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Plasma oscillation of a charge layer at an insulator surface*

D. E. Beck and P. Kumar†

Physics Department and Laboratory for Surface Studies, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201

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The charged layers on a liquid-He surface and the inversion layers on a semiconductor-insulator interface exhibit properties which are treated by assuming that they are a two-dimensional electron gas. We have calculated the dispersion relation for the frequency of plasma oscillations taking into account the finite width and correlation effects. We find important deviations from the corresponding result for the two-dimensional electron gas.

I. INTRODUCTION

The electrons near the surface of a rare-gas liquid are free to move parallel to the surface, but they are trapped near the surface by the long-range attraction due to the image force.[1-3] The electrons in the inversion layer at a semiconductor also move freely parallel to the surface, but their motion perpendicular to the surface is restricted.[4] In both of these systems the electron motion perpendicular to the surface can be described by considering the states of an electron trapped in a one-dimensional potential well at the surface. Since the electron is localized in the direction perpendicular to the surface, many properties of these layers have been studied by assuming that they are a two-dimensional electron gas[5-7] (2-DEG). In Sec. II of this paper, we analyze the importance of the finite extension and the zero-point motion of the trapped electrons on the dynamical response of these systems. In particular, we obtain the dispersion relation for the plasma oscillations (LPO) in these systems and analyze the deviations from the corresponding relation for a 2-DEG.

The existence of the localized states on rare-gas liquids has been verified experimentally by noting the nature of the cyclotron resonance[8] and Stark effect.[9] Fetter[5] has calculated the thermodynamic properties of these layers and Platzman and Fukuyama[6] have studied their stability with respect to a Wigner transition by treating them as a 2-DEG.

Stern[7] and Fetter[5] have calculated the dynamic response of degenerate and nondegenerate 2-DEG. The long-wavelength dispersion of LPO in these layers is[10]

$$\omega_L^2 = (2\pi\bar{n}\, e_*^2/m)q(1+3\lambda q) + O(q^3)\ , \tag{1}$$

where $\bar{n}$ is the density per unit area of the electrons, m is the electron mass (effective mass parallel to surface for semiconductor), and $e_*^2 = 2e^2/(\epsilon_1+\epsilon_2)$. Here ϵ_1 and ϵ_2 are the dielectric constants of the rare-gas liquid and vacuum, respectively, or of the insulator and semiconductor, respectively. The two-dimensional momentum in the plane of the charge layer is q, and λ is a parameter which depends on the "statistical" properties of the system. For a system of degenerate electrons λ is proportional to the Thomas-Fermi screening length,[7] $2\lambda = \lambda_{\text{TF}} = E_F/2\pi\bar{n}e_*^2$, where E_F is the Fermi energy. For a classical system λ is the Debye screening length,[5] $\lambda = \lambda_D = k_BT/2\pi\bar{n}e_*^2$, where k_B is Boltzmann's constant and T is the temperature.

In Sec. II we consider the response function for an electron layer confined in a one-dimensional potential well and obtain the dispersion relation for the plasma oscillations of this layer. We are able to show that the leading term in Eq. (1) is exact and independent of the electron density profile of the layer, since it results from the localization of the electrons. (This distinguishes the dispersion relation from that found for an infinite or semi-infinite[11] system.)

In Sec. III we present a calculation of the effects of electron-electron correlation on the LPO dispersion in these electron layers. We use the correlation-hole approximation[12,13] to obtain an estimate of these correlation effects. Section IV contains a discussion and summary of our results.

II. PLASMA DISPERSION RELATION

We consider a charge layer which has a finite extension perpendicular to the surface (z direction) and is uniform parallel to the surface (x, y direction). It is then convenient to Fourier transform in the directions parallel to the surface and use a mixed representation for our calculations. In this representation the self-consistent equation for the density fluctuations is

$$\delta\rho_\omega(q,z) = \int_0^\infty dz' \int_0^\infty dz''\, \chi_\omega(q,z,z') \times v(q,z',z'')\,\delta\rho_\omega(q,z'')\ . \tag{2}$$

The inclusion of an external field requires that an additional term proportional to the field be added to the right-hand side of this equation.

