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Phonon-induced increase in the energy gap of superconducting films*

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We have observed large increases in the critical current of Josephson junctions under 10-GHz phonon excitation. Eliashberg has predicted that absorption of microwaves in a superconducting film will increase the energy gap by creating a nonthermal quasiparticle distribution. We modify this theory for *phonon* absorption and calculate the increase in the critical current of a Josephson junction corresponding to the increase in the energy gap. We obtain good agreement between our data and the modified Eliashberg theory. We conclude that absorption of phonons of energy $\hbar\omega < 2\Delta(T)$ increases the superconducting energy gap.

I. INTRODUCTION

In 1970 Eliashberg[1] suggested that absorption of microwave electromagnetic radiation in a superconducting film would increase the energy gap of the superconductor through creation of a nonthermal quasiparticle distribution. Ivlev and Eliashberg,[2] Eliashberg,[3] and Ivlev, Lisitsyn, and Eliashberg[4] (ILE) later derived expressions from which the microwave-induced increase of the energy gap could be calculated. Eliashberg suggested that if a Josephson junction were used as a probe of this new state, the critical current of the junction would be increased.

A number of investigators[5-13] have reported small increases in the critical current of microbridge Josephson junctions under low-power microwave excitation. However, there is wide variation in the reported magnitudes, temperature, and rf power dependence of the enhanced critical current. Several of these authors have suggested the ILE gap-enhancement mechanism as the explanation for their results. However, no detailed comparison has been made between the observed enhancement of the critical current and the enhancement calculated on the basis of the ILE theory. Such a comparison would be of questionable value owing to the presence in these experiments of other effects which significantly influence the critical current of a Josephson junction. These effects include the microwave-induced suppression of fluctuations,[13-15] which increases the measured critical current, and the direct effect of rf on the Josephson junction,[16] which decreases the critical current and also produces current steps in the I-V characteristic. The critical current of a Josephson junction under microwave excitation thus represents a superposition of these effects, making isolation of the ILE mechanism difficult. This may account for the wide variation in the magnitude, and temperature and power dependences of the enhancement of the critical current in microbridges reported by different observers and for the lack of observations of any rf-induced enhancement of the critical current in point-contact or tunnel junctions. In the Appendix we examine further observations of rf-induced enhancement of the critical current along with the relevant theory.

We have conducted a series of experiments in which we investigated the response of both point-contact and microbridge Josephson junctions to pulsed 10-GHz microwave *phonons*. In a previous publication[17] we reported large increases in the critical current of both types of junctions under phonon excitation. The measured increases in the critical current were compared with those calculated on the basis of the ILE theory modified to reflect the phonon coherence factor. Good agreement was obtained between theory and experiment. Owing to the absence of other effects which influence the critical current of the Josephson junction,[18,19] such as those mentioned in connection with microwave excitation, a direct comparison between the observed enhancement of the critical current and the ILE prediction is possible for phonon excitation.

In the present article we give details of calculations of the increase of the energy gap of a superconducting film under phonon excitation and the resulting enhancement of the critical current of the Josephson junctions used as probes of the film. We also describe the experimental methods used in obtaining these data. In Sec. II the ILE theory modified for the phonon coherence factor is outlined, and expressions are obtained for the increase in the energy gap, the relaxation time of the excited quasiparticles, and the enhancement of the critical current of a Josephson junction corresponding to the increase in energy gap. In Sec. III the experimental arrangement is discussed and data presented on the response of the junction to phonons and to microwaves. In Sec. IV the temperature dependence, phonon power dependence, and the magnitude of the phonon-induced enhancement of the critical current is compared to the predictions of the ILE theory. In Sec. V we present our conclusions. Observations of the rf-induced enhancement of the critical current by other workers are discussed in the Appendix along with the relevant theory.

II. THEORY

A. Physical basis of ILE theory

The ILE theory considers the effect of *microwaves* on the quasiparticle distribution function and energy gap of a thin superconducting film. The experiments reported in this publication concern the effect of *phonon* absorption on a superconducting film. However, the ILE theory may readily be modified to account for phonon absorption. In the BCS theory the energy gap Δ is given by

$$\Delta = \lambda \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{(\epsilon^2 - \Delta^2)^{1/2}} [1 - 2f(\epsilon)] \ , \qquad (1)$$

where λ is a coupling coefficient depending on the superconductor, ω_D is the Debye frequency, and $f(E)$ is the quasiparticle distribution function.

In equilibrium $f(\epsilon)$ is given by the Fermi function. However, if radiation of frequency $\omega < 2\Delta/\hbar$ is applied to the film, the quasiparticles will be excited to higher energies while leaving the total number of quasiparticles unchanged. The quasiparticles will then relax in a time τ ($\sim 10^{-7}$ sec for aluminum[20]) by phonon emission, with these relaxation phonons readily escaping from the film. For small deviations of $f(\epsilon)$ from thermal equilibrium we can approximate this state of dynamic equilibrium by a nonthermal distribution function $f'(\epsilon)$. The new distribution function $f'(\epsilon)$ will have the same total number of quasiparticles as $f(\epsilon)$ but the center of gravity of $f'(\epsilon)$ will be shifted to higher energies relative to $f(\epsilon)$. At these higher energies the term $1/(\epsilon^2 - \Delta^2)^{1/2}$ in the gap integral (1) is smaller, hence the energy gap will be increased. The maximum gap of the excited film, obtained by neglecting $f(\epsilon)$ in the gap integral, is the zero-temperature gap Δ_0. However, many other effects would intrude at the power levels required to increase the gap to a value approaching Δ_0.

If $\hbar\omega > 2\Delta(T)$, the radiation will not only excite quasiparticles but also will break Cooper pairs. The pair-breaking process increases the number of quasiparticles, thereby tending to destroy the superconducting state. If a distribution $f'(\epsilon) > f(\epsilon)$ is substituted in the gap integral, it can be seen that the gap will tend to decrease.[21]

B. Calculation of energy gap

In principle one could determine the energy gap of the superconducting film under microwave excitation by using the gap equation (1) with an appropriate distribution function $f'(\epsilon)$. In practice, the equation is difficult to solve. However, for temperatures near T_c and for small deviations from thermal equilibrium the problem may be simplified. ILE assume that the new distribution $f'(\epsilon)$ is a small perturbation on the original distribution $f(\epsilon)$:

$$f'(\epsilon) = f(\epsilon) + f_1(\epsilon), \quad f_1(\epsilon) \ll f(\epsilon) \ . \qquad (2)$$

For temperatures in the vicinity of T_c, ILE rewrite the gap integral:

$$0 = \frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8\pi^2} \frac{\Delta^2}{T_c^2} + \frac{\Delta}{T_c} F \ ,$$

where (3)

$$F = -\frac{2T}{\Delta} \int_{\Delta}^{\infty} \frac{dE}{(E^2 - \Delta^2)^{1/2}} f_1(\epsilon) \ .$$

Thus, to calculate the change in the energy gap the change in the distribution function $f_1(\epsilon)$ must be determined.

