

## Free carriers' effective mass and relaxation-time analysis by high-pulsed-field Faraday oscillations in III-V compounds

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The high-pulsed-magnetic-field technique offers a convenient means for observing a large number of Faraday oscillations in a semiconductor, in the far infrared ( $10.6 \mu\text{m}$ ). The known values of the electron effective mass in InSb and GaAs are deduced from the period, but within the frame of the classical theory the decay of the oscillations as a function of the magnetic field is stronger than can be expected from dc conductivity and infrared absorption at the same frequency, in zero field. This is interpreted as due to a marked influence of the magnetic field on the relaxation. The preliminary results of a quantum-mechanical treatment are discussed.

### I. INTRODUCTION

The Faraday effect of free carriers in semiconductors has been extensively studied, mainly because it has proved to be a particularly convenient means of measuring the effective mass,<sup>1-4</sup> its energy and temperature dependence,<sup>5,6</sup> the free-carrier density, and the relaxation time.<sup>3,7,8</sup>

With the advent of submillimeter and far-infrared lasers, new fields of investigation have been opened up in magneto-optics of semiconductor materials. Particularly, the CO<sub>2</sub> laser is a very interesting tool for studying the Faraday effect of free carriers, because it yields an intense and plane-polarized radiation. At  $10.6 \mu\text{m}$  the interband contribution to the Faraday effect is often negligible in the usual semiconductors.<sup>4-6</sup> With a source of high magnetic field, one can observe a large number of oscillations, corresponding to complete rotations of the polarization plane. In addition, the damping of these oscillations with the magnetic field provides us with information as to the relaxation time.

Care is necessary, however, to eliminate the spurious effect of Fabry-Perot interferences, which may appreciably perturb the interpretation of the experimental results.<sup>9,10</sup>

The values of the electron effective mass which have been measured in moderately degenerate InSb and GaAs samples are in satisfactory agreement with previously reported values and Kane's theory<sup>11</sup> of nonparabolic effects in these materials. But the values of the relaxation time which are deduced from a comparison of the experimental results with the classical theory are markedly lower than the values given by dc conductivity and infrared absorption, at the same frequency, in zero field. This discrepancy suggests that the relaxation processes are appreciably influenced by the field. Recently much interest has been centered on this subject,<sup>12</sup> and the theoretical results are usually tested by studying the behavior

of the cyclotron resonance line width.<sup>13-17</sup>

Thus the type of experiment presented here offers an alternative means of investigating the field dependence of the imaginary dielectric constant in semiconductors. The decrease in the relaxation time, observed in the Faraday effect, can be understood with the help of Kubo's theory of transport processes.

Finally the good quality of the oscillograms shows that, in spite of the development of superconducting magnets, the high-pulsed-field technique retains the interest of its early days.

In Sec. II there is described an apparatus permitting measurements of transmission rate in a given polarization plane at  $10.6 \mu\text{m}$ , as a function of the magnetic field varying from 0 to 35 T. Typical results obtained in InSb and GaAs samples are reported.

In Sec. III a classical theory of Faraday oscillations is elaborated. Nonparabolic effects are discussed. In Sec. IV this theory is used for the interpretation of the experimental results. Preliminary results of a quantum-mechanical treatment are given and discussed.

### II. APPARATUS AND EXPERIMENTAL RESULTS

The schematic diagram of the experimental setup is shown in Fig. 1. A 0.5W CO<sub>2</sub> laser source yields a plane-polarized radiation, of  $10.6 \mu\text{m}$ . The useful mode is selected with a 75-grooves/mm grating. The pulsed field generator consists of a 2.2-mF condenser bank, charged with a 3-kV power supply, and a Cu solenoid immersed in a liquid-nitrogen bath. During the discharge of the bank the field increases from 0 to 30 T, over about 1 msec in a volume of a few cm<sup>3</sup>. The cryostat is passed through by an evacuated stainless-steel pipe of 8 mm i. d., along the coil axis. This pipe is closed at both ends by two optical windows, made of Irtran IV. The sample is stuck on the end of a nylon pipe, located inside the stainless-steel pipe, at the maximum and

