Low-field magnetoresistance in metals

W. Kesternich
Institut für Festkörperforschung der Kernforschungsanlage Jülich, 517 Jülich, Germany
(Received 13 October 1975)

Low-field magnetoresistance and Hall coefficient are calculated in the anisotropic-relaxation-time approximation. Simple results are obtained for Fermi-surface models which can be composed of spherical, cylindrical, and planar surfaces. Influences of the Fermi-surface geometry and the scattering anisotropy on the low-field magnetoresistance are discussed. With small modifications the method is applied to polyvalent metals with nearly-free-electron Fermi-surfaces. Simultaneous magnetoresistance and Hall-coefficient measurements combined with a three-group model calculation for the electronic mean free path are suggested as a means to determine the anisotropy of the electronic scattering in nearly-free-electron-like polyvalent metals. In an Appendix the results are extended to the longitudinal magnetoresistance.

I. INTRODUCTION

Magnetoresistance and Hall effect depend on the geometry of the Fermi surface and on the scattering behavior of the electrons at lattice imperfections. Many efforts have been made in the past two decades to investigate the Fermi surfaces of metals. Since attention recently has turned more towards the influence of the lattice defects on the electronic properties, the interest in the low-field galvanomagnetic coefficients has increased relative to the previously dominant interest in the high-field properties. However, while the low-field Hall coefficient has been used successfully in several investigations[1-4] of the anisotropy of the electronic mean free path, the behavior of the more sophisticated magnetoresistance is barely understood qualitatively.

For practical applications, the assumption of a relaxation time[5] or mean free path as an approximate description of the scattering processes has proved to be very useful, and several theoretical methods[6-11] for the calculation of the galvanomagnetic coefficients have been developed on the basis of this assumption. These theories have led to the useful Tsuji formula of the low-field Hall coefficient[12,13] applicable in the case of cubic symmetry as well as to a great number of model calculations[11,14-21] assuming simple geometric shapes of the Fermi surface and simple functions for the variation of the relaxation time $\tau(\vec{k})$. But with few exceptions these efforts were also restricted to Hall-coefficient calculations.

Using the method of Jones and Zener,[6] a new type of model calculation is presented in this paper. Simple formulas for the transverse magnetoresistance are derived in Sec. II, and the qualitative understanding of the magnetoresistance is discussed in Sec. III from the standpoint of Fermi surface and relaxation-time anisotropy. In Sec. IV the method is applied to nearly-free-electron metals. Approximate formulas for the Hall coefficient and magnetoresistance are derived for Fermi surfaces of the nearly-free-electron type, and the influence of scattering anisotropy is discussed in a three-group model for the mean free path.

II. COMPUTATIONAL METHOD

If the approximation of the electronic scattering by an anisotropic relaxation time is justified, the Boltzmann equation can easily be solved for low magnetic fields by expanding the solution in a power series of the magnetic field strength B.[6] Then the components of the conductivity tensor σ are also obtained as a power series of B:

$$\sigma_{ij}(B) = \sum_{n=0}^{\infty} \sigma_{ij}^{(n)}(B), \quad i, j = x, y, z$$

$$\sigma_{ij}^{(n)} = \frac{e^{n+2}}{4\pi^3\hbar^n} B^n \int \left(-\frac{\partial f_0}{\partial \epsilon}\right) v_i \left[\tau\left(v_x \frac{\partial}{\partial k_y} - v_y \frac{\partial}{\partial k_x}\right)\right]^n \times (\tau v_j)\, d^3k, \tag{1}$$

where the z axis is chosen parallel to $\vec{B}$. $\vec{v}(\vec{k})$ is the velocity, $\epsilon(\vec{k})$ the energy, and $\tau(\vec{k})$ the relaxation time of an electron at position $\vec{k}$ in reciprocal space. $f_0(\vec{k})$ is the equilibrium electron distribution in k space in tne absence of external fields.

Introducing the coefficients

$$Q_{ij}^{(n)}(\vec{k}) = \frac{3}{\tau^{n+1} v^{n+2}} v_i \left[\tau\left(v_x \frac{\partial}{\partial k_y} - v_y \frac{\partial}{\partial k_x}\right)\right]^n (\tau v_j) \tag{2}$$

and the electronic mean free path

$$l(\vec{k}) = v(\vec{k})\tau(\vec{k})$$

yields for the components $\sigma_{ij}^{(n)}$

$$\sigma_{ij}^{(n)} = \frac{e^{n+2}}{12\pi^3\hbar^{n+1}} B^n \int_{FS} l^{n+1} Q_{ij}^{(n)}\, dS. \tag{3}$$

In this expression the integral is reduced to a surface integration using $-\partial f_0/\partial\epsilon = \delta(\epsilon - \epsilon_F)$, where ϵ_F is the Fermi energy.

In the limit of low magnetic fields and for the case that the tensor $\sigma_{ij}^{(0)}$ is diagonal, the resistivity in zero magnetic field is given by

$$\rho_{xx}^{(0)} = 1/\sigma_{xx}^{(0)}, \tag{4}$$

the Hall coefficient by

$$R_H = \frac{\rho_{xy}(B) - \rho_{yx}(B)}{2} = \frac{1}{B}\,\frac{\sigma_{xy}^{(1)}}{\sigma_{xx}^{(0)}\,\sigma_{yy}^{(0)}}, \tag{5}$$

and the transverse magnetoresistance by

$$\frac{\Delta\rho}{\rho_0} \equiv \frac{\rho_{xx}(B) - \rho_{xx}(0)}{\rho_{xx}(0)} = -\frac{1}{\sigma_{xx}^{(0)}}\left(\sigma_{xx}^{(2)} + \frac{(\sigma_{xy}^{(1)})^2}{\sigma_{yy}^{(0)}} + \frac{(\sigma_{xz}^{(1)})^2}{\sigma_{zz}^{(0)}}\right), \tag{6}$$

where the current $\vec{j}$ is perpendicular to $\vec{B}$ and by convention parallel to the x coordinate. Equations (5) and (6) are obtained if one uses in each tensor component only $\sigma_{ij}^{(0)}$ and the next-higher-order term which does not vanish under consideration of the Onsager relation, $\sigma_{ij}(B) = -\sigma_{ij}(-B)$.

