Critical behavior of the electrical resistivity in magnetic systems*

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The effect of critical fluctuations on the resistivity near magnetic and order-disorder phase transitions is discussed. It is shown that in all magnetic and electronic systems the asymptotic high-momentum spin correlation functions dominate the temperature dependence of the resistivity sufficiently close to the transition. The dependence on the parameters of the system of the critical behavior and the way in which this behavior is approached is discussed in detail. In particular, the importance of the transition from a classical to a proper critical behavior, for the interpretation of experimental results, is emphasized, and recent renormalization-group results for the form of the correlation functions are utilized. For semiconductors the effects of fluctuations on the band gap are also calculated. It is suggested that the Fisher-Langer relation between the temperature derivative of the resistivity and the specific heat should be valid over a considerable temperature range outside the critical region. Theoretical predictions are compared with available experimental results on the resistivity and band gaps. It is shown that these results can all be understood, at least qualitatively, in the Born approximation by using only the most general known properties of the system.

I. INTRODUCTION

In the last five years a substantial amount of experimental data has been accumulated concerning resistivity anomalies in materials that exhibit magnetic and order-disorder second-order phase transitions.[1] These anomalies are usually characterized by the temperature derivative of the resistivity, $d\rho/dT$, in the vicinity of the critical temperature T_c, and the data are fitted to a power-law behavior of the form $d\rho/dT = At^{-\lambda}$. Here, A is a constant and $t = |T - T_c|/T_c$. The resistivity anomalies of elemental ferromagnetic metals for $T > T_c$ were explained by Fisher and Langer,[2] while all other systems where critical fluctuations interact with charge carriers are not well understood,[1] and no adequate theoretical description which incorporates current understanding of critical fluctuations is available. In particular the accepted analysis by Suezaki and Mori[3] of the resistivity in antiferromagnetic metals involves unjustified approximations, and therefore has to be corrected. In other cases it is sufficient to include in the accepted expressions the correct form for the correlation functions.

Magnetic metals and semiconductors show a variety of behaviors. In some cases the experimental results are consistent with existing theoretical descriptions of the systems considered, while in other cases the resistivity behavior is very different from predictions. In order to demonstrate how severe the confusion is, let us outline some experimental examples of the anomalies exhibited by the four possible magneto-electronic systems.

a. Ferromagnetic metals. Some of the rare-earth Laves phases[4] that are expected to show a Fisher-Langer[2]-type behavior ($A > 0$, $\lambda = \alpha$, where α is the critical index of the specific heat) exhibit

this behavior while others exhibit an Ornstein-Zernike[5] (or a de Gennes–Friedel[6]) type behavior (e.g., $d\rho/dT < 0$ for $T > T_c$).

b. Antiferromagnetic metals. Experimental results on rare earths seem to be in accord with the predictions of Suezaki and Mori[3] as far as the sign of A is concerned,[7–10] but the critical exponents are not always in accord with this theory.[7] On the other hand, for cubic antiferromagnets[11,12] such as PrB_6, GdSb, and HoSb, both the sign of A and the critical exponents are different from those predicted by Suezaki and Mori, and fit a specific-heat-type behavior of $d\rho/dT$.

c. Ferromagnetic semiconductors. Here only the classical Ornstein-Zernike-type theory of Hass[13] exists. If only critical scattering is considered, this theory cannot account for the experimental observations[14,15] on $CdCr_2Se_4$ and EuO that show a peak in $d\rho/dT$ at T_c rather than a peak in ρ at this temperature.

d. Antiferromagnetic semiconductors. For both MnTe and FeO, an $A > 0$ was observed[16,17] and for the latter a $\lambda \approx 0.4$ critical behavior was found. This is clearly in contrast with the classical theory of Hass[13] that does not predict any divergence of $d\rho/dT$ at $T = T_c$.

In view of all this, a unified theoretical approach that can explain, at least qualitatively, the variety of critical behaviors described above seems to be needed. The first attempt to examine the critical behavior of $d\rho/dT$ for all four systems was carried out by Kasuya and Kondo.[18] By using the energy and spin sum rules, they were able to conclude that in the critical region $\lambda = \alpha$ for all four systems. This is in contrast to expectations on the basis of the theories mentioned above. Kasuya and Kondo were also the first to note that a normalized Ornstein-Zernike function can be obtained by using the

spin sum rule, and that this function has a qualita-
tively correct asymptotic behavior above T_c and
can therefore be used to describe the fluctuations
just outside the critical region. However, since
their results were based on qualitative arguments,
they were not able to determine the critical be-
havior of antiferromagnetic metals, and in partic-
ular, to explain the discrepancy between their re-
sults and the results of Suezaki and Mori.[3] More-
over, the critical behavior outside the critical re-
gion, which is of prime importance in understand-
ing the experimental data, was not determined for
any system.

In this paper we go further than Kasuya and
Kondo[18] by using a similar but more uniform and
rigorous approach, and we determine in detail the
behavior of the resistivity for the above four sys-
tems. Using the recent understanding of the spin
correlation function[19] we show the way in which the
$t^{-\alpha}$ divergence is obtained. To determine the be-
havior of the resistivity outside the critical region,
we use our explicit normalized Ornstein-Zernike
functions.[17] In particular, significant improve-
ment of the analysis of Kasuya and Kondo is
achieved here for antiferromagnetic metals and
ferromagnetic semiconductors.

In addition to this, we have used our approach
to determine, for the first time, the critical shift
of the bottom of the conduction band in semicon-
ductors. This shift can affect the number of car-
riers (and thus the critical behavior of the resis-
tivity) and the optical-absorption edge. By a de-
tailed comparison with experimental data, we show
that the present theory can explain all of them with-
out introducing any effects other than critical scat-
tering and band shifts.

The present problem is simpler than most dis-
cussions of transport coefficients near phase-
transition critical points because a static approxi-
mation for the fluctuations is usually adequate. On
the other hand, the important scattering processes
involve relatively high momentum transfers (con-
sistent with energy conservation) so that the as-
ymptotic, high-momentum, correlation function
should be important even in semiconductors. Con-
sideration of this correlation function leads to the
weak, energylike, divergence in the temperature
derivative of the resistivity. We shall show that
close enough to T_c (i.e., in the critical region) the
$t^{-\alpha}$, specific-heat-type divergence of $d\rho/dT$ should
always occur. Outside the critical region a nor-
malized Ornstein-Zernike[17] (OZ) type correlation
function is adequate, yielding a $t^{-1/2}$ type diver-
gence of $d\rho/dT$.

In Sec. II we describe the relevant properties of
the spin correlation functions which are needed for
our purposes. We discuss recent[19] renormaliza-
tion-group results on the asymptotic high-momen-

tum correlation functions. We also use the sum
rules to obtain reasonable forms for these corre-
lation functions beyond the range of validity of the
above techniques. In Sec. III the effect on the re-
sistivity of carriers scattering by critical fluctua-
tions is considered, and we show that close enough
to T_c a Fisher-Langer,[2] energylike divergence is
predicted in all cases, both above and below T_c.
Outside the critical region, i.e., where scaling
does not apply, we can only use the known OZ cor-
relation function that is adequate for fluctuations
with small wave number q, and we account for the
large momentum transfer by normalizing this func-
tion using the spin sum rule[17] which must always
be obeyed.