The new distribution function represents a dynamic balance between the rate at which quasiparticles are excited to a state ϵ and the rate at which the number of quasiparticles in state ϵ relaxes to thermal equilibrium. The rate at which the number $f'(\epsilon)$ of quasiparticles in state ϵ returns to the thermal equilibrium number $f(\epsilon)$ is proportional to the deviation from equilibrium $f_1(\epsilon)$ times a relaxation rate $\gamma(\epsilon)$. Expressing the rate at which quasiparticles in ϵ return to equilibrium as $I(\epsilon)$, we have

$$I(\epsilon) = 2\rho(\epsilon)\, \gamma(\epsilon) f_1(\epsilon) \ , \qquad (4)$$

where $\rho(\epsilon)$ is the superconducting density of states and $\gamma(\epsilon)$ the relaxation rate, to be discussed in Sec. II C. In some instances the relaxation rate $\gamma(\epsilon)$ may not vary greatly over the range of quasiparticle energies and over the temperature range of interest. In such a case the relaxation rate may be approximated by a constant relaxation rate $\gamma = 1/\tau$. This approximation is made by ILE and will be discussed in Sec. II C.

The relaxation rate is to be balanced against the rate at which quasiparticles are excited into the state ϵ by the applied radiation. Three such processes are possible with absorption of radiation of energy $\hbar\omega$. The first process is excitation from $\epsilon - \hbar\omega$ to ϵ, the second is excitation out of ϵ to $\epsilon + \hbar\omega$, and the third is the breaking of a Cooper pair with the creation of quasiparticles in states ϵ and $\hbar\omega - \epsilon$. This last process is possible only for $\hbar\omega > 2\Delta(T)$.

ILE use transition rates for absorption of *microwaves*. In the present experiment we are concerned with *phonon* absorption and so we modify the ILE theory by using the BCS coherence factor appropriate to absorption of longitudinal *phonons*, i.e., $(\epsilon\epsilon' - \Delta^2)/\epsilon\epsilon'$. We obtain the total transition rate into ϵ by summing the three processes:

$$I(\epsilon) = 2\Gamma_0\left(\frac{\epsilon(\epsilon+\omega)\pm\Delta^2}{(\epsilon^2-\Delta^2)^{1/2}[(\epsilon+\omega)^2-\Delta^2]^{1/2}}[f(\epsilon+\omega)-f(\epsilon)]\,\theta(\epsilon-\Delta)+\frac{\epsilon(\epsilon-\omega)\pm\Delta^2}{(\epsilon^2-\Delta^2)^{1/2}[(\epsilon-\omega)^2-\Delta^2]^{1/2}}\right.$$
$$\left.\times[f(\epsilon-\omega)-f(\epsilon)]\,\theta(\epsilon-\omega-\Delta)+\frac{\epsilon(\epsilon-\omega)\pm\Delta^2}{(\epsilon^2-\Delta^2)^{1/2}[(\epsilon-\omega)^2-\Delta^2]^{1/2}}[f(\omega-\epsilon)+f(\epsilon)-1]\,\theta(\omega-2\Delta)\right), \tag{5}$$

where $\hbar$ has been suppressed for clarity. The plus sign is for microwaves and the minus for phonon absorption. For phonon excitation Γ_0 is the coefficient of the normal-state transition rate for phonon absorption, and is proportional to the total phonon power.

Equating (4) and (5) to obtain $f_1(\epsilon)$ and substituting for $f_1(\epsilon)$ in (3), we obtain the following expression for the term F in (2):

$$F = -\frac{2T\Gamma_0}{\Delta}\left(\int_{\Delta}^{\infty}\frac{d\epsilon}{\gamma(\epsilon)}\,\frac{\epsilon(\epsilon+\omega)\pm\Delta^2}{\epsilon(\epsilon^2-\Delta^2)^{1/2}[(\epsilon+\omega)^2-\Delta^2]^{1/2}}[f(\epsilon+\omega)-f(\epsilon)]\right.$$
$$+\int_{\Delta+\omega}^{\infty}\frac{d\epsilon}{\gamma(\epsilon)}\,\frac{\epsilon(\epsilon-\omega)\pm\Delta^2}{\epsilon(\epsilon^2-\Delta^2)^{1/2}[(\epsilon-\omega)^2-\Delta^2]^{1/2}}[f(\epsilon-\omega)-f(\epsilon)]$$
$$\left.+\int_{\Delta-\omega}^{-\Delta}\frac{d\epsilon}{\gamma(\epsilon)}\,\frac{\epsilon(\epsilon+\omega)\pm\Delta^2}{\epsilon(\epsilon^2-\Delta^2)^{1/2}[(\epsilon+\omega)^2-\Delta^2]^{1/2}}[1-f(\epsilon)-f(\epsilon+\omega)]\right). \tag{6}$$

The integrals can be reduced to forms suitable for numerical integration by using straightforward substitutions to eliminate the singularities.[22] Equations (1) and (6) can then be solved by successive approximations for the energy gap of the superconducting film under phonon excitation.

The use of the phonon coherence factor in place of the photon coherence factor introduces substantial differences in the predicted increase of the energy gap. The phonon coherence factor $(1-\Delta^2/\epsilon\epsilon')$ is very large for pair breaking ($\epsilon<0$, $\epsilon'>0$). As a result, for $\hbar\omega>2\Delta(T)$ the decrease in the gap due to the pair-breaking term in (6) dominates the effect of the increase in the gap due to the nonthermal distribution created by the quasiparticle excitation terms in (6) and hence, even at relatively low phonon power levels, drives the film normal. On the other hand, the photon coherence factor $(1+\Delta^2/\epsilon\epsilon')$ is small for pair breaking at frequencies not too much greater than $2\Delta(T)$. Consequently, for low microwave frequencies ($\sim<35$ GHz) the gap is increased for temperatures up to T_c, and even for a small range of temperatures above T_c.

Several approximations have been used in obtaining Eq. (6). It was assumed that the thermal distribution $f(\epsilon)$ could be used in the calculation of the new distribution $f'(\epsilon)$; this approximation will hold only for the low-power limit $f_1(\epsilon)\ll f(\epsilon)$. The condition for the low-power limit is $\Gamma_0\tau\ll 1$. Multiple excitations were also neglected; this assumption is also valid in the low-power limit. Finally, the direct effect of the sound wave on the gap was neglected; this effect is small in comparison to that of a nonthermal distribution since $\omega\tau\gg 1$, where τ is the quasiparticle relaxation time.

