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Nonreciprocal attenuation of magnetoelastic surface waves

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The attenuation of Rayleigh-like surface waves on an insulating magnetic medium is calculated, particularly for waves propagating parallel with the magnetic field along a [111] direction in materials such as YIG; this attenuation results from the radiation of spin waves into the bulk rather than from magnetic resonance. Propagation is generally nonreciprocal (i.e., the attenuations of forward- and of backward-traveling waves differ) and the ratio of forward to backward attenuations can be large. The calculation makes allowance for a small elastic anisotropy and for small deviations from colinearity between the wave vector and the magnetic field.

I. INTRODUCTION

This paper presents a theoretical investigation of the attenuation of Rayleigh surface waves on insulating magnetic materials of cubic structure, such as yttrium iron garnet (YIG). It has long been known[1] that the propagation of surface magnetostatic waves on a magnetic medium can be nonreciprocal, i.e., that the properties of forward- and backward-traveling waves can differ. Similarly, magnetoelastic waves need not be reciprocal if the magnetization is at an angle to the direction of propagation of the waves, or if the magnetization and the propagation direction are parallel and are not normal to a mirror plane of the system. Nonreciprocal attenuation has been reported by Lewis and Patterson[2] for the case of waves traveling at right angles to the field on a thin film of YIG.

In the following inquiry it is taken that the waves go parallel with the field along a $\langle 111\rangle$ axis in the crystal; the theoretical results can be used also for a $\langle 100\rangle$ direction, and in this case the calculated attenuation is similar to that computed by Parekh and Bertoni[3,4] for Ga-doped YIG. The $\langle 111\rangle$ direction is, however, more interesting than a $\langle 100\rangle$ direction since the (111) plane is not a mirror plane in cubic crystals, and nonreciprocal propagation occurs.

It will be found that the attenuation of Rayleigh waves is very different from that of bulk waves. In the latter case it is possible to find spin waves and bulk waves that have the same frequency and spatial dependence; at resonance the two contribute equally to the total motion, so the strong damping of spin waves results in a strong attenuation of the bulk wave. For Rayleigh waves, by contrast, the boundary conditions prohibit the buildup of any large spin wave, and resonance damping is small. However, for frequencies above the resonant frequency, $\omega=\gamma H$, spin waves can travel away from the surface, and the Rayleigh wave is attenuated through radiation of spin waves into the magnetic medium.

II. THEORY

A. Preamble

We start by showing that the attenuation comes primarily from the radiation of spin waves into the medium rather than from magnetic resonance. A surface wave on a magnetic material must in general be constructed from four modes, three predominantly elastic modes and one predominantly magnetic mode, or spin wave. If, say, one satisfied the three elastic boundary conditions with the elastic modes alone, then the magnetoelastic coupling would result in a field and flux within the medium; this forced magnetization need not fit the magnetic boundary conditions at the surface, so a spin wave must be added to take care of the extra boundary condition.

The frequency ω of a spin wave traveling at an angle θ to the direction of the magnetization M is given by[5]

$$\omega^2=\gamma^2H(H+4\pi M\sin^2\theta), \tag{1}$$

wherein γ is the gyromagnetic ratio and H is the total internal field, being the sum of external, demagnetizing, and anisotropy fields

$$H=H_{\text{ext}}+H_{\text{anis}}-H_{\text{demag}}. \tag{2}$$

In a cubic crystal $H_{\text{anis}}=2K_1/M$ for fields along the $\langle 100\rangle$ axis, or $H_{\text{anis}}=-4K_1/3M$ for fields along the $\langle 111\rangle$ axis, K_1 being the anisotropy constant.

When ω and H are such that θ is real, spin waves can propagate away from the surface; these waves are the only magnetic waves admissible in constructing the surface waves, and they extract energy from the neighborhood of the surface. The surface wave is therefore leaky in the range

$$\gamma^2H^2<\omega^2<\gamma^2H(H+4\pi M). \tag{3}$$

Exchange effects must be taken into account if higher frequencies (or lower fields) are to be considered; i.e., the spin wave now travels almost vertically into the material and its wavelength becomes so small that exchange effects serve to increase its frequency. It will be found that the attenuation is low when the angle of the spin wave is steep, so these effects will be ignored.

At low frequencies, $\omega<\gamma H$, the spin wave has the spatial form

$$m \sim \exp(ik_3 z - q_m x)\,, \tag{4}$$

propagation being along the z axis with the x axis the inward normal to the surface. Such a wave may be regarded as traveling at an imaginary angle $i\psi_m$ to the surface, and its frequency is given by

$$\omega^2 = \gamma^2 H(H - 4\pi M \sinh^2\psi_m), \quad \tanh\psi_m = q_m/k_3\,. \tag{5}$$

The spin wave is now localized on the surface, as are the elastic waves, and no loss can occur. We expect losses through radiation of spin waves only in the range of frequencies given in Eq. (3).

