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# Free-carrier absorption due to electron-plasmon interaction: Effect of impurity scattering*

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The effect of impurity scattering on plasmon-assisted optical absorption in semiconductors is studied using Green's-function techniques. The interaction $H_{eE}$ of the electrons with the electric field of the photon, the interaction $H_{ep}$ of electrons and plasmons, and the change $H_{epE}$ in $H_{ep}$ caused by the electric field are all considered. For momentum-conserving final states the amplitudes for processes linear in the electric field $E$ exactly cancel just as they do in the absence of collisions. For final states that are not momentum conserving, this cancellation no longer occurs, and a finite absorption is obtained.

## I. INTRODUCTION

Experimental studies of the infrared magneto-optical properties of InSb have revealed interesting structure near harmonics of the cyclotron resonance frequency for both the cyclotron-resonance-active and inactive circular polarizations in the Faraday configuration.[1–4] The origin of these cyclotron "harmonics" has been attributed to plasmon-assisted magneto-optical transitions.[4–6] Diagrams depicting these transitions are shown in Figs. 2(b) and 2(c). The absorption of the incident photon (wavy line) is accompanied by the creation of an electron-hole pair (solid line). One of the single-particle excitations then emits a plasmon and scatters to its final state. The position of the fundamental cyclotron-resonance-inactive absorption peak as a function of carrier concentration and magnetic field strength has been studied carefully,[4] and it agrees well with the model of plasmon-assisted transitions. The amplitude of the "harmonics" has been estimated by applying second-order perturbation theory to a plasma of free carriers in a perfect crystal,[4–6] and order-of-magnitude agreement with experiment seemed to further justify the model.

The interpretation of these experiments in terms of plasmon-assisted absorption has been questioned in a recent paper of Blinowski and Mycielski[7] (BM). The fundamental point raised by these authors is that for an unbounded plasma of carriers with energy-independent effective mass in a perfect crystal, the center-of-mass degrees of freedom are completely independent of and separable from the internal degrees of freedom of the plasma. Furthermore, only center-of-mass degrees of freedom can be excited by a uniform electric field. Since plasmons are associated with internal degrees of freedom, they cannot affect the absorption of long-wavelength radiation which couples only to the center of mass of the plasma. Blinowski and Mycielski demonstrate explicitly that, in the absence of a dc magnetic field, the modification of the electron-plasmon interaction due to the presence of a spatially uniform electric field contributes a term to the transition amplitude which exactly cancels the contribution from plasmon-assisted processes considered by previous authors.[8,9] The argument in no way depends on the presence or absence of a dc magnetic field, so it should be equally applicable to the case of magnetoplasmon-assisted cyclotron "harmonics." These conclusions rest on a rather general argument which has previously been invoked in connection with cyclotron resonance[10] and spin resonance[11] in conducting materials.

In spite of the point raised by BM, the interpretation of the infrared absorption at "harmonics" of the cyclotron frequency appears to be plausible,[12] though the amplitude of the effect may be reduced somewhat from the initial estimates.[4–6] The reason for this is that the complete separability of the center of mass and internal degrees of freedom does not hold in the actual experimental situation. The separability depends upon there being no preferred frame of reference for the plasma of free carriers. In an actual InSb sample the effect of nonparabolicity of the conduction band and the presence of a random array of fixed impurities make the lattice frame of reference a preferred frame, and the separability no longer occurs. In a previous paper[12] it was pointed out that the modification of the electron-plasmon interaction due to the electric field is reduced in the presence of impurity by a factor $(1+i/\omega\tau_{tr})^{-1}$, where $\tau_{tr}$ is a transport collision time. It was suggested that even when momentum-relaxing collisions of the single-quasiparticle states with impurities were included, the exact cancellation found by BM would not hold. The object of the present paper is to include the effects of impurity scattering on both the single-quasiparticle states and on the center-of-mass motion, and to demonstrate that the exact cancellation of BM does not occur unless $\omega\tau\to\infty$. Since the validity of the BM argument is unrelated to the strength of the applied magnetic field, we consider for simplicity, only the case where no dc magnetic field is present.

In Sec. II we introduce the model Hamiltonian used to describe the electrons, their interaction with the spatially uniform ac electric field (the photon field), and their interaction with the plasmon degrees of freedom. The modification of the electron-plasmon interaction caused by the uniform electric field $E$ is evaluated to first order in $E$.

In Sec. III we consider the well-studied problem of free-carrier absorption due to particle-hole excitations (i.e., plasmon effects are ignored). In the absence of impurity scattering there is no absorption due to this process. When impurity scattering is included, the imaginary part of the susceptibility is found to be

$$\mathrm{Im}\chi = Ne^2[m\omega^3\tau_{tr}(1+\omega^{-2}\tau_{tr}^{-2}]^{-1},$$

where $N$ is the carrier concentration, $\tau_{tr}$ is transport relaxation time. This result agrees with the classical Drude formula for free-carrier absorption. This section is intended as an introduction to the notation and methods used later in the paper for less familiar problems. The question of plasmon-assisted free-carrier absorption is treated in Sec. IV. Here we find that if only momentum-conserving final states of the electronic system are considered, the BM cancellation still holds. However, when non-momentum-conserving states are included, the resulting plasmon-assisted absorption no longer vanishes. In Sec. V we consider the excitation of a single plasmon as a source of optical absorption. We find that the absorption due to a single-plasmon excitation is much smaller than plasmon-assisted free-carrier absorption. The processes discussed in both Secs. IV and V will have a more complicated frequency dependence than the simple Drude formula. However, the main objective of the paper is not to obtain numerical estimates of the strength of the plasmon processes, but to demonstrate that the BM cancellation holds only if $\omega\tau\to\infty$. Section VI contains a summary and discussion of our results.

