Quasi-one-dimensional fluctuation contribution to ultrasonic attenuation in clean type-II superconductors above $T_c(B)$†

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We have calculated the effect of fluctuations on the ultrasonic attenuation in a clean bulk type-II superconductor at temperatures above the transition temperature and in magnetic fields near the zero-temperature upper critical field, where the fluctuations are effectively one dimensional. The result is a decrease in the attenuation which should be observable experimentally. The effect increases with increasing mean free path.

The phase transition between the normal and superconducting states produces sharp experimental features in both thermodynamic and transport properties which agree quite well with mean-field-theory predictions.[1] However, for temperatures T near the transition temperature T_c, fluctuations of the nonequilibrium phase produce both a reduction in T_c and a broadening of the transition.[2,3] The most spectacular effect of fluctuations is seen in the enhancement of the diamagnetism above the superconducting transition, a precursor of the perfect diamagnetism of the superconducting state. Due to a large mean-field-theory discontinuity and the accuracy with which susceptibility measurements can be taken, this effect can be detected over a wide range of temperatures and magnetic fields.[4] A great deal of attention has also been paid to the fluctuation conductivity in zero magnetic field, which is extremely small and can be detected only in dirty samples of reduced dimensionality.[5,6] The effect on other zero-field transport properties is even less significant and is thought to be inaccessible to present experimental techniques.

However, it has been pointed out by several authors that close to the upper critical field of a type-II superconductor fluctuations in the number of electron pairs are essentially one dimensional in nature.[7] In particular, Lee and Shenoy[8] have shown that they give rise to a specific heat well above $T_c(B)$, the transition temperature in the presence of a magnetic field, proportional to $[T - T_c(B)]^{-3/2}$, characteristic of one-dimensional behavior. The physical picture is of fluctuating electron pairs moving in Landau orbitals characterized by k_z and n. The transition temperature $T_c(B)$ is the temperature at which the $n=0$ Landau orbital becomes stable, giving rise to the vortex state. Just above $T_c(B)$ the lowest, $n=0$ orbital dominates the fluctuation contributions and only one degree of freedom, k_z, remains. A bulk superconductor behaves like an array of one-dimensional rods parallel to the field with the number of rods per unit area given by $eB/\pi c$, the Landau degeneracy factor for particles of charge $2e$; c is the speed of light. More recently, the fluctuation specific heat has been calculated below the transition temperature by Thouless,[9] who also provided a scaling formula giving the dependence of the specific heat through the critical region. The theoretical results agree well with the experimental data of Farrant and Gough,[10] who measured the specific heat of a bulk clean ($l \gg \xi_0$; l is the electronic mean free path and ξ_0 is the zero-temperature coherence length) type-II superconductor in a magnetic field through the critical region. This important result is the only measurement of a significant fluctuation effect, other than diamagnetism, in a clean three-dimensional superconductor.

In this paper, motivated by this work and preliminary results of Farrant and Gough,[11] we examine theoretically the fluctuation contribution to the attenuation of longitudinal sound in a clean type-II superconductor in a magnetic field near its upper critical field and at a temperature $T > T_c(B)$. As we have already discussed, in such fields the fluctuations can be assumed to be one dimensional. We will assume that interactions between fluctuations can be neglected and furthermore that the fluctuation propagator has Ginzburg-Landau[12] form; that is, we assume that long-wavelength fluctuations provide the dominant contribution to physical quantities near the transitions. This is not true in general, as can be seen from the detailed calculations of the magnetization,[13,14] where for high fields, $H \gg H_{c_2}(0)$, the bulk critical field at $T=0$, or for high temperatures, $T \gg T_c(0)$, high-energy fluctuations are important, and a more complete form of the free-energy functional is required, which turns out to be nonlocal in character. It is also assumed that only lowest-order fluctuation contributions to the attenuation need be considered. By lowest order we mean only those terms in the density-density correlation function which originate, in a Φ-derivable approximation,[15] from the lowest-order fluctuation contribution to the free-energy functional (see Fig. 1).