The potential $v(q,z,z')$ in Eq. (1) results from the direct Coulomb interaction between the electrons and the contribution from image charges. In the mixed notation it is given by

$$v(q,z,z') = \frac{2\pi e^2}{\epsilon_2 q}\left(e^{-q|z-z'|} - \beta e^{-q|z+z'|}\right)\ , \tag{3}$$

where $\beta = (\epsilon_1 - \epsilon_2)/(\epsilon_1 + \epsilon_2)$. The density-density correlation function $\chi_\omega(q, z, z')$ is to be computed within the random-phase approximation and is given by

$$\chi_\omega(q,z,z') = \sum_{\vec{k}} \sum_{j,j'} \frac{f(\vec{k}+\vec{q}, j') - f(\vec{k}, j)}{E_{\vec{k}+\vec{q}, j'} - E_{\vec{k}, j} + \omega + i\eta} \times \phi_j(z)\phi^*_{j'}(z)\phi^*_j(z')\phi_{j'}(z') \; . \qquad (4)$$

Here $E_{\vec{k}, j} = E_{\vec{k}} - E_j$ ($\hbar = 1$) is the energy of an electron with momentum $\vec{k}$ parallel to the channel (all the vectors used in this paper are in the x-y plane) whose motion perpendicular to the channel is quantized with an eigenvalue E_j. The wave function for an electron with this energy is $\psi_{\vec{k}, j}(\vec{r}, z) = e^{i\vec{k}\cdot\vec{r}}\phi_j(z)/2\pi$, i.e., we have $E_{\vec{k}} = k^2/2m$ and the $\phi_j(z)$'s are the orthonormal eigenfunctions of the one-dimensional Schrödinger's equation corresponding to the eigenvalues E_j. The distribution of particles with energy $E_{\vec{k}, j}$ is given by the Fermi-Dirac distribution function $f(\vec{k}, j)$ (for the classical case this reduces to the Maxwell-Boltzmann distribution). It is also convenient for later calculations to separate χ_ω, Eq. (4), into two terms: A diagonal term which contains the intraband contributions ($j = j'$) χ^D_ω and a nondiagonal term which contains the interband contributions ($j \neq j'$) χ^N_ω.

The plasma dispersion relation is obtained by integrating both sides of Eq. (2) over z and using the orthogonality of the ϕ_j's. The only terms that contribute are those in χ^D_ω and one finds the dispersion relation

$$1 = \frac{1}{q} \sum_{\vec{k}, j} \frac{f(\vec{k}+\vec{q}, j) - f(\vec{k}, j)}{E_{\vec{k}+\vec{q}} - E_{\vec{k}} + \omega + i\eta} C_{\omega, j}(q) \; . \qquad (5)$$

Here we have set

$$C_{\omega, j}(q) = \int_0^\infty dz \, |\phi_j(z)|^2 \int_0^\infty dz' \, q v(q, z, z') \frac{\delta\rho_\omega(q, z')}{\bar{\delta}\rho_\omega} \, , \qquad (6)$$

where v is given by Eq. (3) and $\bar{\delta}\rho_\omega = \int_0^\infty dz \, \delta\rho_\omega(q, z)$. For a given q, Eq. (5) allows only specific values of $\omega = \omega_L(q)$ (in the remainder of this paper we require ω to have these values and drop the ω subscript on our functions), and in particular for a 2-DEG both $|\phi_j(z)|^2$ and $\delta\rho(z)$ are porportional to $\delta(z)$. Hence, $C_j(q) = 4\pi e^2/(\epsilon_1 + \epsilon_2)$ which gives the dispersion relation Eq. (1) obtained by Fetter[5] and Stern[7] for the nondegenerate and degenerate 2-DEG, respectively.

Note that $C_j(0) = 4\pi e^2/(\epsilon_1 + \epsilon_2)$ independently of $\phi_j(z)$ and ω. Hence to lowest order in q the plasma dispersion relation is the same as the 2-DEG result Eq. (1), where the surface density $\bar{n}$ is given by

$$\bar{n} = \int_0^\infty dz \, n(z) = \int_0^\infty dz \sum_{\vec{k}, j} f(\vec{k}, j) \, |\phi_j(z)|^2 \; .$$

The corrections due to the finite width of the layer appear only in the terms of higher order in q. This result is equivalent to the surface-plasmon result[14] that, as $q \to 0$, $\lim \omega_s(q) = \omega_p/\sqrt{2}$ independent of the electronic surface-density profile, where ω_p is the bulk plasmon frequency. However, in the surface-plasmon calculations the functions corresponding to the $\phi_j(z)$'s cannot be normalized, and the limit $q \to 0$ (which must be performed with great care[15]) results in a constant rather than a q-dependent leading term.