## II. INTERACTION HAMILTONIAN

We use the model of BM and describe the free electrons in first quantization. The Hamiltonian for a single electron in the presence of an ac electric field can be written

$$H=(1/2m)[\vec{p}-(e/c)\vec{A}]^2\,, \tag{1}$$

where $\vec{p}$ is the momentum operator for the electron of mass $m$ and charge $e$, and $\vec{A}$ is the vector potential of the ac electric field. We have chosen a gauge in which the scalar potential $\phi$ is equal to zero; therefore the electric field $\vec{E}$ is given by $E=(i\omega/c)\vec{A}$, where $\omega$ is the angular frequency of the ac field. This Hamiltonian can be divided into two parts: the free-electron Hamiltonian $H_e=p^2/2m$, and the interaction (only the linear term in $E$ is retained) of the electron with the ac electric field

$$H_{eE}=-(e/m\omega)\vec{E}\cdot\vec{\nabla}\,. \tag{2}$$

Plasmons are described in second quantization. The free-plasmon Hamiltonian is given by $H_p=\sum_{\vec{q}}\hbar\,\omega_{\vec{q}}b_{\vec{q}}^\dagger b_{\vec{q}}$, where $b_q$ is the annihilation operator for a plasmon of wave vector $q$ and frequency $\omega_q$. The electron-plasmon interaction is given by

$$\overline{H}_{ep}=\sum_{\vec{q}}A(\vec{q})e^{i\vec{q}\cdot(\vec{r}-\vec{R}_{c.m.})}b_{\vec{q}}+\text{H.c.} \tag{3}$$

Here $\vec{r}$ is the electron coordinate and $\vec{R}_{c.m.}$ is the position of the center of mass of the unbounded carriers. In the absence of an electric field $\vec{R}_{c.m.}$ may be chosen to be zero, but in the presence of a spatially uniform electric field $\vec{E}\exp(i\omega t)$, $\vec{R}_{c.m.}$ oscillates as $\vec{R}_{c.m.}=e\vec{E}\exp(i\omega t)/m(\omega+i/\tau_{tr})$, where $\tau_{tr}$ is the transport scattering time due to electron-impurity scattering. It should be noted, however, that the electron-plasmon interaction $H_{ep}$ is modified in the presence of an electric field. This modification[7] (absent in the electron-phonon interaction) results from the fact that the plasma as a whole is not at rest but oscillates with $\vec{R}_{c.m.}$. By expanding $H_{ep}$ in powers of $\vec{E}$ or $\vec{R}_{c.m.}$ and neglecting all the nonlinear terms in $\vec{E}$ (dipole approximation), we can write $\overline{H}_{ep}=H_{ep}+H_{epE}$; here $H_{epE}$ is the modification interaction term due to the external electric field. In second-quantization notation Eqs. (2) and (3) can be rewritten

$$\begin{aligned} H_{eE}&=-\sum_{\vec{p}}\frac{ie\vec{\epsilon}\cdot\vec{p}}{m\omega}a_{\vec{p}}^\dagger a_{\vec{p}}+\text{H.c.},\\ H_{epE}&=-\sum_{\vec{p},\vec{q}}\frac{ie\vec{\epsilon}\cdot\vec{q}A(q)}{m\omega(\omega+i/\tau_{tr})}a_{\vec{p}+\vec{q}}^\dagger a_{\vec{p}}b_{\vec{q}}+\text{H.c.},\\ H_{ep}&=\sum_{\vec{p},\vec{q}}A(q)a_{\vec{p}+\vec{q}}^\dagger a_{\vec{p}}b_{\vec{q}}+\text{H.c.} \end{aligned} \tag{4}$$

Here $\vec{\epsilon}$ is a unit vector parallel to $\vec{E}$. The photon operators are deliberately excluded from $H_{eE}$ and $H_{epE}$. $a_{\vec{p}}^\dagger$ and $a_{\vec{p}}$ are, respectively, the electron creation and annihilation operators. In order to study the effect of impurities, we neglect the plasmon-impurity interaction and only consider the electron-impurity scattering $H_{ei}$. $H_{ei}$ has the form

$$H_{ei} = \sum_{a} \sum_{\vec{p}, \vec{q}} e^{i(\vec{p} - \vec{q})\cdot \vec{r}_a} U(\vec{p} - \vec{q}) a_p^\dagger a_q ,$$

where $\vec{r}_a$ labels the position of the impurity and $U(\vec{p})$ is the coupling between electrons and impurity atoms. In the next few sections we shall investigate the dissipative part of the following frequency-dependent dielectric function

$$\chi(\omega) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} \langle T[H'(t)H'(0)]\rangle dt , \tag{5}$$

where $H' = H_{eH} + H_{epE}$ and $T(--)$ is the $T$ ordering defined in Ref. (13). In the following calculation we shall always put the volume of the system $V = 1$.

## III. ABSORPTION DUE TO PARTICLE-HOLE EXCITATIONS

The response function whose dissipative part yields a particle-hole final state is shown in Fig. 1(a), where the single wavy line represents the incoming or the outgoing photon, the solid line represents the electron Green's function. The shaded triangle appearing in Fig. 1(a) is the photon-electron vertex $H_{eE}$ in Eq. (4), renormalized by electron-impurity scattering. In treating the electron-impurity scattering we shall

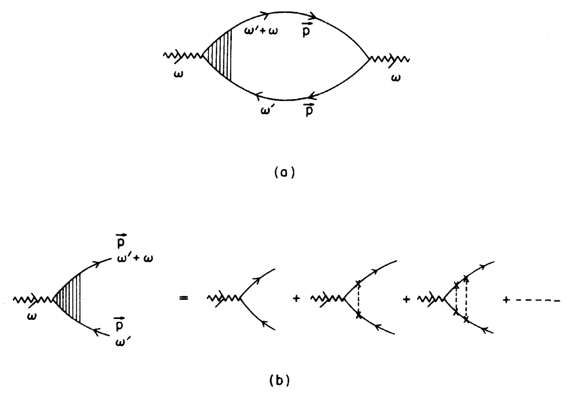

FIG. 1. (a) Diagram showing one of the dielectric response functions. The final state of the dissipative part of the response function is a particle-hole pair. (b) Photon-electron vertex renormalized by electron-impurity scattering. The cross means electron-impurity atom interaction and the dotted line between two crosses means electrons scattering from the same impurity atom.

adopt the method described in Ref. (13). The "average" electron Green's function is defined as[13]

$$G(\vec{p}, \omega) = [\omega - \xi_p + (i/2\tau)(\omega/|\omega|)]^{-1} , \tag{6}$$

where

$$\tau^{-1} = \frac{nmp_0}{(2\pi)^2} \int |U(\theta)|^2 d\Omega .$$

The energy $\xi_p$ is defined by $\xi_p = p^2/2m - p_0^2/2m$; $p_0$ is the Fermi momentum, and $n$ is the number of impurities per unit volume. The renormalized photon-electron vertex is shown in Fig. 1(b). Its evaluation is explicitly given at p. 328 of Ref. 13. We need to write only the result and denote it by $V(p_0, \omega, \omega')$

$$V(\vec{p}_0, \omega, \omega') = -\frac{i\vec{\epsilon}\cdot\vec{p}_0}{m\omega} \left(1 + \frac{i}{\tau_1} \frac{\Theta(-\omega')\Theta(\omega'+\omega)}{\omega + i/\tau_{tr}}\right), \tag{7}$$

where $\vec{p}_0 = p_0\vec{p}/|p|$, $\vec{p}$, $\omega'$ and $\omega$ are the momenta and frequencies appearing in Figs. 1(a) and 1(b). $\Theta(\omega) = 1$ for $\omega > 0$ and $\Theta(\omega) = 0$ for $\omega < 0$. In terms of the electron-impurity coupling $U(p)$, the transport time $\tau_{tr}$ and $\tau_1$ are defined, respectively, as

$$\frac{1}{\tau_{tr}} = \frac{nmp_0}{(2\pi)^2} \int |U(\theta)|^2 (1 - \cos\theta) d\Omega ,$$
$$1/\tau_1 = 1/\tau - 1/\tau_{tr} . \tag{8}$$