As a starting point, we note that if $\omega\tau \ll 1$ and $ql \gg 1$ the attenuation coefficient of longitudinal sound is simply related to the density-density correlation function; ω and q are the frequency and wave vector of the sound wave, respectively. The attenuation rate is given by[16]

$$\alpha = \mathrm{Im}(q^2/\omega\rho_{\mathrm{ion}}v_s)(p_F^2/3m)^2 \mathfrak{D}(\vec{q}, \omega). \quad (1)$$

Here p_F is the Fermi momentum and v_s is the velocity of sound. The function $\mathfrak{D}(\vec{q}, \omega) = \langle [n, n] \rangle$ is the Fourier transform of the retarded density-density correlation function and is obtained by analytic continuation of the thermal product $\mathfrak{D}(\vec{q}, \omega_0)$ from the set of points $i\omega_0 = 2n\pi T$ to $z = \omega + i\delta$; n is an integer. The diagrams contributing to the lowest-order correction to the attenuation are found by attaching in all possible ways two density vertices to the electron lines in the two-particle vacuum polarization diagram, Fig. 1. The components of these diagrams are a free-electron Green's function,

$$G_{\omega_n}(\vec{p}) = (i\tilde{\omega}_n - \xi_p)^{-1}, \quad (2)$$

where $\tilde{\omega}_n = \omega_n + (1/2\tau)(|\omega_n|/\omega_n)$, $\omega_n = (2n+1)\pi T$, $\xi_p = p^2/2m - \mu$, μ is the chemical potential, and the fluctuation propagator

$$t_{\omega_k}(\vec{k}) = [N(0)(\eta + \pi|\omega_k|/8T + v_F^2\sigma k_z^2/6\pi^2T^2)]^{-1}, \quad (3)$$

where $\eta = [T - T_c(B)]/T_c(B)$ is the reduced temperature, $\omega_k = 2\pi nT$, and $\sigma = \frac{7}{8}\zeta(3)$, where ζ is the Riemann ζ function. Since the magnetic field restricts variation of k to one dimension, it is assumed that $\eta < 2H/H_{c2}(0)$,[8] and the phase-space element is $dk_z/2\pi A$, where the area $A = (eB/\hbar c)^{-1}$ is the inverse of the Landau degeneracy factor. In the clean limit the electron-impurity scattering time τ appears only in the propagator $G_{\omega_n}(\vec{p})$. The impurity renormalization of the electron-fluctuation propagator vertex is given in general by

$$\Gamma(\omega_n, k) = \{1 - [\tan^{-1}(kl/2\tau|\tilde{\omega}_n|)]/kl\}^{-1}, \quad (4)$$

where ω_n is the frequency associated with a single electron entering the propagator and where we have neglected the two-particle frequency. In the calculation of the density-density correlation function the discrete frequencies $i\omega_n$ are analytically continued to real frequencies ω, which are then integrated over, weighted by the convergence factor $\cosh^{-2}(\omega/2T)$. In the clean limit, $T_c(0)\tau \gg 1$, and for $T \approx T_c(0)$ the dominant contribution to the frequency integral is from $\omega \lesssim 1/\tau$; hence the vertex function $\Gamma_\omega(k)$ is of order 1, provided $kl \gg 1$, that is, $[B/H_{c_2}(0)](l/\xi_0)^2 \gg 1$, a condition which is easily met in the clean limit. On the other hand, in the dirty limit the vertex function takes on the well-known form

$$\Gamma_{\omega_n}(k) \simeq (2\omega_n\tau + \tfrac{1}{3}k^2l^2)^{-1}. \quad (5)$$

After this preamble we proceed to the calculation of the diagrams of Fig. 1 under the conditions outlined above. In the zero-field case diagrams (a) and (b) were first studied by Aslamazov and Larkin,[17] who found a sharp peak in the attenuation which diverged as $[T - T_c(0)]^{-3/2}$ in three dimensions and $[T - T_c(B)]^{-5/2}$ in one dimension. However, the numerical prefactor of the fluctuation component was so small, $\alpha_{AL}/\alpha_n \sim (T_c/E_F)^4$, in the clean limit that experimental observation of this effect is impossible; α_n is the attenuation in the normal state. In fields near $H_{c_2}(0)$ we have verified that, as expected, the attenuation due to these diagrams diverges as $[T - T_c(B)]^{-5/2}$, characteristic of one dimension, although again the coefficient is extremely small.