Since we give here the first discussion on criti-
cal resistivity in semiconductors and since in many
cases it is the critical effect of the carrier con-
centration that dominates the resistivity anomaly,
we have included a derivation of the critical be-
havior of the bottom of the conduction band. Our
findings do predict the way in which the optical-
absorption edge will shift. In the case of a ferro-
magnet, critical fluctuations will always introduce
a red shift with decreasing temperature, while in
antiferromagnets a blue shift is expected (except
for helical magnetic structures with very long pe-
riods). The critical behavior of the bottom of the
conduction band will be discussed in Sec. IV.

The experimental results mentioned above that
could not be explained before will be discussed in
Sec. V. It will be shown that within the frame-
work of the present approach these results can be
explained and that one can even get some ideas
about the magnetic interaction or the electronic
structure from the experimental observations.

II. SPIN CORRELATION FUNCTIONS FOR CRITICAL SCATTERING

The qualitative features of the temperature and
wave-vector dependence of the spin correlation
function $\Gamma(q, T) = \langle \vec{S}_q \cdot \vec{S}_{-q} \rangle$ have been applied to the
calculations of the resistivity in ferromagnetic
metals by Fisher and Langer[2] and more recently
by Kasuya and Kondo.[18] Recently, detailed renor-
malization-group computations[19] have also become
available. These will enable us to make the argu-
ment more quantitative and also less model depen-
dent.

For magnetic-spin models the energy is bilinear
in the spins while these have a constant magnitude.
Thus, one obtains the two sum rules:

$$\sum \Gamma(q, T) = S(S+1) , \tag{2.1}$$

$$-\sum J_q \Gamma(q, T) = U . \tag{2.2}$$

Here S is the spin magnitude, J_q is the Fourier

transform of the exchange interaction, U is the internal energy, and the sum is over the Brillouin zone. J_q is, in general, a regular function of q which falls off over a range κ_0 in q. The value of κ_0 is determined by the physical range of the spin interaction. Expanding J_q in q and using Eq. (2.1), one obtains a temperature-independent term $[J_0\sum\Gamma(q, T)]$ in U. It follows that close to T_c, where the inverse correlation length κ is small, i.e., where

$$\kappa \ll \kappa_0 , \qquad (2.3)$$

the temperature-dependent part of U,

$$\delta U = -\sum (J_q - J_0)\Gamma(q, T) \qquad (2.4)$$

is dominated by the asymptotic high-q correlation functions $(q/\kappa \gg 1)$. One therefore expects the correlation functions to have energylike leading divergence close to the critical point.[2,19] This implies that

$$\Gamma_{as}(q, t) = D_0(q) + D_1(q)\,\mathrm{sgn}(T - T_c)t^{1-\alpha} - D_2(q)t , \qquad (2.5)$$

where $D_0(q)$, $D_1(q)$, and $D_2(q)$ are positive functions of q both below and above T_c. Thus, for $T > T_c$, $\Gamma_{as}(q, T)$ has, at fixed q, a maximum at some $T_{max}(q) > T_c$. On the other hand, since the second term in (2.5) is negative below T_c, $\Gamma_{as}(q, T)$ is a monotonic increasing function of T. This actually follows from Eq. (2.1) and the requirement that the internal energy is a monotonic increasing function of T.

The energy is dominated by the true critical indices only for very small t. However, condition (2.3) holds over a wider range, and the asymptotic correlation function can therefore be expected to have the same "apparent" indices as the energy quite far from T_c. Explicitly, one expects the Fisher-Langer[2] relation,

$$\frac{d\Gamma_{as}}{dT} \propto \frac{dU}{dT} = C_p , \qquad (2.6)$$

where C_p is the specific heat, to hold well beyond the temperature range where the critical index α is relevant. The only assumption is actually that for high q, the temperature dependence of Γ can be approximated by a leading, q-independent, apparent index. We believe that the surprising experimental confirmation of Eq. (2.6) over a wide temperature range[20,21] has probably an explanation of this nature. Since in the following we will be concerned with the temperature derivative of the resistivity, we show in Fig. 1 the schematic behavior of $d\Gamma_{as}/dT$.

In discussing the resistivity we will need an explicit form for the correlation function. Above the critical point when $t > t_G$, where t_G is the Ginzburg temperature,[22,23] it seems reasonable to use a
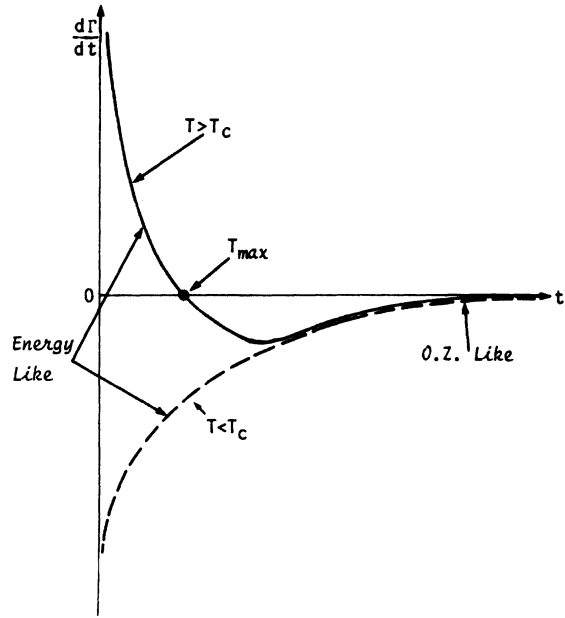

FIG. 1. Temperature dependence of the derivative of the correlation function above and below T_c.

normalized OZ form[17]:

$$\Gamma(q, t) = C/(\kappa^2 + q^2) , \qquad (2.7)$$

where $\kappa = \kappa_0 t^{1/2}$. The normalization constant C is determined from Eq. (2.1) to be

$$C^{-1} = C_0 - C_1\kappa , \qquad (2.8)$$

where the exact values of C_0 and C_1 depend on the boundary conditions (and the exact form of J_q) but the leading temperature dependence, $t^{1/2}$, is not sensitive to this. One notes that explicit diagrammatic expansions also lead to similar corrections.

Sufficiently close to T_c one can use renormalization-group results.[19] From scaling, one has

$$G(q, T) = G_0 t^{-\gamma} D(x^2) , \qquad (2.9)$$

where G_0 is a constant, γ is the susceptibility critical index, and $x = q/\kappa$. Fisher and Aharony[19] find (to order ϵ^2, where $\epsilon = 4 - d$, and d is the number of dimensions) for the Ising model:

$$D(x^2) = [1 + x^2 + O(x^4)]^{-1} \quad \text{for } x \ll 1 \qquad (2.10a)$$

and

$$D(x^2) = (1/x^{2-\eta})(0.962 + 2/x^{(1-\alpha)/\nu} - 3/x^{1/\nu})$$

$$\text{for } x \gg 1 , \qquad (2.10b)$$

where γ, η, ν, and α are defined as usual.[5] As will be shown in Appendix A, the asymptotic expression (2.10b) is valid for $x \gtrsim 10$. Thus the strong OZ-type infrared divergences are canceled out in integration over this range. Only the asymptotic temperature behavior (2.10b) for high q's

can show up beyond this range.