Then it is straightforward to show that after averaging over the positions of the randomly distributed impurities the response function in Fig. 1(a) can be written

$$\chi_1(\omega) = -\frac{1}{2\pi i} \int d\omega' \sum_{\vec{p}} V(\vec{p}_0, \omega, \omega') \times G(\vec{p}, \omega' + \omega) G(\vec{p}, \omega') .$$

Replacing the summation over $\vec{p}$ by an integration,

$$\sum_{\vec{p}} = \frac{mp_0}{(2\pi)^3} \int d\Omega_{\vec{p}_0} \int d\xi_p , \tag{9}$$

and we integrate first over $\xi_p$ and then $\omega'$. We obtain the imaginary part of $\chi_1(\omega)$

$$\text{Im}\chi_1(\omega) = \frac{e^2 N}{m\omega(\omega^2 + 1/\tau_{tr}^2)} \frac{1}{\tau_{tr}} , \tag{10}$$

where $N$ is the total number of electrons. In the clean limit $1/\tau_{tr} \to 0$, $\text{Im}\chi_1(\omega) = 0$. This fact, that the free carriers cannot absorb light in the absence of impurities, is well known. In other words, the final state, in the absence of impurities, does not simultaneously satisfy both the momentum and the energy conservation laws. However, by including the lifetime effect for the electrons, the conservation law is relaxed and therefore a nonzero absorption is obtained. It should be noted Eq. (10) agrees completely with the classical Drude formula.

## IV. ABSORPTION DUE TO PARTICLE-HOLE AND PLASMON EXCITATIONS

In this section we shall discuss the optical absorption process in which the final state is made of particle-hole and a single plasmon. We shall show that unless the final states do not conserve momentum, the absorption coefficient will vanish.

### A. Momentum-conserving final states

The calculation of the response function from Eq. (5) involving the excitation of a particle-hole plus a plasmon is rather cumbersome, because nine diagrams must be evaluated. Another way to calculate the imaginary part of the response function is to use the open-diagram method. This method seems to be much simpler and is going to be applied below. The imaginary part of the response function in the present case can be written

$$\mathrm{Im}\chi_2(\omega)=\sum_{\vec{p},\vec{q}}|T(\omega,\vec{p},\vec{q})|^2 S(\omega,\vec{p},\vec{q})\ , \tag{11}$$

where $T(\omega,\vec{p},\vec{q})$ is the $T$ matrix which is the matrix element between the initial state and the final state. The initial state is a photon and the final state is a plasmon plus a particle-hole pair. $S(\omega,\vec{p},\vec{q})$ is the spectrum density of the final state. $\vec{p}$ and $\vec{q}$ are the momenta associated with electron and plasmon, respectively. Diagrams which represent the $T$ matrix are shown in Fig. 2, where the double wavy line is the plasmon propagator. Its expression is given in Appendix A. The shaded circle in Fig. 2(a) comes from $H_{epE}$ of Eq. (4). It represents a process in which a photon is absorbed, and at the same point in space and time a single plasmon and a particle-hole pair are created. The contribution from Fig. 2(a) is of the form

$$T_{2a}(\omega,\vec{q})=-A(q)\,\frac{ie\vec{\epsilon}\cdot\vec{q}}{m\omega(\omega+i/\tau_{tr})}\ . \tag{12}$$

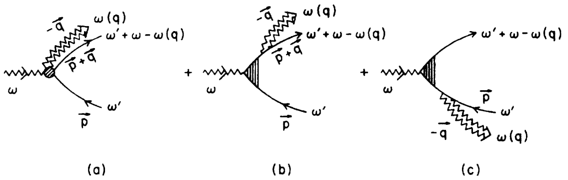


FIG. 2. Diagrams showing the transition matrix amplitudes with an electron-hole pair and a single plasmon as final state. The momentum is conserved in the final state of these processes.

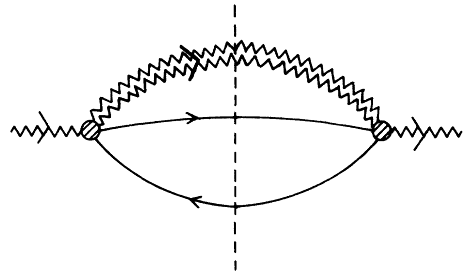

FIG. 3. Diagram showing one of the dielectric response functions. The final state of its dissipative part is a particle-hole pair plus a single plasmon. Momentum is conserved in the final state. The vertical dashed line indicates where the imaginary part should be taken.

The spectrum density is not so easy to determine in the present case, since the lifetime effect of the electrons has to be included. However, we note that the contribution of Fig. 2(a) to $\mathrm{Im}\chi_2(\omega)$ is simply the imaginary part of the diagram as shown in Fig. 3. The evaluation of Fig. 3 should provide the explicit form for $S(\omega,\vec{p},\vec{q})$. Let us use Eq. (9) and integrate over $\xi_p$ first in Eq. (11). Then $\mathrm{Im}\chi_2(\omega)$ can be rewritten[14]

$$\mathrm{Im}\chi_2(\omega)=\frac{mp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0}\sum_{\vec{q}}|T(\omega,\vec{p}_0,\vec{q})|^2 S(\omega,\vec{p}_0,\vec{q})\ . \tag{13}$$

The graph in Fig. 3 has been evaluated in Appendix A. It yields

$$S(\omega,\vec{p}_0,\vec{q})=\tfrac{1}{2}\omega(q)[\omega-\omega(q)]\Theta(\omega-\omega(q)) \times\frac{1}{[\omega-\omega(q)-\epsilon(\vec{p}_0,\vec{q})]^2+1/\tau^2}\,\frac{1}{\tau}\ , \tag{14}$$

where $\epsilon(\vec{p}_0,\vec{q})=\vec{p}_0\cdot\vec{q}/m-q^2/2m$, and $\omega(q)$ is the plasmon frequency. The matrix element represented in Fig. 2(b) represents a process in which

