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Comparison of the major magnetic dimensional resonances in single-carrier and in compensated two-carrier magnetoplasma spheres*

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(Received 9 February 1976)

Employing the perturbation approach to the interaction of an electromagnetic wave with a gyrotropic sphere recently developed by Ford, Furdyna, and Werner, we discuss the differences in the behavior of resonance field, resonance strength, and resonance width for single-carrier magnetoplasma spheres in the helicon regime and for compensated two-carrier magnetoplasma spheres in the Alfvén regime.

I. INTRODUCTION

Recently, Ford and Werner[1] solved the boundary-value problem associated with the interaction of time-varying electric and magnetic fields with a spherical plasma in an external static magnetic field. Their solution is valid in the regime in which the sphere radius is much less than the wavelength of the time-varying field in the medium surrounding the sphere and applies to any nonmagnetic system which can be described by a dielectric tensor of the form

$$\overleftrightarrow{\kappa}=\begin{pmatrix} \kappa_{xx} & \kappa_{xy} & 0 \\ -\kappa_{xy} & \kappa_{xx} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}. \tag{1}$$

However, since the general solution is formal, not analytic, and requires extensive numerical calculations, the problem was also solved, under more restricted conditions, via a perturbation expansion in powers of a dimensionless parameter proportional to the square of the sphere radius.[2] The perturbation expansion yields analytic expressions which provide an adequate description of the major magnetic dipole resonance observed in InSb spheres by Evans, Furdyna, and Galeener.[3,4] Such analytic expressions, in addition to their quantitative validity in the case of very small spheres, can, in fact, lend insight into the general solution, bringing out trends which resonances in larger spheres exhibit.

An understanding of dimensional resonances in multicarrier spherical plasmas situated in a magnetic field is required for interpretation of recent microwave studies of electron-hole drops in Ge.[5–7] To aid in the understanding of these phenomena, we contrast the behavior of the major magnetic dimensional resonances for one-carrier spherical plasmas (which have been experimentally investigated in the microwave regime[4,8,9]) with that for compensated, two-carrier spherical plasmas. Herein, we deal only with carriers described by an isotropic band structure and employ the perturbation theory exclusively. The comparison of the two types of plasmas highlights certain basic and surprising differences in their response, specifically, in the dependence of resonance strength on radius and on carrier density. Such differences have not been fully appreciated to date, and an awareness of them should help in understanding the more complex situation found in electron-hole drops in Ge.

In this paper, we first review the general formulation of the expressions for the power absorbed by a spherical plasma consisting of n types of carriers. We then discuss single-carrier and compensated, two-carrier spherical plasmas under conditions appropriate for the microwave regime.

II. POWER ABSORBED BY n-CARRIER SPHERICAL PLASMA

An incident time-varying magnetic field, $\vec{H}^{ac}e^{-i\omega t}$, can be resolved into three components $\vec{H}^{ac}_m$—two orthogonal circular polarizations transverse to the dc magnetic field $\vec{B}$ (designated by the subscripts $m=+$ and $-$) and a linear polarization parallel to $\vec{B}$ (designated by the subscript $m=\parallel$). As shown by Ford, Furdyna, and Werner,[2] these polarizations correspond to normal-mode excitations of the gyrotropic sphere, i.e., each field component $\vec{H}_m^{ac}$ elicits a linearly independent response, which can be considered separately from the others. In mks units, the mean power absorbed P from any of the linearly independent components of the time-varying magnetic field by a sphere having a complex magnetic dipole polarizability α is

$$P_m = \tfrac{1}{2}\omega\,\mathrm{Im}(\vec{M}_m \cdot \vec{H}_m^{ac}) = (\omega/2\mu_0)(\mathrm{Im}\,\alpha_m)|\vec{H}_m^{ac}|^2 , \quad (2)$$

where $\vec{M}_m \equiv \alpha_m \vec{H}_m^{ac}$ is the induced magnetic dipole moment of the sphere. Throughout this paper, Re and Im indicate the real and imaginary parts of complex quantities. According to Ford, Furdyna, and Werner,[2] the polarizability corresponding to the mth mode of excitation can be written

$$\alpha_m = \frac{2\pi}{15}\frac{a^5\omega^2}{c^2}\frac{\kappa_m^{eff}}{1-\frac{2}{21}(\omega a/c)^2\kappa_m^{eff}} , \quad (3)$$

where a is the radius of the sphere,

$$\kappa_\pm^{eff} = 2\kappa_\pm\kappa_{zz}/(\kappa_\pm+\kappa_{zz}) , \quad \kappa_\parallel^{eff} = 2\kappa_+\kappa_-/(\kappa_+ + \kappa_-) , \quad (4)$$