The calculations also assumed that the phonons emitted in the relaxation of the excited quasiparticles are not reabsorbed, and hence do not lead either to additional excitation of quasiparticles or to creation of new quasiparticles. Because the experiments described herein were conducted at temperatures close to T_c, the frequencies of the relaxation phonons are low and so the phonons have a long mean free path (~30 000 Å for relaxation phonons of frequency 30 GHz). Thus, for thin aluminum films the relaxation phonons readily escape from the film into the substrate and helium bath. For thicker films or for materials with a stronger electron-phonon interaction this assumption would break down. We have performed experiments with 60 000-Å-thick aluminum films and observed a *decrease* in the critical current under 10-GHz phonon excitation caused by partial thermalization of the phonon pulse.

C. Relaxation time

The new distribution and the corresponding increase in energy gap depend not only on the rate at which quasiparticles are excited into state ϵ by the incoming phonons, but also on the rate at which they decay out of ϵ. This decay rate is the relaxation rate $\gamma(\epsilon)$ in (6). The relaxation rate $\gamma(\epsilon)$ is mainly due to processes involving phonon emission or absorption.[23] There are three processes by which the quasiparticle may leave state ϵ. In the first, the quasiparticle recombines with another quasiparticle to form a pair, with emission of a phonon $\hbar\Omega > 2\Delta(T)$. In the second, the quasiparticle relaxes to a lower energy (usually close to the gap edge $\epsilon = \Delta$) with emission of a phonon $\hbar\Omega < \epsilon - \Delta(T)$. In the third, the quasiparticle absorbs a thermal phonon, thus being excited to $\epsilon + \hbar\Omega$.

Expressions have been given for the relaxation rate $\gamma(\epsilon)$ due to these processes by Tewordt.[24] Suppressing the $\hbar$ for clarity,

$$\gamma(\epsilon) = \frac{e^2 m}{2p\omega_p^2}[f(-\beta\epsilon)]^{-1}\Bigg[\int_{\epsilon+\Delta}^{\infty}\frac{d\Omega\,\Omega^2(\Omega-\epsilon)}{[(\Omega-\epsilon)^2-\Delta^2]^{1/2}}\left(1+\frac{\Delta^2}{\epsilon(\Omega-\epsilon)}\right)f(\beta(\Omega-\epsilon))[1+N(\beta\Omega)]$$
$$+\int_0^{\epsilon-\Delta}\frac{d\Omega\,\Omega^2(\epsilon-\Omega)}{[(\epsilon-\Omega)^2-\Delta^2]^{1/2}}\left(1-\frac{\Delta^2}{\epsilon(\epsilon-\Omega)}\right)[1-f(\beta(\epsilon-\Omega))][1+N(\beta\Omega)]$$
$$+\int_0^{\infty}\frac{d\Omega\,\Omega^2(\epsilon+\Omega)}{[(\epsilon+\Omega)^2-\Delta^2]^{1/2}}\left(1-\frac{\Delta^2}{\epsilon(\epsilon+\Omega)}\right)f(-\beta(\epsilon+\Omega))N(\beta\Omega)\Bigg], \quad (7)$$

where ω_p is the ion-plasma frequency, p the electron momentum, and $N(\beta\Omega)$ the phonon distribution function $(e^{\beta\Omega}-1)^{-1}$. These integrals can be reduced to forms suitable for numerical integration by substitutions similar to those used for (6). Since the thermal distribution function $f(\epsilon)$ has been used in calculating the relaxation rate, the expression is valid only in the low-power limit, $f_1(\epsilon) \ll f(\epsilon)$.

ILE have assumed a relaxation time which is independent of energy and temperature in their calculations. Our calculations for aluminum indicate that the use of a constant relaxation rate $\gamma = 1/\tau$ yields approximately the same temperature and phonon-power dependences of the critical current (within 15%) as the use of the full relaxation rate (7) over the temperature range of these experiments except for those temperatures very near T_c ($T/T_c > 0.99$). This approximation may not be valid for other materials, such as tin, owing to the larger range of quasiparticle energies at the higher temperature.

D. Enhancement of critical current in Josephson junctions

The ILE theory gives expressions for the increase in the superconducting energy gap in a thin film due to applied radiation. In the experiments described here, a Josephson junction is used as a probe of the film under *phonon* excitation.[25] The increase in the energy gap of the film is observed as an increase in the critical current in the Josephson junction.

The data in these experiments consist of measurements of the enhancement of the critical current in microbridge and point-contact Josephson junctions as functions of temperature and phonon power. In comparing these data to the modified ILE theory the dependence of the critical current on the energy gap must be determined. The Josephson junctions employed in our work exhibited a critical current which depended on temperature as $(1-T/T_c)^\alpha$, where $1 \leq \alpha \leq \frac{3}{2}$. Many of the junctions displayed a temperature dependence which was close to one of the two extremes, $\alpha = 1$ or $\alpha = \frac{3}{2}$. The two cases have been treated theoretically by Christiansen, Hansen, and Sjöstrom[14] (CHS) and have been observed in tin microbridges by Jahn and Kao,[9] and by Song and Rochlin.[26] The CHS calculations were based on the one-dimensional Ginzburg-Landau equations for a thin-film weak link, where the length of the weak link is less than the coherence length ξ_0. This condition is easily satisfied in the aluminum junctions used in our experiments. The two temperature dependences of the critical current arise from the boundary conditions placed on the order parameter at the ends of the weak-link region. CHS found the critical current to be

$$I_c = \frac{k\sigma}{2e\mu_0}\left(\frac{\lambda}{\sqrt{2}}\right)^{-2\alpha}(kL)^{2\alpha-3}, \quad (8)$$

where k is the Ginzburg-Landau parameter, σ is the effective cross-sectional area, L is the length of the bridge, and λ is the penetration depth. The dependence of the penetration depth on the energy gap for an impure ($l \ll \xi_0$) superconductor is given by Miller[27] as

$$\lambda(T) = \lambda_L(0)\,(2\hbar v_F k T_c/\pi l)^{1/2}\,\Delta(T)^{-1}\ , \qquad (9)$$

where l is the mean free path and $\lambda_L(0) = (mc^2/4\pi ne^2)^{1/2}$. Thus, for temperatures near T_c, I_c increases as $(\Delta/\Delta_0)^{2\alpha}$. For junctions in which I_c is proportional to $(1 - T/T_c)$ the critical current should increase as Δ^2 while for junctions in which I_c is proportional to $(1 - T/T_c)^{3/2}$ the critical current should increase as Δ^3.[28]

III. EXPERIMENTAL RESULTS

A. Experimental arrangement

The experimental arrangement, shown in Fig. 1, comprised on x-cut quartz rod (3 cm length, 0.4 cm diam), one end of which was inserted into a reentrant 10-GHz microwave cavity; the opposite end contained either a point-contact or weak-link microbridge Josephson junction. Phonons were generated by the piezoelectric conversion of pulsed microwaves from a magnetron oscillator. The sound pulse, of about 1-μsec duration, was reflected back and forth by the rod end faces. The echoes gave rise to microwave pulses in the cavity by the reverse piezoelectric effect and were monitored by a superheterodyne receiver. Maximum microwave power input into the cavity was about 300 W, although the data reported here were taken at considerably lower power levels. The conversion efficiency of microwave to sound power was about 0.1%.