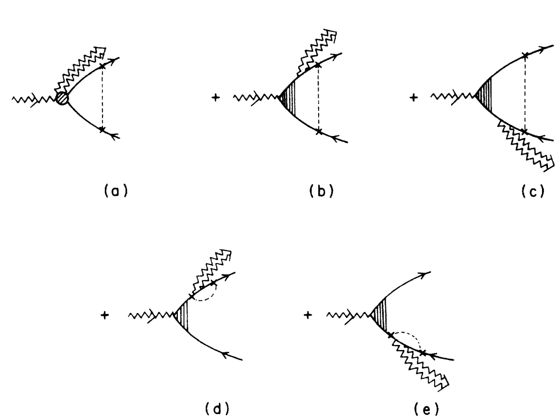


FIG. 4. Lowest-order corrections to graphs in Fig. 2 due to electron-impurity scattering.

a photon is absorbed and an electron-hole pair is created (by $H_{eE}$); subsequently a plasmon is emitted by the electron (by $H_{ep}$). Its contribution to the $T$ matrix is given by

$$T_{2b}(\omega,\vec{p}_0,\vec{q}) = -A(q)\frac{ie\vec{\epsilon}\cdot\vec{p}_0}{m\omega}\left(1+\frac{i}{\tau_1}\frac{1}{\omega+i/\tau_{tr}}\right)\times\left(\frac{1}{\omega'+\omega-\xi_p+i/2\tau}\right)_{\xi_p=\omega'-i/2\tau},$$

$$= -A(q)\frac{ie\vec{\epsilon}\cdot\vec{p}_0}{m\omega(\omega+i/\tau_{tr})}. \quad (15)$$

Similarly the contribution from Fig. 2(c) is given by

$$T_{2c}(\omega,\vec{p}_0,\vec{q}) = A(q)\frac{ie\vec{\epsilon}\cdot(\vec{p}_0+\vec{q})}{m\omega(\omega+i/\tau_{tr})}. \quad (16)$$

It is easy to see that the sum $T_{2a}+T_{2b}+T_{2c}=0$. This means that the system cannot absorb light even though the scattering effect due to impurities is included in the Green's function and the vertex functions. The lowest-order corrections to diagrams in Fig. 2 caused by electron-impurity scattering are shown in Fig. 4. It is rather straightforward to show that, after averaging over the positions of the randomly disttributed impurities, the contribution of Fig. 4(a) to the $T$ matrix appearing in Eq. (13) is given by

$$T_{4a}(\omega,\vec{p}_0,\vec{q}) = n\sum_{\vec{p}''} T_{2a}(\omega,\vec{p}_0'',\vec{q})\,|U(\vec{p}_0-\vec{p}_0'')|^2\times G(\vec{p}''+\vec{q},\omega'+\omega-\omega(q))G(\vec{p}'',\omega').$$

Integrating over $\xi_{p''}$ we obtain

$$T_{4a}(\omega,\vec{p}_0,\vec{q}) = \frac{nmp_0}{(2\pi)^2}\int d\Omega_{\vec{p}_0''}\,T_{2a}(\omega,\vec{p}_0'',\vec{q})\times\frac{|U(\vec{p}_0-\vec{p}_0'')|^2}{\omega-\omega(q)-\epsilon(\vec{p}_0'',\vec{q})+i/\tau}. \quad (17)$$

The contribution of Fig. 4(b) is of the form

$$T_{4b}(\omega,\vec{p}_0,\vec{q}) = n\sum_{\vec{p}''} A(q)V(\vec{p}_0'',\omega,\omega')\,|U(\vec{p}_0-\vec{p}_0'')|^2\,G(\vec{p}'',\omega'+\omega)G(\vec{p}''+\vec{q},\omega'+\omega-\omega(q))G(\vec{p}'',\omega'),$$

where $V(\vec{p}_0'',\omega,\omega')$ is defined in Eq. (7). The integration over $\xi_{p''}$ yields

$$T_{4b}(\omega,\vec{p}_0,\vec{q}) = \frac{nmp_0}{(2\pi)^2}\int d\Omega_{\vec{p}_0''}\,T_{2b}(\omega,\vec{p}_0'',\vec{q})\times\frac{|U(\vec{p}_0-\vec{p}_0'')|^2}{\omega-\omega(q)-\tau(\vec{p}_0'',\vec{q})+i/\tau}. \quad (18)$$

Similarly we find the contribution from Fig. 4(c) to be

$$T_{4c}(\omega,\vec{p}_0,\vec{q}) = \frac{nmp_0}{(2\pi)^2}\int d\Omega_{\vec{p}_0''}\,T_{2c}(\omega,\vec{p}_0'',\vec{q})\times\frac{|U(\vec{p}_0-\vec{p}_0''|^2}{\omega-\omega(q)-\epsilon(\vec{p}_0'',\vec{q})+i/\tau}. \quad (19)$$

Since $T_{2a}+T_{2b}+T_{2c}=0$, we see immediately from Eqs. (17), (18), and (19) that $T_{4a}+T_{4b}+T_{4c}=0$. The last two graphs in Fig. 4 come from the electron-plasmon interaction renormalized by electron-impurity scattering. The renormalized interaction in Fig. 4(d) can be written

$$A(q)\frac{nmp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0''}\,|U(\vec{p}_0-\vec{p}_0'')|^2\times\int_{-\infty}^{\infty} d\xi_{p''}\,G(\vec{p}'',\omega+\omega')\times G(\vec{p}''+\vec{q},\omega+\omega'-\omega(q)).$$

Since the final state requires $\omega+\omega'-\omega(q)>0$ $(\omega+\omega'>0)$ and $\omega'<0$, the pole associated with each of the Green's functions in the above expression is in the lower half-plane. The integration over $\xi_{p''}$ is thus equal to zero. The same conclusion is also true for Fig. 4(e).

The basic conclusion of this section has already been obtained previously in the absence of impurity scattering.[7] What we have demonstrated here is that, by including the effect of impurity scattering in a well-defined approximation, the system cannot absorb energy from external field so long as only optical processes which conserve the momentum in the final states have been taken into account.