In the special case of cubic symmetry, the resistivity in zero magnetic field is $\rho_0 = 1/\sigma^{(0)}$, with[22]

$$\sigma^{(0)} = \frac{e^2}{12\pi^3\hbar}\int_{FS} l\,dS\,. \tag{7}$$

The Hall conductivity and Hall coefficient in cubic symmetry reduce to[13]

$$\sigma_{xy}^{(1)} = \frac{e^3B}{12\pi^3\hbar^2}\int_{FS}\frac{l^2}{\kappa_m}\,dS \tag{8}$$

and

$$R_H = \frac{12\pi^3}{e}\,\frac{\int_{FS}(l^2/\kappa_m)\,dS}{(\int_{FS} l\,dS)^2}\,. \tag{9}$$

$1/\kappa_m$ is the mean curvature defined by

$$\frac{1}{\kappa_m} = \frac{1}{2}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right),$$

where κ_1 and κ_2 are the two principal radii of curvature. The zero-field resistance and the Hall coefficient are independent of crystal orientation. Since $\sigma_{xz}^{(1)}$ vanishes for cubic symmetry, the low-field magnetoresistance equation for cubic metals can be expressed with the aid of Eqs. (7) and (8) as

$$\frac{\Delta\rho}{\rho_0} = -\frac{\int_{FS} l\,dS\int_{FS} l^3Q_{xx}^{(2)}\,dS + [\int_{FS}(l^2/\kappa_m)\,dS]^2}{(\int_{FS} l\,dS)^2}\times\left(\frac{e}{\hbar}B\right)^2, \tag{10a}$$

or with Eq. (9) as

$$\frac{\Delta\rho}{\rho_0} = -\left[\left(\frac{12\pi^3}{e}\right)^2\frac{\int_{FS} l^3Q_{xx}^{(2)}\,dS}{(\int_{FS} l\,dS)^3} + R_H^2\right]\left(\frac{B}{\rho_0}\right)^2. \tag{10b}$$

According to the definition (2), the Q coefficients required for the evaluation of Eq. (10) or its general form (6) are given by the expressions

$$Q_{ii}^{(0)}(\vec{k}) = 3\alpha_i^2, \quad i = x, y, z \tag{11a}$$

where α_i is the direction cosine of v with respect to the i axis,

$$Q_{ij}^{(1)}(\vec{k}) = \frac{3}{\hbar^3v^3}\,\epsilon_i\left((\epsilon_x\epsilon_{yj} - \epsilon_y\epsilon_{xj}) + \frac{\epsilon_j}{\tau}(\epsilon_x\tau_y - \epsilon_y\tau_x)\right), \quad ij = xy, xz \tag{11b}$$

and

$$Q_{xx}^{(2)}(\vec{k}) = -\frac{3}{\hbar^4v^4}\Big\{[\epsilon_x^2(\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}^2) - \epsilon_x(\epsilon_x^2\epsilon_{xyy} - 2\epsilon_x\epsilon_y\epsilon_{xxy} + \epsilon_y^2\epsilon_{xxx})] + \frac{\epsilon_x}{\tau}[\tau_x(\epsilon_x^2\epsilon_{yy} - \epsilon_y^2\epsilon_{xx}) + (2\tau_x\epsilon_y - 4\tau_y\epsilon_x)(\epsilon_x\epsilon_{xy} - \epsilon_y\epsilon_{xx})] - \frac{\epsilon_x^2}{\tau}\left[\epsilon_x^2\left(\frac{\tau_y^2}{\tau} + \tau_{yy}\right) - 2\epsilon_x\epsilon_y\left(\frac{\tau_x\tau_y}{\tau} + \tau_{xy}\right) + \epsilon_y^2\left(\frac{\tau_x^2}{\tau} + \tau_{xx}\right)\right]\Big\}, \tag{11c}$$

where the relation $\hbar v_i = \partial\epsilon/\partial k_i$ and the abbrevations $\epsilon_i = \partial\epsilon/\partial k_i$ and $\tau_i = \partial\tau/\partial k_i$ are used.

A. Q on spherical energy surfaces

First we consider the case in which the energy surfaces in the vicinity of ϵ_F can be represented by

$$\epsilon = f(\kappa)\,, \tag{12}$$

where f is an arbitrary differentiable function, and $\kappa = |\vec{k} - \vec{k}_0|$ is the radius of a sphere with center $\vec{k}_0$. Then the derivatives of ϵ at the position $\vec{k}$ are

$$\epsilon_i \equiv \frac{\partial\epsilon}{\partial k_i} = \frac{df}{d\kappa}\,\alpha_i\,,$$

$$\epsilon_{ij} \equiv \frac{\partial\epsilon}{\partial k_i\partial k_j} = \left(\frac{d^2f}{d\kappa^2} - \frac{1}{\kappa}\frac{df}{d\kappa}\right)\alpha_i\alpha_j + \frac{1}{\kappa}\frac{df}{d\kappa}\,\delta_{ij}\,,$$

and

$$\epsilon_{ijl} \equiv \frac{\partial \epsilon}{\partial k_i \partial k_j \partial k_l} = \left(\frac{d^3 f}{d\kappa^3} - \frac{3}{\kappa}\frac{d^2 f}{d\kappa^2} + \frac{3}{\kappa^2}\frac{df}{d\kappa}\right)\alpha_i\alpha_j\alpha_l + \frac{1}{\kappa}\left(\frac{d^2 f}{d\kappa^2} - \frac{1}{\kappa}\frac{df}{d\kappa}\right)(\alpha_i\delta_{jl} + \alpha_j\delta_{il} + \alpha_l\delta_{ij}) .$$

Using these relations and $v = |d\epsilon/d\vec{k}|$, the Q coefficients given by Eqs. (11b) and (11c), become

$$Q_{ij}^{(1)}(\vec{k}) = \frac{3}{\kappa}(\alpha_i\alpha_x\delta_{yj} - \alpha_i\alpha_y\delta_{xj}) - 3\alpha_i\alpha_j\left(\alpha_y\frac{\tau_x}{\tau} - \alpha_x\frac{\tau_y}{\tau}\right) \tag{13}$$

and

$$Q_{xx}^{(2)}(\vec{k}) = -\frac{3}{\kappa^2}\alpha_x^2 - \frac{3}{\kappa}\left(\alpha_x(\alpha_x^2 - 3\alpha_y^2)\frac{\tau_x}{\tau} + 4\alpha_x^2\alpha_y\frac{\tau_y}{\tau}\right) + 3\alpha_x^2\left[\alpha_x^2\left(\frac{\tau_y^2}{\tau^2} + \frac{\tau_{yy}}{\tau}\right) - 2\alpha_x\alpha_y\left(\frac{\tau_x\tau_y}{\tau^2} + \frac{\tau_{xy}}{\tau}\right) + \alpha_y^2\left(\frac{\tau_x^2}{\tau^2} + \frac{\tau_{xx}}{\tau}\right)\right] . \tag{14}$$