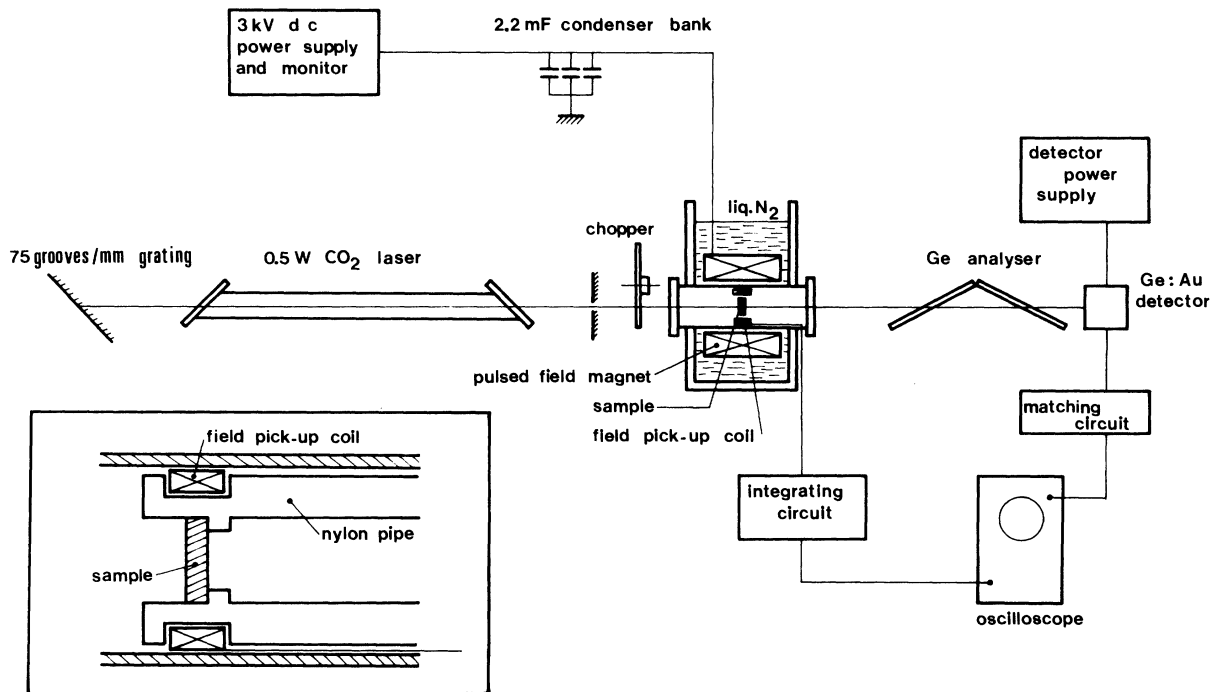


FIG. 1. Schematic diagram of the experimental set used for the measurement of Faraday oscillations, at  $10.6 \mu\text{m}$ . The detail of the device used for the mounting of the sample and the measurement of the magnetic field is shown in the inset.

homogeneous field position. The field is measured by integration of the induced emf in a pickup coil, placed around the sample, in a notch of the nylon pipe (Fig. 1).

The polarization of the emergent radiation is analyzed with two antiparallel Ge plates, at Brewster incidence, and the intensity is measured with a Ge gold-doped detector (Raytheon QKN 1568). The behavior of the intensity as a function of the magnetic field is observed on the oscilloscope.

Measurements have been performed on  $n$ -type InSb and GaAs samples of various thicknesses and electron concentrations. The samples of 6 mm diam were mechanically polished, then mounted on the nylon holder and placed inside the coil. Elimination of the Fabry-Perot interference ef-

fects was achieved by a slight grinding of the back face. At thermal equilibrium in the cryostat, the samples temperature was around 100 K.

The geometrical and electrical characteristics of the investigated samples are listed in Table I;  $\omega_p$  was derived from the infrared reflectivity minimum. The electron concentration  $N$  was calculated from the Hall coefficient, and found to be in agreement with the reflectivity minimum, by using the results published by Spitzer and Fan<sup>18</sup> for InSb, and by Spitzer and Whelan<sup>19</sup> and Okada and Oku<sup>20</sup> for GaAs.

The oscillograms of two of the investigated samples are given in Figs. 2-4. Note the rather large number of oscillations in InSb, in connection with the small value of the effective mass.

TABLE I. Thickness ( $d$ ), conductivity ( $\sigma$ ), plasma frequency ( $\omega_p$ ), and electron concentration ( $N$ ) of the investigated InSb and GaAs samples, along with the measured values of the effective mass and the relaxation time, as described in Sec. IV.