and $\kappa_\pm = \kappa_{xx} \pm i\kappa_{xy}$. In particular, the imaginary part of the magnetic polarizability, which is directly proportional to the power absorbed by the sphere, is

$$\mathrm{Im}\,\alpha_m = \frac{2\pi}{15}\frac{a^5\omega^2}{c^2} \times \frac{\mathrm{Im}\,\kappa_m^{eff}}{[1-\frac{2}{21}(\omega a/c)^2\mathrm{Re}\,\kappa_m^{eff}]^2 + [\frac{2}{21}(\omega a/c)^2\mathrm{Im}\,\kappa_m^{eff}]^2} . \quad (5)$$

The Drude expressions for the dielectric tensor elements of a semiconductor with n types of carriers and an isotropic band structure are

$$\kappa_\pm = \kappa_l - \sum_{i=1}^{n}\frac{\omega_{pi}^2}{\omega}\frac{\omega \pm \omega_{ci} - i\tau_i^{-1}}{(\omega\pm\omega_{ci})^2 + \tau_i^{-2}}$$

and (6)

$$\kappa_{zz} = \kappa_l - \sum_{i=1}^{n}\frac{\omega_{pi}^2}{\omega}\frac{\omega - i\tau_i^{-1}}{\omega^2+\tau_i^{-2}} ,$$

where

$$\omega_{ci} = q_i B/m_i , \quad \omega_{pi}^2 = n_i e^2/\epsilon_0 m_i ,$$

τ_i is the ith carrier relaxation time, q_i is the charge (including sign) of the ith carrier, n_i is the ith carrier concentration, m_i is the effective mass of the ith carrier, and κ_l is the dielectric constant of the lattice.

III. SINGLE-CARRIER SYSTEM

For a single-carrier plasma, the preceding equations can be put into particularly enlightening forms whenever κ_l can be neglected with respect to the free-carrier term (e.g., in the helicon limit, where $\omega_p^2 \gg \kappa_l\omega\omega_c$ and $\omega_c^2 \gg \omega^2, \tau^{-2}$). Neglecting κ_l, we obtain

$$\kappa_\pm^{eff} = -2\frac{\omega_p^2}{\omega}\frac{(2\omega\pm\omega_c) - i2\tau^{-1}}{(2\omega\pm\omega_c)^2 + 4\tau^{-2}} \quad (7)$$

and

$$\kappa_\parallel^{eff} = \kappa_{zz} .$$

It is immediately apparent that, since κ_{zz} is field independent, no resonances (as a function of B) will be observed in the longitudinal field configuration. But, as the free-carrier term is reduced to a point where κ_l is not negligible (by, say, reducing the carrier concentration), a broad shoulder will develop in the power absorption of the longitudinal component of the incident time-varying field at low dc magnetic fields.[10]

Returning to Eq. (7) and substituting the result into Eqs. (2) and (5), we get

$$P_\pm = \frac{\pi}{15}\frac{a^5\omega^2 ne^2}{m} \times \frac{\tau}{[(\omega+\frac{2}{21}a^2\omega\mu_0 ne^2/m)\pm\frac{1}{2}\omega_c]^2\tau^2+1}|\vec{H}_m^{ac}|^2 . \quad (8)$$

Resonant absorption (as a function of B) occurs in the cyclotron-resonance-active transverse polarization at $|\omega_c| = 2\omega + \frac{4}{21}a^2\omega\mu_0 ne^2/m$. The line shape of the resonance is Lorentzian in B, having a half-width at half-power of

$$(\Delta B)_{1/2} = \frac{2}{e\tau/m} = \frac{2}{\mu} , \quad (9)$$

where μ is the carrier mobility. For very small radii, the resonance field is dimension independent, occurring when $|\omega_c| = 2\omega$. But, as the sphere radius increases, the resonance condition gradually acquires a completely dimensional character. The strength of the resonance varies with the fifth power of the radius and is directly proportional to the relaxation time and the carrier concentration. Reducing the carrier concentration so that κ_l is a non-negligible fraction of the free-carrier term skews the resonance and shifts the position of the resonance from that given by Eq. (8).