B. Q on cylindrical energy surfaces

Cylindrical surfaces are treated in an analogous way. We start out from Eq. (12) again, where κ is now the cylinder radius. With the aid of an orthogonal coordinate system $k_1k_2k_3$ (with k_1 equal to the cylinder axis and k_2, k_3 arbitrary) which is related to the $k_xk_yk_z$ system by the rotation matrix α_{li} ($l = 1, 2, 3$; $i = x, y, z$), a few algebraic steps lead to

$$\epsilon_i = \frac{df}{d\kappa}\alpha_i ,$$

$$\epsilon_{ij} = \left(\frac{d^2 f}{d\kappa^2} - \frac{1}{\kappa}\frac{df}{d\kappa}\right)\alpha_i\alpha_j + \frac{1}{\kappa}\frac{df}{d\kappa}(\alpha_{2i}\alpha_{2j} + \alpha_{3i}\alpha_{3j}) ,$$

and

$$\epsilon_{ijl} = \left(\frac{d^3 f}{d\kappa^3} - \frac{3}{\kappa}\frac{d^2 f}{d\kappa^2} + \frac{3}{\kappa^2}\frac{df}{d\kappa}\right)\alpha_i\alpha_j\alpha_l + \frac{1}{\kappa}\left(\frac{d^2 f}{d\kappa^2} - \frac{1}{\kappa}\frac{df}{d\kappa}\right)[\alpha_i(\alpha_{2j}\alpha_{2l} + \alpha_{3j}\alpha_{3l}) + (\text{cyclic perms.})]$$

Inserting these relations in Eqs. (11b) and (11c), and considering that

$$\alpha_{2x}\alpha_{3y} - \alpha_{3x}\alpha_{2y} = \alpha_{1z} \tag{15}$$

and

$$\alpha_{1i}\alpha_{1j} + \alpha_{2i}\alpha_{2j} + \alpha_{3i}\alpha_{3j} = \delta_{ij} , \tag{16}$$

yields for the Q coefficients

$$Q_{ij}^{(1)}(\vec{k}) = \frac{3}{\kappa}[\alpha_i\alpha_x(\delta_{yj} - \alpha_{1y}\alpha_{1j}) - \alpha_i\alpha_y(\delta_{xj} - \alpha_{1x}\alpha_{1j})] - 3\alpha_i\alpha_j\left(\alpha_y\frac{\tau_x}{\tau} - \alpha_x\frac{\tau_y}{\tau}\right) \tag{17}$$

and

$$Q_{xx}^{(2)}(\vec{k}) = -\frac{3}{\kappa^2}\alpha_x^2\alpha_{1z}^2 - \frac{3}{\kappa}\left(\alpha_x[\alpha_x^2(1 - \alpha_{1y}^2) - 3\alpha_y^2(1 - \alpha_{1x}^2) - 2\alpha_x\alpha_y\alpha_{1x}\alpha_{1y}]\frac{\tau_x}{\tau} + 4\alpha_x^2[\alpha_x\alpha_{1x}\alpha_{1y} + \alpha_y(1 - \alpha_{1x}^2)]\frac{\tau_y}{\tau}\right) + 3\alpha_x^2\left[\alpha_x^2\left(\frac{\tau_y^2}{\tau^2} + \frac{\tau_{yy}}{\tau}\right) - 2\alpha_x\alpha_y\left(\frac{\tau_x\tau_y}{\tau^2} + \frac{\tau_{xy}}{\tau}\right) + \alpha_y^2\left(\frac{\tau_x^2}{\tau^2} + \frac{\tau_{xx}}{\tau}\right)\right] . \tag{18}$$

C. Q on planar energy surfaces

For a set of planar energy surfaces again represented by Eq. (12), where κ is now the component of $\vec{k}$ normal to the surface, Eqs. (11b) and (11c) become

$$Q_{ij}^{(1)}(\vec{k}) = -3\alpha_i\alpha_j\left(\alpha_y\frac{\tau_x}{\tau} - \alpha_x\frac{\tau_y}{\tau}\right) \tag{19}$$

and

$$Q_{xx}^{(2)}(\vec{k}) = 3\alpha_x^2\left[\alpha_x^2\left(\frac{\tau_y^2}{\tau^2} + \frac{\tau_{yy}}{\tau}\right) - 2\alpha_x\alpha_y\left(\frac{\tau_x\tau_y}{\tau^2} + \frac{\tau_{xy}}{\tau}\right) + \alpha_y^2\left(\frac{\tau_x^2}{\tau^2} + \frac{\tau_{xx}}{\tau}\right)\right] . \tag{20}$$

Only terms with derivatives of τ contribute on planar Fermi-surface areas.

For special orientations with respect to $\vec{j}$ and $\vec{B}$, equations of similar simplicity can be derived for certain other Fermi-surface models (e.g., ellipsoids with parabolic energy-momentum relation). In general, however, the second and third derivatives of ϵ introduce considerable complexity into Fermi-surface models other than the ones discussed here.

D. Small-$\Delta\tau$ approximation

It is assumed in the preceding calculations that the Fermi surface consists of spherical, cylindrical, or planar surfaces, and that the energy surfaces in the vicinity of the Fermi surface are spherical with the same center, cylindrical with

TABLE I. Summary of the Q coefficients on planar, spherical, and cylindrical energy-surface areas for $\vec{k}$-independent relaxation time.

	$Q_{xy}^{(1)}$		$Q_{xz}^{(1)}$		$Q_{xx}^{(2)}$	
Surface area	General	Cubic symmetry	General	Cubic symmetry	General	Cubic symmetry
planar	0	0	0	0	0	0
spherical	$\frac{3\alpha_x^2}{\kappa}$	$\frac{1}{\kappa}$	0	0	$-\frac{3\alpha_x^2}{\kappa^2}$	$-\frac{1}{\kappa^2}$
cylindrical	$\frac{3}{\kappa}[\alpha_x^2+(\alpha_x\alpha_y\alpha_{1x}-\alpha_x^2\alpha_{1y})\alpha_{1y}]$	$\frac{1}{2\kappa}$	$\frac{3}{\kappa}[\alpha_x\alpha_y\alpha_{1x}-\alpha_x^2\alpha_{1y}]\alpha_{1z}$	0	$-\frac{3\alpha_x^2\alpha_{1z}^2}{\kappa^2}$	$-\frac{3\alpha_x^2\alpha_{1z}^2}{\kappa^2}$

the same axis, or planar with the same normal direction as the corresponding Fermi-surface section. As a consequence, the models cannot account for gradual variations of the length of the velocity vector. Otherwise, the functional dependence $\epsilon=f(\kappa)$ is arbitrary; in particular, the calculation is not restricted to a parabolic energy-momentum relation.