The above is important in discussing the resistivity. The renormalization-group results that we use neglect boundary effects and the interaction between different points of instability in the Brillouin zone. For a ferromagnetic transition, this implies a q that is small compared to the dimension of the Brillouin zone. For antiferromagnets or spiral structures, the separation of the points of instability may impose even more stringent restrictions on the validity of Eq. (2.10). [The wave vector of the fluctuation ($\vec{k}$) has to be much closer to one point of instability than to all others.] Sufficiently close to T_c, the transition to asymptotic behavior does, however, occur in the region where the renormalization-group arguments are valid. One can then use qualitative arguments, based on Eq. (2.2), to deduce a similar *temperature* dependence over the rest of the Brillouin zone.

Below T_c the situation is a little more subtle. The strongly divergent contribution of the order parameter $\langle \vec{S} \rangle$,

$$\Gamma(0, t) = \langle \vec{S} \rangle^2 \propto t^{2\beta} , \qquad (2.11)$$

where β is the corresponding critical index, is somehow canceled out by the finite terms in the sum (2.1). We have already noted in connection with Eq. (2.5) that $dU/dT > 0$. This implies that

$$\frac{d\Gamma_{as}}{dT} \propto - \frac{d\Gamma_{as}}{dt} > 0 \qquad \text{for } T < T_c . \qquad (2.12)$$

Thus the OZ approximation has the correct qualitative temperature dependence. If one tries[17] to correct for the sum rule (2.1), as we did above T_c, one violates Eq. (2.12). We were not able to construct a simple approximation (which could replace the normalized OZ approximation) below T_c. For sufficiently low temperatures one can of course use explicit low-temperature expressions for Γ. Thus, between this low-temperature region and the critical region, only the qualitative temperature dependence of the resistivity will be given.

III. CRITICAL-SCATTERING EFFECTS ON THE RESISTIVITY

In this section we will discuss the effects on the resistivity of carriers scattering by critical fluctuations in the four magnetoelectronic systems mentioned in Sec. I. We assume throughout that the lowest-order Born approximation is adequate and that the time dependence of the fluctuations is sufficiently slow that a static approximation can be used. In semiconductors the resistivity depends also on the number of carriers and therefore on the effect of fluctuations on the band gap. This contribution will be discussed in Sec. IV. On the other hand, for antiferromagnetic metals, the effects of the superzones below T_c are only discussed

briefly.

It is convenient to write a general expression for the resistivity in the form[18]

$$\rho_{\alpha\beta} \propto \sum_{\alpha\beta} \int f_{\alpha\beta}(\vec{k}) \Gamma(\vec{k}, T) d\vec{k} , \qquad (3.1)$$

where for the isotropic part ($\frac{1}{3} \sum_\alpha \rho_{\alpha\alpha}$) and for small wave vectors k we can make the approximation $\sum_\alpha f_{\alpha\alpha}(\vec{k}) \propto |\vec{k}|$. The limits of the integration are determined by energy conservation. We write $\vec{k}$ in Eq. (3.1) for momenta measured from the origin (while the $\vec{q}$'s are measured from the relevant point of instability). We will retain this notation below.

A. Ferromagnets

In this case, $\vec{q} = \vec{k}$, and for a spherical Fermi surface we can write

$$\rho \propto \int_0^{2k_F} dq\, q^3 \Gamma(q, T) , \qquad (3.2)$$

where k_F is the Fermi wave vector. For semiconductors one should replace k_F by the thermal wave vector k_{th}. As was first shown by Fisher and Langer,[2] and as we show explicitly in Appendix A, for the Γ given in (2.10), this always leads for sufficiently small t to

$$\frac{d\rho}{dt} \propto t^{-\alpha} \propto C_p . \qquad (3.3)$$

This assumes, however, that the asymptotic expressions (2.10) are valid, i.e.,

$$2k_F \gtrsim 10\kappa . \qquad (3.4)$$

On the other hand, the classical normalized OZ correlation function[17] (2.7) can be used in Eq. (3.2) for $t > t_G$. This yields

$$\rho \propto [1/(C_0 - C_1\kappa)]\{2k_F^2 - \tfrac{1}{2}\kappa^2 \ln[1 + (2k_F/\kappa)^2]\} . \qquad (3.5)$$

Several possibilities are predicted by Eqs. (3.3) and (3.5). If Eq. (3.4) is still valid at t_G, one has a transition from a Fisher-Langer[2]-type behavior [Eq. (3.3)] to a classical behavior:

$$\frac{d\rho}{dT} \propto t^{-1/2} . \qquad (3.6)$$

This is very important experimentally, because one expects an intermediate "apparent index," $\alpha \lesssim \lambda \lesssim \tfrac{1}{2}$, in the transition region. One notes that the specific heat depends on the same correlation function, so that the proportionality of $d\rho/dt$ and C_p may still be valid beyond t_G.

If k_F (or k_{th}) is relatively small, there are two other possibilities, as can be seen from Eq. (3.5). For large t this equation leads to a de Gennes–Friedel-type behavior,[6] i.e., ρ increases with decreasing t as $t(\ln t)$. This predicts a maximum in

ρ at some temperature t_M, and below $t_{M,\rho}$ decreases with decreasing t as

$$\rho \propto (C_0 - C_1\kappa)^{-1} . \qquad (3.7)$$

Some values of t_M for interesting values of k_F/Λ, where Λ is the effective radius of the Brillouin zone, are given in Table I. These values were derived by using the dimensionless parameter $\tau = (\kappa_0/2k_F)^2 t$. For metals one expects $k_F \approx \Lambda$ and thus, as can be seen from Table I, the de Gennes–Friedel-type behavior is not expected to be observed and the temperature dependence will be the one shown in Fig. 2(a); however, it should be observed in semiconductors, where $k_{\text{th}} \ll \Lambda$. If $(k_{\text{th}}/\kappa_0)^{1/\nu} \equiv t_{\text{th}} < t_G$, requirement (3.4) does not hold at t_G and this implies a nonclassical region that has a de Gennes–Friedel type of temperature dependence, $t^{2\nu}(\ln t)$. If $t_{\text{th}} > t_G$ one expects, with increasing t, a transition from a (3.3) type of behavior to a (3.6) type of behavior and then a transition to a de Gennes–Friedel type of behavior. This situation is shown in Fig. 2(b).

We have already noted that the divergence due to the $t^{2\beta}$ dependence of $\langle \vec{S} \rangle^2 = \Gamma(0, t)$ is also canceled out, at relatively small q, below T_c, and it does not affect the high-q correlation function. Thus the indices below T_c are also energylike. A detailed discussion is, however, complicated because of the possibility of transverse modes (spin waves) and also because the correlation functions are not as well understood. Thus the dashed curves in Fig. 2 describe the regions where the critical behavior of the resistivity below T_c is known, as far as their qualitative temperature dependence [from consideration of the monotonic increase of $\Gamma(T)$] is concerned, but not as far as their exact critical indices are concerned.