If the charge fluctuations are such that, as $q \to 0$, $\lim \bar{\delta}\rho = 0$, the analysis above fails. Fluctuations of this type correspond to higher "multipole" modes, and they have been studied for thin films[16] and surfaces.[17] The symmetry of the thin films allows the fluctuations to be separated according to their spatial character (even or odd) with respect to the midpoint of the film. The dispersion of the even modes is given by Eq. (1) with $\bar{n}$ replaced by $n_0 L$, where n_0 is the uniform bulk density of the film and L is its thickness. The odd modes correspond to a sloshing motion across the film and the limiting value of their dispersion corresponds to the bulk plasmon result $\omega_p^2 = 4\pi n_0 e^2/m$. We plan to report a microscopic study of the higher modes for a charge layer at a later date.

The remainder of this section is devoted to an evaluation of the corrections to the quadratic term in Eq. (1) for a layer electron gas. To accomplish this we need to expand Eq. (5) in powers of q. For $\omega/q > v_F + q/2m$ we have

$$\frac{f(\vec{k}+\vec{q}, j) - f(\vec{k}, j)}{E_{\vec{k}+\vec{q}} - E_{\vec{k}} + \tilde{\omega}} = f(\vec{k}, j) \frac{q^2}{m\tilde{\omega}^2} \left[1 + \frac{3q^2}{\tilde{\omega}^2 m^2} (\vec{k} \cdot \hat{q})^2 + O(q^4/\tilde{\omega}^4) \right] \, , \qquad (7)$$

where $\tilde{\omega} = \omega + i\eta$ and $\hat{q}$ is a unit vector parallel to $\vec{q}$. Expanding Eq. (6), one finds

$$C_j(q) = \frac{4\pi e^2}{\epsilon_1 + \epsilon_2} - \frac{2\pi e^2 q}{\epsilon_2} \int_0^\infty dz \, |\phi_j(z)|^2 \times \int_0^\infty dz' (|z - z'| - \beta |z + z'|) D(0, z) + O(q^2) \, , \qquad (8)$$

where

$$D(q, z) = \delta\rho_{\omega_L(q)}(q, z)/\bar{\delta}\rho_{\omega_L(q)} \; .$$

This function $D(q, z)$ satisfies an integral equation which is obtained from Eq. (2) by replacing ω by $\omega_L(q)$.

To further simplify the calculation, we shall assume that the electrons occupy only the lowest subband ($j = 1$) which limits our considerations to systems whose characteristic energies ($k_B T$ and E_F for the classical and quantum cases, respectively) are less than the level separation, Δ_{12} ($\Delta_{1j} = |E_j - E_1|$). Using Eq. (5) to rewrite the diagonal term in Eq. (2), we then have

$$D(0,z)=\frac{n(z)}{\bar{n}}-\frac{2\pi e^2}{\epsilon_2}\int_0^\infty dz'\,g(z,z')\times\int_0^\infty dz''\left(|z'-z''|-\beta|z'+z''|\right)D(0,z'') , \tag{9}$$

where

$$\begin{aligned}g(z,z')&=\lim_{q\to 0}\chi^N_{\omega_L}(q,z,z')\\&=\bar{n}\phi_1(z)\phi_1^*(z')[\tilde{G}_{E_1}(z,z')+\tilde{G}_{E_1}(z',z)] .\end{aligned}$$

Here $\tilde{G}_E(z,z')$ is the nonsingular part of the Green's function corresponding to the ϕ_j's when $E=E_1$, i.e.,

$$\tilde{G}_E(z,z')={\sum_j}'\frac{\phi_j^*(z)\phi_j(z')}{E-E_j} ,$$

where the prime on the sum indicates the exclusion of the term $j=1$. The calculation of the dynamic response of the system to an external charged disturbance also requires χ_ω, Eq. (4). Hence, it is important to evaluate the Green's function for this model of a layer electron gas and this is carried out in Appendix A.

The model which we use to describe the density profile is discussed in detail in Refs. 2–4. This model has been used very successfully by Brown and Grimes[8,9] to explain their experimental data for electrons on a liquid-He surface. The eigenfunctions are $\phi_j(z)=zR_{j0}(z)$, where $R_{j0}(z)$ is the jth radial solution of the hydrogen atom with nuclear charge $e\alpha$ and $l=0$. The energy levels in eV corresponding to these eigenfunctions are $E_n=-13.6(\alpha/j)^2$ and the scale of variation of $\phi_j(z)$ is $a_0=a_H/\alpha$, where a_H is the Bohr radius ($a_H=0.528$ Å). For liquid He one has $\alpha=\frac{1}{4}\beta$, and for a semiconductor a_0 is treated as a variational parameter.[18,19]