Under this assumption, the higher-order derivatives of ϵ cancel on spherical, cylindrical, and planar Fermi-surface regions. If in addition τ is constant over the Fermi surface, or if the terms containing derivatives of τ can be neglected, the equations reduce and the calculation of the magnetoresistance for these models becomes extremely simple. In general, only small variations of τ across the Fermi surface are permitted if the terms in Q_{ij} containing derivatives of τ are to be neglected. A practical rule is that the mean variation $\Delta\tau$ be small compared to the mean value $\bar{\tau}$. This assumption of small $\Delta\tau$ is used in the following discussion. The accordingly reduced equations are summarized in Table I.

In the case of cubic symmetry, further simplifications can be introduced in the equations by averaging the $Q_{ij}(\vec{k})$ over symmetry-equivalent points of the Fermi surface. The resulting average Q's are also listed in Table I.

III. DISCUSSION

The relations between Fermi-surface topology and Q coefficients are considered in this section, and a few general consequences of Fermi-surface geometry, of crystal orientation, and of the scattering anisotropy on the magnetoresistance are discussed. For specific Fermi-surface geometries, the qualitative behavior of the magnetoresistance may be estimated with the aid of a few rules of thumb for the coefficients $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$.

A. Topological coefficients $Q_{ij}^{(n)}(\vec{k})$

The preceding calculations were based on the local coefficients $Q_{ij}(\vec{k})$ which finally are to be inserted into the integral equation (3). Alternatively, the integral equation could have been partly simplified by making use of the possible partial integrations.[23] This would have avoided the third-order derivatives in Eq. (11c) which, however, cancel anyway in the cases considered in Sec. II, as do the second-order derivatives. The advantage of the method used here is that the illustrative meaning of the $Q_{ij}(\vec{k})$ is maintained: The products $l^{n+1}(\vec{k})\,Q_{ij}^{(n)}(\vec{k})$ directly reflect the local contribution of a point $\vec{k}$ on the Fermi surface to the conductivity $\sigma_{ij}^{(n)}$[see Eq. (3)]. In the approximation that the derivatives of τ can be neglected, the scattering anisotropy enters in $l^{n+1}(\vec{k})$ only, while the $Q_{ij}^{(n)}(\vec{k})$ contain the topological contribution of the Fermi surface.[24] Hence Fermi-surface and scattering contributions can be separated in the equations for $\Delta\rho/\rho_0$ and R_H in the small-$\Delta\tau$ approximation.[25]

Extending the considerations of Sec. II to general Fermi-surface geometries leads to the following qualitative rules for the topological coefficients $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$ (a) *The coefficients $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$ assume high values at positions $\vec{k}$ of strong Fermi-surface curvature.* (On spherical and cylindrical Fermi-surface sections, $Q_{xy}^{(1)}$ is proportional to $1/\kappa$, $Q_{xx}^{(2)}$ to $1/\kappa^2$.) (b) Considering the orientation of the Fermi surface with respect to $\vec{B}$, *the curvature along the cyclotron orbit is the crucial one, while the curvature perpendicular to the cyclotron orbit is of minor importance.* (c) Regarding the orientation of the Fermi surface with respect to $\vec{j}$, it is clear that the *component of the Fermi velocity in the current direction is important for the electric transport.* Rules (b) and (c) relate to the orientation dependence of R_H for noncubic symmetry and of $\Delta\rho/\rho_0$ for both the cubic and noncubic cases. The validity of these rules for spherical and cylindrical Fermi-surface regions is directly obvious from the equations of Table I. They are, in general, also valid for Fermi surfaces with more complicated curvature (e.g., ellipsoidal, parabolic, or hyperbolic surface regions). Then of course a simple dependence on κ or κ_m cannot be established.

B. Influence of the Fermi surface on $\Delta\rho/\rho_0$

1. Compensation of the positive and negative terms in the magnetoresistance equation

In the magnetoresistance equation (6), the value of the first term is always positive, while the second and third terms are always negative. The negative terms tend to compensate the positive term but can never exceed the positive term. For a few singular cases of Fermi-surface geometries, viz., a single sphere, a single cylinder, or a single ellipsoid, this compensation is complete. The magnetoresistance of these Fermi-surface models is zero. The simplest Fermi-surface models with nonvanishing magnetoresistance are *two-band* models[26] consisting of two spheres with *different carrier mobilities* or *multivalley* models consisting of cylinders[27] or ellipsoids[28] with *different axis directions*.

The other extremal case is that the negative terms in $\Delta\rho/\rho_0$ may be neglected compared to the positive term. This occurs on Fermi surfaces with strongly varying values of κ_m since the ratio of the positive and negative terms is of the order $\langle 1/\kappa_m^2\rangle/\langle 1/\kappa_m\rangle^2$, where $\langle\ \rangle$ denotes the average over the Fermi surface. Examples are the Fermi surfaces of polyvalent metals in the nearly-free-electron approximation as discussed in Sec. IV.

2. Consequences of the dependence on curvature

A simple single-band model which has been frequently used in discussions of the properties of the Hall coefficient is the planar-faced-energy-surface model.[11,19–21,29–33] To obtain proper low-field results, a modified version of the model with rounded edges[31] must be used. For one special case (a cube with rounded edges) Allgaier also calculated the magnetoresistance.[20] Since the planar-faced-energy-surface models with rounded edges can be composed of planar areas joined by cylindrical areas along the edges and by spherical areas at the corners, the computational method described in Sec. II is quite suitable for this type of model. With the aid of the equations of Table I, $\Delta\rho/\rho_0$ and R_H can readily be calculated for any such model and for arbitrary orientation with respect to $\vec{j}$ and $\vec{B}$. A remarkable result which was found by Allgaier, and which is also valid for any other planar-faced-energy-surface calculation, is the increase of $\Delta\rho/\rho_0$ towards infinity as the cylinder radius of the edges tends towards zero.[34] This apparent contradiction requires a reconsideration of the low-magnetic-field condition.