The point-contact junctions were formed between an aluminum film evaporated on the end face of the quartz rod and an adjustable tin point mounted normal to the film. Phonon-induced enhancement of the critical current was observed in film thicknesses from 600 to 20 000 Å; however, film thickness of 6400 Å $\sim \lambda_{\text{sound}}$ were normally used. Contact areas between point and film were typically a few microns in diameter.

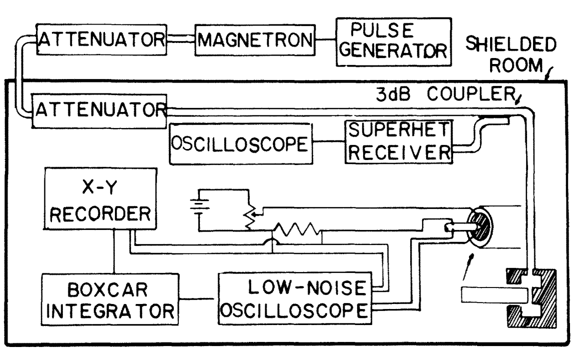

FIG. 1. Block diagram of experimental arrangement.

The microbridge junctions were formed from 1500- to 2000-Å-thick aluminum films evaporated on the rod end face. The fabrication technique was essentially that of Gregers-Hansen and Levinsen.[7] Microbridge dimensions were 1 μm long by $\frac{3}{4}$ μm wide, and normal-state junction resistances were ~1 Ω. Junctions made of thinner films were impractical as they burned out easily, whereas junctions formed from films thicker than 2000 Å proved difficult to fabricate.

Both types of junction were constant-current biased. Thus, the arrival of the phonon pulses at the junction was seen as a series of voltage pulses due to the switching of the junction's I-V curve from the unperturbed state to the I-V curve under phonon excitation. The voltage across the junction was monitored by a preamplifier and a boxcar integrator. The gate of the boxcar integrator could be timed to coincide with the arrival of a phonon pulse, thereby permitting the I-V curve of the junction to be measured under phonon excitation.

B. Junction response to phonons

Results are presented here for two microbridge junctions and two point-contact junctions. Data on phonon-induced enhancement of the critical current have been obtained for seven point-contact and five microbridge junctions; all show behavior similar to the junctions reported in this publication.

A portion of a typical longitudinal echo train is

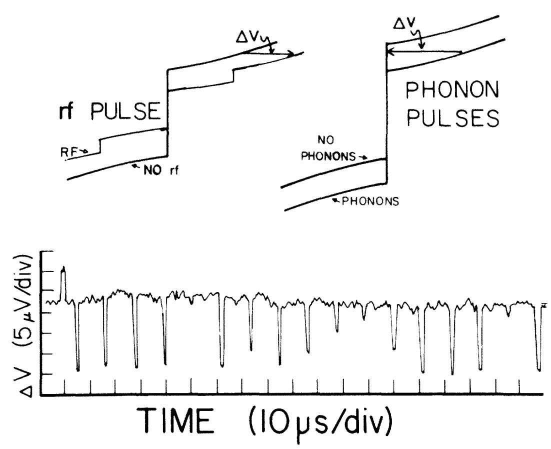

FIG. 2. Echo pattern obtained by monitoring junction voltage as a function of time. First pulse is due to leakage of rf from the cavity during the initial microwave pulse; remaining pulses are phonon echoes. Inserts show schematic I-V curves of unperturbed junction and junction under phonon and under rf excitation. Arrows show the voltage change observed in the echo pattern.

shown in Fig. 2. The data were obtained from a point-contact junction at $T/T_c = 0.96$. The junction was constant-current biased such that $V(I_{\text{bias}}) = 20$ μV. The first pulse is rf feedthrough, caused by leakage of rf from the cavity during the microwave pulse. As indicated in the insert, the microwave pulse decreased the critical current, producing a positive voltage pulse. The latter pulses are due to the phonon echoes. As indicated in the insert, the phonons increased the critical current, producing a negative voltage pulse as the junction switched from $V(I_{\text{bias}})$ to $V=0$. The data in this paper were obtained by timing the gate of the boxcar integrator to coincide with the first phonon pulse.[29]

The temperature dependence of the response was as follows. For $T - T_c < 0.005$ °K the phonons caused both types of junction to become normal. Abruptly at $T/T_c = 0.996$ the critical current, instead of vanishing, was observed to increase substantially under phonon excitation. This phonon-induced enhancement of the critical current was observed to the lowest temperature attainable in this experiment ($T = 0.9$ °K).

Figure 3 shows current-voltage characteristics of a point-contact junction at $T = 0.97 T_c$ with and without longitudinal phonon excitation. The enhancement of the critical current is about 450%; larger relative enhancements were observed at higher temperatures and higher phonon power levels. Microbridge Josephson junctions displayed phonon-induced enhancements of the critical current of about the same magnitude.

It was also possible to explore the response of the junctions to transverse phonons at 10 GHz by

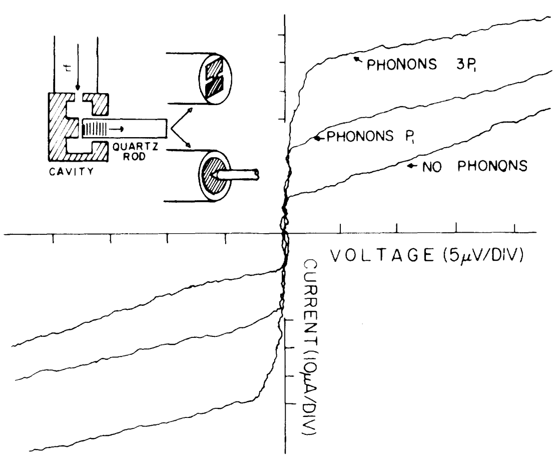

FIG. 3. I-V characteristics of a point-contact Josephson junction at $T = 0.97\ T_c$ for two phonon power levels, and without phonons. The rounding of the critical current is a result of variations in phonon power during the pulse. Insert shows experimental arrangement.

using a reentrant cavity with a wedge-shaped post. Substantial increases of the critical current were likewise observed for transverse-phonon excitation. This observation is consistent with the ILE mechanism, although a detailed comparison between the ILE theory and experiment would have to take into account the proper coherence factor for transverse-phonon absorption.