It will be seen a magnetic resonance appears to occur in the low-frequency range, $\omega<\gamma H$, when the elastic wave can have an envelope similar to that of the magnetic wave. The elastic wave is a superposition of terms localized near the surface,

$$\vec{R} = \sum_\alpha \vec{U}^\alpha \exp(ik_3 z - q_\alpha x)\,, \tag{6}$$

where $\vec{R}$ is the displacement and α denotes the elastic mode; α stands for longitudinal or transverse. The forced magnetization has the same form as $\vec{R}$ so the mode α is associated with a magnetization whose natural frequency is

$$\omega_\alpha^2 = \gamma^2 H(H - 4\pi M \sinh^2\psi_\alpha), \quad \tanh\psi_\alpha = q_\alpha/k_3\,. \tag{7}$$

Resonance occurs when the forced frequency of the driven wave approaches its natural frequency, $\omega\to\omega_\alpha$; from Eqs. (5) and (7) one then has $q_\alpha\to q_m$. When this occurs, the magnetization associated with mode α diverges as $(\omega^2-\omega_\alpha^2)^{-1}$. However, neither the forced magnetization nor the spin wave can by itself satisfy the magnetic boundary conditions, and since they have now the same form, $q_\alpha=q_m$, no combination of them can fit the boundary conditions. The divergence in the first term must therefore be cancelled by an opposite divergence in the second term, so that the total magnetization remains small; this point is demonstrated at the end of the next section. It follows that no magnetic resonance occurs at any frequency. Therefore, the magnetoelastic interaction, which is weak, can always be treated as a perturbation and the theory can be developed to successive orders in this interaction.

The magnetoelastic energy in a cubic crystal is[6]

$$E_{\rm int} = \frac{b_1}{M^2}\,(M_{x_c}^2 S_{x_c x_c} + M_{y_c}^2 S_{y_c y_c} + M_{z_c}^2 S_{z_c z_c})$$
$$+ 2\,\frac{b_2}{M^2}\,(M_{x_c} M_{y_c} S_{x_c y_c} + M_{y_c} M_{z_c} S_{y_c z_c} + M_{z_c} M_{x_c} S_{z_c x_c})\,, \tag{8}$$

where x_c, y_c, and z_c are the crystal coordinates, b_1 and b_2 are the magnetoelastic coupling constants, M_{x_c}, etc., are the components of the magnetization $\vec{M}$, and $S_{x_c x_c}$, etc., are the components of the strains. The strains are defined through the displacement $\vec{R}$ by

$$S_{xy} = \frac{1}{2}\left(\frac{\partial R_x}{\partial y} + \frac{\partial R_y}{\partial x}\right), \text{ etc.}$$

We shall consider surface waves traveling along a $\langle 111\rangle$ direction on a $\langle 110\rangle$ face, and must therefore transform the above expression to new coordinates sketched in Fig. 1 with x, y, and z defined as follows: z is parallel with $[111]$; x, the inward normal to the surface, is parallel with $[1\bar{1}0]$, and y is parallel with $[11\bar{2}]$. In transforming Eq. (8) we take it that the magnetization is predominantly along the new z axis, $M_z^2 = M^2 - M_x^2 - M_y^2 \simeq M^2$. To first order in M_x and M_y one finds

$$E_{\rm int} = 2\,\frac{b'}{M}\,(M_x S_{xz} + M_y S_{yz})$$
$$+ \frac{b''}{M}[M_y(S_{xx} - S_{yy}) + 2M_x S_{xy}]\,, \tag{9}$$

where

$$b' = \tfrac{1}{3}(2b_1 + b_2), \quad b'' = \tfrac{1}{3}\sqrt{2}\,(b_1 - b_2)\,.$$

In crystal coordinates an equivalent result holds with $b' = b_2$ and $b'' = 0$.

Finally, we remark on the magnetic boundary conditions, which will be introduced through a surface permeability μ_s; the method is analogous to Ingebrigtsen's[7] introduction of an effective dielectric constant in his discussion of the propagation of surface waves on piezoelectric media. The external field, $x<0$, may be represented as the gradient of a potential of the type

$$\phi \sim \exp(ik_3 z - i\omega t + |k_3| x)\,.$$

If the external medium has permeability μ_e, then just above the surface at $x=-0$ one has

$$\mu_e = i\,\frac{k_3}{|k_3|}\,\frac{B_x(-0)}{H_z(-0)}\,.$$

Within the medium let $\vec{B}$ and $\vec{H}$ be found for some values of ω and k_3; a surface dielectric constant μ_s can then be defined through