It was pointed out in the discussion of the topological coefficients $Q_{ij}(\vec{k})$ that the *local curvature* at each position $\vec{k}$ is the Fermi-surface quantity which mainly determines the qualitative properties of the low-field magnetoresistance and Hall coefficient (for comparison, the high-field galvanomagnetic properties are determined mainly by the *topology of the total electron orbit*, e.g., whether the orbit is closed or open). Consequently, a *"local" low-field condition*[20] must replace the usually applied *"orbital" low-field condition*, $\omega\tau \ll 1$, where ω is the cyclotron frequency. The true low-field condition is realized when

$$B \ll \left|\hbar\kappa_m(\vec{k})/el(\vec{k})\right| \tag{21}$$

for any position $\vec{k}$ of the Fermi surface. The application of the Jones-Zener approximation or any other *low-field* solution of the Boltzmann equation requires the observation of this relation. The contradiction of infinitely increasing magnetoresistance occurs if $\Delta\rho/\rho_0$ is considered as a function of κ_m for constant B. According to relation (21), however, B must be decreased simultaneously when the minimal values of κ_m are decreased, if the condition of low magnetic field is to be maintained.[20]

In the "local" low-field limit, the magnetoresistance of a Fermi-surface containing regions with small values of κ_m generally exceeds the value of $\Delta\rho/\rho_0$ of a Fermi surface with only slightly curved regions, with the exception of the cases where compensation is effective. Comparing, for instance, a polyvalent metal such as Al and a noble metal, one finds in fact that the experimental values of $\Delta\rho/\rho_0$ in [35,36]Al are about 10 to 15 times larger than in [37–39]Cu for equal values of $\omega\tau$.

Another remarkable consequence of the influence of the local curvature is the different *relative* contributions to $\Delta\rho/\rho_0$ and R_H of various Fermi-surface regions. In many polyvalent metals, edgelike regions of strong curvature alternate with regions of relatively slight curvature. Though the area covered by the edgelike regions is generally small, these regions contribute significantly to R_H, and may even predominate in determining the value of $\Delta\rho/\rho_0$. This will be demonstrated clearly in the nearly-free-electron (NFE) model considered in Sec. IV.

3. Orientation dependence of the low-field magnetoresistance

The values of the galvanomagnetic coefficients depend on the orientation of the Fermi surface with respect to $\vec{j}$ and $\vec{B}$. In the equations of Table I, the direction cosines α_i and α_{1i} describe the orientation dependence. For more general Fermi surfaces, the influence of orientation is expressed qualitatively by rules (b) and (c) for the topological coefficients $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$. As can be seen from the orientation dependence of these local coefficients, $\Delta\rho/\rho_0$ and R_H can be sensitively dependent on orientation. The effects of general Fermi-surface shapes are to diverse to be discussed here. We only want to give some consideration

to the special case of cubic symmetry, where the situation is simplified by averaging of the orientation dependences: Resistivity in zero magnetic field and Hall coefficient are isotropic, and the magnetoresistance exhibits only a small dependence on crystal orientation. Nevertheless, even in cubic metals the orientation dependence of $\Delta\rho/\rho_0$ may be not as small as first expected. Many Fermi surfaces consist of a major almost-spherical area and smaller areas which are strongly distorted (noble metals, nearly-free-electron metals). The orientation-insensitive contributions of the spherical part almost cancel in the positive and negative term of $\Delta\rho/\rho_0$ so that the distorted areas essentially determine the magnetoresistance, and thus may create a still considerable anisotropy. This is indeed confirmed by measurements of (110)-rotation diagrams in Cu and[40] Ag where relative anisotropies $(\Delta\rho^{\max} - \Delta\rho^{\min})/\Delta\rho^{\min}$ of 0.6 to 0.9 were observed.

C. Influence of the electron scattering on $\Delta\rho/\rho_0$

If the Fermi surface is spherical and $l(\vec{k})$ is cubically symmetric, Eq. (10) reduces to

$$\frac{\Delta\rho}{\rho_0} = \frac{1}{(ne)^2}\frac{\langle l\rangle\langle l^3\rangle - \langle l^2\rangle^2}{\langle l\rangle^4}\left(\frac{B}{\rho_0}\right)^2, \tag{22}$$

where $\langle l^n\rangle$ denotes the average of l^n over the Fermi surface. In the case of free electrons, where $\vec{k} = m\vec{v}/\hbar$, Eq. (22) is equivalent to a similar equation already given by Bethe,[41]

$$\frac{\Delta\rho}{\rho_0} = \frac{\langle\tau\rangle\langle\tau^3\rangle - \langle\tau^2\rangle^2}{\langle\tau\rangle^2}\left(\frac{eB}{m}\right)^2.$$

In order to obtain these equations, the second and third terms of Eq. (14) (containing the derivatives of τ) have to be neglected. This, however, is not a good approximation if the magnetoresistance of the Fermi surface (i.e., for isotropic τ) is zero. As a consequence, Eq. (22) and the Bethe equation may give values of $\Delta\rho/\rho_0$ which are too low, and they do not account for the orientation dependence of $\Delta\rho/\rho_0$ originating from anisotropies in τ.[42] Without this restriction, the Hall coefficient of a spherical Fermi surface and for cubically symmetric $l(\vec{k})$ is obtained as

$$R_H = \frac{1}{ne}\frac{\langle l^2\rangle}{\langle l\rangle^2}, \tag{23}$$

where ne is negative for electrons and positive for holes. $\Delta\rho/\rho_0$ and $|R_H|$ increase with increasing variation of l.

In the general case of anisotropic Fermi surfaces and arbitrary crystal symmetry, the contributions of the Fermi surface, $Q_{ij}^{(n)}(\vec{k})$, have to be weighted by the corresponding functions $l^{n+1}(\vec{k})$. The second and third powers of $l(\vec{k})$ appearing in R_H and $\Delta\rho/\rho_0$ cause the galvanomagnetic coefficients to be sensitively dependent on the anisotropy of the electron scattering. One particular application which may become of interest would be in polyvalent metals, where the contributions of the small but highly curved regions of the Fermi surface are particularly enhanced. Therefore measuring $\Delta\rho/\rho_0$ and R_H in weak magnetic fields provides a method for studying the anisotropy of l selectively on these highly curved regions which, owing to their small size, are less accessible to other methods.

IV. APPLICATION TO NEARLY-FREE-ELECTRON (NFE) METALS

With little modification the results of Sec. II can be applied to NFE Fermi surfaces. In the NFE model, the free-electron sphere is maintained in the major part of the Fermi surface with the exception of the vicinity of the intersection circles where the Brillouin-zone boundaries intersect the free-electron sphere. The Fermi surface is composed of spherical regions with free-electron curvature κ_{FE} and rounded edges along the intersection circles. We call the radius of curvature of the Fermi surface along the edges $\kappa_{\parallel}$ and the radius perpendicular to the edges $\kappa_{\perp}$. If $\kappa_{\perp} \ll \kappa_{\parallel}$, which is usually the case, we approximate the edges of the Fermi surface and the energy surfaces in their vicinity by a set of concentric cylindrical sections, where the cylinder axis changes direction parallel to the edge direction. A suitable constant value $\kappa_{\perp}$ has to be assumed at the Fermi energy. The corners where two or more edges of different orientations join are correspondingly approximated by small spherical regions of radius $\kappa_{\perp}$.