In contrast to the very large enhancements of the critical current observed for point-contact and microbridge junctions under *phonon* excitation, *microwave* excitation produced no enhancement of the critical current in point-contact junctions and only small enhancements in some microbridges. Since both point-contact and microbridge junctions used films, on the basis of the ILE theory alone one would expect to see microwave-induced enhancement of the critical current in both types of junction such as is observed with phonon excitation.

The maximum microwave-induced enhancement of the critical current in the microbridges for $T/T_c = 0.97$ was 4%, as compared to over 400% observed for phonon excitation at the same temperature. This microwave-induced enhancement was observed at low rf power levels. At higher powers the critical current decreased and the I-V characteristics displayed the usual rf-induced steps which oscillated in rf power as described by previous investigators.[7] We would suggest that the decrease in the critical current due to the direct effect of microwaves on a Josephson junction occurs at much lower microwave power levels than the increase in the critical current caused by the absorption of microwaves predicted by the ILE theory, thereby rendering the latter effect difficult to observe. However, by using phonons, the enhancement of the critical current predicted by ILE becomes observable.

In addition to the data obtained with phonon excitation only, data were collected with simultaneous excitation by phonons (pulsed) and microwaves (continuous at low power). A typical I-V curve is shown in Fig. 4(a). Both the critical current and the microwave-induced current steps are enhanced by the phonons. The increase in the amplitudes of the rf steps is a consequence of the increase in the critical current, as shown in the models of a Josephson junction of Fack and Kose[30] and Russer.[31] In these models the junction is treated as a resistively shunted Josephson junction, and the equations are solved by analog-computer simulation. Russer's results indicate that as the critical current increases, the step amplitudes also increase, though the relative increase in the step amplitudes is not as large as that of the critical current. Moreover, the rf voltages at which the step amplitudes go to zero increase.

Figure 4 shows the amplitudes of the $n = 0$, 1, and 2 steps as a function of V_{rf} = (microwave power)$^{1/2}$ for a point-contact junction with and without phonon excitation. The data were taken from a different junction and at a lower temperature than the I-V curve in Fig. 3(a). The critical current is increased by the phonon excitation. Also, the step amplitudes are increased and the zeros of the step amplitudes are moved farther out on the V_{rf} scale. Both these latter effects are a consequence of the increased critical current, as predicted by Russer's calculations.

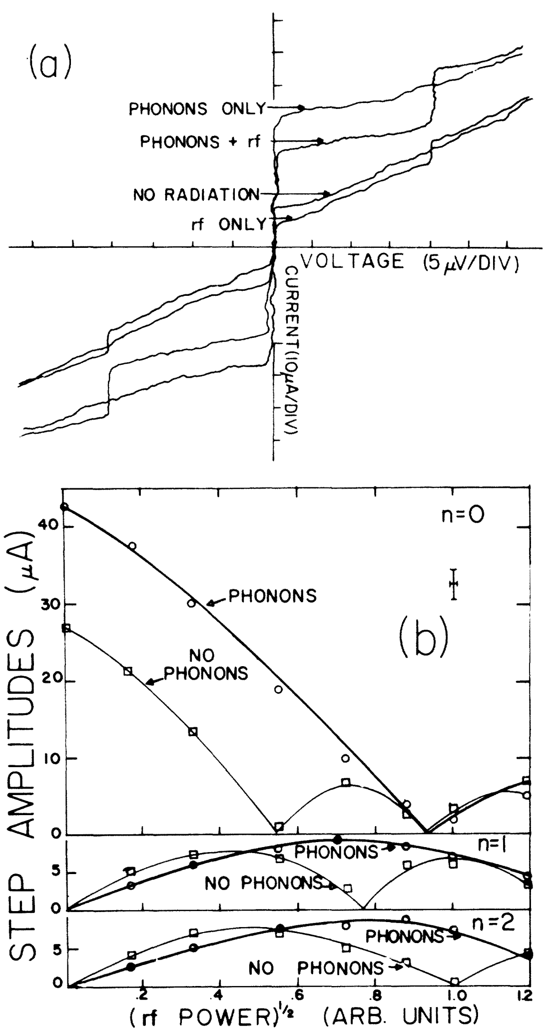

FIG. 4. (a) I-V characteristics of a point-contact junction at $T = 0.97T_c$ under simultaneous phonon (pulsed) and microwave (continuous at low power) excitation. (b) Step amplitudes of the $n = 0$, 1, and 2 microwave-induced steps with and without phonon excitation for a point-contact junction at $T = 0.94T_c$. Solid lines are drawn for clarity.

IV. COMPARISON WITH ILE THEORY

We propose that the phonon-induced increase in the critical current of our junctions can be explained by the ILE theory modified for phonon absorption. We consider the temperature and the phonon-power dependences of the enhancement of the critical current and the magnitude of the enhancement. Since the ILE theory contains assumptions which break down at high power levels, comparisons are made for relatively low phonon power levels.

A. Temperature dependence

Figure 5(a) shows the relative increase in the critical current of an aluminum microbridge as a function of temperature along with the results calculated using the ILE theory modified for phonon excitation. The microbridge had a critical current $I_0 \propto (1 - T/T_c) \propto (\Delta/\Delta_0)^2$. The adjustable parameter is the absolute phonon power present. For $T/T_c > 0.996$ the junction is driven normal. This is due to the onset of pair breaking by the phonons at $\hbar\omega > 2\Delta(T)$, as predicted by the ILE theory for phonon absorption. For $T/T_c < 0.996$