B. Antiferromagnets

General expressions for the resistivity in antiferromagnetic metals were derived by Suezaki and Mori.[3] Their results were recently revised by Kasuya and Kondo.[18] These authors showed that the critical effects of the resistivity are very sensitive to the shape of the Fermi surface and the position of the antiferromagnetic instability in the Brillouin zone ($\vec{Q}$). They also use a more realistic form for the spin correlation function. Their dis-
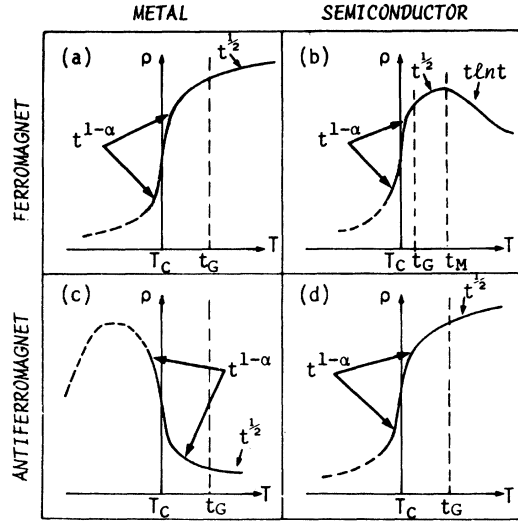

FIG. 2. Temperature dependence of the resistivity as predicted by the present work for: (a) a ferromagnetic metal, (b) a ferromagnetic semiconductor, (c) an antiferromagnetic metal, and (d) an antiferromagnetic semiconductor. The dashed curves below T_c indicate that the qualitative behavior was predicted but not the critical index. t_G corresponds to the Ginzburg temperature, and t_M is the temperature where ρ has its maximum value.

cussion tends to overemphasize the importance of the shape of the Fermi surface in determining the qualitative features of the result and can be misleading below T_c.

For simplicity we assume a spherical Fermi surface and discuss here only the isotropic part of the resistivity ρ_{iso}. The anisotropy is discussed in Appendix B. In analogy with Eq. (3.2) one then has

$$\rho_{\text{iso}} \propto \sum_i \int |\vec{q}+\vec{Q}_i| \Gamma_q d^3 q , \qquad (3.8)$$

where the $\vec{Q}_i$ are the points of instability in the Brillouin zone. Equation (3.8) can also be obtained from the expression of Ref. 3 if one computes the isotropic part of the resistivity ($\rho_{\text{iso}} = \frac{1}{3}\sum_\mu \rho_{\mu\mu}$). Note also that for cubic crystals there is no anisotropy in ρ (above T_c). In Eq. (3.8) the range of integration is limited by the constraints

$$|\vec{q}+\vec{Q}_i| \leq 2k_F , \qquad (3.9)$$

$$|\vec{q}+\vec{Q}_i - \vec{Q}_j| > q , \qquad (3.10)$$

and the requirement that $\vec{q}+\vec{Q}_i$ is in the (first) Brillouin zone. In Eq. (3.8) we have associated the fluctuations at each point in the reciprocal antiferromagnetic lattice with the nearest instability. This is the condition described by Eq. (3.10). The inequality (3.9) is the condition for elastic scattering and will at most exclude some corners of the

TABLE I. t_M as a function of k_F/Λ for $\kappa_0/\Lambda = \frac{1}{2}$.

k_F/Λ	τ	t_M
0.01	2×10^{-6}	3.2×10^{-9}
0.1	5×10^{-4}	8×10^{-5}
0.5	4×10^{-2}	0.16

zone when $2k_F > |\vec{Q}_i|$. For proper antiferromagnets, $\vec{Q}_i$ is on the zone boundary, and the inequalities (3.9) and (3.10) restrict the integration over q to the region $q < |\vec{Q}_i|$. This amounts to a restriction on the angular integration of $\vec{q}$. Performing the angular integration, one can write

$$\rho \propto 2pQ \int F_0(q/Q)\Gamma_q q^2 dq ,\qquad (3.11)$$

where p is the number of distinct points $\vec{Q}_i$ and $Q = |\vec{Q}_i|$. Here the function $F_0(q/Q)$ describes the angular integration and is thus given by

$$F_0(q/Q) = (1/Q) \int d\Omega_q |\vec{Q}_i + \vec{q}| ,\qquad (3.12)$$

where $d\Omega_q$ is the solid-angle element.

For example, if one considers the MnO-type magnetic structure where the $\vec{Q}_i$'s are on the faces of the Brillouin zone, the integration can be performed over the range $-1 < \cos\theta < 0$. A section of the Brillouin zone for this structure is shown in Fig. 3. Integration of Eq. (3.12) in this case yields

$$F_0(x) = [(1 + x^2)^{3/2} - (1 - x)^3]/3x .\qquad (3.13)$$

The temperature dependence of Eq. (3.11) when (3.13) is used can be easily seen by expanding (3.13) around $x = 0$,

$$F_0(x) = 1 - \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{8} + O(x^4) .\qquad (3.14)$$

This expansion is quite adequate in the range of interest $(x < 1)$. Suezaki and Mori[3] derived their results by retaining only the leading term (1) in Eq. (3.14). They then evaluated the integral (3.11) in the OZ approximation. This is clearly inadequate.
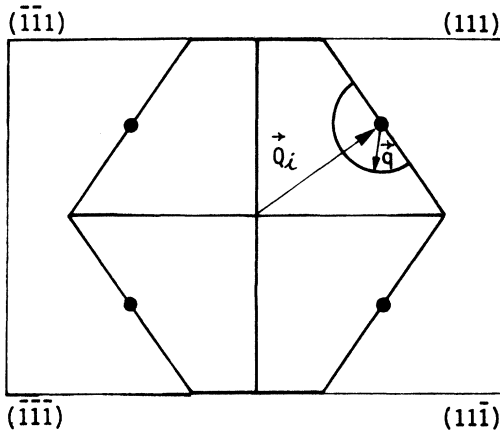

FIG. 3. A section of the Brillouin zone of a MnO-type magnetic structure, along the (110) plane. $\vec{Q}_i$ is one of the magnetic reciprocal-lattice vectors.

Above T_c, the leading term of (3.14), in the integral (3.11), leads just to the spin sum rule and is therefore temperature independent. The temperature dependence of ρ is therefore determined by the higher-order terms. In the present example, and in fact whenever Q is large enough (see below) and $2k_F > Q$, the next-to-leading term has the de Gennes–Friedel form[5]

$$\int \Gamma_q q^3 dq ,\qquad (3.15)$$

but with a negative sign. The other terms will yield higher powers of q in the integrand. As in the ferromagnetic metals, it follows that the diverging small-q region in Γ_q does not contribute for any interesting value of κ. Thus, the resistivity reflects the critical behavior of the asymptotic high-q correlation function. Since the leading term (3.15) has a negative sign for $x \leq 1$, the sign is reversed compared to the ferromagnetic cases, in agreement with the sign of Suezaki and Mori. The resistivity will thus be a decreasing function of temperature, with

$$\frac{d\rho}{dT} \propto -t^{-\alpha} \qquad \text{for } T > T_c .\qquad (3.16)$$

The generalization of this result is straightforward. In general,

$$\rho_{iso} \propto \int f(\vec{k})\Gamma(\vec{k}, T) d\vec{k} = \sum_i \int f(\vec{Q}_i + \vec{q})\Gamma(\vec{q}) d\vec{q} ,\qquad (3.17)$$

where $f(\vec{k})$ includes both Fermi-surface and interaction effects. Angular integration then leads to the form (3.11) where the only assumption is that $\Gamma(\vec{q})$ can be considered isotropic, at least for small q. Thus, quite generally,

$$\rho_{iso} \propto p \int F(q/Q)\Gamma_q q^2 dq ,\qquad (3.18)$$

where

$$F(q/Q) = \int d\Omega_q f(\vec{Q}_i + \vec{q}) ,\qquad (3.19)$$

and the temperature dependence is always determined by the asymptotic Γ_q sufficiently close to T_c. One notes that the sign of the divergence at T_c depends on the sign of $F(x) - F(0)$ for $x \leq 1$.