	$d$ ( $\mu\text{m}$ )	$\sigma$ ( $\Omega^{-1} \text{m}^{-1}$ )	$\omega_p$ ( $10^{13} \text{sec}^{-1}$ )	$N$ ( $10^{23} \text{m}^{-3}$ )	$m^*/m$	$\tau_F$ ( $10^{-13} \text{sec}$ )	$\tau_0$ ( $10^{-13} \text{sec}$ )	$\tau_{1r}$ ( $10^{-13} \text{sec}$ )
InSb 1	340	$2.4 \times 10^5$	5,44	5,4	$0.023 \pm 0.002$	5,0	5,8	
2	405	3,0	6,26	5,0	$0.026 \pm 0.002$	3,35	5,5	5,9
3	260	2,65	6,40	5,8	$0.029 \pm 0.002$	3,3	4,7	
GaAs 1	455	3,5	5,30	7,0	$0.072 \pm 0.002$	0,55	1,3	1,7
2	455	6,6	7,33	14,3	$0.077 \pm 0.002$	0,47	1,3	

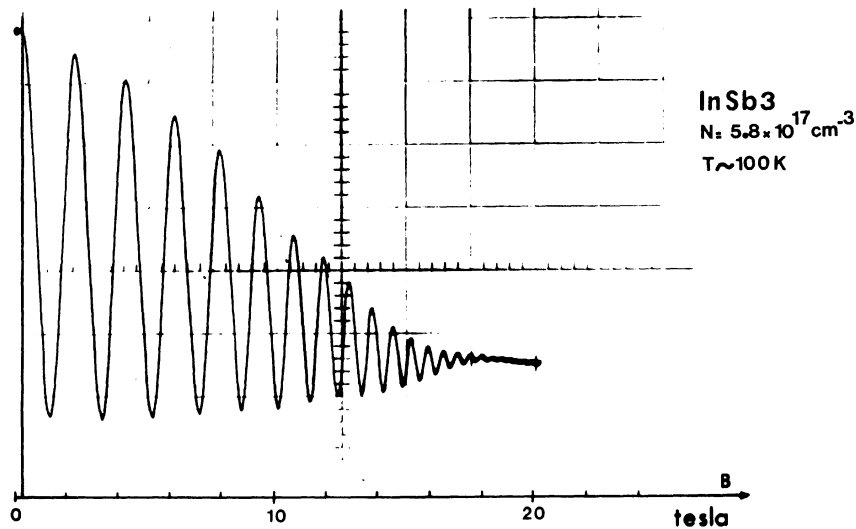


FIG. 2. Faraday oscillations as a whole, observed in InSb, sample No. 1.

### III. CLASSICAL THEORY OF FARADAY OSCILLATIONS

Each of the rotating components of the electric field of the plane-polarized incident electromagnetic wave, propagating along  $z$ , are represented, as usual, by the real part of the complex vector

$$\vec{E}_\nu = \frac{1}{2}(E_0, -i\nu E_0, 0) \exp[i(\omega t - k_0 z)],$$

where  $E_0$  is the field amplitude,  $\omega$  the circular frequency,  $k_0 = \omega/c$  the propagating constant, and  $\nu = +1$  or  $-1$  according to whether the field is rotating to the right or to the left, with respect to  $z$ .

In the presence of a magnetic field  $B$ , parallel to  $z$ , the classical Boltzmann theory leads to the following expression of the conductivity  $\sigma_\nu$  of the  $\nu$  component, at frequency  $\omega$

$$\sigma_\nu = -\frac{e^2}{4\pi^2} \int \frac{\partial f_0}{\partial \epsilon} \frac{\tau}{1 + i(\omega - \nu\omega_c)\tau} v_t^2 k_t dk_t dk_z, \quad (1)$$

where  $e$ ,  $\tau$ ,  $k_t$ ,  $k_z$ ,  $v_t$ ,  $v_z$ , respectively, denote the charge, the relaxation time of the electrons, the transverse and longitudinal components of the wave vector  $\vec{k}$ , and the velocity  $\vec{v}$ ;  $\epsilon$  is the energy of the carrier and  $f_0(\epsilon)$  the Fermi function;  $\omega_c$  is the cyclotron frequency defined as follows, for an isotropic band  $\epsilon(k)$ :

$$\omega_c = \frac{eB}{\hbar^2 k} \frac{d\epsilon}{dk}. \quad (2)$$

The dielectric constant  $\epsilon_\nu$  is deduced from  $\sigma_\nu$  and the lattice dielectric constant  $\epsilon_l$  by

$$\epsilon_\nu = \epsilon_l - i\sigma_\nu/\epsilon_0\omega, \quad (3)$$

where  $\epsilon_0$  is the vacuum dielectric constant (mks units).