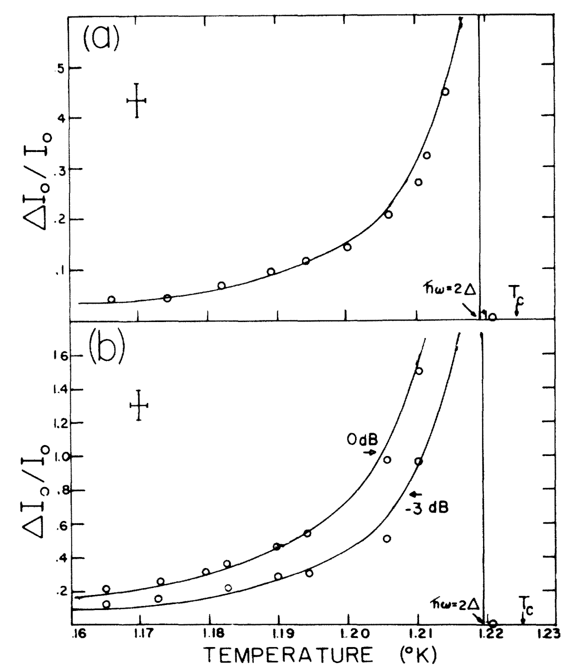

FIG. 5. (a) Relative increase in the critical current as a function of temperature for a microbridge with $I_0 \propto (1-T/T_c)$. Solid lines show prediction of ILE theory modified for phonon excitation. (b) Same as (a) for a microbridge with $I_0 \propto (1-T/T_c)^{3/2}$ and two phonon power levels, where P (0 dB) = $2P$ (− 3 dB). Fitting parameter was the absolute phonon power present.

pair breaking no longer takes place and the critical current is enhanced. At lower temperatures both the relative and the absolute increase in the critical current are smaller. The data follow the predictions of the ILE theory within experimental error.

A few of the microbridges displayed a critical current proportional to $(1 - T/T_c)^{3/2}$ for temperatures $T > 0.92\ T_c$. For this type of junction the critical current should increase as Δ^3 at a given temperature. Results for such a junction under phonon excitation appear in Fig. 5(b) for two phonon power levels 3 dB apart, along with results of calculations based on the ILE theory.

B. Power dependence

Figure 6 shows the power dependence of the relative increase in critical current $\Delta I_0/I_0$ along with results of calculations based on the ILE theory. The data were obtained from a point-contact junction at $T = 0.975 T_c$. At lower power levels the critical current increases linearly with power, as predicted by the calculations. At higher power levels the increase in critical current saturates.

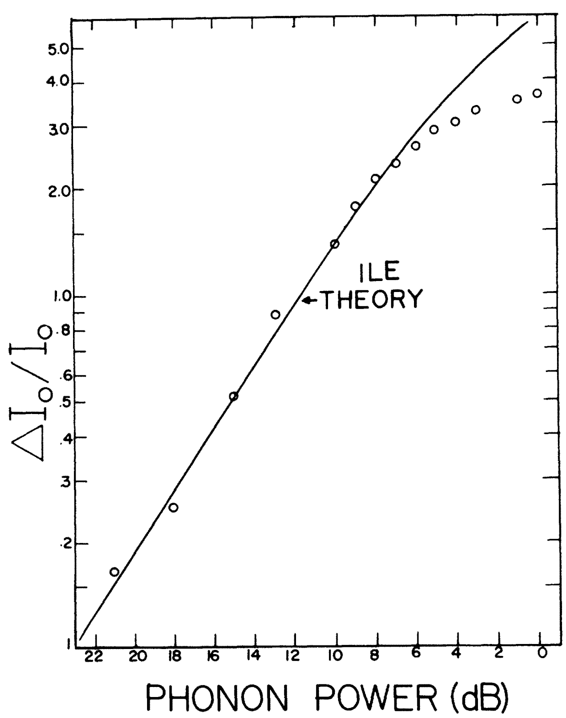

FIG. 6. Relative increase in critical current as a function of relative phonon power for a point-contact junction at $T = 0.97 T_c$. Solid line shows prediction of ILE theory. The fitting parameter was the absolute phonon power present at a low power level. Error of measurement on the current scale is ± 0.06.

At these high power levels the assumptions underlying the calculations are no longer valid.

The fitting parameter in matching the theoretical and experimental curves was the absolute phonon power present; this was matched to the data point at −15 dB. Similar agreement between the observed and calculated phonon-power dependence was obtained for other temperatures.

C. Magnitude of enhancement

It is possible to estimate the magnitude of the phonon-induced enhancement of the critical current predicted by the modified ILE theory and compare it with the enhancement obtained from the experiments. In comparing the theoretical results to the experimental data, the fitting parameter was the absolute phonon power present, which was proportional to $\Gamma_0 \tau_{av}$ (where Γ_0 is the coefficient of the transition rate for the absorption of the 10-GHz phonons and τ_{av} the average quasiparticle relaxation time). We wish to estimate Γ_0 from the acoustic power and compare this to the values of Γ_0 indicated by the observed enhancement of the critical current. We do this for the lower power curve of Fig. 3 as an illustration; other junctions showed similar agreement between the calculated and the observed enhancement of the critical current. We write Γ as the number of phonons per second absorbed in the film:

$$\Gamma = 4N(0) V \Gamma_0 \int_\Delta^\infty \left(1 - \frac{\Delta^2}{\epsilon(\epsilon + \omega)}\right) \rho(\epsilon)\rho(\epsilon + \omega) \times [f(\epsilon) - f(\epsilon + \omega)] d\epsilon ,$$

where $N(0)$ is the single spin density of states, V the volume of superconductor irradiated, and $\rho(\epsilon)$ the superconducting density of states. By estimating Γ from the acoustic power absorbed in the film, Γ_0 can be determined.

The I-V curve at the lower phonon power in Fig. 2 was taken at an rf power level 30 dB below the 9-kW output of the magnetron, or 9 W. Using a figure of 30 dB for the conversion efficiency of the cavity gives an acoustic power of 9 mW. Eagan and Garfunkel[32] measured the ultrasonic attentuation in aluminum. Using their value of the attenuation at $T = 0.97 T_c$, the sound power absorbed in the 6400-Å film is thus 0.457 mW or 0.61×10^{20} phonons/sec. Using this value for Γ, Eq. (10) can be solved for Γ_0, giving $\Gamma_0 = 3.9 \times 10^6$/sec.

The relaxation time can be estimated from the measurements of the recombination time in aluminum by Levine and Hsieh.[20] The data of Levine and Hsieh give a recombination time of 1.4×10^{-7}/sec at 1.21°K. About 70% of the total relaxation rate is due to recombination, so we estimate $\tau \sim 10^{-7}$ sec. Thus the value of $\Gamma_0 \tau$ estimated from the acoustic power and measurements of the recombination time is about 0.39.