While this argument is quite general, the range for which the exponents are those of the asymptotic correlation function can be very small. This would, for example, happen in the highly anisotropic case considered by Kondo and Kasuya,[18] which has features analogous to those of the ferromagnetic semiconductor. A discussion of this case is given in Appendix C.

The case we have considered above is that of the "proper" antiferromagnetic metal, where Q is close to the zone boundary and smaller than $2k_F$.

There are, of course, other possibilities. When $Q > 2k_F$ the integration in Eq. (3.8) is restricted to a region of relatively large q. This is the case for semiconductors, where this region is given by $|\vec{Q} + \vec{q}| \sim 2k_{th} \ll Q$. In this region the correlation function is a slowly varying function of q and therefore $\rho \propto \Gamma_Q$ and ρ has an "energylike" divergence proportional to C_p. Another case is that of a small Q close to the center of the zone. Since Eq. (3.8) approaches the expression for the ferromagnets as $Q \to 0$, it is clear that the small-Q behavior is at least qualitatively ferromagnetic. Thus $d\rho/dT$ changes its sign as a function of Q for small Q's and as a function of k_F for large Q's.

Outside the critical region we can again use the normalized Ornstein-Zernike correlation function [Eq. (3.5)]. The first term in Eq. (3.11), when the expansion (3.14) is used, leads to the spin-sum-rule integral and gives a constant contribution to the resistivity. The next term in the resistivity is proportional to

$$\frac{-1}{\Lambda - \frac{1}{2}\pi\kappa} \int_0^Q dq \, \frac{q^3}{q^2 + \kappa^2} \, , \qquad (3.20)$$

and thus yields a $(-t^{1/2})$ dependence of the resistivity (see above). It should be noted that the sign of $t^{1/2}$ will follow the sign of $t^{1-\alpha}$ in all the cases considered above. Thus, above T_c the temperature dependence of the resistivity for "proper" antiferromagnetic metals will be as shown in Fig. 2(c), and for antiferromagnetic semiconductors it will be as shown in Fig. 2(d).

The behavior of the resistivity below T_c is by far more complicated mainly because of the complications introduced by the t^β temperature dependence of the order parameter and the appearance of new gaps due to the reduction of translational symmetry. Sufficiently close to the transition temperature the order parameter $\langle \vec{S} \rangle$ is small and can be treated by perturbation theory. Therefore,

$$\rho \propto Q\langle \vec{S} \rangle^2 + \int' |\vec{q} + \vec{Q}| \, \Gamma_q \, d^3q \, , \qquad (3.21)$$

where the prime indicates exclusion of the point $q = 0$ from the integral. Thus,

$$\rho \propto Q\left(\langle \vec{S} \rangle^2 + \int' \Gamma_q \, d^3q\right) + \int \left(|\vec{q} + \vec{Q}| - Q\right)\Gamma_q \, d^3q \, . \qquad (3.22)$$

The first term is a constant due to the spin sum rule, and we have already shown that the second integral involves only the asymptotic correlation function. The sign of this integral depends again on the magnitude of Q. It is important to note that it is always the same below and above T_c. Thus the resistivity has a $t^{1-\alpha}$ continuous behavior at T_c, and there is no cusp in the resistivity. This behavior is described in Figs. 2(c) and 2(d).

Treating the order parameter in the Born approx-

imation is no longer legitimate when $\langle \vec{S} \rangle$ becomes large. Even though $d\rho/dT < 0$ for $T < T_c$, it is necessary that $\rho \to 0$ when $T \to 0$. This comes from the monotonic temperature dependence of the correlation function (see Sec. II). We could not, however, find any simple approximation (such as the normalized OZ approximation for $T > T_c$) for Γ_q when $T < T_c$ which would describe this behavior. The behavior in this region is indicated in Figs. 2(c) and 2(d) by the solid curves in the range where $\langle \vec{S} \rangle$ can be treated as a perturbation, and by the dashed curve where this treatment is not legitimate.

IV. CRITICAL BAND SHIFTS

In this section we deal with the effect of the critical spin fluctuations, on the conduction band in a magnetic semiconductor. This effect has bearing on the critical behavior of the resistivity, since the number of carriers may depend on the position of the bottom of the conduction band. For example, if the donor level is not affected by the critical fluctuations, the number of carriers is related to the energy shift ΔE of the bottom of the conduction band by

$$n = n_0 \exp\{[-(E_a + \Delta E)/2k_B T]\} \, , \qquad (4.1)$$

where n_0 is the concentration of donors and E_a is the activation energy for $T \ll T_c$. Experimentally, however, it is the temperature dependence of the optical-absorption edge that can reveal the critical behavior of ΔE.

A. Ferromagnetic semiconductors

The problem of the band shift of a ferromagnetic semiconductor due to spin fluctuations was already studied although without emphasizing its critical behavior.[24] Above T_c and in the second Born approximation, the energy shift of the level corresponding to the momentum $\hbar k$ is[24]

$$\Delta E = \left[\frac{(G^2/\Omega)}{4(2\pi)^3}\right] \int_0^\Lambda d^3q \left[\frac{\Gamma_q}{(E_{\vec{k}} - E_{\vec{k}-\vec{q}})}\right], \qquad (4.2)$$

where $E_{\vec{k}} = \hbar^2 k^2/2m^*$, m^* is the carrier's effective mass, Ω is the volume per spin, and G is the de Gennes–Friedel[6] interaction constant between the carrier and the ion spin. Angular integration in (4.2) gives

$$\Delta E = \frac{-m^*(G^2/\Omega)}{4(2\pi)^2\hbar^2 k} \int_0^\Lambda dq \, q\Gamma_q \ln\left|\frac{q + 2k}{q - 2k}\right| \, . \qquad (4.3)$$

Using the spin sum rule (2.1), this yields

$$\Delta E = -\frac{m^*(G^2/\Omega)}{8\hbar^2 k^2} S(S+1) + \frac{m^*(G^2/\Omega)}{4(2\pi)^2\hbar^2 k}$$
$$\times \int_0^\Lambda dq \, q\left(\frac{q}{k} - \ln\left|\frac{q + 2k}{q - 2k}\right|\right)\Gamma_q \, . \qquad (4.4)$$

Separating the integral into the two regions $q < 2k$

and $q > 2k$ allows the following approximation. In the first region, we can write

$$\ln\left|\frac{q+2k}{q-2k}\right| = 2\left[\frac{q}{2k} + \frac{1}{3}\left(\frac{q}{2k}\right)^3 + \cdots\right] , \quad (4.5)$$

and thus the leading term in the integral will yield a $-t^{1-\alpha}$ type dependence of ΔE. In the second region, considering that $\Gamma_q \propto +t^{1-\alpha}$ for finite q and that the integral is positive except for a small region around $2k$, the integral is proportional to $+t^{1-\alpha}$. This contribution is with a larger coefficient than the corresponding coefficient of the first region. All this is true for $k \ll \Lambda$, which is the case for semiconductors. Thus, in the critical region for $T > T_c$ we obtain

$$\frac{d(\Delta E)}{dt} \propto +t^{-\alpha} . \quad (4.6)$$

Outside the critical region and for $T > T_c$ we substitute the normalized OZ correlation function (2.7) into Eq. (4.2), obtaining

$$\Delta E = -H\left(\int_0^\infty dx\, \frac{x}{1+x^2}\ln\left|\frac{x+2k/\kappa}{x-2k/\kappa}\right| \right.$$

$$\left. -\int_{\Lambda/\kappa}^\infty dx\, \frac{x}{1+x^2}\ln\left|\frac{x+2k/\kappa}{x-2k/\kappa}\right| \right), \quad (4.7)$$

with $H = m^*(G^2/\Omega)S(S+1)/[8\hbar^2 k(\Lambda - \frac{1}{2}\pi\kappa)]$. The first integral is just $\pi \arctan(2k/\kappa)$. Since $\Lambda \gg 2k$ and $\Lambda/\kappa \gg 1$, we can, for the second integral, use Eq. (4.5) and the expression $(1+x^2)^{-1} \approx 1/x^2 - 1/x^4 + \cdots$. Thus

$$\Delta E = -H[\pi \arctan(2k/\kappa) + O(k/\Lambda) + O(k^2/\Lambda^2)] . \quad (4.8)$$

