Low-frequency noise in tin and lead films at the superconducting transition*

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We have measured the noise power spectra $S_V(f)$ of tin and lead films at the superconducting transition in the frequency range 0.1 Hz to 5 kHz. Two types of samples were made: Type A were evaporated directly onto glass substrates, while type B were evaporated onto glass and sapphire substrates with a 5-nm aluminum underlay. For type A samples the spectra varied approximately as f^{-1} and were proportional to $\bar{V}^2\beta^2/\Omega$, where $\bar{V}$ was the mean voltage across the sample, β was the temperature coefficient of resistance, and Ω was the sample volume. For type-A tin samples, the noise in two regions a distance d apart was correlated at frequencies $\lesssim D/\pi d^2$. These results are consistent with a thermal diffusion model of Clarke and Voss. The magnitude of the noise in type-A tin samples was accurately predicted by the semiempirical formula $S_V(f) = \bar{V}^2\beta^2 k_B T^2/C_V[3 + 2\ln(l_1/l_2)]$ where l_1 and l_2 are the length and width of the film; for lead samples, this formula overestimates the observed noise by a factor of 5. As the magnetic field perpendicular to the tin films was increased from zero to about 2 G, β was reduced by about a factor of 5, and $S_V(f)$ was found to be proportional to β^2. The spectra of type-B films were markedly different from those of type-A films. In the case of the tin samples, the spectra became flat below about 30 Hz. The degree of spatial correlation of the noise was markedly reduced. In the case of the lead films, the spectra varied as about $f^{-1.1}$ for type A and as about $f^{-0.8}$ for type B. These changes are ascribed to the enhancement of the thermal contact between the films and the substrate by the aluminum underlay. Separate experiments confirmed that the underlay decreased the thermal boundary resistance between the film and the substrate. Implications of this work for device applications are briefly discussed.

I. INTRODUCTION

A model involving equilibrium temperature fluctuations has been used by Voss and Clarke[1,2] to quantitatively predict the $1/f$ voltage noise observed in metal films at room temperature. The temperature fluctuations generate resistance fluctuations if the temperature coefficient of resistance, $\beta = (1/R)\,dR/dT$, is nonzero. In the presence of a steady current the resistance fluctuations in turn give rise to voltage fluctuations. In this paper, we report measurements of $1/f$ noise in tin and lead films at the superconducting transition. These measurements provide strong additional evidence to support the thermal-fluctuation model.[3]

There are few reported measurements of low-frequency noise at the superconducting transition when the applied magnetic field and current are sufficiently low that the transition temperature is not significantly suppressed from its zero-field value. Maul, Strandberg, and Kyhl[4] found excess noise at the superconducting transition of tin films, but did not demonstrate explicitly that the power spectrum was $1/f$ at low frequencies. They analyzed their results using a model in which equilibrium temperature fluctuations of the film gave rise to resistance fluctuations. They assumed that the temperature of the film was uniform at any instant of time, and that the substrate was a constant-temperature reservoir. In this model the decay of a fluctuation was exponential, characterized by a single relaxation time, and the power spectrum was a Lorentzian. The voltage power spectrum was shown experimentally to be proportional to $(dR/dT)^2 I^2$, where I was the current flowing in the film. Katz and Rose[5] showed that the low-frequency noise at the transition of tin films has a $1/f$ power spectrum, but they did not propose a mechanism for this behavior.

There have been extensive measurements of low-frequency noise generated by flux flow in superconducting films and foils.[6-16] Most of these measurements were made in the presence of high magnetic fields and/or currents and at temperatures well below the zero-field transition temperature. The samples were therefore in an intermediate state (type-I superconductors) or a mixed state (type-II superconductors) in which flux motion was induced by the current. In our own measurements, the low-frequency noise is dominated by the thermal-fluctuation mechanism. We shall compare the results expected from the flux-flow and thermal mechanisms in Sec. VII.

In Sec. II we briefly review the theory relevant to our measurements, and in Sec. III we describe the experimental apparatus and techniques. Sec-

tions IV–VI contain the experimental results. A discussion of these results and our conclusions appear in Sec. VII.

II. THEORETICAL CONSIDERATIONS

We briefly review the essential features of the theory of $1/f$ noise proposed by Voss and Clarke.[1,2] They suggest that $1/f$ noise in metal films arises from equilibrium temperature fluctuations that modulate the resistance of the film and thus generate voltage fluctuations in the presence of a steady current. Consider a model system in which a rectangular metal bar of dimensions l_1, l_2, l_3 ($l_1 \gg l_2 \gg l_3$) is buried in a infinite medium that is an electrical insulator. The metal and the insulator are assumed to have the same thermal diffusivity D and to have no thermal boundary resistance. We define a local temperature $T(\vec{r}, t)$ in an element of volume as the average energy divided by the heat capacity. In the original theory it was assumed that $T(\vec{r}, t)$ obeys a diffusion equation driven by a random energy flux. The resulting temperature fluctuations are spatially uncorrelated, that is, $\langle \Delta T(\vec{r}+\vec{s}, t)\, \Delta T^*(\vec{r}, t)\rangle \propto \delta(\vec{s})$. One can then calculate the frequency-dependent correlation function $c_T(\vec{s}, f) \equiv \langle T(\vec{r}+\vec{s}, f)\, T^*(\vec{r}, f)\rangle$:

$$c_T^{(3)}(\vec{s}, f) \propto |\vec{s}|^{-1} \cos(|\vec{s}|/\lambda)\, e^{-|\vec{s}|/\lambda}, \tag{2.1}$$

where $\lambda = (D/\pi f)^{1/2}$ is the frequency-dependent correlation length. For a one-dimensional system, i.e., an infinitely long isolated rod in which heat flows only along the rod,

$$c_T^{(1)}(\vec{s}, f) \propto f^{-1/2} \cos[(\tfrac{1}{4}\pi) + |\vec{s}|/\lambda]\, e^{-|\vec{s}|/\lambda}. \tag{2.2}$$

The spatial correlation of the noise at low frequencies implied by Eqs. (2.1) and (2.2) was observed experimentally.