$$\mu_s = -i\,\frac{k_3}{|k_3|}\,\frac{B_x(+0)}{H_z(+0)}\,. \tag{10}$$

The magnetic boundary conditions are $H_z(+0) = H_z(-0)$, and $B_x(+0) = B_x(-0)$. These conditions are satisfied if

$$\mu_s(\omega, k) = -\mu_e\,. \tag{11}$$

B. Surface wave propagation

The calculation is carried out for a nonconducting magnetic medium in which exchange effects can be ignored. The material is taken to be elastically isotropic for the first part of the calculation, and elastic anisotropy is inserted as a correction at the end. This procedure is justified in YIG by the small elastic anisotropy[8] $c_{11} - c_{12} - 2c_{44} = 26.9 - 10.77 - 15.28 = 0.85\times 10^{11}$ dyn/cm^2.

The equations of motion are developed in the manner of Kittel[6] and Schlöman.[5] Let the magnetization $\vec{M}(\vec{r})$ and field $\vec{H}(\vec{r})$ be

$$\vec{M}(\vec{r}) = \vec{M} + \vec{m}e^{i\vec{k}\cdot\vec{r}}, \quad \vec{H}(\vec{r}) = \vec{H} + \vec{h}e^{i\vec{k}\cdot\vec{r}},$$

where the unperturbed fields $\vec{H}$ and $\vec{M}$ are along the z axis. The small field $\vec{h}$ is determined from the conditions $\mathrm{curl}\vec{H}(\vec{r}) = \mathrm{div}[\vec{H}(\vec{r}) + 4\pi\vec{M}(\vec{r})] = 0$, and is

$$\vec{h} = -4\pi\vec{k}(\vec{k}\cdot\vec{m})/k^2\,. \tag{12}$$

If all quantities are independent of y and the displacement at $\vec{r}$ is $\vec{R}(\vec{r})$, then from the magnetoelastic interaction of Eq. (9) the equations of motion are

$$\begin{aligned}
\dot{m}_x &= -\gamma\left(Hm_y - Mh_y + b'\,\frac{\partial R_y}{\partial z} + b''\,\frac{\partial R_x}{\partial x}\right),\\
\dot{m}_y &= \gamma\left[Hm_x - Mh_x + b'\left(\frac{\partial R_z}{\partial x} + \frac{\partial R_x}{\partial z}\right) + b''\,\frac{\partial R_y}{\partial x}\right],\\
\ddot{R}_x &= s_t^2\nabla^2 R_x + (s_l^2 - s_t^2)\,\frac{\partial}{\partial x}\,(\vec{\nabla}\cdot\vec{R})\\
&\quad + \frac{b'}{\rho M}\,\frac{\partial m_x}{\partial z} + \frac{b''}{\rho M}\,\frac{\partial m_y}{\partial x},\\
\ddot{R}_z &= s_t^2\nabla^2 R_z + (s_l^2 - s_t^2)\,\frac{\partial}{\partial z}\,(\vec{\nabla}\cdot\vec{R}) + \frac{b'}{\rho M}\,\frac{\partial m_x}{\partial x}\,.
\end{aligned} \tag{13}$$

Here s_t and s_l are the velocities of transverse and of longitudinal modes, respectively, and ρ is the density of the material. The equation of R_y has been omitted, since R_y can be dropped when the material is isotropic, provided that the calculation is taken only to first order in b^2.

To construct a surface wave we seek those solutions to Eqs. (13) that are localized near the surface, i.e., that are of the form

$$(\vec{R}, \vec{m}) = (\vec{R}^\alpha, \vec{m}^\alpha)\exp(ik_3 z - q_\alpha x - i\omega t)\,. \tag{14}$$

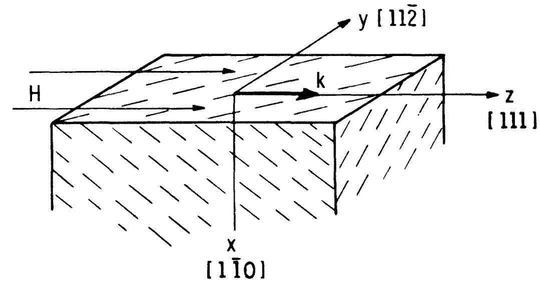

FIG. 1. Coordinates used in the calculation.

When R_y is zero, there are three solutions to Eqs. (13) giving the three modes α for a given ω and k_3. Two modes are predominantly elastic, $\alpha = l$ or t (longitudinal or transverse), and one is predominantly magnetic, $\alpha = m$. When $\omega < \gamma H$ the magnetic mode is localized near the surface, Eq. (5), and q_m is real; when $\omega > \gamma H$ the spin wave travels away from the surface, Eq. (3), and q_m is imaginary (small corrections to the spin-wave spectrum through the magnetoelastic coupling, which were taken into account by Parekh and Bertoni,[3,4] are ignored here).