IV. COMPENSATED TWO-CARRIER SYSTEM

Now we turn to a compensated system, consisting of equal concentrations of electrons (e) and holes (h). In the limit in which $(\omega_p)_{e,h} \gg |\omega_c|_{e,h} \gg \omega \gtrsim \tau_{e,h}^{-1}$, we have $\kappa_+ \simeq \kappa_-$ and

$$\mathrm{Re}\,\kappa_+ = \frac{n(m_e+m_h)}{\epsilon_0 B^2} \quad \text{and} \quad \mathrm{Im}\,\kappa_+ = \frac{n(m_e+m_h)}{\epsilon_0 B^2}\frac{1}{\omega\tau_{av}} , \quad (10)$$

where

$$\tau_{av} = \tau_e \tau_h (m_e + m_h)/(m_e \tau_h + m_h \tau_e) \ .$$

Also, $|\text{Re}\kappa_{zz}| \gg \text{Re}\kappa_+$ and $\text{Im}\kappa_{zz} \gg \text{Im}\kappa_+$ so that

$$\kappa_{\pm}^{eff} = 2\kappa_+ \quad \text{and} \quad \kappa_{\parallel}^{eff} = \kappa_+ \ . \tag{11}$$

Substituting these expressions into Eqs. (2) and (6), we get

$$P_m = \frac{7\pi}{10\mu_0} a^3 \omega^2 \tau_{av} \frac{(B/B_m^{Alf})^2}{[(B/B_m^{Alf})^2 - 1]^2 \omega^2 \tau_{av}^2 + 1} |\vec{H}_m^{ac}|^2 \ , \tag{12}$$

where

$$(B_m^{Alf})^2 = \begin{cases} \frac{4}{21}(\omega a/c)^2 n(m_e + m_h)/\epsilon_0 & (m = \pm) \ , \\ \frac{2}{21}(\omega a/c)^2 n(m_e + m_h)/\epsilon_0 & (m = \parallel) \ . \end{cases} \tag{13}$$

Setting the derivative of the absorbed power with respect to B equal to zero yields the resonance condition

$$B_m^{Res} = B_m^{Alf}\left(1 + \frac{1}{\omega^2 \tau_{av}^2}\right)^{1/4} = \begin{cases} \frac{a\omega}{c}\left[\frac{4}{21}\ \frac{n(m_e + m_h)}{\epsilon_0}\left(1 + \frac{1}{\omega^2\tau_{av}^2}\right)^{1/2}\right]^{1/2} & (m = \pm) \ , \\ \frac{a\omega}{c}\left[\frac{2}{21}\ \frac{n(m_e + m_h)}{\epsilon_0}\left(1 + \frac{1}{\omega^2\tau_{av}^2}\right)^{1/2}\right]^{1/2} & (m = \parallel) \ . \end{cases} \tag{14}$$

On eliminating B in Eq. (12) by using Eq. (14), we obtain the magnitude of the resonance

$$P_m^{Res} = (7\pi/10\mu_0) a^3 \omega^2 \tau_{av} f |\vec{H}_m^{ac}|^2 \ , \tag{15}$$

where

$$f = \frac{(1 + 1/\omega^2\tau_{av}^2)^{1/2}}{1 + \omega^2\tau_{av}^2[1 - (1 + 1/\omega^2\tau_{av}^2)^{1/2}]^2} \ . \tag{16}$$

In the Alfvén limit,[11] in which $(\omega_p)_{e,h} \gg (\omega_c)_{e,h} \gg \omega \gg \tau_{e,h}^{-1}$, the quantity f becomes unity when terms of the order of $1/\omega^2\tau_{av}^2$ are neglected with respect to 1.

Using Eq. (12), we can show that the separation of the squares of the upper (B^{UH}) and lower (B^{LH}) half-power points is

$$(B_m^{UH})^2 - (B_m^{LH})^2 = (2/\omega\tau_{av})(B_m^{Alf})^2 \times (2/f - 1 + 1/f^2\omega^2\tau_{av}^2)^{1/2} \ . \tag{17}$$

Since, in the Alfvén limit,

$$(B_m^{UH})^2 - (B_m^{LH})^2 = 2(\Delta B_m) B_m^{Res} \ , \tag{18}$$

where $\Delta B_m = B_m^{UH} - B_m^{LH}$, we have

$$\Delta B_m = B_m^{Res}/\omega\tau_{av} \ . \tag{19}$$

Thus, as expected, the resonant field varies directly with the sphere radius a rather than a^2, as for single-carrier systems. Unlike the single-carrier case, the width of the resonance is not independent of the resonance field but increases in a manner directly proportional to the resonance field. Furthermore, quite surprisingly, the magnitude of the resonance is *independent* of the carrier concentration. Also, the magnitude of the resonance absorption varies as the cube of the sphere radius,[12] not as the fifth power, as for single carriers. As the radius is reduced, the resonances will gradually move into a field range where $|\omega_c|_{e,h}$ is not much greater than ω. As the resonances do so, their character changes smooth-