Using Eq. (6) calculations were made to determine the enhancement of the critical current by the junction of Fig. 2. The value of $\Gamma_0 \tau_{av}$ estimated by matching these calculations to the observed enhancement was 0.20, well within the range of error of the estimates for τ and for the acoustic power present. The magnitude of the enhanced critical current of other junctions, both point contact and microbridge, showed agreement within a factor of 6 between the observed and predicted values. We conclude that the observed enhancement of the critical current is of the same magnitude as that predicted on the basis of the ILE theory.

V. CONCLUSIONS

In summary, we have observed large increases in the critical current in microbridge and point-contact Josephson junctions under phonon excitation. The magnitude, temperature dependence, and phonon-power dependence of this enhancement has been compared to that predicted on the basis of the Eliashberg theory modified for phonon excitation. The good agreement between theory and experiment indicates that the enhancement of the critical current is due to the phonon-induced enhancement of the energy gap. The gap-enhancement mechanism is the phonon analog of that predicted by Eliashberg for microwave absorption in a thin superconducting film. While microwave-induced enhancements of the critical current in microbridges have been reported by many investigators, the widely varying results suggest that the Eliashberg mechanism, if present, is difficult to isolate from the direct effect of rf on the junction. The use of phonons to create the nonthermal distribution has made it possible to isolate the ILE mechanism and to compare the ILE theory to experiment.

ACKNOWLEDGMENT

We thank Professor Sidney Shapiro for many encouraging and illuminating discussions concerning experimental and theoretical aspects of Josephson junctions.

APPENDIX: MICROWAVE-INDUCED ENHANCEMENT OF CRITICAL CURRENT

In this appendix we consider the observations of microwave-induced enhancement of the critical current. Enhancement of the critical current

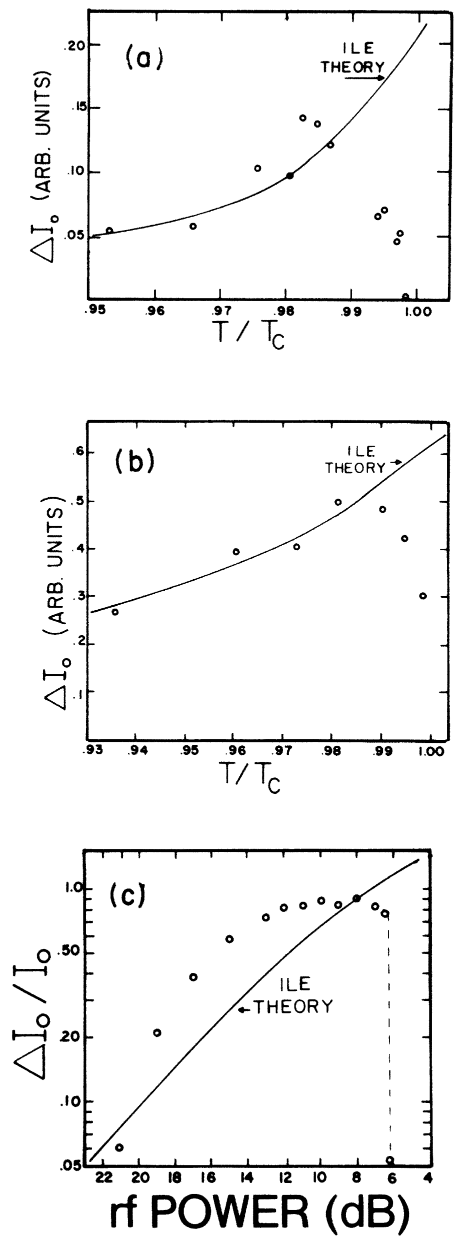

FIG. 7. (a) Change in critical current as a function of temperature for tin microbridge under 10-GHz microwave excitation. The change in critical current is normalized to the total critical current at $0.98T_c$. Data are from Levinsen (Ref. 11); solid line shows results of calculations based on the ILE theory. (b) Change in critical current as a function of temperature for tin microbridge under 10-GHz microwave excitation. Change in critical current is normalized to critical current at $0.98T_c$. Data are from Latyshev and Nad (Ref. 12); solid line shows results of calculations based on the ILE theory. (c) Relative change in critical current as a function of power for a tin microbridge under 53.5-GHz microwave excitation. Data are from Latyshev and Nad (Ref. 12) at $T = 0.978T_c$; solid line shows results of calculations based on the ILE theory.

was first observed by Wyatt *et al.*[5] in tin microbridges under 10-GHz microwave excitation. The enhancement was explored as a function of temperature, frequency, and bridge dimensions by Dayem and Wiegand.[6] Dayem and Wiegand observed enhancement of the critical current up to a temperature slightly below T_c, while Wyatt observed an enhancement in the critical current both below T_c and for a small range of temperatures above T_c.

Hunt and Mercereau[13] proposed that the enhancement of the critical current and transition temperature could be explained in terms of a suppression-of-fluctuations model. In this model the critical current is found to be suppressed below its thermodynamic equilibrium value by the presence of fluctuations due to noise, either k_BT noise or noise from the external circuit. The applied rf phase locks the junction, thereby negating the effect of noise on the critical current. Hunt and Mercereau obtained good agreement between their data on tin bridges and this model. The model was further developed by Dmitriev *et al.*,[16] who also obtained good agreement with data on tin microbridges. Christiansen, Hansen, and Sjöstrom[15] applied the one-dimensional Ginzburg-Landau equations to a microbridge and obtained expressions for the decrease in T_c due to fluctuations. Jahn and Kao[9] compared data from their experiments on enhancement of T_c in tin microbridges to the CHS theory and obtained good agreement.

In recent experiments Levinsen[11] and Latyshev and Nad[12] have observed increases in the critical current of microbridges under rf excitation which appear too large in magnitude to be explained by rf-induced suppression of fluctuations. Both authors have suggested the ILE mechanism as the explanation for their results. Figure 7(a) shows the increase in critical current as a function of temperature observed by Levinsen at 10 GHz and Fig. 7(b) shows that observed by Latyshev and Nad. Both sets of data were obtained from tin microbridges. The data are inconsistent in that Levinsen found that the critical current decreased to zero somewhat below T_c while Latyshev and Nad observe an increase in T_c. Similar disagreement exists between the two at higher frequencies; Latyshev and Nad observed a slight enhancement of T_c at 35 GHz while Levinsen found that 35-GHz microwaves drove the junction normal for a range of temperatures below T_c.

We have carried out calculations to determine the microwave-induced enhancement of the critical current predicted by the ILE theory. The calculations were similar to the calculations used for phonon excitation, except that the photon coherence factor was used instead of the phonon coherence factor in (6). Results of these calculations are shown as solid lines in Figs. 7(a) and 7(b). The calculations indicate that the critical current should increase at temperatures up to T_c and even for a range of temperatures above T_c. This is in contrast to the results for phonon excitation and is due to the difference between the phonon and photon coherence factors.