### B. Final states that do not conserve momentum

Up to the present moment the type of response function shown in Fig. 5 has not been considered. The matrix element between the initial state and the final state can be represented by the part of the diagram which is on the left-hand side of the dashed vertical line in Fig. 5. This part of the diagram has been resketched as graph (a) of Fig. 6. From this graph it is quite clear that the photon is the initial state and the final state is made of a particle-hole pair and a single plasmon. The momentum of the final state is not conserved because the impurity can absorb momentum from the final state in the particular absorption process under consideration. There are, all together,

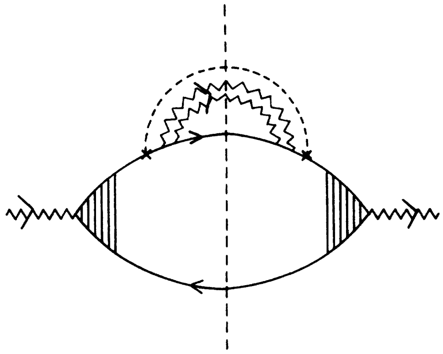

FIG. 5. Diagram showing one of the dielectric response functions. The final state of its dissipative part is a particle-hole pair plus a single plasmon. The momentum of the electron system is not conserved in the final state.

eight different $T$ matrices and these matrices are shown by graphs in Fig. 6. By using similar analysis to that described in Sec. IV A, it can be shown that contributions from graphs 6(c) + 6(d) + 6(e) and 6(f) + 6(g) + 6(h) are separately equal to zero. The only contribution of Fig. 6 is from graphs 6(a) and 6(b). We have examined the contribution from these two graphs by neglecting the impurity-scattering effects in the Green's function ($\tau = \infty$) and also in the vertex function ($\tau_{\mathrm{tr}} = \infty$). Unfortunately, the result does not converge.

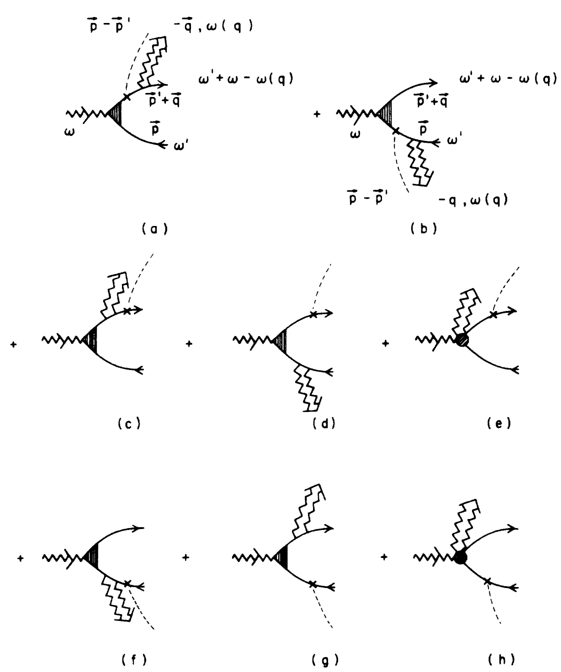


FIG. 6. Diagrams showing the transition matrix amplitudes with an electron-hole pair and a single plasmon as final state. The electronic momentum is not conserved in the final state of these processes.

Therefore the inclusion of the lifetime effect, particularly in the Green's function, is essential. It is noted that the external frequency $\omega > \omega(q)$, the plasmon frequency of the free carrier. Therefore the condition $\omega\tau \sim \omega\tau_{\mathrm{tr}} \gg 1$ is always satisfied; thus the vertex corrections due to the impurity scatterings can be neglected [see Eqs. (4) and (7)]. In order to simplify the arithmetic, let us use the approximate Green's function to evaluate Figs. 6(a) and 6(b),

$$G(k,\omega) = \left(\omega - \xi_k + \frac{i}{2\tau}\frac{\xi_k}{|\xi_k|}\right)^{-1} . \tag{20}$$

Compare it with the Green's function defined in Eq. (6); it is easy to see that they are identical in the limit $\tau \to \infty$. Therefore we expect Eq. (20) to be a good approximation to the Green's function in the low impurity-concentration limit. It is easy to show that Figs. 6(a) and 6(b) have the following form:

$$\mathrm{Im}\chi_6(\omega) = n \sum_{\vec{p},\vec{p}',\vec{q}} |T(\omega,\vec{p},\vec{p}',\vec{q})|^2 \, |U(\vec{p}-\vec{p}')|^2 \theta(-\xi_p)\theta(\xi_{p'+q}) \times \tfrac{1}{2}\,\omega(q) \frac{1}{[\omega - \omega(q) - \xi_{p'+q} + \xi_p]^2 + 1/\tau^2}\frac{1}{\tau}, \tag{21}$$

where $T = T_{6a} + T_{6b}$, $T_{6a}$ comes from Fig. 6(a), and $T_{6b}$ comes from Fig. 6(b). They are, respectively, given by

$$T_{6a} = -A(q)(ie\hat{\epsilon}\cdot\vec{p}/m)[G(\vec{p},\omega'+\omega)G(\vec{p}',\omega'+\omega)]_{\omega'},$$

with $\omega' = \xi_p + i/2\tau$

$$T_{6a} = -A(q)\frac{ie\vec{\epsilon}\cdot\vec{p}}{m\omega^2}\frac{1}{\omega + \xi_p - \xi_{p'} + i/\tau}, \tag{22}$$

$$T_{6b} = A(q)\frac{ie\vec{\epsilon}\cdot(\vec{p}'+\vec{q})}{m\omega^2}\frac{1}{\omega + \xi_{\vec{p}+\vec{q}} - \xi_{\vec{p}'+\vec{q}} + i/\tau}. \tag{23}$$

If we make the variable change $\vec{p}' + \vec{q} \to \vec{p}'$ and assume that spectrum density appearing in Eq. (21) behaves approximately like the $\delta$ function in the low impurity-concentration limit, then Eq. (21) can be written more explicitly like

$$\mathrm{Im}\chi_6(\omega) = (1/\tau)(e^2\epsilon_F N/2m\omega^4) \times [T_1(\omega) + T_2(\omega) - T_3(\omega)], \tag{24}$$

where $T_1$, $T_2$, and $T_3$ depend upon $\omega$ and $N$ through the Fermi energy $\epsilon_F$ and are dimensionless quantities. Their explicit expressions are given in Appendix B. In obtaining Eq. (24), we have neglected the momentum dependence of the impurity potential $U(p)$, or we have assumed the impurity potential is a $\delta(\vec{r})$ like function. It is easy to see that $T_1(\omega)$ and $T_2(\omega)$ are not convergent if $\tau = \infty$.