The spatially averaged temperature of the bar fluctuates with a power spectrum

$$S_T(f) \propto \begin{cases} \text{const}, & f \ll f_1 \\ \ln(\text{const}/f), & f_1 \ll f \ll f_2 \\ f^{-1/2}, & f_2 \ll f \ll f_3 \\ f^{-3/2}, & f_3 \ll f \end{cases} \tag{2.3}$$

The spectrum has changes of slope near the characteristic frequencies of the bar, $f_i = D/(4\pi l_i^2)$. The spectrum does not contain an explicit $1/f$ region. However, in the experimental systems studied, the assumptions of thermal homogeneity are far from realistic and the spectrum is expected to deviate from Eq. (2.3). Voss and Clarke[1,2] proposed that Eq. (2.3) should be replaced with a semiempirical model spectrum in which $S_T(f) \propto$ const for $f \ll f_1$, $S_T(f) \propto f^{-1}$ for $f_1 \ll f \ll f_2$, and $S_T(f) \propto f^{-3/2}$ for $f_2 \ll f$. This assumption together with the normalization condition

$$\langle (\Delta T)^2 \rangle = k_B T^2 / C_V = \int_0^\infty S_T(f)\, df$$

yields the result $S_T(f) = k_B T^2 / C_V [3 + 2\ln(l_1/l_2)]f$ for $f_1 \ll f \ll f_2$, where C_V is the heat capacity of the bar.

In the frequency range $f \ll f_2$ we have $\lambda(f) = (D/\pi f)^{1/2} \gg l_2 \gg l_3$, so that the temperature is uniform over the cross section of the bar. In this limit, a steady current flowing along the bar generates a voltage that fluctuates with a power spectrum $S_V(f) = \overline{V}^2 \beta^2 S_T(f)$, where $\overline{V}$ is the mean voltage across the bar. The predicted power spectrum for the voltage fluctuations is

$$\frac{S_V(f)}{\overline{V}^2} = \frac{\beta^2 k_B T^2}{C_V [3 + 2\ln(l_1/l_2)]f}. \tag{2.4}$$

Voss and Clarke[2] further demonstrated that an explicit $1/f$ spectrum was predicted by a model in which the temperature fluctuations were correlated, with $\langle \Delta T(\vec{r}+\vec{s}, t)\, \Delta T^*(\vec{r}, t)\rangle \propto |\vec{s}|^{-1}$. The predicted spectrum was $S_T(f) \propto f^{-1/2}$ for $f \ll f_1$, $S_T(f) \propto f^{-1}$ for $f_1 \ll f \ll f_2$, $S_T(f) \propto f^{-3/2}$ for $f_2 \ll f \ll f_3$, and $S_T(f) \propto f^{-2}$ for $f_3 \ll f$. The magnitude of the $1/f$ region differed from Eq. (2.4) only by a factor close to unity. The frequency-dependent correlation function in three dimensions becomes

$$c_T^{(3)}(\vec{s}, f) \propto \frac{e^{-|\vec{s}|/\lambda}}{f^{1/2}} \frac{\sin(|\vec{s}|/\lambda)}{|\vec{s}|/\lambda}. \tag{2.5}$$

In both model calculations, it was assumed that the subvolume and its surrounding medium had the same values of D and were separated by a negligible thermal boundary resistance. It is difficult to quantitatively assess the effect of a finite boundary resistance and of a discontinuity in D.[17] One can show, however, that if the film is strongly thermally coupled to a substrate of much higher diffusivity the spectrum at low frequencies will be flatter than if the film is weakly coupled and/or if the substrate has a low thermal diffusivity.

The observed spectra in metal films were always close to $1/f$ down to the lowest frequency measured, typically 10^{-3} Hz. This frequency is two orders of magnitude below f_1 for typical samples. Nevertheless, Eq. (2.4) quantitatively predicted the magnitude of the noise over the entire frequency range. It should be noted that the magnitude of Eq. (2.4) is very insensitive to the value of $f_1 = D/4\pi l_1^2$: If f_1 is decreased by several orders of magnitude, the effect on $S_V(f)$ is insignificant.

It was not possible to test in detail the dependence of Eq. (2.4) on β, sample volume (Ω), and

temperature by studying the room-temperature samples. The purpose of the present paper is to report a detailed study of $1/f$ noise at the superconducting transition of tin and lead, and to show that $S_V(f)$ depends on $\bar{V}$, β, Ω, and T as predicted by Eq. (2.4). We also show the effects on the power spectrum of varying the thermal coupling to the substrate.

The fact that the superconducting transition is smeared out in temperature is a consequence of fluctuations in the order parameter. A recent review of the microscopic fluctuation theories and of their experimental verification has been given by Tinkham.[16] These fluctuations occur on a time scale that is extremely short compared with the times of interest in the present work. We are concerned only with the resultant average shape of the transition curve, and the relatively slow thermal fluctuations that take place due to the exchange of energy between the superconductor and its substrate.

In order to apply Eq. (2.4) to our results, we require a value for the heat capacity of the film C_V. As the temperature of the film is lowered through the superconducting transition there is an increase in the electronic heat capacity that we assume to be smeared out in a way that reflects the shape of the resistive transition. Thus, as energy is exchanged between the film and the substrate, the heat capacity of the film fluctuates. However, it may be shown[18] that the correction introduced is of order $\langle(\Delta T)^2\rangle^{1/2}/\delta T$, where δT is the width of the transition. For our samples, $\langle(\Delta T)^2\rangle^{1/2} \sim 10^{-7}$ K and $\delta T \sim 3$ mK, so that the correction is of order 10^{-4}, and negligible compared with other errors in the measurements. Since most of our measurements were made with the films near the midpoint of the transition, we shall take as the heat capacity the sum of the phonon heat capacity at T_C and the average of the superconducting and normal electronic heat capacities just below and just above the transition.

III. EXPERIMENTAL PROCEDURES

A. Film preparation and cryostat

Tin and lead films, typically 0.1 μm thick, were evaporated at about 5×10^{-5} Torr onto glass slides or single crystal C-cut sapphire plates at room temperature. These will be referred to as type-A samples. Other samples (type-B) were prepared with a 5-nm aluminum film deposited on the substrate prior to the tin or lead evaporation. A period of about 1 min elapsed between the completion of the aluminum deposition and the beginning of the tin or lead evaporation, so that the aluminum was largely oxidized. Type-A and type-B films had comparable resistivity ratios (~5 for tin and ~30 for lead), superconducting transition temperatures (~3.8 K for tin and ~7.2 K for lead), and transition widths (~3 mK for tin and ~30 mK for lead).

