presence of a direct current I the relative power spectrum of the voltage fluctuations is $S(f) = S_V(f)/\overline{V}^2 = S_{R_0}(f)/\overline{R}^2$. The solid line in Fig. 11 shows $S(f)$ for a 20-MΩ bridge of InSb measured with a direct current.

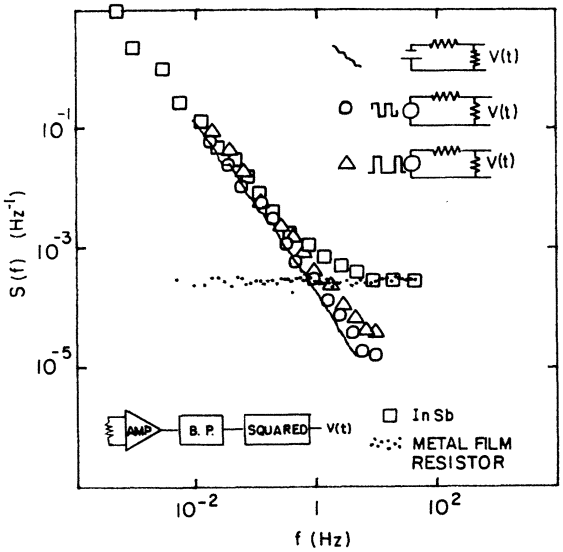

FIG. 11. InSb bridge: $S_V(f)/\overline{V}^2$ using dc bias (solid line), ac bias (○), pulsed current bias (△); Johnson noise measurement, $S_P(f)/\overline{P}^2$(□). Background $S_P(f)/\overline{P}^2$ from metal film resistor (dotted line).

We also measured the $1/f$ noise in two other ways. The spectrum was remeasured using an ac technique in which a square-wave current of zero mean value was applied to the sample. The frequency was about 10^3 times higher than the lowest frequency to be measured in the spectrum. The PDP-11 was used as a digital lock-in detector to measure the spectrum of the amplitude fluctuations of the induced voltage. The relative spectrum is plotted with open circles in Fig. 11. In a third technique the current was supplied as a series of positive-going pulses to reduce the power dissipated in the sample. The pulses were typically 1 msec long with a period of 10 msec. The relative spectrum is shown in Fig. 11 as open triangles. All three techniques measure the same resistance spectrum $S_{R_0}(f)/\overline{R}^2$. The agreement of the three spectra demonstrates that neither a direct current nor a constant dissipation of power is the cause of the $1/f$ spectrum.

For the measurement of $P(t)$, the sample was capacitively coupled to a preamplifier to prevent any leakage current flowing through the sample. The input capacitance produced a knee frequency, $1/2\pi Rc \approx 500$ Hz, in the Johnson noise spectrum. After amplification the noise was filtered with a 10–300-kHz bandpass filter, squared with an analog multiplier, and filtered to remove frequencies above the digitizing frequency. Since the bandpass is above the knee frequency the calculated relative spectrum of this signal is given by Eq. (8.2) (with the minus sign), while the measured relative spectrum is shown as the open squares in Fig. 11. The white spectrum above 1 Hz represents $S_{P_0}/\overline{P}^2$. The $1/f$ spectrum below 1 Hz closely matches the current-biased measurements. To ensure that the $1/f$ spectrum was generated by fluctuations in the sample rather than by spurious effects from our electronics, the InSb was replaced by a metal film resistor (which did not exhibit $1/f$ noise) of the same resistance. This relative spectrum is shown dotted in Fig. 11. The spectrum is white down to the lowest frequency measured, and represents only the term $S_{P0}/\overline{P}^2$.

It was sometimes necessary to use the digital lock-in technique to eliminate drifts in the zero offset of the analog multiplier. The amplifier output was gated on and off to provide an analog multiplier output that consisted of equal periods of zero offset and zero offset plus squared signal. These periods were digitized and the offset subtracted within the computer before the spectrum was analyzed.

We have made similar measurements on metal films. The three current-biased techniques gave identical relative spectra for continuous metal films in which the resistance fluctuations are temperature induced, confirming that the current serves only as a probe of the equilibrium fluctuations. However, we were unable to make these films small enough for $S_R(f)/\overline{R}^2$ to dominate $S_{P_0}/\overline{P}^2$ at frequencies down to 10^{-3} Hz. We therefore used very thin (< 100 Å) films in which temperature induced fluctuations are not dominant. In Fig. 12, the continuous curve is the relative spectrum of a very-thin Nb film ($R \approx 200$ kΩ) measured with an ac current bias. The open squares are a Johnson noise measurement with a bandwidth of 100–200 kHz, above the knee frequency of 40 kHz. The agreement below 10^{-2} Hz is excellent. The dotted spectrum was obtained from the same sample using a bandwidth of 5–200 kHz, which includes the knee frequency and most of the Johnson noise power. Although the low-frequency spectrum is substantially reduced [as expected when $P(t)$ is no longer sensitive to resistance fluctuations], it is still above the background spectrum of a large metal film resistor. This residual noise is possibly due to the temperature fluctuation term $S_T(f)/T_0^2$. Indeed, the assumption of a $1/f$ spectrum for $S_T(f)$ [Fig. 5(b)] for a sample of 10^6 atoms atoms yields

$$S_T(f)/T_0^2 \sim 3\times 10^{-7}/f \text{ Hz}^{-1},$$

a value that is consistent with the observed spectrum.