A surface wave is a linear superposition of the three modes of Eq. (14) such that the mechanical and magnetic boundary conditions are satisfied. The mechanical boundary conditions are

$$T_{11} = T_{13} = 0\,. \tag{15}$$

The magnetic boundary condition is used to define a surface permeability μ_s, Eq. (10),

$$B_x - i\mu_s H_z = 0\,. \tag{16}$$

In terms of the displacements and magnetizations, the quantities that enter the boundary conditions are

$$\begin{aligned}
T_{11} &= \rho\left(s_l^2\,\frac{\partial R_x}{\partial x} + (s_l^2 - 2s_t^2)\,\frac{\partial R_z}{\partial z}\right) + \frac{b''}{M}\,m_y\,,\\
T_{13} &= \rho s_t^2\left(\frac{\partial R_x}{\partial z} + \frac{\partial R_z}{\partial x}\right) + \frac{b'}{M}\,m_x\,,\\
B_x &= 4\pi m_1 - 4\pi k_1(\vec{k}\cdot\vec{m})/k^2\,,\\
H_z &= -4\pi k_3(\vec{k}\cdot\vec{m})/k^2\,,
\end{aligned} \tag{17}$$

where $k_1 = iq_\alpha$.

Choose, say, as variables the components R_z^l, R_z^t of the elastic waves, and m_x^m of the magnetic wave, and find the stresses and fields associated with each mode. The first boundary condition has the form

$$T_{11}^l R_z^l + T_{11}^t R_z^t + T_{11}^m m_x^m = 0\,,$$

and so on for the others. The three boundary conditions give

$$\begin{vmatrix} T_{11}^l & T_{11}^t & T_{11}^m \\ T_{13}^l & T_{13}^t & T_{13}^m \\ B^l - i\mu_s H^l & B^t - i\mu_s H^t & B^m - i\mu_s H^m \end{vmatrix} = 0 . \quad (18)$$

The solution to this is

$$\mu_s = -i\Delta_B/\Delta_H , \quad (19)$$

where Δ_B and Δ_H are determinants that contain the B^α and the H^α alone.

When the magnetostrictive coupling is zero, T_{11}^m, T_{13}^m, B^l, B^t, H^l, and H^t are all zero; Eq. (18) is then just the Rayleigh determinant. When there is a finite magnetostriction the determinants Δ_B and Δ_H have the form

$$\Delta_B = B^m \Delta_R + \delta_B, \quad \Delta_H = H^m \Delta_R + \delta_H , \quad (20)$$

where δ_B and δ_H are of second order in the magnetoelastic coupling, and Δ_R is the Rayleigh determinant. The boundary condition at a free surface, $\mu_s = -1$, is then satisfied by

$$\Delta_R = -(\delta_B + i\delta_H)/(B^m + iH^m).$$

On expanding about the Rayleigh velocity v_R, defined by $\Delta_R(v_R) = 0$, one obtains

$$v - v_R = -(\delta_B + i\delta_H)/(B^m + iH^m)\,\frac{\partial \Delta_R}{\partial v} . \quad (21)$$

This expression is intrinsically of second order in the magnetoelastic coupling, since B^m, H^m, and Δ_R are of zero order. It follows that the magnetoelasticity can be dropped in finding the fundamental modes l, t, and m from Eqs. (13). For example, in the elastic modes one chooses some ω, k_3, and sets $b' = b'' = 0$ in the last two members of Eqs. (13); the displacements $\vec{R}^\alpha$ and the wave number q_α are then found from these two equations alone, $\alpha = l$ or t. The displacement so found is then treated as a source term in the first two members of Eqs. (13), which yield the magnetization associated with the elastic waves.

Ignoring R_y and approaching the magnetic resonance will be discussed briefly before the attenuation is derived. It was shown by Parekh and Bertoni[4] that R_y, though small at the surface, need not be very small. The other transverse mode t' involves R_y alone; when it is included, the extra boundary condition $T_{12} = 0$ must also be included. Equation (18) then becomes a fourth-order determinant; the new diagonal term is of order unity, while T_{12}, T_{12}^t, $T_{11}^{t'}$, and $T_{13}^{t'}$ are of order b^2. The correction due to R_y is therefore of order b^4, and has little effect.