The films were cut with a diamond point into the geometry shown in the inset of Fig. 4, with separate current and voltage leads. (Films with cut edges tend to have much sharper transition temperatures than those with the tapered edges left by evaporation through a narrow mask.[19]) Manganin wires were attached to the films with cold-pressed indium. The reverse side of the substrate was greased with Apiezon-N grease, and the substrate clamped to a copper plate. A 68-Ω Allen Bradley carbon resistor, used as thermometer, and a noninductively wound heater were glued to the reverse side of the copper plate. The plate was supported by a stainless steel rod, and the whole assembly mounted in a vacuum can. The can was immersed in liquid helium whose temperature was stabilized by a mechanical manostat to ± 100 μK. The temperature of the copper plate could be raised above that of the bath by the heater. The thermal time constant of the copper plate was about 75 sec. A superconducting solenoid wound around the can was used in some experiments to produce a persistent magnetic field perpendicular to the plane of the film. The cyrostat was surrounded by two concentric μ-metal cylinders that reduced the ambient magnetic field in the can to about 10 mG. The complete cryostat was placed in a shielded room that effectively eliminated rf pickup.

B. Measurement techniques

A steady current was supplied to the sample from a mercury battery in series with a large wire-wound resistor. The fluctuating voltage across the sample was amplified by a room-temperature transformer with a turns ratio of 100:1 (PAR-190 below 100 Hz and Triad JZ-5 above 100 Hz) dc coupled to a battery operated PAR-113 preamplifier inside the shielded room. The overall gain of the system was calibrated with an ac signal of variable frequency and a white noise source for each sample resistance. The output from the preamplifier was filtered and Fourier analyzed by a PDP-11 computer using a fast Fourier transform. The preamplifier noise was subtracted out from each measured spectrum.

C. Spurious noise sources

There are several sources of extraneous noise.

(i) 60-Hz pickup. A 60-Hz signal was observed in most measurements as an easily recognizable peak in the spectrum that could be subtracted out.

(ii) Current supply noise. The noise in the dc current supplied to the sample was investigated by taking a noise spectrum with the sample in the normal state: The noise was indistinguishable from that obtained with the current turned off. This noise source was thus insignificant.

(iii) Nonequilibrium fluctuations in the temperature of the substrate. Slow drifts in the temperature of the helium bath and the heater current induced low-frequency fluctuations in the temperature of the copper plate. We investigated these fluctuations by measuring the $1/f$ noise in a film whose area was about 100 times greater than that of our typical size samples. Above about 0.1 Hz, $S_V(f)/\overline{V}^2$ was about 100 times smaller for the larger sample than for the typical sample, as predicted by Eq. (2.4). If the noise had originated from fluctuations in the temperature of the copper plate, the measured spectrum would have been independent of sample volume. As the frequency was lowered below 0.1 Hz, the power spectra of the two films approached each other, and their slope steepened. (With our measuring technique, a linear change of voltage with time produces a $1/f^2$ power spectrum.)

We concluded that temperature fluctuations in the substrate were significant at or below 0.1 Hz, while at or above 0.1 Hz, the measured noise for our typical sample size was dominated by intrinsic equilibrium fluctuations in the film. We generally regarded our measured spectra to be reliable down to about 1 Hz.

(iv) Microphonic noise. Vibration of the cryostat could induce a substantial amount of noise. The effects were minimized by rigidly mounting the sample and can, and by taking care not to disturb the cryostat during measurements.

(v) Preamplifier noise. At frequencies above 1 Hz preamplifier noise was usually the dominant background noise source. The noise was measured by switching off the dc bias supply or by having the sample in the normal or superconducting state with the bias current switched on. The preamplifier noise spectrum was measured immediately after the power spectrum of each sample was measured, and subtracted out. The preamplifier power spectrum was typically 1–5 orders of magnitude below the sample power spectrum. For a combined sample and lead resistance of 22 Ω, the noise was typically 5×10^{-18} V^2 Hz^{-1} referred to the input of the transformer.

IV. THERMAL COUPLING OF TYPES-*A* AND -*B* FILMS

Before describing the measured noise spectra, it is convenient to discuss the thermal coupling between the films and the substrates with and without an aluminum underlay. The original motivation for preparing type-*B* samples was as follows. In an attempt to investigate the effect of the substrate properties on the noise spectrum we prepared type-*A* tin samples on single-crystal sapphire plates whose thermal diffusivity was several orders of magnitude higher than that of glass. We found that the film was very poorly anchored to the sapphire, and could easily be removed. In order to improve the bonding of the film to the substrate we predeposited a 50-Å layer of aluminum. This layer greatly enhanced the bonding. We briefly describe two methods by which we qualitatively investigated the effect of the underlay on the thermal contact of the films to the substrate.

At sufficiently high bias currents all of our samples became unstable due to self-heating effects, and could not be maintained at the superconducting transition. Low-frequency instabilities set in when βP becomes comparable with the thermal conductance between the film and the substrate,[20] where P is the power dissipated in the film. For type-*A* tin samples of size 2.5 mm×15 μm×0.1 μm and with $\beta \sim 150$ K^{-1} thermal instability occurred with a glass substrate at a power dissipation of order 1 μW. This power is at least an order of magnitude greater than the power dissipated in any of the noise measurements, during which the samples had no tendency to become unstable. Type-*A* lead samples on glass substrates of comparable size and with $\beta \sim 30$ K^{-1} became thermally unstable at a power level of around 4 μW. Since $\beta_{Sn} \sim 5\beta_{Pb}$, these results tend to suggest that the thermal coupling of tin and lead films to glass substrate is of the same order. Type-*A* tin samples on sapphire substrates were subject to thermal runaway at relatively low-power dissipation ($\ll 1$ μW), indicating a poor thermal contact with sapphire. For type-*B* samples of the same size, thermal instability occurred at typically 10 μW for tin films and 4 μW for lead films for both glass and sapphire substrates. It appears that the aluminum underlay greatly enhances the thermal contact of tin to glass and sapphire, but has relatively little effect on lead deposited on glass.