The integral on the left-hand side of Eq. (9) scales as $\gamma=2\pi e_*^2\bar{n}a_0/\Delta_{12}$ compared to the leading term. For a nondegenerate system, it is given by $\gamma=a_0k_BT/\lambda_D\Delta_{12}$. For liquid He $\epsilon_1=1.05$ and $\epsilon_2=1$ giving $a_0\approx 80$ Å, and for[9] $\bar{n}=10^6$ cm^{-2} and $T=1$ °K one has $\lambda_D\approx 10^4$ Å and $\gamma\approx 2\times 10^{-3}<1$. We shall, therefore, use the approximation $D(0,z)\approx n(z)/\bar{n}$ in Eq. (8) in order to estimate the finite size corrections to the dispersion relation. Since $\phi_1(z)=zR_{10}$ is a known function, we can then evaluate the integral in Eq. (8) and obtain from Eqs. (5) and (7),

$$\omega_L^2=\frac{2\pi\bar{n}e_*^2}{m}q\left\{1+3q\left[\lambda+\frac{a_0}{32}\left(11-21\frac{\epsilon_1}{\epsilon_2}\right)\right]+O(q^2)\right\} \tag{10}$$

for the nondegenerate electron gas $\lambda=\lambda_D$, and we find for this case that the corrections are small (25 Å$/\lambda_D=2.5\times 10^{-3}$). The level spacing here is $\Delta_{12}/k_B\approx 5.6$ °K.

For a degenerate system $\lambda=2\lambda_{\mathrm{TF}}$ and λ_{TF} is independent of the density, $\lambda_{\mathrm{TF}}=(2me_*^2)^{-1}$. Using the values of the parameters given by Stern and Howard[18] for a (100) Si-SiO$_2$ surface at low temperature ($m=0.19m_e$, $\epsilon_1=3.8$, $\epsilon_2=11.8$, $a_0=22$ Å, $k_F=1.25\times 10^{-2}$ Å^{-1}, $\bar{n}_{\mathrm{inv}}=5\times 10^{11}$ cm^{-2}, and $\Delta_{12}=12.4\times 10^{-3}$ eV), one finds $\lambda_{\mathrm{TF}}=0.4$ Å and $\gamma\approx 1.0$. However, the experimental value of the level spacing is larger than the theoretical value by about a factor of 2.[19] Hence, it is reasonable to use the same approximation for $D(0,z)$ that we used for the liquid-He calculation. The LPO dispersion is then given by Eq. (10) and $\lambda=2\lambda_{\mathrm{TF}}$ so that the correction to the 2-DEG result is large for this system (2.9 Å$/2\lambda_{\mathrm{TF}}\approx 3.6$). The Fermi energy ($E_F=6\times 10^{-3}$ eV) is less than the level spacing in agreement with our assumption.

III. CORRELATION EFFECTS

The effect of electronic correlation can be included in a random-phase-approximation calculation by employing the correlation-hole approximation.[12,13] In this approximation the electrons are assumed to be correlated within a range of $1/\kappa$, and the effective potential in Eq. (2) is replaced by the mean-field potential,[13,20,21]

$$\int dz'\int d\vec{r}'\,v(\vec{r}-\vec{r}',z,z')K(\vec{r}-\vec{r}',z,z')\delta\rho_\omega(\vec{r}',z') . \tag{11}$$

Here $K(\vec{r}-\vec{r}',z,z')$ describes the Fermi correlation hole, it approaches 1 for $\kappa|\vec{r}-\vec{r}'|\gg 1$ and has a value $0<K<\frac{1}{2}$ when $|\vec{r}'-\vec{r}|\kappa\to 0$.

We will follow the usual practice of using the pair distribution function to approximate K.[22,23] After accounting for spin the pair-distribution function is given by

$$P(\vec{r},z,\vec{r}',z')=\rho(0,z)\rho(0,z')-\tfrac{1}{2}|\rho(\vec{r}-\vec{r}',z,z')|^2 . \tag{12}$$

Here ρ is the electron density matrix,

$$\rho(\vec{r}-\vec{r}',z,z')=\frac{1}{\Omega}\sum_{\vec{k},j}f(\vec{k},j)\psi^\dagger_{\vec{k},j}(\vec{r}',z')\psi_{\vec{k},j}(\vec{r},z) ,$$

which for our system is given by

$$\rho(\vec{r}-\vec{r}',z,z')=\frac{1}{\Omega}\sum_{\vec{k},j}f(\vec{k},j)\,e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}\phi_j^*(z')\phi_j(z) . \tag{13}$$