At  $10.6 \mu\text{m}$  ( $\omega = 1.78 \times 10^{14} \text{ sec}^{-1}$ ), we have  $(\omega\tau)^2 \gg 1$ , which enables us to write, in the integrand

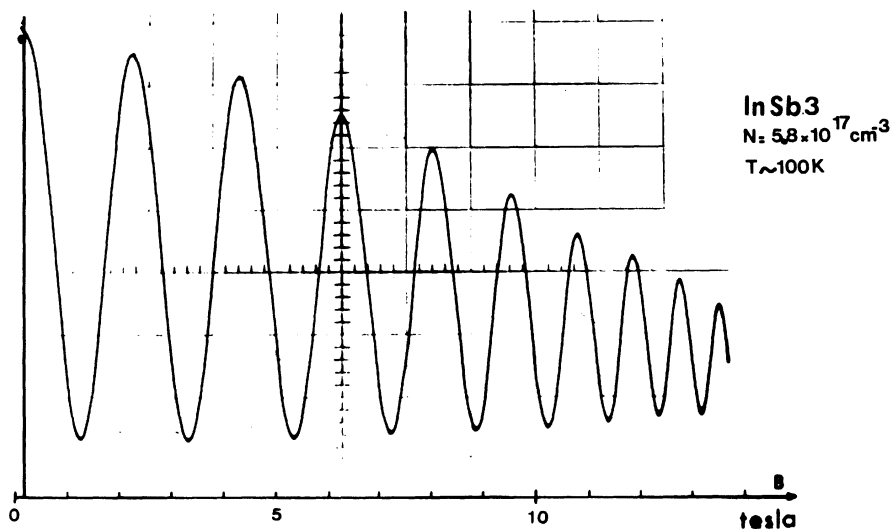


FIG. 3. Faraday oscillations observed in InSb, sample No. 1, in the range 0-14 T.

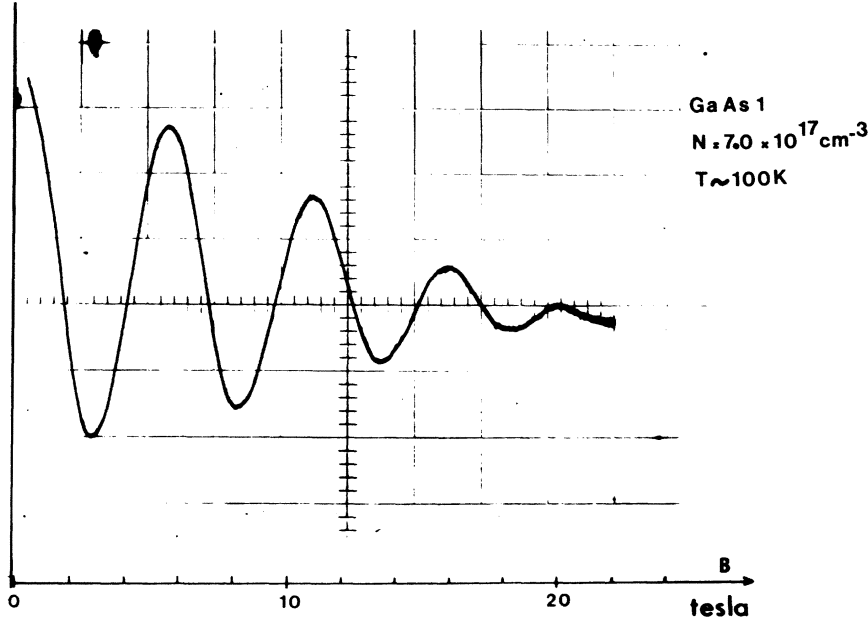


FIG. 4. Faraday oscillations observed in GaAs, sample No. 1, in the range 0–21 T.

of (1),

$$\frac{\tau}{1 + i(\omega - \nu\omega_c)\tau} = \frac{1}{(\omega - \nu\omega_c)^2\tau} - \frac{i}{\omega - \nu\omega_c}.$$

Hence, assuming a constant value of the relaxation time, the following expressions of the real and imaginary parts of the dielectric constant are obtained

$$\epsilon_{1\nu} = \epsilon_1 + \frac{e^2}{3\pi^2\hbar^2\epsilon_0\omega} \int \frac{\partial f_0}{\partial \epsilon} \frac{d\epsilon}{dk} \frac{k^2 d\epsilon}{\omega - \nu\omega_c}, \quad (4)$$

$$\epsilon_{2\nu} = \frac{e^2}{3\pi^2\hbar^2\epsilon_0\omega\tau} \int \frac{\partial f_0}{\partial \epsilon} \frac{d\epsilon}{dk} \frac{k^2 d\epsilon}{(\omega - \nu\omega_c)^2}, \quad (5)$$