In this approximation the formulas of Table I can be applied using the equations for spherical surfaces with $\kappa = \kappa_{FE}$ for the free-electron-like areas and $\kappa = \kappa_{\perp}$ for the corners, and the equations for cylindrical surfaces with $\kappa = \kappa_{\perp}$ for the edges. These relations are inserted into Eqs. (3) and (6). Using $\kappa_{FE} \gg \kappa_{\perp}$, the second and third terms of Eq. (6) can be neglected, and in the first term, the integrals over the free-electron-like areas and the corner areas can be neglected compared to integrals over the cylindrical areas along the edges. The remaining expression is

$$\frac{\Delta\rho}{\rho_0} = -\frac{\int_{\text{edge}} Q_{xx}^{(2)}(\vec{k})\, l^3(\vec{k})\, dS}{\int_{FS} l(\vec{k})\, dS}\left(\frac{e}{\hbar}B\right)^2$$

$$= -\frac{\int_{\text{edge}} Q_{xx}^{(2)}(\vec{k})\, l^3(\vec{k})\, dS}{\left[\int_{FS} l(\vec{k})\, dS\right]^3}\left(\frac{12\pi^3}{e}\frac{B}{\rho_0}\right)^2, \tag{24}$$

$$Q_{xx}^{(2)}(\vec{k}) = -\frac{3\,\alpha_x^2(\vec{k})\,\alpha_{1_E}^2(\vec{k})}{\kappa_{\perp}^2},$$

where α_{1_z} is now $\vec{k}$ dependent and equal to the cosine of the angle between B and the tangent to the intersection circle of the sphere and Brillouin-zone boundary. Equation (24) is also valid for noncubic symmetry since in the NFE model $\sigma_{xx}^{(0)}$ is approximately equal to the isotropic value of $\sigma^{(0)}$ for the free-electron sphere.

In the same approximation one obtains for the Hall coefficient

$$R_H = R_H^{FE} + \frac{12\pi^3}{e} \frac{\int_{edge} Q_{xy}^{(1)}(\vec{k}) l^2(\vec{k})\, dS}{(\int_{FS} l\, dS)^2}\,,$$
$$Q_{xy}^{(1)}(\vec{k}) = (3/\kappa_\perp)\{\alpha_x^2(\vec{k})[1-\alpha_{1_y}^2(\vec{k})] + \alpha_x(\vec{k})\alpha_y(\vec{k})\alpha_{1_x}(\vec{k})\alpha_{1_y}(\vec{k})\}\,, \tag{25}$$

where $R_H^{FE} = 1/ne$ is the free-electron value of R_H.

For NFE Fermi surfaces of polyvalent metals, the low-field magnetoresistance is dominated by the small cylindrical areas along the edges. In the Hall coefficient, the contributions of the edges and the free-electron-like areas are of the same order of magnitude, and in the resistivity the contributions are proportional to the surface areas, i.e., the free-electron-like areas dominate. This is due to the enhancement of the contributions to the Hall coefficient and magnetoresistance by $1/\kappa$ and $(1/\kappa)^2$, respectively.

A. Three-group model for $l(\vec{k})$

The function $l(\vec{k})$ depends on both the electronic structure of the metal and on the scattering potentials of the defects present in the lattice. The lack of quantitative information about mean free paths makes it as yet practically impossible to evaluate Eqs. (24) and (25) for any combination of defects and host metal. On the other hand, if simplifying assumptions regarding $l(\vec{k})$ are made, low-field magnetoresistance and Hall effect can be applied to give a coarse measure of the anisotropy of l or τ.

This has been done in the past with Hall-coefficient measurements alone. If it is reasonable to divide the k states on the Fermi surface into two groups so that $l(\vec{k})$ in each of the groups can be described by a constant average value, then from the relatively simple measurement of the scalar quantity R_H alone, the quotient of the relaxation times can be determined. Dugdale and Fifth,[2] for instance, have determined τ_{neck}/τ_{belly} in Cu and Ag for different types of lattice defects.

For the polyvalent metals, such a two-group model is not realistic. The wave functions in the NFE model are qualitatively different on the free-electron-like regions of the Fermi surface, along the edges with electronlike curvature, and along the edges with holelike curvature[43] (the curvature of the Fermi surface is defined as electronlike if the electron states on the inner side of the curvature are occupied, and holelike if the electron states on the outer side of the curvature are occupied). Thus at least three groups of electrons have to be distinguished, even in the simplest polyvalent metals. If such a three-group model is used, and l is named l_- on the free-electron-like Fermi surface areas, l_{--} on the edges with electronlike curvature, and l_{++} on the edges with holelike curvature, $\Delta\rho/\rho_0$ in the NFE approximation is

$$\frac{\Delta\rho}{\rho_0} = \frac{g_m}{\kappa_{FE}\kappa_\perp} \frac{l_{--}^3 + l_{++}^3}{2l_-} \left(\frac{e}{\hbar}B\right)^2 = g_m \frac{\kappa_{FE}}{\kappa_\perp} \frac{l_{--}^3 + l_{++}^3}{2l_-^3} \left(\frac{B}{ne\rho_0}\right)^2\,, \tag{26}$$
$$g_m = \frac{1}{4\pi\kappa_{FE}\kappa_\perp} \int_{edge} 3\alpha_x^2(\vec{k})\alpha_{1_x}^2(\vec{k})\, dS\,.$$

Equation (26) is obtained by approximating the integral in the denominator of Eq. (24) by the integral over the free-electron sphere and by taking the length of the edges equal to the sum of the circumferences of the intersection circles of zone boundaries and the free-electron sphere. In the application to real metals, this assumption may need a correction.[44]

The analogous equation for R_H is

$$R_H = \frac{1}{ne}\left(1 + g_h \frac{l_{--}^2 - l_{++}^2}{2l_-^2}\right), \tag{27}$$

where

$$g_h = \frac{1}{8\pi\kappa_{FE}\kappa_\perp} \int_{edge} 3\{\alpha_x^2(\vec{k})[1-\alpha_{1_y}^2(\vec{k})] + \alpha_x(\vec{k})\alpha_y(\vec{k})\alpha_{1_x}(\vec{k})\alpha_{1_y}(\vec{k})\}\, dS$$

for noncubic symmetry and

$$g_h = \frac{1}{8\pi\kappa_{FE}\kappa_\perp} \int_{edge} dS$$

for cubic symmetry.

In Eqs. (26) and (27), the Fermi-surface geometry and the anisotropy of l enter separately in two independent factors. The geometry factors g_m and g_h are of the order of unity. Thus the magnetoresistance is directly proportional to the curvature $1/\kappa_\perp$ of the edges while the Hall coefficient is independent of $\kappa_\perp$ in this approximation.