We obtained another measure of the effective thermal conductance between film and substrate as follows. With the temperature in the transition range, the voltage across the film generated by a given bias current was measured. The value of the current was then changed, and the voltage remeasured when the sample had again reached a steady state. The change in the temperature of the film was deduced from the change in its resistance. We then calculated an effective thermal conductance (K_e) from the change in temperature result-

ing from a given change in power dissipation. For tin samples of size 2.5 mm×15 μm×0.1 μm on glass substrates we find for type A, $K_e \approx 1.5$ μWK^{-1} (~40 μWK^{-1} for 1 mm^2), while for type B, $K_e \approx 12$ μWK^{-1}. For lead films of size 25 mm ×20 μm×0.12 μm on glass substrates we find for type A, $K_e \approx 6$ μWK^{-1} (~120 μWK^{-1} for 1 mm^2) while for type B, $K_e \approx 12$ μWK^{-1}. These results confirm that the aluminum underlay greatly enhances the thermal contact between tin films and a glass substrate, but has relatively little effect on lead films. It is far from clear what "effective" thermal conductivity is measured by this technique. When power is dissipated in the film, a temperature gradient is established in the substrate in the vicinity of the film. Consequently, the measured thermal conductivity cannot be interpreted as the ratio of the power dissipation to the temperature difference between the film and a constant temperature substrate.

We believe that the aluminum underlay enhances the thermal contact by improving the bonding of the films to the substrate. It is known that aluminum[21] adheres strongly to glass whereas tin[22] does not. As mentioned in Sec. III the aluminum film is certainly partially oxidized, and it is known that an oxide layer can greatly enhance the bonding of a metal film to glass.[22-24] As evidence for our hypothesis, in Fig. 1 we show scanning electron micrographs of A and B tin films on the same glass substrate. In Fig. 1(a), the type-A film shows considerable clumping of the tin, while in Fig. 1(b), the tin grains are much more uniformly nucleated. It is therefore likely that the effective contact area between film and substrate is substantially higher in type-B samples than in type A.

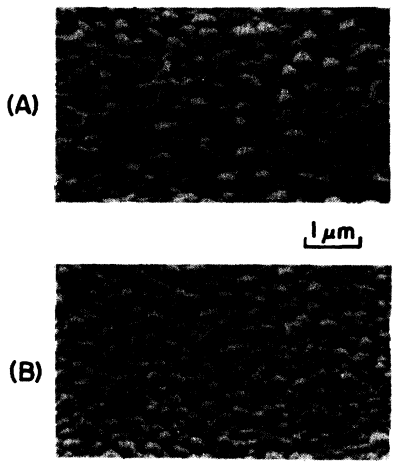

FIG. 1. Scanning electron micrographs of tin films on a glass substrate: (a) type A; (b) type B.

The fact that the transmission probability of phonons from a film to its substrate depends strongly on the nature of the films probably also plays a role in the reduction of the thermal contact resistance by the aluminum underlay. The fraction[25] of ballistic phonons transmitted from an aluminum film to a glass substrate is about 0.8, while the fraction for a tin film is only about 0.1. These values are calculated[25] on the assumption that the films are ideally bonded to the substrate. Thus one expects the boundary resistance between aluminum film and a glass substrate to be substantially lower than that between a tin film and a glass substrate.

Despite the highly qualitative nature of our experiments, it is apparent that the underlay makes a marked improvement in the thermal contact between tin films and glass or sapphire substrates, but has little effect in the case of lead films. We shall see in Secs. V and VI that the change in the thermal contact can have a marked effect on the noise spectrum.

V. EXPERIMENTAL RESULTS FOR TIN

A. Noise power spectra

A typical power spectrum for a type-A tin sample on a glass substrate is shown in Fig. 2(a) with a dc current bias of 50 μA. The sample resistance

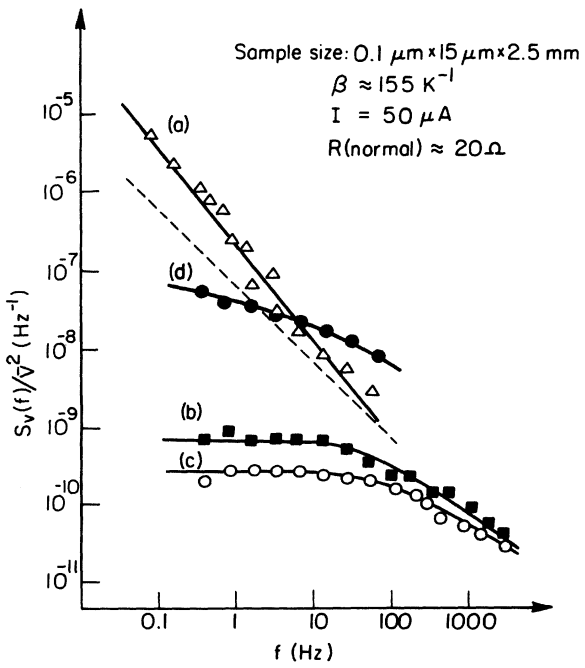

FIG. 2. Typical noise power spectra for tin films: (a) type A on glass [dashed line is calculated from Eq. (2.4)]; (b) type B on sapphire; (c) and (d) type B on glass.

in the normal state was about 20 Ω. The dashed line indicates the spectrum obtained from Eq. (2.4) using the following parameters: $\Omega = 3.8 \times 10^{-9}\,\mathrm{cm}^3$, $\beta = 155\,\mathrm{K}^{-1}$, and an average $C_V/\Omega = 1.44 \times 10^{-3}\,\mathrm{JK}^{-1}\mathrm{cm}^{-3}$.[26] Although the noise power spectrum is a little steeper than f^{-1}, the general agreement between the two curves is good.

We have investigated the dependence of $S_V(f)$ on Ω, β, and $\overline{V}$. In Fig. 3(a) we plot S_V vs Ω^{-1} for six samples of different volumes, each biased to give the same value of $\overline{V}$. Five of these samples (solid circles) were 0.1 μm thick, and each was biased at a point on the transition to give a comparable value of β, about $155\,\mathrm{K}^{-1}$. The sixth sample [a triangle in Fig. 3(a)] had a thickness of 0.6 μm, and its superconducting transition was broadened to give $\beta = 62\,\mathrm{K}^{-1}$ at the midpoint. In Fig. 3(a), the value of $S_V(f)$ has been multiplied by a factor of 6.25 to account for the lower β. The fact that $S_V(f)$ scales with Ω^{-1} for variations in both the area and the thickness of the films provides strong evidence that the $1/f$ noise is a bulk effect. If the noise were a surface effect, $S_V(f)/\overline{V}^2$ would scale inversely as the area of the films, but would presumably be independent of the thickness of the films.