The approach to resonance occurs at frequencies $\omega < \gamma H$; consider, for example, the neighborhood of the frequency ω_t defined in Eq. (7). It is readily found, from the above prescription, that the magnetization m_x^t associated with the transverse mode diverges as

$$m_x^t \sim (\omega^2 - \omega_t^2)^{-1} \sim (\sinh^2\psi_t - \sinh^2\psi_m)^{-1} .$$

The magnetization associated with the spin wave can be found from the boundary condition preceding Eq. (21); it contains a variety of finite terms and a divergent term proportional to m_x^t. The ratio of the two divergent terms is $-\tanh\psi_t/\tanh\psi_m$; the total divergence in the magnetization is

$$m_x = m_x^t + m_x^m \sim \frac{\tanh\psi_t - \tanh\psi_m}{\sinh^2\psi_t - \sinh^2\psi_m} ,$$

which remains finite as $\psi_m \to \psi_t$. Magnetic resonance therefore does not occur in surface waves.

C. Attenuation

We have used Eq. (21) to find the attenuation of Rayleigh waves; before giving the result some quantities must be defined. The inward decay of the elastic modes is written as a fraction β_α of the wave number, $q_\alpha = \beta_\alpha k_3$; then

$$\beta_t^2 = 1 - v^2/s_t^2, \quad \beta_l^2 = 1 - v^2/s_l^2 . \quad (22)$$

A dimensionless form for the Rayleigh determinant is

$$D_R = 1 - (2 - v^2/s_t^2)^2/4\beta_l\beta_t . \quad (23)$$

The attenuation is then given by

$$\frac{1}{v}\,\mathrm{Im}\,v = A\,\frac{b'^2}{4\pi\rho s_t^2 M^2}\,\sin\theta\cos\theta\,\frac{[B - C\cos^2\theta + (\omega/\gamma H)(b''/b')(D - E\cos^2\theta)]^2}{[1-(v^2/s_t^2)\cos^2\theta]^2[1-(v^2/s_l^2)\cos^2\theta]^2} , \quad (24)$$

wherein θ is the angle at which spin waves go into the bulk, Eq. (1), and A, B, C, D and E are numbers of order unity. These numbers are

$$A = 4\,\frac{s_t^2}{v^2}\Big/\beta_l v\,\frac{\partial D_R}{\partial v}, \quad B = \beta_l(\beta_l - \beta_t),$$

$$C = \beta_l(\beta_l - \beta_t)(1+\beta_l)(1+\beta_t),$$

$$D = \tfrac{1}{2}(\beta_l - \beta_t) + \frac{1}{4}\,\frac{v^2}{s_l^2}\,(1+\beta_t) - \frac{1}{2}\,\frac{v^2}{s_l^2}\,(1+\beta_l), \quad (25)$$

$$E = \tfrac{1}{2}(1+\beta_l)(1+\beta_t)\left(\beta_l - \beta_t + \frac{1}{2}\,\frac{v^2}{s_l^2}\,(1-\beta_l) - \frac{v^2}{s_l^2}\,(1-\beta_t)\right),$$

wherein v is the Rayleigh velocity throughout. In YIG, where[8] $s_l^2/s_t^2 = 3.5$, one has $v^2/s_t^2 = 0.86$, and hence: $A = 1.55$, $B = 0.431$, $C = 1.106$, $D = 0.315$, and $E = 0.512$.

At small angles θ the attenuation is governed by the term $\sin\theta/[1-(v^2/s_t^2)\cos^2\theta]^2$ in Eq. (24). The ratio v^2/s_t^2 is not much less than unity, about 0.86 in YIG, so this factor reaches a maximum when θ is small; the maximum of the attenuation occurs when $\sin\theta \simeq 0.2$. From Eq. (1) it follows that the peak attenuation occurs when ω is slightly greater than γH.

Nonreciprocal propagation comes from the term proportional to $\omega/\gamma H$ in square brackets in the numerator of Eq. (24). If either the field or the sound direction is reversed (ω or H negative), the term in question reverses sign; if both are reversed, the term retains its sign. Near the peak attenuation, $\cos^2\theta \simeq 1$, this part of the numerator is

$$[(B-C) + (\omega/\gamma H)(b''/b')(D-E)]^2 .$$

The numbers $(B-C)$ and $(D-E)$ are both negative; if the ratio b''/b' is positive the attenuation is greatest when the field and the sound have the same direction. This conclusion holds for sound propagating along the [111] axis when the $[1\bar{1}0]$ axis is the inward directed normal to the surface; when the $[1\bar{1}0]$ axis is the outward normal the behavior is reversed, and the attenuation is greatest when the field and sound are oppositely directed. We see that one can change attenuations by reversing the field, or the direction of the sound, or by interchanging top and bottom of the sample.

It can be shown that the choice of a $(1\bar{1}0)$ plane gives the greatest possible nonreciprocity for waves traveling along the [111] axis; if the plane on which the waves travel is rotated through an angle ϕ about the [111] axis, then Eq. (24) holds with a factor $\cos 3\phi$ multiplying the term containing $\omega/\gamma H$. Propagation is therefore reciprocal on the $(11\bar{2})$ plane.

