Comment on "Hydrodynamics of ^{3}He in anisotropic A phase"

Mario Liu
Physik-Department der Technischen Universität, München, Germany
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It is shown that two elastic constants in the hydrodynamic theory of ^{3}He-A as formulated by Graham are not independent in a bulk system. Also, the symmetry permits additional terms to the fluxes and fourth sound has a second dissipative part with a different angular dependence.

Starting from the thermodynamic identity, Eq. (6) of Ref. 1, one may proceed by expanding $\vec{\lambda}^s$, the new conjugate variable of the less-symmetric phase:

$$\lambda_i^s = \rho_{ij}^s v_j^s + C_{ij} v_j^r + \tilde{\rho}_{ij} v_j^n \ , \tag{1}$$

with all the three tensors again of the form

$$\rho_{ij}^s = \rho_\parallel^s l_i l_j + \rho_\perp^s(\delta_{ij} - l_i l_j)$$

to comply with the axial symmetry. But $C_\parallel$ and $C_\perp$ are not independent in the bulk system. Neglecting the surface energy, i.e., requiring the thermodynamic identity, especially the term $C_{ij}\nabla_i\varphi(\nabla\times\vec{l})_j$, to be invariant under partial integration, we have the additional relation

$$C_\perp = - C_\parallel \equiv C \ . \tag{2}$$

More precisely one may state that a generally uniaxial $C_{ij} = C_\parallel l_i l_j + C_\perp(\delta_{ij} - l_i l_j)$ can be rewritten as a sum of $C_{ij}^B = \frac{1}{2}(C_\perp - C_\parallel)(\delta_{ij} - 2 l_i l_j)$ and $C_{ij}^S = \frac{1}{2}(C_\parallel + C_\perp)\delta_{ij}$. C_{ij}^S only contributes to surface energy, whose neglect is expressed by Eq. (2). Dealing with a bulk system in the present hydrodynamic theory, it is a convenient way to eliminate surface effects. The elastic coefficients K_1 and K_2 have been treated correctly along this line in Ref. 1. C_{ij} has been calculated microscopically to be proportional to $\delta_{ij} - 2 l_i l_j$ near T_c and at zero temperature.[2] This seems to indicate a generally vanishing surface contribution to the energy at these temperature ranges. $\tilde{\rho}_{ij} = -\rho_{ij}^s$ can now be deduced by letting Graham's first relation of Eq. (7) to undergo a uniform Galilean transformation.

The following terms are permitted by the symmetry and may be added to the reactive part of the stress tensor and the superflux, i.e., to σ_{ij}^R and $\mathcal{J}_\varphi^R$, respectively,

$$\Delta\sigma_{ij}^R = -\lambda_{ijkl}\nabla_k\lambda_l^s - \varphi_{ijkl}\nabla_k v_l^n \ , \tag{3}$$

with

$$\lambda_{ijkl} = \lambda l_p[\epsilon_{pik}\delta_{jl} + (i,j \to j,i)] \ ,$$

$$\varphi_{ijkl} = \{\varphi_1[(\delta_{ik} - l_i l_k) l_p \epsilon_{pjl} + (\delta_{jk} - l_j l_k) l_p \epsilon_{pil} + (\delta_{il} - l_i l_l) l_p \epsilon_{pjk} + (\delta_{jl} - l_j l_l) l_p \epsilon_{pik}] + \varphi_2(l_i l_k l_p \epsilon_{pjl} + l_j l_k l_p \epsilon_{pil} + l_i l_l l_p \epsilon_{pjk} + l_j l_l l_p \epsilon_{pik})\} \ ,$$

and

$$\Delta\mathcal{J}_\varphi^R = \lambda l_p \epsilon_{pij}\nabla_i v_j^n \ . \tag{4}$$

Since any σ_{ij} leading to the same $\nabla_j\,\sigma_{ij}$ is correct, one can show the equivalence of the antisymmetric $\Delta\sigma_{ij} = -\lambda l_p \epsilon_{pij}\nabla_k\lambda_k^s$ and the symmetric $\Delta\sigma_{ij} = -\lambda[l_p\epsilon_{pik}\nabla_k\lambda_j^s + (i,j \to j,i)]$. Because of this relationship, it is erroneous to require a vanishing λ, on the grounds that, for a system of conserved orbital angular momentum, σ_{ij} is always symmetric. In fact $\lambda = \frac{1}{2}$. This is obvious from the definition[3] $\vec{v}^s = -\nabla\Omega_3$, Ω_3 being the rotation angle around the preferred z axis and for a rigid rotation $\dot{\Omega}_3 = \frac{1}{2}(\nabla\times\vec{v}^n)_3$. The invariance under reflection at a plane which contains the preferred axis is a symmetry element of the nematic crystals[4] and not of the axial phase of ^{3}He. Therefore φ_1 and φ_2 are vanishing coefficients in the hydrodynamics of liquid crystals.[5]

With Eqs. (7)–(11) of Ref. 1, fourth sound has been recalculated and found to reflect the dissipative coupling to the orbital variables with an additional damping term

$$\omega_4^2 = \omega_0\{\omega_0 - ik^2[\zeta\rho^s + \eta(2C\sin\Theta\cos\Theta)^2/\rho^s]\} \ , \tag{5}$$

with $\rho^s = \rho_\parallel^s\cos^2\Theta + \rho_\perp^s\sin^2\Theta$ and Θ as the angle between $\vec{k}$ and $\vec{l}$.

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