In an isotropic, but non parabolic band, the function  $\epsilon(k)$  can be represented by the following expansion, in the vicinity of the Fermi energy  $\epsilon_F$ :

$$\epsilon - \epsilon_F = (\hbar^2/m^*)[k_F(k - k_F) + \frac{1}{2}(1 - \xi)(k - k_F)^2 + (\eta/k_F)(k - k_F)^3], \quad (6)$$

where  $m^*$ ,  $\xi$ ,  $\eta$  are the band parameters. For a parabolic band  $\xi = \eta = 0$ . It is straightforward to show that the Fermi wave vector  $k_F$  is related to the electron density  $N$  by the equation

$$k_F = (3\pi^2 N)^{1/3} \left( 1 - \frac{(\pi m^* k_B T)^2 (1 + \xi)}{6\hbar^4 (3\pi^2 N)^{4/3}} \right), \quad (7)$$

including the first Sommerfeld correction in  $(k_B T)^2$  for a moderately degenerate electronic gas;  $k_B$  is the Boltzmann constant.

By expanding the coefficient of  $\partial f_0/\partial \epsilon$  in the integrand of (4) and (5) up to the second order in  $(\epsilon - \epsilon_F)$ ,  $\epsilon_{1\nu}$  and  $\epsilon_{2\nu}$  can be calculated at the order  $(k_B T)^2$ . Then, by eliminating  $k_F$  with the help of

Eq. (7) and the reverse of the expansion (6), we obtain

$$\frac{\epsilon_{1\nu}}{\epsilon_1} = 1 - \frac{\omega_p^2}{\omega^2(1 - \nu b)} \left\{ 1 - \frac{\pi^2 (k_B T)^2 m^{*2}}{3\hbar^4 (3\pi^2 N)^{4/3}} \left[ \frac{1}{4} \xi (3 + \xi) + \frac{3\nu b \xi}{2(1 - \nu b)} - \frac{1}{2} \nu b (1 + \nu b) \left( \frac{\xi}{1 - \nu b} \right)^2 - 3\eta \right] \right\}, \quad (8)$$

$$\frac{\epsilon_{2\nu}}{\epsilon_1} = -\frac{\omega_p^2}{\omega\tau(1 - \nu b)} \left\{ 1 - \frac{2\pi^2 (k_B T)^2 m^{*2}}{3\hbar^4 (3\pi^2 N)^{4/3}} \left[ \frac{1}{4} \xi (3 + \xi) + \frac{3\nu b \xi}{1 - \nu b} - \frac{1}{2} \nu b (1 + 2\nu b) \left( \frac{\xi}{1 - \nu b} \right)^2 - \frac{3}{2} \eta \right] \right\}, \quad (9)$$

where  $\omega_p = (Ne^2/m^*\epsilon_0\epsilon_1)^{1/2}$  is the electron plasma frequency and  $b$  a reduced magnetic field defined by  $b = eB/m^*\omega_0$ .

It is seen that the Sommerfeld corrections vanish in the parabolic limit ( $\xi = \eta = 0$ ). Equations (8) and (9) then reduce to the usual Drude-Zener expressions. This could have been expected since, in this limit, the Drude-Zener formulas can be derived from (4) and (5) by straightforward integration.

The smaller the ratio  $k_B T/\epsilon_F$ , the greater the validity of the Sommerfeld-nonparabolic correction in (8) and (9). On the other hand, the non-parabolic parameters  $\xi$ ,  $\eta$  are increasing functions of the energy  $\epsilon$ . It follows that we obtain an upper limit of the magnitude of these corrections in InSb by evaluating them in the most heavily doped sample, InSb 3.  $\xi$  and  $\eta$  can be calculated from Kane's equation [Eq. (10) of Ref. 11]. With the values quoted in Ref. 21 for the band gap  $\epsilon_g$ , the spin-orbit splitting  $\Delta$ , and the effective-mass ratio at the bottom of the conduction band

$\epsilon_g = 0.24$  eV,  $\Delta = 0.83$  eV,  $m_0^*/m_0 = 0.014$ ,  
we obtain for  $N = 5.8 \times 10^{23} \text{ m}^{-3}$

$$\xi = 0.31, \quad \eta = 0.09,$$

whence the corrections are found to amount to 1.5% in  $\epsilon_{1\nu}$  and 1.2% in  $\epsilon_{2\nu}$ .