The dependence of attenuation on field at constant frequency is shown in Fig. 2 for some values of b_2/b_1. The frequency chosen is $\omega = 0.2 \times 4\pi\gamma M$, which corresponds to about 1 GHz in YIG; the elastic contants are related through $s_l^2 = 3.5 s_t^2$, as in YIG, but the nature of the results is insensitive to reasonable alterations in this ratio. It will be noted that propagation is reciprocal in the cases $b_1 = b_2$ and $b_1 = -0.5 b_2$; the first of these corresponds to magnetoelastic isotropy, $b'' = 0$, and the second to $b' = 0$. When b'' is not zero, there is a pronounced structure in the low-field part of the attenuation as well as at the main peak near resonance; this structure comes from the factor $\omega/\gamma H$, which multiplies terms involving b'' and is large at low fields.

It is easy to obtain the attenuation when $\vec{k}$ and $\vec{H}$ are along the [001] axis from the above calculation: one simply sets $b' = b_2$ and $b'' = 0$; the at-

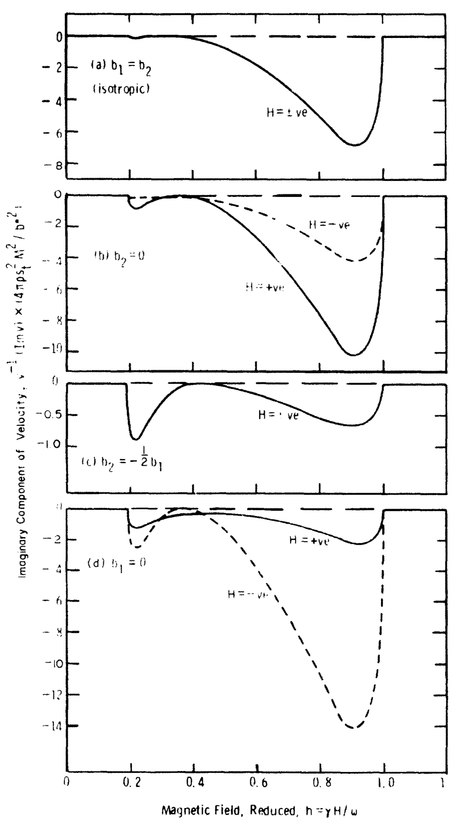

FIG. 2. Attenuation vs magnetic field at a frequency $\omega = 0.2 \times 4\pi\gamma M$, for various ratios $b_1 : b_2$. To obtain v^{-1} Imv, the ordinate must be multiplied by the coupling constant $(b^{*2}/4\pi M^2 c_{44})$, where for curves (a), (b), and (d) $b^* = b' = \frac{1}{3}(2b_1 + b_2)$, and for curve (c) $b^* = b'' = \frac{1}{3}\sqrt{2}\,(b_1 - b_2)$.

tenuation is then reciprocal. The effect of different boundary conditions is not too difficult to find in this case. When the exterior permeability is μ_e, the term in square brackets in the numerator of Eq. (24) is modified as follows:

$$[\quad]^2 \to [\mu_e(B - C' \cos^2\theta) - C'' \cos^2\theta]^2/(\cos^2\theta + \mu_e^2 \sin^2\theta) \tag{26}$$

wherein

$$C' = \beta_l(\beta_l - \beta_t)(1 + \beta_l\beta_t), \quad C'' = \beta_l(\beta_l^2 - \beta_t^2).$$

When $s_l^2 = 3.5 s_t^2$, one finds $C' = 0.571$, $C'' = 0.535$.

Parekh and Bertoni[4] have computed the dependence of attenuation on frequency for the [100] direction in Ga-doped YIG, where the attenuation is about ten times stronger than in YIG; the computation considered both a free surface, $\mu_e = 1$, and a surface covered with a thin layer of a perfect conductor, $\mu_e = 0$. The dependence of the computed attenuation on frequency is indistinguishable from that given by Eq. (24). For Ga-doped YIG the coupling constant $b_2^2/4\pi\rho s_t^2 M^2$ is 1.51×10^{-3}; setting this in Eq. (24) one finds a peak attenuation given by $v^{-1} \operatorname{Im} v = 0.98 \times 10^{-2}$, cf. the computed value of 1.0×10^{-2}. The attenuation near the peak is reduced on a metallized surface; from Eq. (26) the peak attenuation then falls to 0.69 of its value on a free surface, cf. a computed ratio of 0.67. This level of agreement with Parekh and Bertoni's computation is very satisfactory.