The power-law behavior of ΔE can be easily deduced from Eq. (4.8) for the two cases $\kappa/2k \ll 1$ and $\kappa/2k \gg 1$. Since ΔE is proportional to $(1 + \frac{1}{2}\pi\kappa/\Lambda)(\pi/2 - \kappa/2k)$, and in semiconductors $\Lambda \gg k$, we will get in the first case $\Delta E \propto +t^{1/2}$ and in the second case $\Delta E \propto -k/\kappa \propto -t^{-1/2}$.

It should be noted that the bottom of the band itself ($k = 0$) is a singular point where the second Born approximation diverges when T approaches T_c. This can be handled by a series summation.[24] For our purpose it is enough to consider a finite k. In particular, for the resistivity, we are interested in the shift of the energy level associated with k_{th}. Below T_c, the leading contribution to ΔE comes from the first Born approximation[24] and it is proportional to the magnetization. Thus for $T < T_c$,

$$\frac{d(\Delta E)}{dT} \propto +t^{\beta-1} . \quad (4.9)$$

The temperature behavior of ΔE as a function of T for the ferromagnetic semiconductor is sketched in Fig. 4(a).
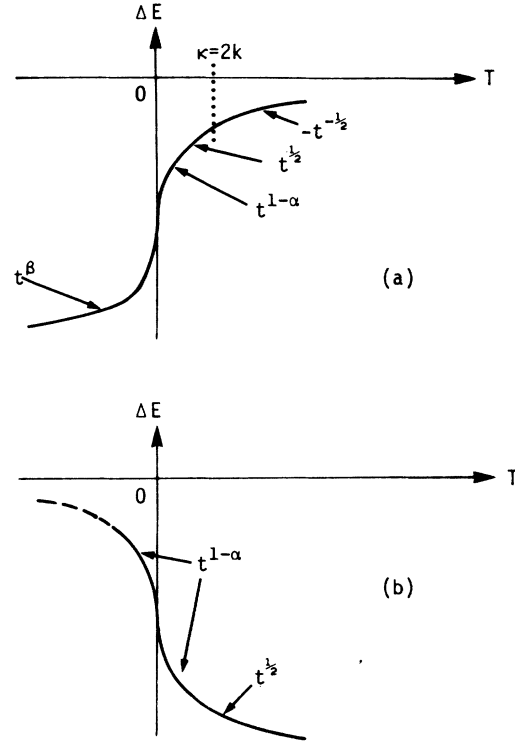

FIG. 4. Temperature dependence of the shift of the bottom of the conduction band in the vicinity of T_c: (a) for a ferromagnetic semiconductor, and (b) for an antiferromagnetic semiconductor.

B. Antiferromagnetic semiconductors

In antiferromagnetic semiconductors the bottom of the conduction band is not a singular point, and so we restrict ourselves to the case $k = 0$. Thus, for $T > T_c$, the energy shift in the second Born approximation is given by

$$\Delta E = -\frac{2m^*(G^2/\Omega)}{4(2\pi)^3\hbar^2}\sum_i \int d^3q\, \frac{\Gamma_q}{|\vec{Q}_i + \vec{q}|^2} . \quad (4.10)$$

The integral here is taken with the constraint (3.10), and the condition that $\vec{Q}_i + \vec{q}$ is in the first Brillouin zone. Thus

$$\Delta E \approx L\int_0^Q dq\, q\Gamma_q \ln\frac{(Q-q)^2}{Q^2+q^2} , \quad (4.11)$$

where $L = m^*(G^2/\Omega)p/4(2\pi)^2\hbar^2 Q$ (see Sec. III B). Using the spin sum rule we now get

$$\Delta E = -(4\pi^2 L/Q)[S(S+1)]$$

$$+ L\int_0^Q dq\, q\left(\ln\frac{(Q-q)^2}{Q^2+q^2} + \frac{2q}{Q}\right)\Gamma_q . \quad (4.12)$$

For $q < Q$ one can use an expansion of $\ln[(Q-q)^2/(Q^2+q^2)]$ in powers of $2qQ/(q^2+Q^2)$. This produces negative terms of order q^3 or higher. Hence, in the critical region the integral will be proportional

to $-t^{1-\alpha}$. We do not have to worry about the region $q \approx Q$ since Γ_q is almost a constant there and thus the integral (4.11) does not diverge. For t outside the critical region we can substitute again the normalized OZ function into (4.11). If we keep the leading term in the integral we now get

$$\Delta E = L\left(-\frac{4\pi^2 S(S+1)}{Q} - \frac{2}{Q^2}\int_0^Q dq\, q^3 \Gamma_q\right), \quad (4.13)$$

and thus

$$\frac{d(\Delta E)}{dt} \propto -t^{-1/2}. \quad (4.14)$$

Below T_c in the critical region we can use the considerations used in Sec. III. This yields a divergence of the type $d(\Delta E)/dt \propto -t^{-\alpha}$. It should be noted that below T_c the sum in (4.10) includes only the specific reciprocal-lattice vector $\vec{Q}_i$ which describes the actual magnetically ordered state.

The behavior of ΔE for an antiferromagnetic semiconductor is sketched in Fig. 4(b).

V. COMPARISON OF THE THEORY WITH EXPERIMENTAL DATA

As was clear throughout the paper, the theoretical predictions for the critical behavior of the resistivity are practically limited to the determination of the critical exponent λ and the sign of the leading diverging term in $d\rho/dT$. The bulk of the experimental data contains even less information. Most of the data are concerned with a qualitative characterization of the shape of $\rho(T)$, as being of a de Gennes–Friedel[6] type ($d\rho/dT$ changes sign at T_c) or of a Fisher-Langer[2] type ($d\rho/dT > 0$ above and below T_c). In the relatively few cases where critical indices were derived, attempts were made to correlate them with the theoretical critical indices. In view of the present analysis this procedure should be questioned, since in principle any $\alpha \le \lambda \le \frac{1}{2}$ can be found, depending on the t range over which the measurements were carried out. Rather, it is the relation

$$\frac{d\rho}{dT} \propto C_p \quad (5.1)$$

that must be the criterion for comparing theory with experiment. In this section we shall review the experimental situation in the four magneto-electronic systems, and discuss in particular the "problematic" cases mentioned in the introduction.

A. Ferromagnetic metals

In the elemental ferromagnetic metals over the range $10^{-4} < t < 10^{-1}$, relation (5.1) was found to be in perfect agreement with experiment.[20,21] This relation was obtained even though α was quite different from the common theoretical predictions.