In the GaAs samples, the convergence of the Sommerfeld expansion is poor. However, this material is much more parabolic than InSb,<sup>4</sup> so that the nonparabolic corrections in the Drude-Zener forms are likely to be negligible.

Finally within the accuracy of the present treatment and experiments, these corrections can be disregarded in all of our samples. The Faraday effect is thus sensitive only to the band parameter  $m^* = \hbar k (\partial k / \partial \epsilon)_{\epsilon_F}$ .

We now proceed to calculate the intensity of

$$I \sim |E_x|^2 = \frac{1}{4} E_0^2 \left( \frac{|T_+|^2 \exp(-2\kappa_+ k_0 d)}{1 - 2|R_+| \exp(-2\kappa_+ k_0 d) \cos(2n_+ k_0 d - \phi_{R_+}) + |R_+|^2 \exp(-4\kappa_+ k_0 d)} \right. \\ \left. \times \frac{|T_+ T_-^*| \exp\{-[\kappa_+ + \kappa_- + i(n_+ - n_-)]k_0 d\}}{1 - R_+ \exp[-2(\kappa_+ + i n_+)k_0 d] - R_-^* \exp[-2(\kappa_- - i n_-)k_0 d] + R_+ R_-^* \exp\{-2[\kappa_+ + \kappa_- + i(n_+ - n_-)]k_0 d\}} + \text{p. q.} \right), \quad (10)$$

where  $\phi_+$  is the argument of the complex number  $R_+$ ; p. q. stands for "permuted quantity," i. e., the same quantity after permutation of the subscripts + and -.

In the latter expression, the coefficients  $R_+$ ,  $T_+$  are slowly varying with the magnetic field. The factor  $\exp[-i(n_+ - n_-)k_0 d]$  is responsible for the Faraday rotation, whereas terms in  $\exp(\pm i n_\nu k_0 d)$  correspond to Fabry-Perot interference effects, due to internal reflexions. In practice the latter are smeared out by geometrical imperfections on the faces of the crystal, at the scale of the wavelength. As a result, the expression (10) can be averaged with respect to fluctuations of the thickness  $d$ .<sup>22</sup> Furthermore terms of order  $R^2$  (or  $T^2$ )  $\exp(-\kappa k_0 d)$ , which are very small, will be dropped, as well as the phase angles of  $T_\nu$  and  $R_\nu$ . The resulting expression of the transmitted intensity is then, except for a constant factor

$$I = \frac{T_+^2 \exp(-2\kappa_+ k_0 d)}{1 - R_+^2 \exp(-4\kappa_+ k_0 d)} + \frac{T_-^2 \exp(-2\kappa_- k_0 d)}{1 - R_-^2 \exp(-4\kappa_- k_0 d)} \\ + \frac{2T_+ T_- \exp[-(\kappa_+ + \kappa_-)k_0 d] \cos[(n_+ - n_-)k_0 d]}{1 - R_+ R_- \exp[-2(\kappa_+ + \kappa_-)k_0 d] \cos[2(n_+ - n_-)k_0 d]}. \quad (11)$$

#### IV. INTERPRETATION OF EXPERIMENTAL RESULTS AND DISCUSSION

It is seen on Eq. (11) that  $I$  reaches a maximum value whenever the magnitude of the magnetic

field is such that

$$(n_+ - n_-) k_0 d = 2\pi q,$$

in which  $q$  is a positive integer.  $n_+$  and  $n_-$  are calculated from Eq. (8) and (9) [in which the  $(k_B T)^2$  terms are dropped]. In our samples the condition  $\omega_p^2/\omega^2 < 0.12$  was satisfied, which permits disregard of terms of order  $\omega_p^4/\omega^4$ , whence

$$n_\nu \approx \epsilon_i^{1/2} - \frac{\epsilon_i^{1/2} \omega_p^2}{2\omega^2(1 - \nu b)} \left( 1 + \frac{\omega_p^2}{4\omega^2(1 - \nu b)} \right), \\ \kappa_\nu \approx \frac{\epsilon_i^{1/2} \omega_p^2}{2\omega^3 \tau (1 - \nu b)^2} \left( 1 + \frac{\omega_p^2}{2\omega^2(1 - \nu b)} \right). \quad (12)$$