D. Contributions to nonreciprocal propagation

In many cases the magnetoelastic anisotropy, b''/b', is rather large and can be expected to dominate the nonreciprocal nature of the waves. When this is not the case we must consider also the effects of elastic anisotropy, and of tilting the field at an angle to the direction of propagation; both effects will be presumed small.

Elastic anisotropy can be taken into account by including a displacement R_y in the elastic surface wave. We presume

$$R_y = (R_y^l e^{-q_l x} + R_y^t e^{-q_t x}) \exp i(k_3 z - \omega t). \tag{27}$$

In the coordinates (xyz) the stress T_{12} at the surface is

$$T_{12} = (2c_{44} + \tfrac{1}{3}\delta c) S_{12} + \tfrac{1}{3}\sqrt{2}\, \delta c S_{13},$$

where $\delta c = c_{11} - c_{12} - 2c_{44} = 0$ in an isotropic material. To zero order in δc one has $S_{13} = 0$ on the surface; to first order in δc, therefore, the condition $T_{12} = 0$ is satisfied by $S_{12} = 0$, i.e., one has

$$q_l R_y^l + q_t R_y^t = 0. \tag{28}$$

The displacement R_y^l can be regarded as being forced by the longitudinal component of the surface wave. The equation of motion of R_y is

$$\rho\omega^2 R_y = [(c_{44} + \tfrac{1}{6}\delta c) k_x^2 + (c_{44} + \tfrac{1}{3}\delta c) k_z^2] R_y + \tfrac{1}{3}\sqrt{2}\, k_1 k_3 \delta c R_x + (1/3\sqrt{2}) k_1^2 \delta c R_z$$

for a wave with wave number $(k_1, 0, k_3)$. If R_x^l and R_z^l are the displacements associated with a longitudinal wave, then to first order in δc one obtains

$$R_y^l = \frac{1}{3\sqrt{2}} \frac{\delta c}{c_{44}} \frac{s_t^2/v^2}{1 - s_t^2/s_l^2} \frac{1}{k_3^2} (2k_3 k_1^l R_x^l + k_1^{l2} R_z^l), \tag{29}$$

where R_x^l and R_z^l are the displacements in the longitudinal component of the surface wave on an elastically isotropic medium, with $k_1^l = iq_l$.

The displacement R_y can be found from Eqs. (27)–(29) and plays the major role in altering the attenuation of the surface wave; other terms enter to order $(\delta c/c_{44})^2$. The total correction is shown below in Eqs. (31) and (32).

A further effect comes when the field is at an angle to the direction of propagation. This has been taken into account only for the case when the field remains in the plane of the surface and is rotated counterclockwise through a small angle ψ relative to the [111] axis. The spin waves now travel at an angle θ into the surface when the frequency ω is given by

$$\omega^2 = \gamma^2 H[H + 4\pi M(\sin^2\psi + \sin^2\theta \cos^2\psi)]. \tag{30}$$

The calculation of this effect is tedious in even the simplest case; only the effect on the nonreciprocal terms has been found, and that only to lowest order in ψ. The result is given below.

On collecting the corrections together the following result is obtained: the term in square brackets, [ ], in the numerator of Eq. (24) becomes

$$[\quad] \to \left[(B - C\cos^2\theta) - \frac{\delta c}{c_{44}} \frac{b''}{b'} (B_1 - C_1 \cos^2\theta) + \frac{\omega}{\gamma H}\left(\frac{b''}{b'}(D - E\cos^2\theta) - \frac{\delta c}{c_{14}}(D_1 + E_1\cos^2\theta) - \sin\psi(D_2 - E_2\cos^2\theta)\right)\right], \tag{31}$$

in which B, C, D, and E are given in Eq. (25), and

$$\delta c = c_{11} - c_{12} - 2c_{44},$$

$$B_1 = \frac{1}{2\sqrt{2}} \frac{s_t/v^2}{1 - s_t^2/s_l^2} \beta_l^3(\beta_l - \beta_t),$$

$$C_1 = (1 + \beta_l)(1 + \beta_t) B_1, \quad D_1 = B_1/\beta_l\beta_t,$$

$$E_1 = (1 + \beta_l)(1 + \beta_t)(\beta_l + \beta_t - 1) B_1/\beta_l\beta_t, \tag{32}$$

$$D_2 = 1 + \beta_l(1 - 2\beta_t - \beta_t^2)/(1 + \beta_t^2),$$

$$E_2 = (1 + \beta_l)(1 + \beta_t)[1 - \beta_t - 2\beta_l\beta_t(1 - \beta_l)/(1 + \beta_t^2)].$$

In the above the elastic anisotropy $\delta c/c_{44}$ and the angle of the field to the [111] axis ψ must be small if the result is to hold; the magnetoelastic anisotropy b''/b' may however be large. The quantities B_1, etc. are numbers; for a material with $s_l^2 = 3.5 s_t^2$, as in YIG, one has $B_1 = 0.187$, $C_1 = 0.481$, $D_1 = 0.576$, $E_1 = 0.359$, $D_2 = 1.086$, and $E_2 = 1.417$.