The purpose of introducing the approximate Green's function in Eq. (20) is to remove this divergence and carrying out the momentum summation in Eq. (21) exactly. Equation (24) is the leading contribution to the plasmon-assisted free-carrier absorption coefficient. Comparing with Eq. (10), $\mathrm{Im}\chi_6(\omega)$ is the same order of magnitude as the classical Drude formula.

## V. ABSORPTION DUE TO A SINGLE PLASMON

### A. Momentum-conserving final states

It is easy to show that light cannot excite a single plasmon in the system under consideration if the final state conserves momentum. The optical processes which yield a single plasmon in the final state are shown in the first two graphs of Fig. 7. The $T$ matrix for each of these processes is zero. The contribution from Fig. 7(a) is

$$T_{7a}=-\frac{1}{2\pi i}\int d\omega'\sum_{\vec{p}}\frac{ie\vec{\epsilon}\cdot\vec{p}}{mw}\times\left(1+\frac{i}{\tau_1}\,\frac{\Theta(-\omega')\Theta(\omega'+\omega)}{\omega+i/\tau_{tr}}\right)\times G(\vec{p},\omega'+\omega)G(\vec{p},\omega').$$

Integrating over the angle between $\vec{\epsilon}$ and $\vec{p}$ yields $T_{7a}=0$. The contribution from Fig. 7(b) is

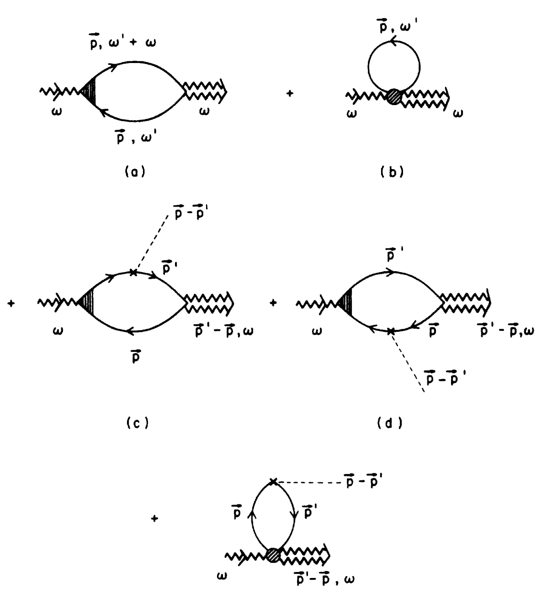


FIG. 7. Diagrams showing the transition matrix amplitudes with a single plasmon as final state. (a) and (b) are momentum-conserving final states. (c), (d), and (e) are final states which do not conserve electron momentum.

$$T_{7b}=-\frac{1}{2\pi i}\int d\omega'\,\frac{ie\vec{\epsilon}\cdot\vec{q}A(q)}{m\omega(\omega+i/\tau_{tr})}\sum_{\vec{p}}G(\vec{p},\omega'),$$

where $\vec{q}$ is the momentum associated with the incident light, and $\vec{\epsilon}\perp\vec{q}$ or $\vec{\epsilon}\cdot\vec{q}=0$. Thus we have $T_{7b}=0$.

### B. Final states that do not conserve momentum

The single-plasmon processes shown in the last three graphs of Fig. 7 do not conserve the final-state momentum. This is due to the ability of the impurity to take away momentum from the final state. After averaging over the positions of impurity atoms. The contribution from these processes to the absorption coefficient is of the form

$$\mathrm{Im}\chi_7(\omega)=n\sum_{\vec{p},\vec{p}'}|T(\omega,\vec{p}',\vec{p})|^2\times|U(\vec{p}-\vec{p}')|^2\,\frac{\omega(\vec{p}-\vec{p}')}{2}\times\pi\delta(\omega-\omega(\vec{p}-\vec{p}')). \tag{25}$$

Since the external frequency $\omega$ has to be larger than the plasmon frequency $\omega(\vec{p}-\vec{p}')$, the condition $\omega\tau_{tr}\gg1$ can be satisfied. We thus may neglect the impurity scatterings in the vertex function. The lifetime effect of the free carrier is taken into account only approximately through Eq. (20) in order to remove any divergence that may occur in Eq. (25). The expression for Fig 7(c) is given by

$$T_{7c}=-A(\vec{p}-\vec{p}')\frac{ie\vec{\epsilon}\cdot\vec{p}}{m\omega}\,\frac{1}{2\pi i}\times\int d\omega'\,G(\vec{p},\omega')G(\vec{p},\omega'+\omega)G(\vec{p}',\omega'+\omega).$$

After integrating over $\omega'$ we have

$$T_{7c}=-A(\vec{p}-\vec{p}')(ie\vec{\epsilon}\cdot\vec{p}/m\omega^2)H(p,p',\omega), \tag{26}$$

where

$$H(p,p',\omega)=n(\xi_p)[1-n(\xi_{p'})]\times\left(\frac{1}{\omega+\xi_p-\xi_{p'}+i/\tau}-\frac{1}{\xi_p-\xi_{p'}+i/\tau}\right)-[1-n(\xi_p)]n(\xi_{p'})\left(\frac{1}{\omega+\xi_p-\xi_{p'}-i/\tau}-\frac{1}{\xi_p-\xi_{p'}-i/\tau}\right).$$

Here $n(\xi)$ is the Fermi function. Similarly, we have the contribution from Fig. 7(d):

$$T_{7d}=A(\vec{p}-\vec{p}')\,\frac{ie\vec{\epsilon}\cdot\vec{p}'}{m\omega^2}H(p,p',\omega). \tag{27}$$

The contribution from Fig. 7(e) is given by

$$T_{7e} = -A(\vec{p}' - \vec{p}) \frac{ie\vec{\epsilon}\cdot(\vec{p}' - \vec{p})}{m\omega^2} \frac{(-1)}{2\pi i} \times \int d\omega' G(\vec{p}, \omega') G(\vec{p}', \omega').$$

Due to the occurrence of a closed loop in Fig. 7(e) an extra minus sign appears as a factor in the above expression. Integration over $\omega'$ is straightforward and yields

$$T_{7e} = A(\vec{p}' - \vec{p}) \frac{ie\vec{\epsilon}\cdot(\vec{p}' - \vec{p})}{m\omega^2} \frac{n(\xi_p)[1 - n(\xi_{p'})]}{\xi_p - \xi_{p'} + i/\tau} - \frac{n(\xi_{p'})[1 - n(\xi_p)]}{\xi_p - \xi_{p'} - i/\tau}. \tag{28}$$