Diagram (e) is the so-called anomalous Maki diagram[18] which, as has been much discussed in conductivity calculations,[19] diverges independent of the value of T in one or two dimensions in the dirty limit with no pair breaking. For example, in the dirty limit at zero field Robinson *et al.*[20] find that the contribution of this diagram to the fluctuation attenuation in one dimension is

$$\frac{\alpha_e}{\alpha_n} = \frac{-\pi}{4N(0)} \sum_k \frac{1}{(Dk^2 + \delta)[(\pi/8T_c)Dk^2 + \eta]} = \frac{-\pi}{8N(0)SD^{1/2}(\delta\eta)^{1/2}[\eta^{1/2} + (\pi\delta/8T_c)^{1/2}]} \cdot \quad (6)$$

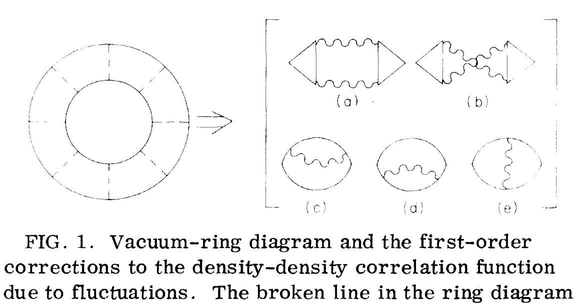

FIG. 1. Vacuum-ring diagram and the first-order corrections to the density-density correlation function due to fluctuations. The broken line in the ring diagram represents a pair interaction, and the solid and wavy lines represent the electron and fluctuation propagators, respectively. At the left- and right-hand vertices in $\langle [n,n] \rangle$ we put density operators.

Here S is the cross-sectional area of the whisker, $D=\frac{1}{3}v_F l$ is the diffusion constant, and δ is the pair pair-breaking energy first introduced by Thompson.[19] The divergence mentioned above has its origin, in the limit $\delta \to 0$, in the $1/k^2$ factor appearing in Eq. (6), which in turn is a result of the k^2 dependence of $\Gamma_{\omega_n}(k)$ in the dirty limit. Notice also that Eq. (6) predicts a decrease in the total attenuation, as would be expected in a broadened transition.

However, in the clean limit (that is, under the condition $l/\xi_0 \gg 1$) diagram (e) leads to an attenuation contribution of the form

$$\frac{\alpha_e}{\alpha_n}=\frac{-\pi\tau}{4N(0)}\frac{eB}{\hbar c}\sum_k \frac{1+O((ql)^{-1})}{\eta+v_F^2\sigma k^2/6\pi^2T^2}\,. \tag{7}$$

Neglecting the correction term of order $(ql)^{-1}$ and performing the one-dimensional sum over k, this becomes

$$\frac{\alpha_e}{\alpha_n}\simeq -\pi^3\left(\frac{6}{\sigma}\right)^{1/2}\frac{B}{H_{c_2}(0)}\frac{l/\xi_0}{14\eta^{1/2}k_F^2\xi_0^2}\,. \tag{8}$$

The remaining two diagrams, (c) and (d), which are simple modifications of the normal attenuation due to emission and absorption of fluctuating pairs, give a contribution to the fluctuation attenuation which is exactly equal in sign and magnitude to the correction term in Eq. (7) of order $(ql)^{-1}$; therefore they are negligible.

Thus the net result is that the effect of fluctuations on the attenuation is dominated by the anomalous Maki diagram and is given by Eq. (8), to lowest order in $(ql)^{-1}$. Again this leads to a decrease in the attenuation, effectively smearing out the transition. The apparent divergence is $\sim\eta^{-1/2}$, which is, of course, cut off in the critical region.

For typical values of the parameters of interest $B/H_{c_2}(0)\simeq 10^{-1}$, $T_c(B)/T_c(0)\approx 1$, $1/k_F\xi_0 \sim 2\times 10^{-3}$, and $l/\xi_0\sim 5\times 10^2$, we find $\alpha_e/\alpha_n \approx -10^{-2}$ at $\eta\approx 10^{-2}$, indicating a 1% effect at temperatures $T/T_c(B) - 1 \approx 10^{-2}$. This result emphasizes clearly the increase in the effect of fluctuations as the sample becomes purer,[21] contrary to the general lore that adding inpurities increases the fluctuation effect. Some insight into the reason why α_e/α_n is proportional to l can be gained by noting that below the superconducting transition, the change in the attenuation is also proportional to l.[16]

Although we feel that in the temperature and field range considered here short-wavelength fluctuations should not be important, a careful investigation of this effect will be carried out. Of further interest is the extension of the theory to the region $ql<1$.

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