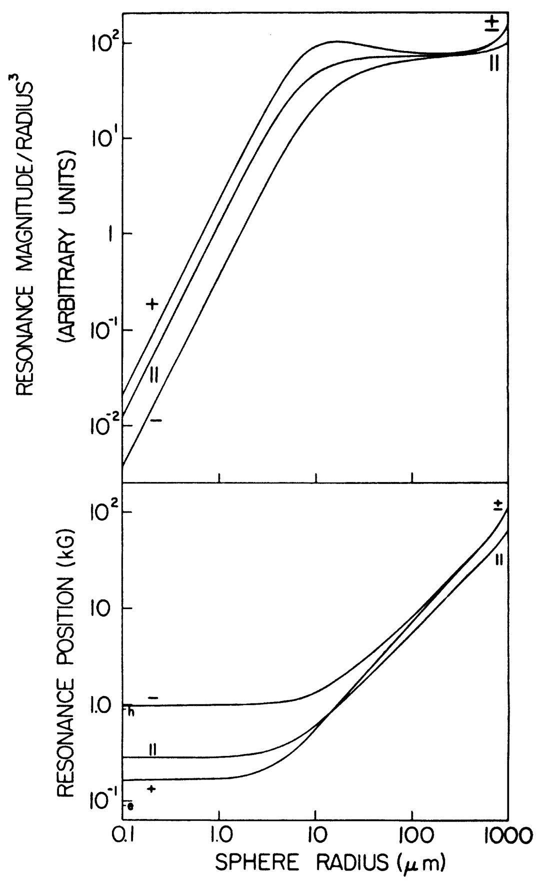

FIG. 1. Resonance strength and resonance position as a function of the radius of a spherical plasma composed of two compensated carriers of opposite sign. The results displayed were calculated using the following parameters: $n_{e,h} = 10^{17}$ cm^{-3}, $m_e = 0.01\ m_0$, $m_h = 0.1\ m_0$, $\tau_{e,h} = 10^{-10}$ sec, $\kappa_l = 16$, and $\nu = 25$ GHz, where m_0 is the free-electron mass and $\omega = 2\pi\nu$. The symbols +, −, and ‖ denote the appropriate mode of excitation of the sphere (see text). The cyclotron-resonance-field positions for electrons and holes are indicated by the letters e and h, respectively.

ly from a dimensional to a radius-independent resonance. Also, their resonance strength changes from a cubic to a fifth-power dependence on radius as the radius grows smaller. Figure 1 illustrates the transition of resonance fields from a dimensional to a nondimensional character.[13]

In the Alfvén limit, no difference exists between the two transverse circular polarizations that are the normal-mode excitations of the sphere. However, the longitudinal resonance has the same strength as the transverse resonances for a given radius but occurs at a field $\sqrt{2}$ smaller than the resonances in the transverse polarizations. Likewise, in keeping with Eq. (19), the width of the longitudinal resonance is also $\sqrt{2}$ smaller than that of the transverse resonances.

In summary, between a single-carrier spherical plasma in the helicon region and a compensated, two-carrier spherical plasma in the Alfvén limit, there exist several important differences:

(a) *Resonance field.* In the single-carrier system, a resonance occurs only for the cyclotron-resonance-active circular polarization, and its position depends on a^2. In the compensated two-carrier system, resonances occur in all three linearly independent modes of excitation, the resonances for the transverse polarizations occurring at the same field while the longitudinal resonance occurs at a field smaller by a factor of $\sqrt{2}$. The positions of all the resonances depend directly on a.

(b) *Resonance strength.* The resonance strength is proportional to a^5 and n for a single-carrier system. It is proportional to a^3 and is independent of n for a compensated two-carrier system.

(c) *Resonance width at half-power.* For a single-carrier system, the resonance width is independent of resonant field. It depends only on m and τ. For a compensated two-carrier system, the resonance width is directly proportional to the field at which resonance occurs.

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*Work supported by the NSF (MRL Program DMR-7203018-A04).

†Purdue Research Foundation David Ross Fellow.

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[13]For the largest radii shown on Fig. 1, there is an upturn of the resonance strength per unit volume and a slightly greater than linear dependence of the resonance field on radius, both arising from an encroachment upon the upper field limit of the Alfvén approximation determined by the condition $(\omega_p)_{e,h} \gg |\omega_c|_{e,h}$.