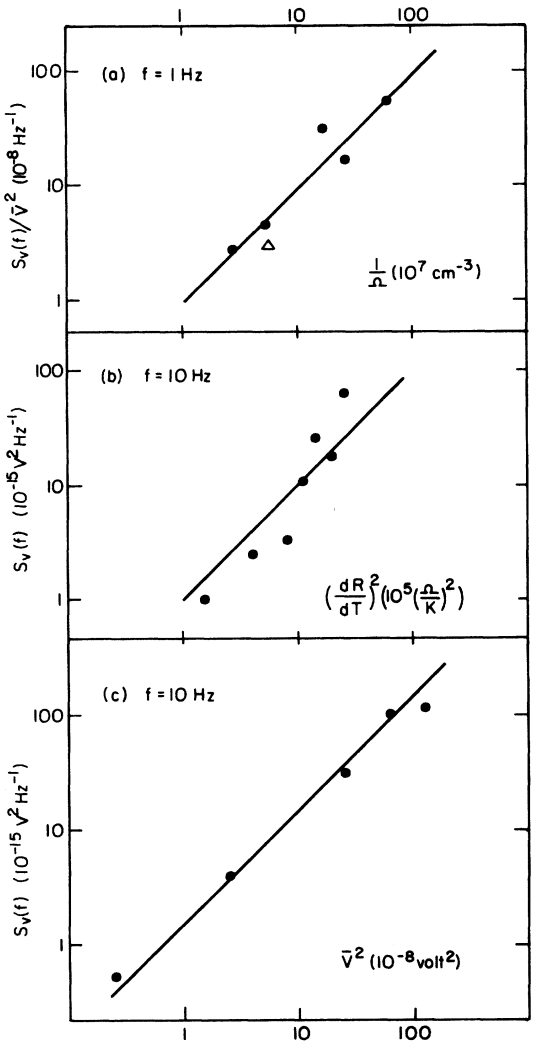

FIG. 3. (a) S_V (1 Hz)/$\overline{V}^2$ vs Ω^{-1} (solid circles, 0.1-μm films; triangle, 0.6 μm film); (b) S_V (10 Hz) vs $(dR/dT)^2$; (c) S_V (10 Hz) vs $\overline{V}^2$. A line of slope unity has been fitted in each figure.

In Fig. 3(b), we plot $S_V(f)$ vs $(dR/dT)^2$ for one sample at constant bias current. Each value of dR/dT was obtained by maintaining the sample at a slightly different temperature on the transition curve. No corrections were made for the variation in T^2/C_V: This error is insignificant compared with the measurement errors. In Fig. 3(c), we plot $S_V(f)$ vs $\overline{V}^2$ for one sample at constant β. The fact that $S_V(f) \propto \overline{V}^2$ indicates that self-heating effects did not contribute measureably to the noise spectra.

In each figure, a line of unity slope was fitted. Bearing in mind the uncertainties in the measurements, we regard the measured dependence of $S_V(f)/\overline{V}^2$ on Ω^{-1}, and $S_V(f)$ on dR/dT and $\overline{V}$ to be in satisfactory agreement with the predictions of Eq. (2.4).

We next measured the power spectra of type-*B* tin samples. Typical noise power spectra for a sapphire substrate ($D \sim 10^5\,\mathrm{cm}^2\,\mathrm{sec}^{-1}$) and a glass substrate ($D \sim 3\,\mathrm{cm}^2\,\mathrm{sec}^{-1}$) are shown in Figs. 2(b) and (c). The spectra flatten below about 30 Hz, and are flatter than $1/f$ at higher frequencies. Since type-*A* and *B* samples had almost identical volumes, resistivities, transition temperatures, and transition widths, it is highly likely that the enormous difference between their noise spectra arises from the different thermal coupling of the films to the substrates. The flattening of the spectra at low frequencies indicates that the heat flow is much more three-dimensional in *B* samples than in *A* samples. A fluctuation at one point on the film has a higher probability of decaying into the substrate (rather than along the film) in a type-*B* sample. It is interesting to note that the type-*B* samples have very similar spectra on both glass and sapphire substrates, suggesting that the thermal boundary resistance, rather than the diffusivity of the substrate, is the dominant factor in determining the heat flow out of the film.

For one type-*B* sample on a glass substrate the power spectrum shown in Fig. 2(d) was obtained. The spectrum is intermediate between the spectra (a) for type-*A* samples and the more typical type-*B* spectra, (b) and (c). Thus the low-frequency end of the power spectrum, varying as

about $f^{-0.35}$, is less depressed than that of the other type-B samples. Apparently the thermal conductance between the film and the substrate was less enhanced by the aluminum underlay than was usual.

We initially supposed that the knees in Figs. 2(b) and 2(c) at 30 Hz corresponded to the frequency f_1. However, we also measured the spectra of type B samples in which the length was varied from 0.6 to 5 mm. We observed no systematic variation in the knee frequency. Consequently, we suspect that the knee frequency corresponds to the inverse of the relaxation time of the film and part of the substrate; this time, about 10 msec, is independent of the length of the sample. It should be noted that this time is much greater than the time defined by the ratio of the heat capacity of the film ($\sim 5\times10^{-12}$ JK^{-1} for 2.5 mm$\times$15 μm $\times$0.1 μm) to K_e ($\sim$10 μWK^{-1}), which is on the order of 1 μsec. This result suggests that part of the substrate adjacent to the film is involved in the thermal relaxation process, and that the heat capacity of this part of the substrate contributes to the relaxation time.