If we again limit our considerations to systems where only the lowest subband ($j=1$) is occupied this expression simplifies to

$$\begin{aligned}\rho(\vec{r}-\vec{r}',z,z')&=\frac{\phi_1^*(z')\phi_1(z)}{\Omega}\sum_{\vec{k}}f(\vec{k},1)\,e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}\\&=\phi_1^*(z')\phi_1(z)\bar{n}P(|\vec{r}-\vec{r}'|) .\end{aligned} \tag{14}$$

For the nondegenerate electron gas the two-dimensional integration gives

$$P(r) = e^{-r^2/4\lambda_T^2} , \qquad (15)$$

where λ_T is the thermal wavelength $\lambda_T = \hbar/(2mk_BT)^{1/2}$. For the degenerate electron gas one obtains

$$P(r) = 2J_1(k_F r)/k_F r , \qquad (16)$$

where $J_1(k_F r)$ is the cylindrical Bessel function of order 1.

The pair-distribution function is given by

$$P(\vec{r}, z, \vec{r}', z') = \bar{n}^2 |\phi_1(z)|^2 |\phi_1(z')|^2 [1 - \tfrac{1}{2} P^2(|\vec{r} - \vec{r}'|)] ,$$

and we have, comparing Eqs. (2), (4), and (11),

$$K(\vec{r} - \vec{r}', z, z') = 1 - \tfrac{1}{2} P^2(|\vec{r} - \vec{r}'|) . \qquad (17)$$

Fourier transforming Eq. (11) with respect to r and replacing the effective potential in Eq. (2) by Eq. (11), we can write the integral equation for the density fluctuation which includes the correlation-hole approximation as

$$\delta\rho_\omega(q, z) = \int_0^\infty dz' \int_0^\infty dz'' \, \chi_\omega(q, z, z') \times [v(q, z', z'') - \tilde{v}(q, z', z'')] \, \delta\rho_\omega(q, z'') , \qquad (18)$$

where $v(q, z, z')$ is given in Eq. (3) and

$$\tilde{v}(q, z, z') = \frac{1}{2} \int e^{i\vec{q}\cdot\vec{r}} dr \, v(\vec{r}, z, z') P^2(r) = \frac{e^2}{2\epsilon_2} [h(q, |z - z'|) - \beta h(q, z + z')] . \qquad (19)$$

After integrating the Coulomb potential over the angular variable, the expression for h is

$$h(q, z) = 2\pi \int_0^\infty r \, dr \, J_0(q, r) P^2(r) (r^2 + z^2)^{-1/2} . \qquad (20)$$

The plasma dispersion relation is again obtained by integrating Eq. (18) over z [see Eq. (5) in Sec. II] as

$$1 = \frac{1}{q} \sum_{\vec{k}} \frac{f(\vec{k} + \vec{q}, 1) - f(\vec{k}, 1)}{E_{\vec{k}+\vec{q}} - E_{\vec{k}} + \omega + i\eta} [C_{\omega,1}(q) - \tilde{C}_{\omega,1}(q)] , \qquad (21)$$

where $C_{\omega,1}(q)$ is given by Eq. (6) and $\tilde{C}_{\omega,1}(q)$ is obtained by replacing v by $\tilde{v}$ in Eq. (6). The term in Eq. (21) containing $C_{\omega,1}$ gives the dispersion relation which was evaluated in Sec. II, and we shall devote the remainder of this section to an evaluation of the term involving $\tilde{C}_{\omega,1}$.

We are only interested in terms in the dispersion relation whose order is less than q^3. Hence, we can expand $\tilde{C}$ and h about $q = 0$ and retain only terms of order q^0 and q,

$$h(q, z) = 2\pi \int_0^\infty r \, dr \, P^2(r) (r^2 + z^2)^{-1/2} + O(q^2) .$$

For the nondegenerate gas $P(r)$ is given by Eq. (15) and this integral is easily evaluated,

$$h(0, z) = \sqrt{2} \, \lambda_T \pi^{3/2} e^{z^2/2\lambda_T^2} \operatorname{erfc}(z/\sqrt{2}\,\lambda_T) + O(q^2) , \qquad (22)$$

where $\operatorname{erfc}(x)$ is the complementary error function. Using the series expansion for $e^{x^2}\operatorname{erfc}(x)$,[24] one has

$$h(0, z) = \sqrt{2} \, \lambda_T \pi^{3/2} \sum_{m=0}^\infty \left(- \frac{z}{\sqrt{2}\,\lambda_T}\right)^m \Big/ \Gamma\left(\frac{m+2}{2}\right) ,$$