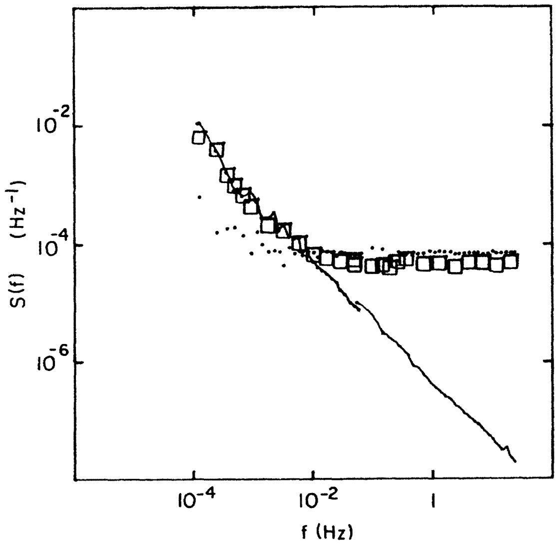

FIG. 12. Nb bridge: $S_V(f)/\overline{V}^2$ using ac bias (solid line); $S_P(f)/\overline{P}^2$ (□). $S_P(f)/\overline{P}$ including knee frequency (dotted line).

Our results strongly suggest that $1/f$ noise in semiconductors and discontinuous metal films arises from equilibrium resistance fluctuations. Current-biased measurements probe these resistance fluctuations, but in no way generate them. This idea is consistent with several current theories of $1/f$ noise that propose various mechanisms for the resistance fluctuations, for example, the McWhorter[9] theory for semiconductors, carrier mobility fluctuations in semiconductions and ionic solutions,[7,8] and the temperature fluctuation model. Our results are obviously inconsistent with theories that involve nonequilibrium processes. For example, turbulence theories,[34] theories that require a long-term steady current or power,[35] and theories involving thermal feedback via the heat generated by an external current.[36]

IX. CONCLUSIONS

We have shown that $1/f$ noise in metal and semiconductor films is an equilibrium process. For continuous metal films the absence of $1/f$ noise in manganin; the scaling of $S_V(f)$ as $\overline{V}^2/\Omega$ for different materials; the general decrease of $S_V(f)/\overline{V}^2$ with decreasing temperature; the observation of frequency-dependent spatial correlation for the $1/f$ noise; the agreement of ac, dc, and pulsed-current resistance-fluctuation spectra; and the ability of equilibrium temperature fluctuations to accurately predict the magnitude of the $1/f$ noise [with an assumed $1/f$ spectrum for $S_T(f)$] indicate that equilibrium temperature fluctuations modulating the resistance are the physical origin of the $1/f$ noise. The same mechanism also accounts for the $1/f$ noise in metal films at the superconducting transition and in Josephson junctions.

Although temperature fluctuations are expected to obey a diffusion equation, the usual calculated spectra for uniform diffusive systems, in which the fluctuations are spatially uncorrelated, do not give a $1/f$ spectrum. Attempts at more accurate models of the complex experimental configuration (in which the diffusive medium is coupled to a substrate) only flatten the spectrum further at low frequencies. Moreover, we have demonstrated experimentally for the metal films (by a measurement of the shape of the autocorrelation function from the temperature response to a δ-function power input) that uncorrelated temperature fluctuations do not produce the $1/f$ spectrum.

On the other hand, we have shown both theoretically and experimentally (by the temperature response to a step function of power) that spatially correlated temperature fluctuations can, in fact, account for the $1/f$ spectrum in the frequency range in which it is observed. The physical origin of the spatially correlated temperature fluctuations remains an unsolved problem. Another possible difficulty is the proper normalization of the spectrum for correlated fluctuations. However, the use of $\langle(\Delta T)^2\rangle = k_B T^2/C_V$ to normalize the spectrum for correlated fluctuations does lead to a result in excellent agreement with the experimental measurements.

A different physical mechanism for the $1/f$ noise dominates in semiconductors and discontinuous metal films: the observed noise is much larger than predicted by the theory, and is not spatially correlated. The agreement of the low-frequency resistance-fluctuation spectrum obtained from Johnson noise measurements with that obtained from current-biased measurements shows, however, that even in these systems the $1/f$ noise is due to *equilibrium* resistance fluctuations.

ACKNOWLEDGMENT

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