The wide variations among the observations of the temperature dependence of the enhancement of the critical current and the considerable departures from the ILE theory suggest that effects in addition to those considered by ILE are taking place.

There is similar disagreement in observations of the power dependence of the enhancement of the critical current. Dayem and Wiegand[6] report that the critical current at first increases with rf power and then decreases smoothly to zero. Latyshev and Nad[12] report an increase followed by a sharp cutoff at which the film is driven normal. The ILE theory predicts enhancement of the critical current at all rf power levels, although saturation occurs at high power levels. In the region of very high rf power one would expect the film to be driven normal. Figure 7(c) shows results on the power dependence of the enhancement in the critical current obtained by Latyshev and Nad for a tin microbridge under 53.5-GHz microwave excitation along with results of calculations based on the ILE theory (solid lines). The lack of agreement suggests that additional effects are taking place.

There is also wide variation in the reported magnitudes of the rf-induced enhancement of the critical current. It has been our experience, as well as that of other investigators, that junctions prepared in the same manner and of approximately the same dimensions display wide variations in the rf-induced enhancement of the critical current, with many junctions displaying no enhancement. Moreover, in the present work no rf-induced enhancement was noted for point-contact junctions using thin aluminum films, while most microbridge junctions did show a slight enhancement. On the basis of the ILE theory one would predict the same enhancement in both cases, such as is observed with phonons.

We would suggest that for microwave excitation the ILE mechanism, if present, is difficult to isolate from other effects of rf excitation of the junctions. Effects which would compete with the ILE mechanism include the direct effect of rf on the junction, which decreases the critical current and produces steps in the I-V characteristic, and the rf-induced suppression of fluctuations, which increases the critical current. These latter two effects are known to depend strongly on junction geometry, perhaps partially explaining the wide variation in the reported magnitude of the rf-induced enhancement of the critical current. We conclude that detailed comparison of the microwave-induced enhancement of the critical current with ILE theory is of limited validity due to the presence of other effects which influence the critical current of Josephson junctions.

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[18]In a previously reported experiment (Ref. 19) we observed a decrease in critical current under phonon excitation of point-contact junctions formed between very thin (< 500 Å $\ll \lambda_s$) aluminum films and niobium points. We suggested that the coupling between sound wave and junction occurred via the modulation of the chemical potential of the film by the sound wave. In the present publication we present results for thicker films ($d \sim \lambda_s \cong 6400$ Å). In these thicker films the absorption of the sound wave is much greater and the modulation of the chemical potential averaged over the film thickness much smaller. Thus we observe an increase in the critical current due to phonon absorption instead of a decrease in critical current caused by modulation of the chemical potential. The absence of steps in the I-V characteristics of these thicker junctions under phonon excitation (see Figs. 3 and 4) is an indication that any residual modulation of the chemical potential of the film must be of rather small magnitude.

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[21]ILE also find that there is a minimum frequency below which gap enhancement will not occur. This frequency is of order 1 GHz in aluminum, and is not of concern in this experiment.

[22]In the first integral we use the substitution $u^2 = \epsilon - \Delta$ and in the second $u^2 = \epsilon - \omega - \Delta$. The third integral is broken into two parts, $\epsilon = \Delta - \omega$ to $\epsilon = -\omega/2$ and $\epsilon = -\omega/2$ to $\epsilon = -\Delta$. In the first part the substitution $u^2 = \epsilon + \omega - \Delta$ is used and in the second $u^2 = -\epsilon - \Delta$. The integrals can then be solved numerically.

[23]Electron-electron interactions provide an additional relaxation mechanism. However, the relaxation rate due to this mechanism appears to be more than an order of magnitude lower than relaxation via phonon emission. Measurements or recombination times in aluminum give $\tau \sim 10^{-7}$ sec for phonon emission (Ref. 20). To our knowledge no measurements of relaxation time via electron-electron interactions have been made for aluminum. While comparisons of the relaxation rate via phonon scattering to that via electron scattering have been made by measuring the temperature dependence of the electrical resistivity, these relaxation times are not directly applicable to the present experiment. In electrical resistivity measurements small-angle electron-phonon scattering is not effective in increasing the resistivity; thus for electrical resistivity measurements at low temperatures the electron-phonon mechanism is considerably reduced and electron-electron interactions predominate. In the present experiment, however, absorption and emission of low-frequency phonons is an important relaxation mechanism. Calculations for the electron-electron relaxation rate have been made by Eliashberg (Ref. 3), with the relaxation time being of the order $E_F/\Delta^2 \sim 10^{-5}$ sec in aluminum. This is two orders of magnitude longer than the electron-phonon relaxation time; thus the use of the relaxation rate by phonon emission alone appears to be a reasonable approximation.

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[25]In principle one could use tunnel junctions or far-infrared transmission measurements as probes of this new state, thereby measuring the increased energy gap directly. However, the presence of phonon-assisted tunneling and rf standing waves in tunnel junctions excited by a sound wave greatly complicates such measurements. Furthermore, far-infrared transmission measurements under pulse conditions present many experimental difficulties.

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[28]The gap dependence of the critical current can be determined from other derivations of the critical current in a Josephson junction. For example, using a pure tunneling model, V. Ambegaokar and A. Baratoff [Phys. Rev. Lett. 11, 104 (1963)] find the critical current between identical superconductors: $J_c(T)=\frac{1}{2}\pi R_n^{-1}\Delta(T)\tanh[\Delta(T)/2k_BT]$, R_n is the normal-state resistance of the junction. For temperatures near T_c, $\Delta(T) << 2k_BT$ and $J_c \sim \Delta(T)^2$.

[29]It should be noted that the pulse heights in the echo pattern are not linear in the phonon power absorbed. The junction was constant-current biased about 20 μV above $V=0$; thus the maximum signal under phonon excitation was 20 μV. As a result the peaks in the echo pattern appear "chopped off" so that the decay of the echo pattern in Fig. 2 is not exponential. Some of the pulses adjacent to each other show considerable differences in amplitude. The reason for this variation is not entirely clear. We would suggest that stress or inhomogeneities in the quartz rod deflect the sound wave in an irregular way so that the acoustic power at the junction can vary considerably from echo to echo. Experiments using small-area bolometers have been carried out and likewise reveal irregular variations in echo height. The gate of the boxcar integrator, usually timed to coincide with the first pulse, has been positioned also on later pulses and I-V curves obtained. The I-V curves on the later pulses are qualitatively identical to those obtained from the first pulse, suggesting that the difference between pulses is one of local acoustic power only.

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