where $\Gamma(k)$ is the gamma or factorial function. The thermal wavelength of the electron on the helium surface at $T = 1$ °K is $\lambda_T \approx 200$ Å which is larger than the scale of variation of $|\phi_1(z)|^2$, $a_0/2\sqrt{2}\,\lambda_T = 0.14$. Again using the approximation for $D(0, z) \approx n(z)/\bar{n}$, which we introduced in Sec. II, we can evaluate the integrals in $\tilde{C}_1(q)$ to obtain

$$\tilde{C}_1(0, q) = 2\pi e_*^2 q \lambda_T (0.98) + O(q^2) . \qquad (23)$$

Using Eqs. (7) and (21), we find that the correction to the q^2 term in the dispersion relation, Eq. (10), due to correlations is small compared to $\lambda_D \approx 10^4$ Å, $0.98\lambda_T \ll 3\lambda_D$ [the correction for a nondegenerate 2-DEG is $\lambda_T(\frac{1}{2}\pi)^{1/2} = 1.25\lambda_T$ within the correlation-hole approximation].

For the degenerate electron gas $P(r)$ is given by Eq. (16) and one finds

$$h(q, z) = \frac{8\pi}{k_F^2} \int_0^\infty \frac{dr}{r} \frac{J_1^2(k_F r)}{(r^2 + z^2)^{1/2}} + O(q^2) .$$

Evaluating this integral one finds (see Appendix B)

$$h(0, z) = \frac{16\sqrt{\pi}}{k_F} \sum_{m=0}^\infty \frac{(-2k_F z)^m}{\Gamma(m+3)} \frac{\Gamma(\frac{1}{2}(m+4))}{\Gamma(\frac{1}{2}(m+5))} . \qquad (24)$$

The scale of variation of $|\phi_1(z)|^2$ is $a_0 \approx 22$ Å and for this system $k_F a_0 = 0.27$. Again approximating $D(0, z)$ by $n(z)/\bar{n}$, we find

$$\tilde{C}_1(0, q) = 2\pi e_*^2 q (1.69)/k_F + O(q^2) , \qquad (25)$$

and this correction for correlations is large $2\lambda_{TF} < 1.7/k_F \approx 1.50$ Å (the correction for a degenerate 2-DEG using the correlation-hole approximation is $16/3\pi k_F = 1.7/k_F$). Hence it is clear that the corrections for correlation are important in this degenerate electron layer.

IV. DISCUSSION

We have presented calculations of the plasmon dispersion relations for electron layers on liquid helium and at the Si-SiO$_2$ interface of a semiconductor surface. These calculations have included the effects owing to the finite thickness of the layer and electron-electron correlations. Our calculations have been confined to the coefficient of the q^2 term λ_2, in the LPO dispersion relation

$$\omega_L^2 = (2\pi\bar{n} e_*^2/m) q (1 + \lambda_2 q) + O(q^3) .$$

Hence q needs to be small but large enough where retardation effects are not important.[10] We have also limited our considerations to systems where only the lowest subband ($j = 1$) is occupied.

For the degenerate electron gas at the Si-SiO$_2$ interface, we obtain

$$\lambda_2 = \tfrac{3}{2}\lambda_{\mathrm{TF}} + 0.13 a_0 - 1.7/k_F \ .$$

The first term on the left-hand side is the 2-DEG result.[7] The second term is the correction for finite thickness and the third term results from electron-electron correlations. For the system considered here with $\bar{n}_{\mathrm{inv}} = 5\times 10^{11}$ cm^{-2}, both of these corrections to the 2-DEG result are important and large.

For the nondegenerate gas on a liquid-helium surface, we find

$$\lambda_2 \approx 3\lambda_D - a_0 - \lambda_T \ ,$$

where the terms are arranged in the same order as they were for the degenerate case above. For $\bar{n} = 10^6$ cm^{-2}, the corrections to the 2-DEG results are unimportant. However, a_0 and λ_T are independent of the electron density and $\lambda_D \sim \bar{n}^{-1}$. The system is still nondegenerate for[8] $\bar{n} = 2\times 10^8$ cm^{-2} and for this density $\lambda_D \approx 50$ Å, which is of the same order of magnitude as $a_0 \approx 25$ Å and smaller than $\lambda_T \approx 200$ Å. Hence the corrections can also be important for this electron gas layer.