From Eqs. (26), (27), and (28) we obtain

$$T(\omega, \vec{p}', \vec{p}) = A(\vec{p} - \vec{p}') \frac{ie\vec{\epsilon}\cdot(\vec{p}' - \vec{p})}{m\omega^2} \times \left( \frac{n(\xi_p)[1 - n(\xi_{p'})]}{\omega + \xi_p - \xi_{p'} + i/\tau} - \frac{n(\xi_{p'})[1 - n(\xi_p)]}{\omega + \xi_p - \xi_{p'} - i/\tau} \right). \tag{29}$$

Substituting Eq. (29) into Eq. (25) and carrying out the angular integrations gives

$$\mathrm{Im}\chi_7(\omega) = \frac{1}{\tau}\left(\frac{e^2}{m\omega^3}\right)\left(\frac{m^*\omega_p^2}{32m\epsilon_F^2}\right) \int_{-\epsilon_F}^{\infty} d\xi_p \int_{-\epsilon_F}^{\infty} d\xi_{p'}\, \Theta\left(\frac{pp'}{m^*} - \left|\omega - \omega_p - \frac{1}{2m^*}(p^2 + p'^2)\right|\right) \Theta(q_c - [2m^*(\omega - \omega_p)]^{1/2}) \times \frac{n(\xi_p)[1 - n(\xi_{p'})] + n(\xi_{p'})[1 - n(\xi_p)]}{(\omega + \xi_p - \xi_{p'})^2 + 1/\tau^2}, \tag{30}$$

where $p = [2m(\epsilon_F + \xi_p)]^{1/2}$, and $\epsilon_F$ is the Fermi energy. In obtaining Eq. (30) we have neglected the momentum dependence of $U(\vec{p} - \vec{p}')$ in Eq. (25) and approximated the plasmon dispersion relation as $\omega(q) = \omega_p + q^2/2m^*$ for $q \leq q_c$ (cut-off momentum). We also use the fact $A(q) = (4\pi e^2/q^2)^{1/2}$ and $\omega_p = 4\pi e^2 N/m$ for a free-electron gas. Compare Eq. (30) with Eq. (24). It is easy to see that $\mathrm{Im}\chi_7(\omega)$ is $1/N$ smaller than $\mathrm{Im}\chi_6(\omega)$. Therefore comparing with the plasma-assisted free-carrier absorption processes the single-plasmon absorption can be neglected.

## VI. CONCLUSION

In this paper we have studied the effect of impurity scattering on plasmon-assisted optical absorption in semiconductors by using Green's-function methods. The interaction $H_{eE}$ of the electrons with the electric field of the photon, the interaction $H_{ep}$ of electrons and plasmons, and the change $H_{epE}$ in $H_{ep}$ caused by the electric field are all considered. If we identify the transport lifetime $\tau_{tr}$ appearing in $H_{epE}$ of Eq. (4) to be given by Eq. (8) for all external frequency, then the method described in Ref. 13 is applied. Within this well-defined approximation,[13] for momentum-conserving final states the amplitudes for processes linear in the electric field $E$ exactly cancel just as they do in the absence of collisions.[7] For final states which are not momentum conserving, this cancellation no longer occurs and a final absorption is obtained. The strength of the absorption due to plasmon-assisted free-carrier processes (Sec. IV) is in the same order of magnitude as the classical Drude formula, but is larger than that from a single-plasmon excitation (Sec. V) by a factor $N$. Although we have written down our results [Eqs. (24) and (30)] explicitly in terms of simple double integrations, we have not attempted to numerically evaluate the integrals because the experimental situation is not sufficiently clear to warrant detailed numerical work. Our method should be easily extended to include a dc magnetic field. The plasmon-assisted free-carrier absorption in the presence of a magnetic field is a more interesting problem and is subjected to future investigation.

It has been pointed out to us that a paper by DuBois and Kivelson[15] addressed the problem of the effect of electron-plasmon interaction on free-carrier absorption and was the first work that demonstrated that this contribution vanished in the dipole approximation if lattice and impurity effects are ignored. In addition, the modification of the electron-plasmon interaction by the ac field was treated first by DuBois, Gilinsky, and Kivelson.[16] These authors also made extensive use of the open-diagram method used throughout the present paper.

## APPENDIX A: EVALUATION OF THE DIAGRAM IN FIG. 3

The corresponding expression for the diagram in Fig. 3 can be written

$$\chi_3(\omega) = \left(\frac{1}{2\pi i}\right)^2 \int\int d\omega' d\omega'' \sum_{\vec{p},\vec{q}} G(\vec{p}, \omega') G(\vec{p} + \vec{q}, \omega' + \omega - \omega'') D(-\vec{q}, \omega'') \left| A(q) \frac{ie\vec{\epsilon}\cdot\vec{q}}{m\omega(\omega + i/\tau_{tr})} \right|^2, \tag{A1}$$

where the electron Green's function is defined in Eq. (7), and the plasmon propagator is given by

$$D(\vec{q},\omega)=\frac{\omega(q)}{2}\left(\frac{1}{\omega-\omega(q)+i\delta}-\frac{1}{\omega+\omega(q)-i\delta}\right). \tag{A2}$$

Here $\omega(q)$ is the plasmon energy with momentum $q$.

It is rather straightforward to show that

$$\left(\frac{1}{2\pi i}\right)^2\int_{-\infty}^{\infty}d\omega''\int_{-\infty}^{\infty}d\omega'\sum_{\vec{p}}G(\vec{p},\omega')G(\vec{p}+\vec{q},\omega'+\omega-\omega'')D(-\vec{q},\omega'')=I_1+I_2, \tag{A3}$$

where

$$I_1=\frac{1}{2\pi i}\frac{mp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0}\int_{-\infty}^{\infty}d\omega''\frac{(\omega-\omega'')\Theta(\omega-\omega'')}{\omega-\omega''-\epsilon(\vec{p}_0,\vec{q})+i/\tau}D(-\vec{q},\omega''),$$

and

$$I_2=\frac{1}{2\pi i}\frac{mp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0}\int_{-\infty}^{\infty}d\omega''\frac{(\omega-\omega'')\Theta(\omega''-\omega)}{\omega-\omega''-\epsilon(\vec{p}_0,\vec{q})-i/\tau}D(-\vec{q},\omega'').$$