B. Spatial correlation of the noise

As a further test of the thermal diffusion mechanism, we studied the spatial correlation of the $1/f$ noise. The experimental configuration is shown inset in Fig. 4. We measured the correlation of the noise generated in two regions of the same film separated by a distance d. The two noise voltages $V_1(t)$ and $V_2(t)$ were separately amplified with PAR-190 transformers and PAR-113 preamplifiers, and the spectrum of their sum or difference measured in the usual way. If $S_+(f)$ is the spectrum of $[V_1(t)+V_2(t)]$ and $S_-(f)$ is the spectrum of $[V_1(t)-V_2(t)]$ the fractional correlation between the strips, $C(f)$, is given by

$$C(f)=[S_+(f)-S_-(f)]/[S_+(f)+S_-(f)]\ . \qquad (5.1)$$

When V_1 and V_2 are independent, $S_+=S_-$ and $C(f)=0$. When $V_1=V_2$, S_- vanishes, and $C(f)=1$. The measured correlation for $d=5$mm is shown as open circles in Fig. 4. At high frequencies, $\lambda(f)\ll d$ and $C(f)\to 0$, while at low frequencies $\lambda(f)\gg d$, and $C(f)\to$ const. The change from correlated to uncorrelated behavior occurs when $\lambda(f)\sim d$. Because of the thermally inhomogeneous nature of the system, we have introduced an *effective* diffusivity D_e that represents an appropriate average for the film and the substrate. The observed changeover from correlated to uncorrelated noise at about 8 Hz corresponds to $D_e \approx 6$ cm^2 sec^{-1}. From the measured electrical resistance of the film, its estimated heat capacity, and the Wiedemann-Franz Law, we estimate $D\approx 50$ cm^2 sec^{-1}. Given the complexity of the metal-on-glass system, it is difficult to quantitatively relate D_e to the diffusivity of the film. One would expect the relatively low diffusivity of the substrate ($D\sim 3$ cm^2 sec^{-1}) to lower the effective diffusivity measured by the correlation experiment, and the low value of D_e is perhaps not too surprising.

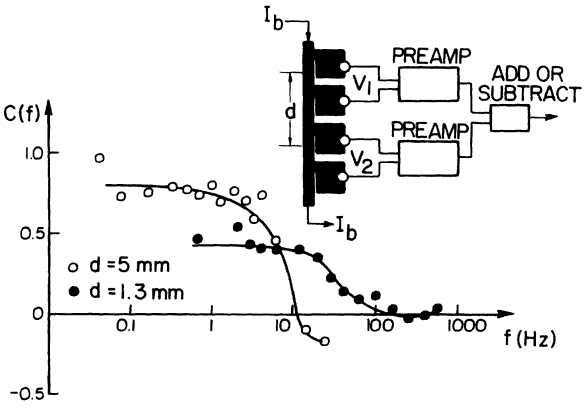

FIG. 4. $C(f)$ vs f for type-A tin films on glass (open circles) and type-B tin films on glass (solid circles). Inset is experimental configuration.

Correlation measurements were performed on type-B samples using the same configuration as for type-A samples. With $d=5$ mm, no convincing correlation was observed in any of several samples tested. With $d=1.3$ mm and a glass substrate, the correlation shown as solid circles in Fig. 4 was observed. The degree of correlation at low frequencies is less than that for type-A samples. This smaller correlation and the lack of correlation for $d=5$ mm are consistent with the idea that type-B samples are much more three-dimensional (3-D) than type A. A fluctuation in the film in a type-A sample tends to decay along the film, whereas in a type-B sample, it has a much greater probability of decaying into the substrate, thereby reducing the correlation along the film. This result may be seen by comparing the 1-D and 3-D correlation functions for uncorrelated temperature fluctuations, Eqs. (2.2) and (2.1). The presence of $1/|\vec{s}|$ in $c_T^{(3)}(\vec{s},f)$ indicates that the correlation between two regions is smaller in a 3-D system than in a 1-D system. The 3-D correlation function for correlated temperature fluctuations, Eq. (2.5), is proportional to $1/|\vec{s}|f^{1/2}$. If the noise originates from correlated fluctuations, one might hope to observe the $f^{-1/2}$ dependence in $C(f)$ provided that the separation d of the two regions is large compared with the length of either region, and $C(f)$ approximates the point correla-

tion function, Eq. (2.5). Unfortunately, the data are too few and too scattered to enable us to distinguish a $f^{-1/2}$ dependence from a f^0 dependence.

For both 1-D and 3-D systems, $C(f)$ is expected to roll off when $\lambda(f) \sim d$. The value of D_e that we deduce, about 4 $cm^2\,sec^{-1}$, is not significantly different from the value for type-A samples.

C. Effect of magnetic field on type-A samples

We investigated the effect of a magnetic field H perpendicular to the plane of the film on T_c, β, and $S_V(f)$ for type-A tin samples. In the inset of Fig. 5 we plot the dependence of T_c and β (measured at the midpoint of the transition) vs H. A field of a few gauss was sufficient to reduce β by an order of magnitude. The noise power spectra were $1/f$ for all values of magnetic field that we investigated.

In Fig. 5 we plot S_V (10 Hz) vs β^2. Open circles and crosses are for bias currents of 150 and 50 μA, respectively. The value of the magnetic field for each measurement is written in parentheses. The points are scattered about a fitted line of slope unity. The observations that $S_V(f) \propto \beta^2(H)$ and that $S_V(f)$ decreases as H increases are strong pieces of evidence for believing that the $1/f$ noise originates from temperature fluctuations and not from flux flow (see Sec. VII).

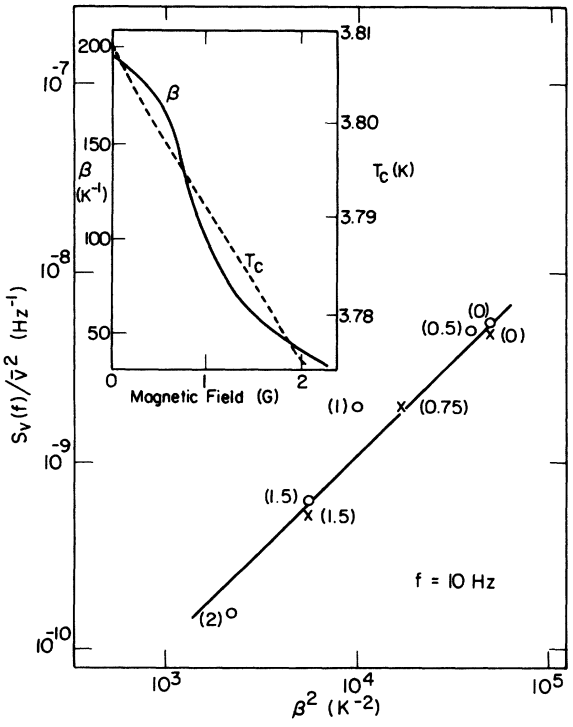

FIG. 5. S_V (10 Hz)/$\bar{V}^2$ vs β^2 for type-A tin samples for several values of magnetic field (in parentheses in G). Inset shows variation of β and T_c with magnetic field.