APPENDIX A: NONSINGULAR GREEN'S FUNCTION

In this Appendix we present the nonsingular part of the Green's function for the electrons in the lowest level $E = E_1$ of the one-dimensional Coulomb potential

$$\tilde{G}_{E_1}(z, z') = \sum_j{}' \frac{\phi_j^*(z)\phi_j(z')}{E_1 - E_j} \ , \qquad \text{(A1)}$$

where the prime on the sum indicates the exclusion of the term $j = 1$. The ϕ_j's are solutions of the Schrödinger equation

$$\left(-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} - \frac{Ze^2}{z}\right)\phi_j(z) = E_j\phi_j(z) \ , \qquad \text{(A2)}$$

with the boundary conditions $\phi_j(0) = 0$ and $\int_0^\infty dz \times |\phi_j(z)|^2 = 1 < \infty$. Here, Ze is the effective strength of the potential, m is the effective mass of the electron, $a_0 = \hbar^2/mZe^2$, and $E_j = -Ze^2/2a_0 j^2$. The lowest-energy eigenfunction is $\phi_1(z) = 2(a_0)^{-3/2} z e^{-a_0 z}$.

Using Eq. (A2), we obtain the differential equation for the Green's function, (A1),

$$\left(\frac{d^2}{dz^2} + \frac{2}{a_0 z} - \frac{1}{a_0^2}\right)\frac{\hbar^2}{2m}\tilde{G}_{E_1}(z, z') = \delta(z - z') - \phi_1(z)\phi_1(z').$$

It is convenient to change variables to $x = 2z/a_0$ and introduce $u_1(x) = (\tfrac{1}{2}a_0)^{1/2}\phi_1(z)$. The differential equation for $\tilde{G}$ is then transformed to

$$\left(\frac{d^2}{dx^2} + \frac{1}{x} - \frac{1}{4}\right) g(x, x') = \delta(x - x') - u_1(x)u_1(x') \ , \qquad \text{(A3)}$$

where $g(x, x') = (\hbar^2/ma_0)\tilde{G}_{E_1}(z, z')$. Solutions to the homogeneous differential equation are $u_1(x) = xe^{-x/2}/\sqrt{2}$ and $u_2(x) = 2u_1(x)\,\mathrm{Ei}(x) - \sqrt{2}\,e^{x/2}$, where[25] $\mathrm{Ei}(x) = P\int_{-\infty}^{x}(e^t/t)\,dt$ and P denotes the principal value. A particular solution to the differential equation when $x \neq x'$ is $f_P(x) = u_P(x)u_1(x')$, where

$$u_P(x) = \sqrt{2}\,e^{-x/2}[(\tfrac{1}{2}x^2) + x\ln x - 1] \ .$$

The Green's function can therefore be written

$$g^>(x, x') = Au_1(x)u_2(x') + u_P(x)u_1(x') + u_1(x)u_P(x') + Bu_1(x)u_1(x')$$

for $x > x'$, and $g^<(x, x') = g^>(x', x)$ for $x < x'$. The constant A is determined so that the derivative of $g(x, x')$ has a finite jump of magnitude 1 at $x = x'$ and the constant B is determined so that $\int_0^\infty dx' \times g(x, x')u_1(x') = 0$. These determinations give

$$g^>(x, x') = -u_1(x)u_2(x') + u_P(x)u_1(x') + u_1(x)u_P(x') - (7 - 4\gamma)u_1(x)u_1(x') \ , \qquad \text{(A4)}$$

where Euler's constant is $\gamma = 0.577\,215\cdots$.

APPENDIX B: INTEGRAL FOR CORRELATION

In this Appendix the integral

$$I(zk_F) = \frac{1}{k_F}\int_0^\infty \frac{dr}{r}\frac{J_1^2(k_F r)}{(r^2 + z^2)^{1/2}}$$

needed for Eq. (24) is evaluated. The notation is simplified if one changes to variables $x = k_F r$ and $b = k_F z$,

$$I(b) = \int_0^\infty \frac{dx}{x}\frac{J_1^2(x)}{(x^2 + b^2)^{1/2}} \ . \qquad \text{(B1)}$$

To evaluate the integral one uses[26]

$$J_1^2(x) = \frac{2}{\pi}\int_0^{\pi/2} d\theta\, J_2(2x\sin\theta)$$

and

$$4J_2(x)/x = J_1(x) + J_3(x) \ .$$

Inserting these two expressions into the integral (B1) and interchanging orders of integration, one finds

$$I(b) = \frac{1}{\pi}\int_0^{\pi/2} d\theta\,\sin\theta[I_{1/2}(b\sin\theta)K_{1/2}(b\sin\theta) + I_{3/2}(b\sin\theta)K_{3/2}(b\sin\theta)] \ . \qquad \text{(B2)}$$

Here $I_\nu(x)$ and $K_\nu(x)$ are modified Bessel functions of order ν, and the integral over x has been performed using[27]

$$\int_0^\infty \frac{J_\nu(ax)}{(x^2 + b^2)^{1/2}}dx = I_{\nu/2}\left(\frac{ab}{2}\right)K_{\nu/2}\left(\frac{ab}{2}\right) \ .$$