From the expression of  $n_\nu$ , the following equation is then obtained, for the maxima of  $I$ :

$$\frac{\epsilon_i^{1/2} \omega_p^2 b d}{c\omega(1 - b^2)} \left( 1 + \frac{\omega_p^2}{2\omega^2(1 - b^2)} \right) = 2\pi q. \quad (13)$$

The effective mass  $m^*$  was then calculated through the relation  $m^* = eB_q/\omega b_q$ , in which  $b_q$  is the root of Eq. (13) and  $B_q$  the value of the field at the maximum  $q$ . The results which were only very slightly dependent on the field are given in Table I. They are in satisfactory agreement with previous data.<sup>4-6, 18, 19</sup>

We now turn to evaluation of the damping of the transmitted intensity. Equation (11) shows that the oscillations are enclosed in the envelope function

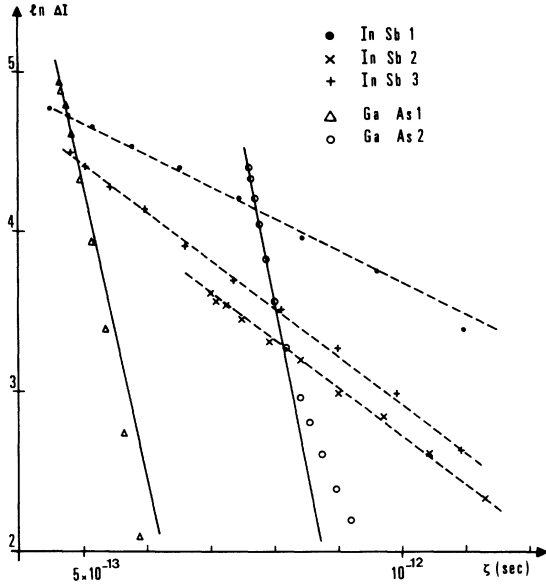


FIG. 5. Plot of the logarithm of the amplitude  $\Delta I$  of Faraday oscillations vs the field function  $\xi$  (b).

$$I_{\text{env}} = \frac{T_+^2 \exp(-2\kappa_+ k_0 d)}{1 - R_+^2 \exp(-4\kappa_+ k_0 d)} + \frac{T_-^2 \exp(-2\kappa_- k_0 d)}{1 - R_-^2 \exp(-4\kappa_- k_0 d)} \pm \frac{2T_+ T_- \exp[-(\kappa_+ + \kappa_-)k_0 d]}{1 - R_+ R_- \exp[-2(\kappa_+ + \kappa_-)k_0 d]}.$$

Thus the amplitude of the oscillations is proportional to

$$\Delta I = \frac{T_+ T_- \exp[-(\kappa_+ + \kappa_-)k_0 d]}{1 - R_+ R_- \exp[-2(\kappa_+ + \kappa_-)k_0 d]}.$$

The main dependence in  $B$  is contained in the factor  $\exp[-(\kappa_+ + \kappa_-)k_0 d]$ . Hence

$$\ln \Delta I \approx -(\kappa_+ + \kappa_-)k_0 d.$$

By using the expression (12) of  $\kappa_v$

$$\ln \Delta I \approx -\frac{d\varepsilon_i^{1/2} \omega_p^2 (1+b^2)}{\tau c \omega^2 (1-b^2)^2} \left(1 + \frac{\omega_p^2}{2\omega^2} \frac{1+3b^2}{1-b^4}\right). \quad (14)$$

The coefficient of  $1/\tau$  in the right-hand side is a known function  $\xi(b)$  of the field. By plotting  $\ln \Delta I$  vs  $\xi(b)$  (Fig. 5), we can see that the linear dependence approximately holds, at least below 13 T in GaAs. The resulting values of the relaxation time  $\tau_F$  are listed in Table I, along with the value  $\tau_0$  deduced from dc conductivity and the carriers concentration. For two of the samples we have quoted the relaxation time  $\tau_{ir}$  deduced from the values of the infrared absorption constant  $\alpha_{ir}$  at the laser frequency.

From Eq. (12), for  $b=0$

$$\tau_{ir} = 2\kappa k_0 d = (d\varepsilon_i^{1/2} \omega_p^2 / \alpha_{ir} c \omega^2) (1 + \omega_p^2 / 2\omega^2).$$

From the results, we can see that the dc and infrared relaxation time, at  $B=0$ , are in satisfac-

tory agreement on account of the difference which may be expected between  $\tau_0$  and  $\tau_{ir}$ , as discussed by Dumke<sup>23</sup> and Baltz and Escher.<sup>24</sup> These values are, however, markedly larger than the Faraday relaxation time. This behavior may be ascribed to an appreciable effect of the magnetic field on the scattering process and requires a more elaborate theoretical treatment. Preliminary calculations, based on Kubo's formalism, have shown, indeed, that whatever the scattering process may be, the imaginary dielectric constant starts to increase with the magnetic field.