For example, in nickel for $T < T_c$ it was found that $\alpha = -0.26 \pm 0.06$ rather than the theoretical value[5] $\alpha \approx 0.125$, but relation (5.1) was confirmed and a value $\lambda = -0.3$ was found.[20] Thus the elemental ferromagnets behave according to previous[2,18] and present expectations.

Among the rare-earth Laves phases[4] (where the sign of $d\rho/dT$ was determined but not λ) some show a behavior expected for a ferromagnetic metal [Fig. 2(a)], while others exhibit the behavior of a ferromagnetic semiconductor [Fig. 2(b)]. In GdNi$_2$, which is the most thoroughly studied material, the semiconductor-type behavior is very apparent, and for $t > 10^{-1}$ a quantitative fit to a de Gennes–Friedel behavior was found.[25] The difference between the behavior of the different phases becomes clear if one examines Table I. Since the change of sign of $d\rho/dT$ for $T > T_c$ depends solely on k_F/Λ (κ_0/Λ should be practically the same for all these phases), the above difference is just due to the difference in the effective k_F that takes part in the critical scattering process. We can thus learn from the present analysis that in the different Laves phases studied, the dominant critical scattering relates to carriers with different k_F. Table I explains further why the elemental ferromagnetic metals, where k_F is large, exhibit the Fisher-Langer-type behavior, while in the degenerate ferromagnetic semiconductor EuB$_6$, where band effects are not important, a behavior of the type shown in Fig. 2(b) was found.[26]

B. Ferromagnetic semiconductors

Apart from EuO, transport and optical studies in ferromagnetic semiconductors so far determined only qualitative critical behaviors of the resistivity and the optical-absorption edge. In the many materials studied, a red shift in the absorption was observed,[27] in accord with Fig. 4(a), and the resistivity behavior was found to be in accord with Fig. 4(a) and Eq. (4.1), or Fig. 2(b), or a combination of both.[28-31]

The critical behavior was studied in more detail in EuO. There, it was found[32] that the red shift above T_c is proportional to $t^{1-\alpha}$. Below T_c it was found[33] that the carriers' activation energy varies as t^β in the range $10^{-2} < t < 2 \times 10^{-1}$. Since the activation is from a tightly bound state to the bottom of the conduction band,[15] it indicates that the optical red shift below T_c also has a t^β dependence. All this is in agreement with Fig. 4(a). The experimental critical behavior of the resistivity due to variations of carrier mobility was not studied quantitatively, but it is in accord with Fig. 2(b). The fact that it is the same for the highly doped and the lightly doped materials,[15] and the fact that above T_c the results are in agreement[30] with relation (5.1), indicate a quantitative agreement with Fig. 2(b).

In addition to these agreements, our results can provide the answer to the question raised by Oliver et al.[15] as to why the peak in the resistivity in EuO does not move with decreasing doping for samples with carrier concentrations of less than 2×10^{18} cm^{-3} while it does move for samples with higher carrier concentrations. Table I shows that for this low carrier concentration the peak should shift by less than 10^{-2} °K (much less than the accuracy of the measurements of Ref. 15), while for the highest concentrations used, a shift of the order of 10 °K should be observed. All this is indeed in agreement with the experimental results.

In CdCr$_2$Se$_4$ the predicted red shift was observed and the transport measurements indicate that the changes in transport coefficients are due to carrier-density variations rather than to mobility variations.[14] Preliminary measurements of the critical resistivity[31] indicate that very close ($t < 10^{-3}$) to T_c the resistivity is dominated by critical scattering, i.e., $\rho \propto t^{1-\alpha}$ both above and below T_c, while further out from T_c the critical resistivity is dominated by the change in carrier concentration and is in agreement with the expectations from Fig. 4(a). Thus, in accord with previous analysis,[14] it can be concluded that the large changes of the transport coefficients are due to the red shift of the bottom of the conduction band.

C. Antiferromagnetic metals

As was pointed out in the introduction, both the qualitative and the quantitative temperature dependence of the resistivity are found to be different for different antiferromagnetic metals. Some[7-10] (rare earths and Cr) exhibit the temperature dependence shown in Fig. 2(c), while others[11,12] (PrB$_6$, GaSb, HoSb) exhibit the temperature dependence shown in Fig. 2(d). It is clear from the analysis of Sec. III that the difference between the two types of $\rho(T)$ can be explained by the difference in the effective k_F of the carriers that participate in the scattering process. Thus, it is not the details of magnetic and electronic structure that determine the resistivity anomaly (at least for $T > T_c$) as suggested by Kasuya and Kondo,[18] but rather this effective k_F.

As far as the values of the critical exponents are concerned, they have to obey relation (5.1) rather than the relation predicted by Suezaki and Mori (i.e., $d\rho/dT \propto t^{-1/3}$). The apparent agreement of the experimental results, say for Dy,[8] with the $t^{-1/3}$ dependence, must come about from a measurement carried out in a transition interval between the classical and critical regions (see Sec. III and Appendix C). That this is indeed the case can be seen from the results obtained for Tb, where a transition from a $t^{-1/2}$ to a $t^{-\alpha}$ behavior was found for $d\rho/dT$ as T_c was approached from above.[7] This

result is in quantitative agreement with the predictions of Fig. 2(c).

The behavior of $\rho(T)$ for $T < T_c$ is explained in our analysis on the basis of the general features of the correlation function and the critical scattering without introduction of superzone effects on the carrier concentration.[3] We cannot, however, exclude the possibility that these effects determine the detailed behavior of $\rho(T)$ for $T < T_c$. Thus the interpretation of experimental results only in terms of carrier-concentration changes[8,34] is not necessarily correct.

Our results for antiferromagnetic metals should also describe the resistivity of binary alloys which undergo order-disorder transitions. Again, for large values of k_F a behavior of the type shown in Fig. 2(c) is expected, while for small k_F a behavior of the type shown in Fig. 2(d) is expected. The order-disorder transition in Fe$_3$Al does indeed exhibit exactly the first type of behavior,[34] while CuZn exhibits the second type of behavior.[35] (It should be noted that in CuZn it is known that a small k_F is formed around a H point.[18]) Thus, to understand the behavior of these two systems there is no need to invoke further details of the systems as was suggested by Kasuya and Kondo[18] for Fe$_3$Al, or to assume that there is something anomalous about CuZn, as was suggested by Thomas et al.[34]

D. Antiferromagnetic semiconductors

Of the four magnetoelectronic systems, this is the least-studied system. The absorption edge was found in general to exhibit a blue shift[27,36] (for example α-MnS, CoO, ZnCr$_2$Se$_4$). In the only case where a red shift was found[27] (MnO) the results were interpreted as due to excitation of localized electrons rather than to the excitation of electrons to the conduction band.[27] It should be noted that the results of Sec. IV were derived for the second case.

The temperature dependence of the resistivity was measured in some detail[16,17] in the vicinity of T_c only for MnTe and FeO. In both materials the results were found to be in qualitative agreement with the behavior shown in Fig. 2(d). In FeO it was found that $\lambda = 0.4 \pm 0.1$ in the range $10^{-3} \le t \le 10^{-1}$ for both $T > T_c$ and $T < T_c$, which indicates that this temperature range is outside the critical region in FeO, since the above value of λ is close to the classical value ($\frac{1}{2}$).