Consider the attenuation in the neighborhood of its maximum when $s_l^2 = 3.5 s_t^2$; the nature of the results is little changed by alteration of the elastic constants. In most cases the attenuation reaches its greatest maximum when $\cos^2\theta \simeq 0.96$, though this need not be the case for the lesser attenuation when nonreciprocal effects are strong. At this maximum the ratio of forward to backward attenuations is

$$R = \frac{[1 + (\omega/\gamma H) f]^2}{[1 - (\omega/\gamma H) f]^2}, \tag{33}$$

where $\omega/\gamma H$ is somewhat greater than unity, and

$$f = \left(0.28\frac{b''}{b'} + 1.46\frac{\delta c}{c_{44}} - 0.434 \sin\psi\right) \Big/ \left(1 - 0.44\frac{b''}{b'}\frac{\delta c}{c_{44}}\right).$$

The ratio of attenuations is clearly greatest when $|f|$ is close to unity. It can be seen that the effect of the elastic anisotropy, $\delta c/c_{44}$, is about five times stronger than that of the magnetoelastic anisotropy, b''/b'.

The dependence of attenuation on magnetic field in YIG is shown in Fig. 3 for a frequency of 1 GHz, assuming $\vec{H}$ parallel with $\vec{k}$; the field is the total internal field, including the anisotropy field.

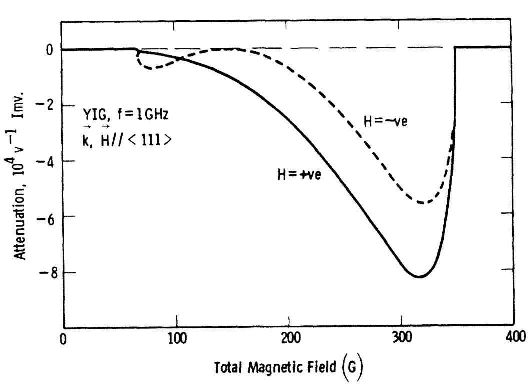

FIG. 3. Attenuation vs magnetic field at 1 GHz in YIG; note that the ordinate should be multiplied by 10^{-4} to obtain $v^{-1}\,\mathrm{Im}v$. The coordinates of Fig. 1 must be used: if the $[1\bar{1}0]$ axis points out of the surface, the curves for $H = +ve$ and for $H = -ve$ are interchanged.

Material parameters used in the calculation are[8] $\delta c/c_{44} = 0.111$ and[9] $b_1 = 3.5 \times 10^6$ erg/cm^3, $b_2 = 6.4 \times 10^6$ erg/cm^3, and hence $b''/b' = -0.306$; also $c_{44} = 7.64 \times 10^{11}$ dyn/cm and $M = 140$ Oe. For the coordinates shown in Fig. 1 the attenuation is greater when H is positive; this results from the elastic anisotropy, which has an effect about twice as strong as the magnetoelastic anisotropy and in the opposite direction.

This result is somewhat unexpected. The elastic anisotropy in YIG is small and often ignored; it happens however that the magnetoelastic anisotropy is also unusually small in this material, so the elastic anisotropy is not ignorable. Most of the rare-earth garnets have large magnetoelastic anisotropies, and the difference between the forward and backward attenuations will be governed by this quantity.

III. CONCLUSION

The attenuation of Rayleigh surface waves on an insulating magnetic medium has been discussed and shown to result primarily from the radiation of spin waves into the bulk; the effect of magnetic resonance is weak, primarily because of the strength of the surface boundary conditions.

When the field and direction of propagation are parallel along a [111] axis in a cubic material, the propagation can be nonreciprocal; i.e., the attenuation can have one of two values depending on whether the field is positive or negative. The attenuation is switched from one value to the other by reversing the field, or the direction of propagation, or by interchanging top and bottom of the sample. This effect is of interest in constructing acoustic surface wave isolators.[2]

The calculation of nonreciprocal propagation is given for arbitrary magnetoelastic anisotropy, and also when a small elastic anisotropy and a small rotation of the field are taken into account. In the case of YIG it is found that the elastic anisotropy, though small, is dominant.

Note added in manuscript: In a recent calculation for a magnetically isotropic medium Scott and Mills[10] find that spin damping smoothes the sharp changes in $v^{-1}\,\mathrm{Im}v$ shown in Fig. 2(a), and also reduces the strength of the peak attenuation. The effect, though small in YIG, can be large in materials with a broad magnetic resonance.

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