After integrating over $\omega''$, $I_1$ can be written

$$I_1=\frac{mp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0}\frac{\omega(q)}{4\pi i}\left[\frac{\omega-\omega(q)}{\omega-\omega(q)-\epsilon(\vec{p}_0,\vec{q})+i/\tau}\left(\tfrac{1}{2}\ln\frac{[\omega-\omega(q)]^2}{[\epsilon(\vec{p}_0,\vec{q})]^2+1/\tau^2}-i\tan^{-1}\frac{\epsilon(\vec{p}_0,\vec{q})}{1/\tau}-i\frac{\pi}{2}-i\pi\Theta(\omega-\omega(q))\right)\right.$$
$$\left.-\frac{\omega+\omega(q)}{\omega+\omega(q)-\epsilon(\vec{p}_0,\vec{q})+i/\tau}\left(\tfrac{1}{2}\ln\frac{[\omega+\omega(q)]^2}{[\epsilon(\vec{p}_0,\vec{q})]^2+1/\tau^2}-\tan^{-1}\frac{\epsilon(\vec{p}_0,\vec{q})}{1/\tau}-\frac{i\pi}{2}+i\pi\Theta(\omega+\omega(q))\right)\right],$$

and $I_2$ can be written

$$I_2=\frac{mp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0}\frac{\omega(q)}{4\pi i}\left[\frac{\omega-\omega(q)}{\omega-\omega(q)-\epsilon(\vec{p}_0,\vec{q})-i/\tau}\left(-\tfrac{1}{2}\ln\frac{[\omega-\omega(q)]^2}{[\epsilon(\vec{p}_0,\vec{q})]^2+1/\tau^2}-i\tan^{-1}\frac{\epsilon(p_0,q)}{1/\tau}+\frac{i\pi}{2}-i\pi\Theta(\omega(q)-\omega)\right)\right.$$
$$\left.-\frac{\omega+\omega(q)}{\omega+\omega(q)-\epsilon(\vec{p}_0,\vec{q})+i/\tau}\left(-\tfrac{1}{2}\ln\frac{[\omega+\omega(q)]^2}{[\epsilon(\vec{p}_0,\vec{q})]^2+1/\tau^2}-i\tan^{-1}\frac{\epsilon(\vec{p}_0,\vec{q})}{1/\tau}+\frac{i\pi}{2}+i\pi\Theta(-\omega(q)-\omega)\right)\right].$$

By substituting $I_1+I_2$ into (A3) and then into (A1), it is easy to show that the imaginary part of $\chi_3(\omega)$ is given by

$$\mathrm{Im}\chi_3(\omega)=\frac{mp_0}{(2\pi)^3}\int d\Omega_{\vec{p}_0}\sum_{\vec{q}}\left|A(q)\frac{ie\vec{\epsilon}\cdot\vec{q}}{m\omega(\omega+i/\tau_{tr})}\right|^2\frac{\omega(q)}{2}\frac{[\omega-\omega(q)]\Theta(\omega-\omega(q))}{[\omega-\omega(q)-\epsilon(\vec{p}_0,\vec{q})]^2+1/\tau^2}\frac{1}{\tau}. \tag{A4}$$

## APPENDIX B: EXPRESSIONS FOR $T_1(\omega)$, $T_2(\omega)$, and $T_3(\omega)$ IN EQ. (24)

$T_1(\omega)$ is the term due to the absolute squared value of Eq. (22). It is given by

$$T_1(\omega)=\frac{3}{8}\int_0^{q_c}dq\frac{q^2\bar{A}^2(q)\omega(q)}{k_F^3\epsilon_F^2}\int_{-\epsilon_F}^{0}d\xi\{(\epsilon_F+\xi)[\epsilon_F+\omega-\omega(q)+\xi]^3\}^{1/2}\frac{1}{\epsilon_F}\Theta(\omega-\omega(q)+\xi)\frac{m\tau}{p'q}$$
$$\times[\tan^{-1}\tau|\omega_-(q)+p'q/m|-\tan^{-1}\tau|\omega_-(q)-p'q/m|],$$

where $\epsilon_F$ is the Fermi energy, $\omega_-(q)=\omega(q)-q^2/2m$, and $p'=\{2m[\epsilon_F+\omega-\omega(q)+\xi]\}^{1/2}$. $\bar{A}^2(q)=NA^2(q)$, and the plasmon-electron interaction $A(q)=(4\pi e^2/q^2)^{1/2}$ for a free-electron gas. $q_c$ is a cutoff momentum for the plasmon. $T_2(\omega)$ is the term due to the absolute squared value of Eq. (23). It is given by

$$T_2(\omega)=\frac{3}{8}\int_0^{q_c}dq\frac{q^2\bar{A}^2(q)\omega(q)}{k_F^3\epsilon_F^2}\int_{-\epsilon_F}^{0}d\xi\{(\epsilon_F+\xi)^3[\epsilon_F+\omega-\omega(q)+\xi]\}^{1/2}\frac{1}{\epsilon_F}\Theta(\omega-\omega(q)+\xi)\frac{m\tau}{pq}$$
$$\times[\tan^{-1}\tau|\omega_+(q)+pq/m|-\tan^{-1}\tau|\omega_+(q)-pq/m|],$$

where $\omega_+(q)=\omega(q)+q^2/2m$, and $p=[2m(\epsilon_F+\xi)]^{1/2}$. $T_3(\omega)$ is due to the cross product of Eq. (22) and

(23) and is given by

$$T_3(\omega)=\frac{3}{64}\int_0^{q_c}dq\,\frac{\bar{A}^2(q)\omega(q)}{k_F\epsilon_F^2}\int_{-\epsilon_F}^{0}d\xi\,\frac{\{(\epsilon_F+\xi)[\epsilon_F+\omega-\omega(q)]\}^{1/2}}{\epsilon_F^2}\,\Theta(\omega-\omega(q)+\xi)$$
$$\times\left\{\left[2-\left(\omega_+(q)-\frac{i}{\tau}\right)\frac{m}{pq}\ln\left(\frac{\omega_+(q)+pq/m-i/\tau}{\omega_+(q)-pq/m-i/\tau}\right)\right]\right.$$
$$\left.\times\left[2-\left(\omega_-(q)+\frac{i}{\tau}\right)\frac{m}{p'q}\ln\left(\frac{\omega_-(q)+p'q/m+i/\tau}{\omega_-(q)-p'q/m+i/\tau}\right)\right]+\text{c.c.}\right\}.$$

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[14]The use of Eq. (9) seems not rigorously applicable in the present case, since the magnitude of the external frequency $\omega$ is larger than the plasmon frequency $\omega(q)$. On the other hand, we have assumed that the transport time $\tau_{tr}$ appearing in $H_{eEp}$ of Eq. (4) is $\omega$ independent, and we also have assumed that $\tau_{tr}$ is given by Eq. (8) for all $\omega$. To be consistent we applied the method described in Ref. (13) to Sec. IV A and to see how the amplitudes cancel among themselves in this approximation.


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