APPENDIX A

In using the Fisher-Aharony[19] result for the correlation function (2.9), it is convenient to use a hybrid form $G(x, y)$ which has the form (2.10a) for $x < y$ and the form (2.10b) for $x > y$. To make this consistent, y has to be chosen so that the energy has the correct critical behavior, i.e., the leading

divergence of dU/dT will not be stronger than $t^{-\alpha}$.

To do this let us first approximate the energy sum rule

$$U = -\int_0^\Lambda J_q G(q, T)q^2 dq \qquad (A1)$$

by

$$U = -J_0 \int_0^{\kappa_0} G(q, T)q^2 dq . \qquad (A2)$$

Here we have approximated the exchange coupling J_q, over the range $0 \le q \le \Lambda$, by a constant J_0 in the interval $0 \le q \le \kappa_0$ and by $J_q = 0$ for $q > \kappa_0$. In Eq. (A1), Λ is the momentum cutoff (that can be taken as the effective radius of the Brillouin zone). Now, U should be of the form

$$U \simeq A_1 + A_2 t^{1-\alpha} \operatorname{sgn}(T - T_c) - A_3 t , \qquad (A3)$$

where the A_i's are constants. If we choose y so that the integral (A2) will yield the result (A3), we can find a good approximation for $G(q, T)$ over the entire range of x. For this purpose consider integrals of the type

$$I_n = \int_0^{Q_0} \Gamma_q q^n dq \qquad (A4)$$

for $n \ge 2$. Here Q_0 is some finite cutoff. Using the approximation[19] for $G(q, T)$, we have

$$I_n = \int_0^{y\kappa} \frac{G_0 t^{-\gamma}}{1 + x^2} q^n dq$$
$$+ \int_{y\kappa}^{Q_0} \frac{G_0 t^{-\gamma}}{x^{2-\eta}} \left(0.962 + \frac{2}{x^{(1-\alpha)/\nu}} - \frac{3}{x^{1/\nu}} \right) q^n dq . \qquad (A5)$$

By substituting $q = x\kappa_0 t^\nu$, carrying out the integration, and recalling the scaling relation[5] $\gamma = (2 - \eta)\nu$, one finds that

$$I_n = E_1(y)t^{(n+1)\nu-\gamma} + E_2 + E_3 t^{1-\alpha} - E_4 t , \qquad (A6)$$

where $E_1(y)$ is a function of y, and E_2, E_3, and E_4 are constants that depend on Q_0 and κ_0. By requiring $E_1(y) = 0$, we have determined y. We find that $y = 10.5$, and that the mismatch of the two parts of the approximate function at $x = y$ is 7%.

From the result (A6) it becomes apparent that for any $n \ge 3$ (and also for $n = 2$ if $Q_0 > y\kappa$), the leading divergence in dI_n/dt is proportional to $t^{-\alpha}$. This is of importance in this paper since in the resistivity calculations we encounter integrals of the form (A4).

APPENDIX B

The isotropic resistivity ρ_{iso} is defined by

$$\rho_{iso} = \frac{1}{3} \sum_\mu \rho_{\mu\mu} \propto \int |\vec{Q} + \vec{q}| q^2 \Gamma_q dq . \qquad (B1)$$

The proportionality follows immediately from the expression of Suezaki and Mori[3] in the static ap-

proximation. The transverse resistivity $\rho_\perp$ is given by

$$\rho_\perp \propto \int \frac{q^4 \sin^2\theta}{|\vec{Q} + \vec{q}|} \Gamma_q dq \, d(\cos\theta) . \qquad (B2)$$

Thus,

$$\rho_\perp \propto + \frac{1}{Q} \int q^4 \Gamma_q dq , \qquad (B3)$$

to lowest order in q/Q. Here Q is the magnetic reciprocal-lattice vector which describes the magnetic structure below T_c. On the other hand,

$$\rho_{iso} = \frac{1}{3}(\rho_{||} + 2\rho_\perp) , \qquad (B4)$$

where $\rho_{||}$ is the longitudinal resistivity. From Eq. (B4) we obtain

$$\rho_{||} = 3\rho_{iso} - 2\rho_\perp . \qquad (B5)$$

From Appendix A it is immediate that the leading divergence of $\rho_\perp$ near T_c is of the form $+ t^{1-\alpha}$, as was first suggested qualitatively by Suezaki and Mori.[3] Thus, it follows here that $\rho_{||}$ also has a $t^{1-\alpha}$ dependence.

APPENDIX C

It is interesting to analyze the highly anisotropic case discussed by Kasuya and Kondo[18] to see how the transition to energylike critical indices comes about.

We consider the longitudinal resistivity $\rho_{||}$ in a metal with a pair of flat and parallel Fermi surfaces where $|\vec{Q}| = 2(k_F)_{||}$. As discussed by Kasuya and Kondo, this situation is relevant to the rare-earth metals, where $\vec{Q}$ is in the c direction of the hcp structure. The general expression of Suezaki and Mori[3] for antiferromagnets will take, for the present case, the form

$$\rho_{||} \propto \int d\vec{p} \, d\vec{k} \Gamma_{\vec{k}} k_{||}^2 f_{\vec{p}}(1 - f_{\vec{p}+\vec{k}})\delta(E_{\vec{p}} - E_{\vec{p}+\vec{k}}) , \qquad (C1)$$

where $f_{\vec{p}}$ is the Fermi function and $E_{\vec{p}} = p^2/2m^*$. From the definition $\vec{k} = \vec{q} - \vec{Q}$, one has to lowest order

$$\rho_{||} \propto \frac{m^*}{(k_F)_{||}} \int d^3p \, d^3q \Gamma_q (q_{||} - Q)^2$$
$$\times f_{\vec{p}}(1 - f_{\vec{p}-\vec{Q}+\vec{q}})\delta(2\delta p_{||} + q_{||}) , \qquad (C2)$$

where $\delta p_{||} = p_{||} - (k_F)_{||}$. The leading term in Eq. (C2) is

$$\rho_{||} \propto |\vec{Q}| \int d^3p \, dq_{||} \, dq_\perp \, q_\perp \Gamma_q f((k_F)_{||} \delta p_{||})$$
$$\times [1 - f((- k_F)_{||}(q_{||} + \delta p_{||}))]\delta(2\delta p_{||} + q_{||}) , \qquad (C3)$$

and thus

$$\rho \propto \tfrac{1}{2} Q \int p_\perp \, dp_\perp \, q_\perp \, dq_\perp \, dq_{\shortparallel} \Gamma_q f(-(k_F)_{\shortparallel} q_{\shortparallel})$$

$$\times [1 - f(-(k_F)_{\shortparallel} q_{\shortparallel})] . \tag{C4}$$

The Fermi function imposes a width on the $q_{\shortparallel}$ integration:

$$q_{\shortparallel} \lesssim 2m^* k_B T/(k_F)_{\shortparallel} = k_{th}^2/(k_F)_{\shortparallel} , \tag{C5}$$

where k_B is the Boltzmann constant.

Thus the integration on $\vec{q}$ is effectively three-

dimensional, and the integral has energylike divergence for

$$\kappa < k_{th}^2/(k_F)_{\shortparallel} . \tag{C6}$$

This defines a much narrower energylike region than the one we encountered for ferromagnetic semiconductors. When Eq. (C6) does not hold, the integral of $\vec{q}$ in Eq. (C4) is essentially two dimensional, and therefore $\rho_{\shortparallel}$ will be proportional to t^ν. Previous authors[18] only consider this regime and disregard the transition to energylike indices.

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