Applying Eqs. (26) and (27) to real metals, the Fermi surface of the metal is approximated by the NFE Fermi surface with rounded edges of curvature $1/\kappa_\perp$, which is taken to be the average curvature of the edges of the real Fermi surface. Then g_m and g_h can be calculated from the NFE Fermi surface, and the anisotropy of l, as given by the ratios l_{++}/l_- and l_{--}/l_-, can be determined from values of R_H and $\Delta\rho/\rho_0$.

Experimental values of low-field Hall coefficient

and magnetoresistance can easily be obtained from simultaneous measurements on a single sample. This suggests such measurements for the determination of the anisotropy of the electronic mean free path in a three-group model. Experimental results in Al will be presented in another paper.

V. CONCLUSION

Low-field magnetoresistance and Hall coefficient are experimentally easily attainable quantities. However, the physical properties of the materials to be studied by these electronic transport coefficients are hidden in integral expressions of uninviting complexity. In particular, little qualitative understanding of the low-field magnetoresistance has been attained up to now. In this paper an attempt has been made, on the one hand, to increase this qualitative understanding, and on the other hand, to stimulate more low-field magnetoresistance experiments by the presentation of simple formulas applicable to nearly-free-electron-like metals.

Based on the series-expansion method by Jones and Zener, it was shown that the higher-order derivatives of ϵ generally appearing in the magnetoresistance equation cancel on Fermi-surface models which can be composed of spherical, cylindrical, and planar areas. Transparent equations in terms of *local* Fermi surface contributions $Q_{ij}^{(n)}(\vec{k})\, l^{n+1}(\vec{k})$ are obtained. The essential results are the following:

(i) The topological coefficients $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$, and hence R_H and particularly $\Delta\rho/\rho_0$, are strongly dependent on the local curvature of the Fermi surface. The value of the low-field magnetoresistance is predominantly determined from Fermi-surface regions of high curvature.

(ii) $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$ are strongly dependent on the crystal orientation with respect to $\vec{j}$ and $\vec{B}$. This can give rise to a sensitive orientation dependence of R_H and $\Delta\rho/\rho_0$ for noncubic symmetry, while for cubic symmetry the orientation dependence of $Q_{xy}^{(1)}$ and $Q_{xx}^{(2)}$ averages out to a large extent.

(iii) R_H depends quadraticly and $\Delta\rho/\rho_0$ cubicly on the mean free path $l(\vec{k})$, and thus they provide a sensitive method for investigations of anisotropic scattering probabilities.

(iv) For the nearly-free-electron model, very simple formulas for R_H and $\Delta\rho/\rho_0$ are obtained, because of the dominating effect of the Fermi-surface edges. The application of a three-group model for $l(\vec{k})$ is suggested as an approach to determine the anisotropy of l in simple polyvalent metals from galvanomagnetic measurements.

Two assumptions have been used throughout this paper: (a) The variation of τ across the Fermi surface is small compared to the average value of τ, and (b) the energy surfaces in the vicinity of the Fermi surface may be approximated by spherical, cylindrical, and planar sections. These are severe restrictions. However, a large computational step is required if these assumptions are not to be applied since this brings us back to the complexity of the general low-field magnetoresistance equation. We therefore hope that a number of semiquantitative investigations may be stimulated on the basis of the formulas presented here.

ACKNOWLEDGMENTS

The author wishes to thank Dr. B. Lengeler and Professor W. Schilling for valuable discussions initiating and supporting this work. A conversation with Professor G. Eilenberger clarifying some theoretical points is gratefully acknowledged.

APPENDIX: LONGITUDINAL MAGNETORESISTANCE

In the limit of low magnetic fields, the longitudinal magnetoresistance ($\vec{j} \parallel \vec{B}$) is

$$\frac{\Delta\rho^{\text{long}}}{\rho_0} = -\frac{1}{\sigma_{zz}^{(0)}}\left(\sigma_{zz}^{(2)} + \frac{(\sigma_{xz}^{(1)})^2}{\sigma_{xx}^{(0)}} + \frac{(\sigma_{yz}^{(1)})^2}{\sigma_{yy}^{(0)}}\right), \qquad \text{(A1)}$$

where the $\sigma_{ij}^{(n)}$ are given by Eq. (4). The calculations of the longitudinal magnetoresistance are analogous to the transverse case. Corresponding to Table I, Table II contains the Q coefficients required for the longitudinal magnetoresistance of Eq. (A1) for planar, spherical, and cylindrical energy surfaces and for isotropic τ. Unlike for the transverse case, all three coefficients $Q_{xz}^{(1)}$, $Q_{yz}^{(1)}$, and $Q_{zz}^{(2)}$ are zero on spherical surface sections, and hence spherical surface sections do not yield a contribution to the longitudinal magnetoresistance.

TABLE II. Q coefficients on planar, spherical, and cylindrical energy-surface areas for the longitudinal magnetoresistance.

	$Q_{xz}^{(1)}$		$Q_{yz}^{(1)}$		$Q_{zz}^{(2)}$	
Surface area	General	Cubic symmetry	General	Cubic symmetry	General	Cubic symmetry
planar	0	0	0	0	0	0
spherical	0	0	0	0	0	0
cylindrical	$\frac{3}{\kappa}(\alpha_x\alpha_y\alpha_{1x} - \alpha_x^2\alpha_{1y})\alpha_{1z}$	0	$\frac{3}{\kappa}(\alpha_y^2\alpha_{1x} - \alpha_x\alpha_y\alpha_{1y})\alpha_{1z}$	0	$\frac{3}{\kappa}(1-\alpha_z\alpha_{1z})\alpha_z\alpha_{1z}$	$\frac{3}{\kappa^2}(1-\alpha_z\alpha_{1z})\alpha_z\alpha_{1z}$

(A special case of this result is that the longitudinal magnetoresistance of the two-band model[22] is zero.) Another remarkable difference is that for cubic symmetry only the first term of Eq. (A1) is nonzero. This simplifies calculations and qualitative discussions of the longitudinal magnetoresistance in cubic metals as compared to the transverse case. Otherwise the qualitative behavior, resulting from the dependence on $\kappa_m(\vec{k})$ and $l(\vec{k})$, can be discussed by a straightforward extension of the properties of the transverse magnetoresistance discussed in Sec. III.