For ionized impurity scattering, which is predominant in our samples, the  $B$  dependence of the dielectric constant is linear, at the lowest order. Calculations lead to the following expression of the relaxation frequency:

$$1/\tau = (e^4 / 48\pi^3 \varepsilon_0^2 \varepsilon_i^2 \hbar^2 \omega) \{-2k'_F k_F + (k_F'^2 + k_F^2 + 4/l^2) \times \ln[(k'_F + k_F)/(k'_F - k_F)]\}$$

where  $l$  is the cyclotron radius,  $l = (\hbar/eB)^{1/2}$ , and  $k'_F = (k_F^2 + 2m\omega/\hbar)^{1/2}$ . Parabolic band, zero absolute temperature, complete ionization of impurities, and infinite screening radius (since  $\omega \gg \omega_p$ ) are assumed. This expression is written in the compact form

$$1/\tau = (1/\tau_{ir})(1 + B/B_0) \quad (15)$$

by putting

$$1/\tau_{ir} = (e^4 / 48\pi^3 \varepsilon_0^2 \varepsilon_i^2 \hbar^2 \omega) \{(k_F'^2 + k_F^2) \times \ln[(k'_F + k_F)/k'_F - k_F] - 2k'_F k_F\}, \quad (16)$$

$$B_0 = \frac{(k_F'^2 + k_F^2) \ln[(k'_F + k_F)/(k'_F - k_F)] - 2k'_F k_F}{4(e/\hbar) \ln[(k'_F + k_F)/(k'_F - k_F)]}. \quad (17)$$

The relaxation time in zero field, as given by (16), can be shown to be a more explicit form of previous results.<sup>24</sup> Calculated values of  $\tau_{ir}$  and of the characteristic field  $B_0$  are listed in Table II. The reason for which the theoretical values of  $\tau_{ir}$  are larger than the experimental one can be explained by the presence of other scattering mechanisms.

TABLE II. Theoretical value ( $\tau_{ir}^{(th)}$ ) and experimental estimate ( $\tau_{ir}^{(F)}$ ) of the electron relaxation time in InSb and GaAs.  $B_0$  is the parameter of the linear field correction of the relaxation time.

		$\tau_{ir}^{(th)}$ ( $10^{-13}$ sec)	$B_0$ (T)	$\tau_{ir}^{(F)}$ ( $10^{-13}$ sec)
InSb	1	9.2	10.3	8-25
	2	6.5	13.6	10-40
	3	5.9	15.9	7-15
GaAs	1	3.7	16.8	8-17
	2	1.9	27.2	11-27

On the other hand, the decrease of  $\tau$  from  $\tau_{1r}$ , owing to the linear term, is typified by the field  $B_0$ . For a quantitative comparison with experiment, one must take care that the linear dependence only holds up to a small fraction of  $B_0$ , typically  $\frac{1}{10} B_0$ . In that range,  $b \ll 1$ , so that Eq. (14) can be simplified. By substituting  $1/\tau$  from (15), we obtain

$$\frac{d(\ln \Delta I)}{dB} \approx -\frac{d\varepsilon_I^{1/2} \omega_p^2}{c\omega^2 B_0} \left(1 + \frac{\omega_p^2}{2\omega^2}\right) \frac{1}{\tau_{1r}}. \quad (18)$$

The present experiment is not suitable for a test of Eq. (18). At low field (a few teslas) the decrease of  $\ln \Delta I$  is only known with a very poor accuracy. Nevertheless, by using the amplitude of the first two or three oscillations, we obtain a rough estimate for the experimental values of  $\tau_{1r}$ , denoted as  $\tau_{1r}^{(P)}$  in Table II. The order of magnitude is good, although the results seem too large.

This could be ascribed to the above mentioned simplifying assumptions of the theory.

In conclusion, the analysis of Faraday oscillations, by means of classical theory, offers a convenient way of determining the effective mass together with some average value of the relaxation time over a wide range of field. The preliminary results of a quantum mechanical treatment agree qualitatively (and quantitatively to some degree) with experiment. This encourages more precise transmission experiments at low field, as well as developments of the theory towards higher fields.

#### ACKNOWLEDGMENTS

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