Expanding the products of modified Bessel's functions, one obtains

$$I(b) = \frac{4}{\pi}\int_0^{\pi/2} d\theta\,\sin\theta\sum_{m=0}^{\infty}\frac{(-2b\sin\theta)^m(m+2)}{\Gamma(m+4)} \ .$$

The integrals over θ in this expression can be evaluated by considering the beta function[28]

$$B(s,r)=2\int_0^{\pi/2}(\sin\theta)^{2s-1}(\cos\theta)^{2r-1}\,d\theta=\frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)},$$

and yield

$$I(b)=\frac{2}{\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{(-2b)^m}{\Gamma(m+3)}\frac{\Gamma(\frac{1}{2}(m+4))}{\Gamma(\frac{1}{2}(m+5))}\,. \qquad \text{(B3)}$$

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[†] Present address: Dept. of Physics, University of Sou Southern California, Los Angeles, Calif. 90007.

[1] W. T. Sommer, thesis (Stanford University, 1964) (unpublished); see also W. T. Sommer and D. J. Tanner, Phys. Rev. Lett. 27, 1345 (1971).

[2] M. W. Cole and M. H. Cohen, Phys. Rev. Lett. 23, 1238 (1969); M. W. Cole, Phys. Rev. B 2, 4239 (1970); Rev. Mod. Phys. 46, 451 (1974).

[3] V. B. Shikin, Zh. Eksp. Teor. Fiz. 58, 1748 (1970) [Sov. Phys.-JETP 31, 936 (1970)].

[4] F. F. Fang and W. E. Howard, Phys. Rev. Lett. 16, 797 (1966); F. Stern, Phys. Rev. B 5, 4891 (1972). An excellent review may be found by F. Stern, in *Critical Review in Solid State Sciences* (Chemical Rubber, Cleveland, Ohio, 1974), Vol. 4, No. 3.

[5] A. L. Fetter, Phys. Rev. B 10, 3739 (1974).

[6] P. M. Platzman and H. Fukuyama, Phys. Rev. B 10, 3150 (1974).

[7] F. Stern, Phys. Rev. Lett. 18, 546 (1967).

[8] T. R. Brown and C. C. Grimes, Phys. Rev. Lett. 29, 1233 (1972).

[9] C. C. Grimes and T. R. Brown, Phys. Rev. Lett. 32, 280 (1974).

[10] Our results are invalidated in the small-q regime by the retardation effects which become important if $q \lesssim 2\pi\bar{n}\,e^2/ms^2$, where s is the velocity of light in the medium surrounding the charged layer. See Ref. 7.

[11] R. H. Ritchie, Surf. Sci. 34, 1 (1973).

[12] J. Hubbard, Proc. R. Soc. Lond. A 243, 336 (1957).

[13] K. S. Singwi, M. P. Tosi, R. H. Land, and A. Sjölander, Phys. Rev. 176, 589 (1968); Phys. Rev. B 7, 1044 (1970).

[14] P. J. Feibelman, Phys. Rev. B 3, 220 (1971).

[15] D. E. Beck and B. B. Dasgupta, Phys. Rev. B 12, 1995 (1975).

[16] D. M. Newns, Phys. Lett. A 39, 341 (1972).

[17] A. J. Bennett, Phys. Rev. B 1, 203 (1970); A. Eguiluz, S. C. Ying, and J. J. Quinn, *ibid.* 11, 2118 (1975).

[18] F. Stern and W. E. Howard, Phys. Rev. 163, 816 (1967).

[19] F. Stern, Phys. Rev. Lett. 30, 278 (1973).

[20] J. Harris, J. Phys. C 5, 1757 (1972).

[21] C. S. Ting, T. K. Lee, and J. J. Quinn, Phys. Rev. Lett. 14, 870 (1975).

[22] W. Jones and N. H. March, *Theoretical Solid State Physics* (Wiley, New York, 1973), Vol. 1, p. 128. A more exact numerical calculation using the kernel function obtained by Singwi *et al.* (Ref. 13) could be performed. However, a good estimate of the effects due to correlations can be obtained analytically by using this approximation.

[23] R. K. P. Zia, J. Phys. C 6, 3121 (1973).

[24] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (U. S. Natl. Bur. Stand., Washington, D.C., 1964), p. 297.

[25] See p. 228, Ref. 24.

[26] See p. 485, Ref. 24.

[27] See p. 488, Ref. 24.

[28] See p. 258, Ref. 24.