VI. EXPERIMENTAL RESULTS FOR LEAD

A typical noise power spectrum for a type-A lead film with a normal resistance of 4 Ω and a dc current bias of 500 μA is shown in Fig. 6(a). The spectrum varies as $f^{-1.1}$. The dashed line is calculated from Eq. (2.4) using $\Omega = 6.0 \times 10^{-9}$ cm^3, $\beta = 32\ K^{-1}$, and $C_V/\Omega = 5.7 \times 10^{-2}\ J\,K^{-1}\,cm^{-3}$.[26] At 1 Hz the theoretical power spectrum exceeds the measured power spectrum by a factor of about 5. Samples evaporated on a sapphire substrate (with no underlay) had similar power spectra. We found that $S_V(f)$ was again proportional to $\bar{V}^2$ and $(dR/dT)^2$. The power spectrum of a sample whose volume was nine times greater than that of Fig. 6(a) was approximately a factor of 10 smaller in magnitude, as expected if $S_V(f) \propto \Omega^{-1}$.

A typical spectrum for a type-B lead sample on a glass substrate is shown in Fig. 6(b). Similar results were obtained using a sapphire substrate. The spectrum varies approximately as $f^{-0.8}$, and is somewhat flatter that that in Fig. 6(a). The spectrum is reminiscent of that obtained for the anomalous type-B tin sample, Fig. 2(d). The fact that the flattening of the spectrum at low frequencies is much less dramatic than that usually observed for tin suggests that the aluminum underlay has much less effect on the thermal contact for lead than for tin.

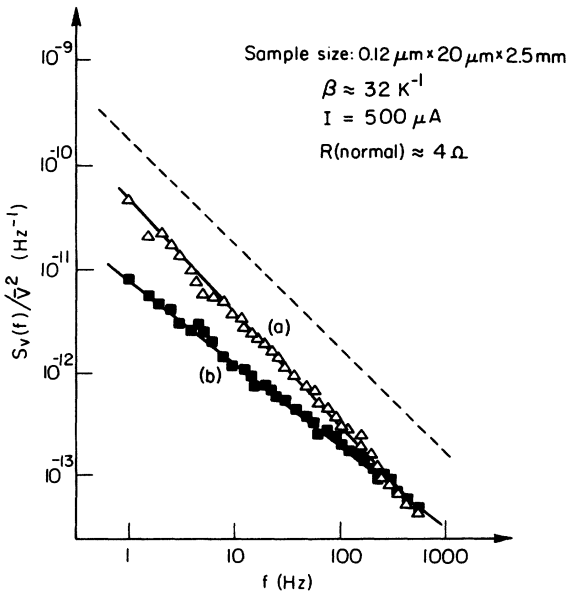

FIG. 6. Typical noise power spectra for lead films: (a) type A on glass [dashed line is calculated from Eq. (2.4)]; (b) type B on glass.

VII. DISCUSSION AND CONCLUSIONS

We first briefly show that the low-frequency noise we observe is inconsistent with that expected from a flux-flow mechanism. The power spectrum of flux-flow noise has been calculated for various models,[6] and is usually not $1/f$. The maximum value of the power spectrum at zero frequency[6] is $2\phi V_f$, where ϕ is the flux contained in each bundle, and V_f is the voltage across the sample due to flux flow. We assume that the maximum area of the flux bundle is $l_2^2 (l_2 \approx 15\mu m$ is the width of the film), and that the maximum field (residual ambient field and self field of the current) is 0.1 G($=10^{-5}$ T). Thus $\phi \sim 2\times10^{-15}$ Wb $\sim \phi_0$, where ϕ_0 is the flux quantum. Let us further suppose that the average voltage across the film is entirely due to flux flow: A maximum value for V_f in a typical sample would then be 1 mV. Thus an upper limit on the zero-frequency power spectrum is 4×10^{-18} $V^2\,Hz^{-1}$, a value typically five orders of magnitude smaller than our observed noise at 1 Hz. In fact, for a sample biased at a voltage of one-half the normal state voltage, the flux-flow voltage V_f is a very small fraction of the total voltage (see Fig. 6 of Ref. 6). Thus the expected flux-flow noise voltage is considerably less than our estimate of 4×10^{-18} $V^2\,Hz^{-1}$. Furthermore, the observed noise spectrum decreases (and is proportional to β^2) as the applied magnetic field is increased, whereas for a constant bias voltage the flux-flow noise would increase. Finally, the observed noise power spectrum is proportional to β^2, and is a maximum close to the midpoint of the transition. In the case of flux-flow noise, the spectrum has a maximum value when the film resistance is much less than one-half the normal-state resistance.[6,8] We conclude that the noise observed in our experiments is quite inconsistent with that expected from flux flow.

The experimental results presented in this paper provide strong support for the thermal diffusion theory of $1/f$ noise. We have shown that for tin films the noise voltage power spectrum $S_V(f)$ is proportional to $\bar{V}^2$, β^2, and Ω^{-1} as predicted by Eq. (2.4). In particular, $S_V(f)$ is inversely proportional to the film thickness, indicating that the noise is a bulk rather than a surface effect. The flattening of $S_V(f)$ at low frequencies when the film is in good thermal contact with the substrate strongly supports a thermal diffusion model. The spatial correlation of the noise at low frequencies observed in both type-A and type-B samples provides further clear evidence for a thermal diffusion process. Finally, the prediction of the semiempirical formula Eq. (2.4) is in excellent agreement with the measured magnitude of the noise. It is particularly noteworthy that although β^2T^2/C_V increases by about nine orders of magnitude when a tin film is cooled from room temperature to the midpoint of its superconducting transition, Eq. (2.4) predicts the magnitude of the noise at each temperature to within a factor of 2 or 3.[1,2] In Fig. 7 we plot measured values of S_V(10 Hz) for type-A tin samples versus the values of S_V (10 Hz) predicted by Eq. (4) for the measurements plotted in Figs. 2, 3, and 5. Although there is considerable scatter of the data, there is a good fit to a line of slope unity over four decades.