In the nearly-free-electron model, the formula analogous to Eq. (24) is

$$\frac{\Delta\rho^{\text{long}}}{\rho_0} = -\frac{\int_{\text{edge}} Q_{gg}^{(2)}(\vec{k})\, l^3(\vec{k})\, dS}{\int_{\text{FS}} l(\vec{k})\, dS}\left(\frac{e}{\hbar}B\right)^2 . \qquad \text{(A2)}$$

In a three-group model, this reduces in the same way as Eq. (26) to

$$\frac{\Delta\rho^{\text{long}}}{\rho_0} = \frac{g_m^{\text{long}}}{\kappa_{\text{FE}}\kappa_\perp}\frac{l_{--}^3 + l_{++}^3}{2l_-^3}\left(\frac{e}{\hbar}B\right)^2 = g_m^{\text{long}}\frac{\kappa_{\text{FE}}}{\kappa_\perp}\frac{l_{--}^3 + l_{++}^3}{2l_-^3}\left(\frac{B}{ne\rho_0}\right)^2 , \qquad \text{(A3)}$$

where

$$g_m^{\text{long}} = \frac{1}{4\pi\kappa_{\text{FE}}\kappa_\perp}\int_{\text{edge}} 3[\alpha_z(\vec{k})\alpha_{1z}(\vec{k}) - 1] \times \alpha_z(\vec{k})\alpha_{1z}(\vec{k})\, dS .$$

Since the scattering factor $(l_{--}^3 + l_{++}^3)/l_-^3$ is the same in Eqs. (26) and (A3), no additional information on the anisotropy of l can be obtained from the longitudinal magnetoresistance within the three-group approximation for the anisotropy of l.

[1] J. N. Cooper, P. Cotti, and F. B. Rasmussen, Phys. Lett. 19, 560 (1965).
[2] J. S. Dugdale and L. D. Firth, Phys. Kondens. Mater. 9, 54 (1969).
[3] W. van der Mark, H. R. Ott, D. Sargent, and F. B. Rasmussen, Phys. Kondens. Mater. 9, 63 (1969).
[4] G. Bergmann, Z. Phys. (to be published).
[5] The problematics of this assumption have been frequently discussed. However, for a number of scattering processes (e.g., point defects and high-temperature phonons), the anisotropic-relaxation-time concept seems to be a useful approximation.
[6] H. Jones and C. Zener, Proc. R. Soc. A 145, 268 (1934).
[7] W. Shockley, Phys. Rev. 79, 191 (1950).
[8] J. W. McClure, Phys. Rev. 101, 1642 (1956).
[9] R. G. Chambers, Proc. R. Soc. A 238, 344 (1957).
[10] A. B. Pippard, Rep. Prog. Phys. 23, 176 (1960).
[11] R. S. Allgaier, Phys. Rev. 165, 775 (1968).
[12] M. Tsuji, J. Phys. Soc. Jpn. 13, 979 (1958).
[13] K. Böning, Phys. Kondens. Mater. 11, 177 (1970).
[14] L. Davis, Phys. Rev. 56, 93 (1939).
[15] R. Olson and S. Rodriguez, Phys. Rev. 108, 1212 (1957).
[16] J. R. A. Cooper and S. Raimes, Philos. Mag. 4, 145 (1959).
[17] B. Abeles and S. Meiboom, Phys. Rev. 101, 544 (1956).
[18] J. M. Ziman, Adv. Phys. 37, 1 (1961).
[19] H. Miyazawa, in *Proceedings of the Conference on the Physics of Semiconductors, Exeter* (The Institute of Physics and The Physical Society, London, 1962), p. 636.
[20] R. S. Allgaier and R. Perl, Phys. Rev. B 2, 877 (1970).
[21] P. H. Cowley and R. S. Allgaier, Philos. Mag. 29, 111 (1974).
[22] J. M. Ziman, *Principles of the Theory of Solids* (Cambridge U.P., Cambridge, England, 1964), Chap. 7.
[23] H. Jones, Handb. Phys. 19, 227 (1956).
[24] It should be remembered that the consideration of local contributions of separate Fermi-surface regions is made possible by the relaxation-time concept and hence is reasonable only insofar as the relaxation-time approximation is reasonable.
[25] In the case of cubic symmetry, the derivatives of τ cancel in the integral over $Q_{xy}^{(1)}$. Thus the Tsuji formula of R_H, Eq. (9), is valid irrespective of the variation of τ for metals of cubic symmetry.
[26] E. H. Sondheimer and A. H. Wilson, Proc. R. Soc. A 190, 435 (1947).
[27] J. M. McClure, Phys. Rev. 112, 715 (1958).
[28] B. Lax, Rev. Mod. Phys. 30, 122 (1958).
[29] C. Goldberg, E. Adams, and R. Davis, Phys. Rev. 105, 865 (1957).
[30] R. S. Allgaier, Phys. Rev. 158, 699 (1967).
[31] R. S. Allgaier, Phys. Rev. B 2, 3869 (1970).
[32] P. H. Cowley and J. Stringer, Phys. Rev. B 8, 2519 (1973).
[33] P. H. Cowley and J. Stringer, Philos. Mag. 29, 99 (1974).
[34] This is not true for the Hall coefficient, where the decrease in area of the edges (proportional to κ) cancels the increase of $Q_{xy}^{(1)}$ (proportional to $1/\kappa$). A slight decrease of R_H towards a constant value is usually found, as shown by Allgaier when the cylindrical edges are converted into sharp edges.
[35] K. Böning, H. J. Fenzl, E. Olympios, J.-M. Welter, and H. Wenzl, Phys. Status Solidi 34, 395 (1969).
[36] W. Kesternich, Ph.D. thesis (report JÜL-851-FF, Jülich, 1972) (unpublished).
[37] J. de Launey, R. L. Dolecek, and R. T. Webber, J. Phys. Chem. Solids 11, 37 (1959).
[38] W. Huppmann and F. Stangler, Z. Metallkd. 60, 204 (1969).
[39] C. M. Hurd and J. E. A. Alderson, Phys. Rev. B 4, 1088 (1971).
[40] W. Neubert, Z. Naturforsch. A 24, 922 (1969).
[41] A. Sommerfeld and H. Bethe, Handb. Phys. 24, 333 (1933).
[42] F. Seitz, Phys. Rev. 79, 372 (1950).
[43] For one type of edge, the standing-wave part of the wave function has a sine, and for the other, a cosine character.

[44]The Fermi surface of Ca for instance can, according to recent de Haas–van Alphen investigations by A. A. Gaertner [J. Low Temp. Phys. 10, 503 (1973)], be well described as NFE-like. However, the (200)-zone boundary is only touched but not penetrated by the Fermi surface, giving rise to holelike edges without corresponding electronlike edges at the (200) boundary. Two different values of g_m for the electronlike and holelike edges are needed in this example.