The small amount of data collected on lead films also supports the thermal diffusion model. The correct dependences of $S_V(f)$ on $\bar{V}^2$, β^2, and Ω^{-1} were observed. The type-A films have $1/f$-like spectra that are within a factor of about 5 or the prediction of Eq. (2.4). However, it should be noted that had it been possible to obtain data at a somewhat higher frequency, the agreement between the experimental data and the semiempirical model would have been improved. Clearly, if the observed $S_V(f)$ does not vary precisely as f^{-1}, there will be good numerical agreement between the measured and calculated spectra only over a limited frequency range. (In the same way, the good fit to the theory obtained for the power spectra of the type-A tin films may be regarded as somewhat fortuitous: The fit would have been poorer at a substantially higher or lower frequency.)

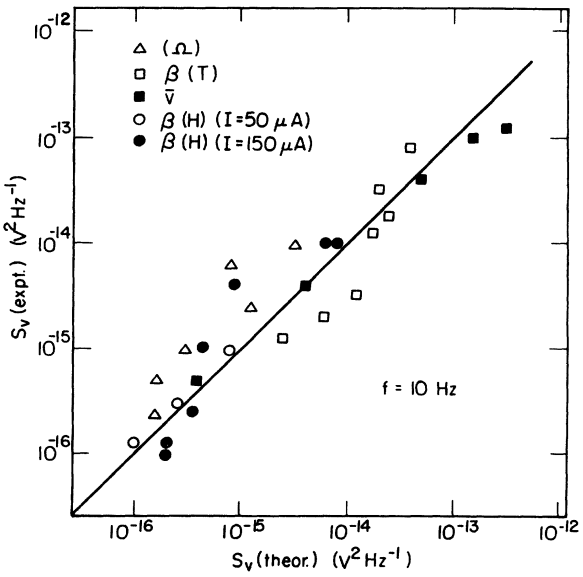

FIG. 7. Comparison of measured S_V (10 Hz) from Figs. 2, 3, and 5 with S_V (10 Hz) calculated from Eq. (2.4) for type-A tin samples.

For type-B lead samples, $S_V(f)$ varied approximately as $f^{-0.8}$. Although this was a significantly flatter slope than that of the type-A samples (for which $S_V(f) \propto f^{-1.1}$), the change was much less marked than for most of the tin samples, presumably because of the relatively small change in the thermal boundary resistance.

The major unresolved problem in this work is the microscopic origin of the $1/f$ slope for the type-A samples. Voss and Clarke[2] showed that for a thermally homogeneous system a model involving correlated temperature fluctuations produced a $1/f$ spectrum in the low-frequency limit of a two-dimensional system. In the low-frequency limit of a one-dimensional system $S_V(f) \propto f^{-3/2}$. The present measurements do not provide overwhelming evidence in favor of either correlated or uncorrelated temperature fluctuations. However, one qualitative comment is in order. In the type-A samples, the poor coupling of the films to their substrates implies that most of the heat flow is along the film rather than into the substrate, i.e., that the film is more one-dimensional than two-dimensional. In the room-temperature measurements of Voss and Clarke,[2] the spectra also tended to be somewhat steeper than f^{-1}. This slope is thus consistent with the prediction of the model involving correlated temperature fluctuations. It is not consistent with the model involving uncorrelated fluctuations in which the low-frequency spectra for two- and one-dimensional systems are, respectively, $\ln(1/f)$ and $f^{-1/2}$. Thus there is some further evidence for the correlated fluctuations, albeit of a qualitative nature. One would also like to understand quantitatively the effects of strong thermal coupling between the film and the substrate. Evidently, if the coupling is sufficiently strong, the power spectrum becomes flat at low frequencies. An intermediate case also exists, in which $S_V(f) \propto f^{-\alpha}$, where $0 < \alpha < 1$ [Figs. 2(d) and 6(b)]. A calculation taking into account the proper boundary conditions is urgently needed.

Our results have an immediate practical application to superconducting bolometers. Our understanding of the role of the various parameters in determining the magnitude of the $1/f$ noise has enabled us to design a superconducting bolometer with significantly lower noise than has been achieved previously.[27] The use of an aluminum underlay to improve the thermal coupling of tin films to glass or sapphire substrates would be of interest in other experiments, for example, measurements of the quasiparticle recombination time using tunnel junctions.[28]

ACKNOWLEDGMENTS

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[18]Assume that the electronic heat capacity near T_c can be written in the form $C_e = C_{en} + (C_{es} - C_{en})(T_n - T)/\delta T$, where C_{en} and C_{es} are the heat capacities just above and below the transition at temperatures T_n and T_s. From Ref. 16 we find $C_{es} - C_{en} = 1.43\ C_{en}$. Writing

$T = \bar{T} + \Theta(t)$, where $\bar{T}$ is the average temperature, and where the mean square value of the fluctuating quantity $\Theta(t)$ is $k_B T^2/C$, we find $C_e(t) = \bar{C}_e[1 - 1.43(C_{en}/\bar{C}_e)\Theta(t)/\delta T]$. The rms correction to C_e is thus of order $\langle(\Delta T)^2\rangle^{1/2}/\delta T$.

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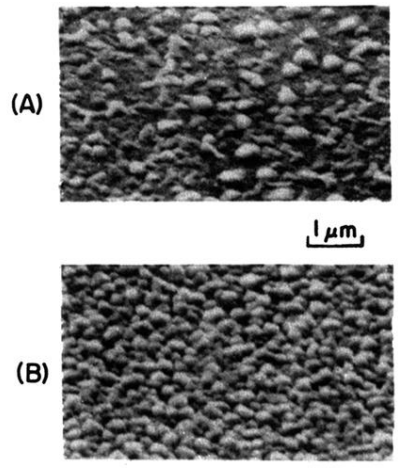

FIG. 1. Scanning electron micrographs of tin films on a glass substrate: (a) type A; (b) type B.