Thermodynamic behavior of the charged Bose gas around $T = 0$

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We have used a low-temperature excitation spectrum to calculate the thermodynamic behavior of a charged Bose gas for $T \gtrsim 0$. These results are compared with the thermodynamic behavior obtained using Foldy's $T = 0$ excitation spectrum.

In 1970 Fetter[1] used the Landau[2] quasiparticle model together with Foldy's[3] zero-temperature ($T=0$) excitation spectrum to calculate the thermodynamic functions of a dense charged Bose gas for $T \gtrsim 0$.

In a previous paper the authors[4] have used a dielectric-constant formalism in the random-phase approximation to calculate the quasiparticle energy spectrum of a dense charged Bose gas for $T \gtrsim 0$. We have used this generalized low-temperature spectrum to calculate the thermodynamic functions of the dense charged Bose gas around $T=0$. These results can then be compared with Fetter's to determine whether or not the presence of temperature corrections in the energy spectrum changes the basic character of the thermodynamic functions for $T \gtrsim 0$ obtained using Foldy's $T=0$ excitation spectrum.

We have done this and it is found that the temperature corrections to the energy spectrum affected the thermodynamic functions only to the extent that they changed the coefficients of the third- and higher-order terms in the expansions for the thermodynamic functions. To illustrate this we here exhibit the results for the Helmholtz free energy. We find

$$F = -\left(\frac{m^3\omega_p^5}{2\sqrt{2}\pi^4\hbar}\right)^{1/2}\left(\frac{kT}{\hbar\omega_p}\right)^{7/4} e^{-\hbar\omega_p/kT}\left\{\Gamma(\tfrac{3}{4}) + \tfrac{1}{2}\Gamma(\tfrac{11}{4})\left(\frac{kT}{\hbar\omega_p}\right) - \left[\tfrac{1}{32}\Gamma(\tfrac{15}{4}) + \frac{3\sqrt{2}\,\zeta(\frac{5}{2})\Gamma(\frac{5}{4})}{\zeta(\frac{3}{2})}\left(\frac{\hbar\omega_p}{kT_c}\right)^{3/2}\right]\left(\frac{kT}{\hbar\omega_p}\right)^2 + \cdots\right\},$$

whereas Fetter finds

$$F = -\left(\frac{m^3\omega_p^5}{2\sqrt{2}\pi^4\hbar}\right)^{1/2}\left(\frac{kT}{\hbar\omega_p}\right)^{7/4} e^{-\hbar\omega_p/kT}\left[\Gamma(\tfrac{3}{4}) + \tfrac{1}{2}\Gamma(\tfrac{11}{4})\left(\frac{kT}{\hbar\omega_p}\right) - \tfrac{1}{32}\Gamma(\tfrac{15}{4})\left(\frac{kT}{\hbar\omega_p}\right)^2 + \cdots\right],$$

where T_c is the transition temperature of an ideal Bose gas, ω_p is the plasma frequency of the charged Bose gas, $\Gamma(x)$ is the Γ function, and $\zeta(x)$ is the Riemann ζ function.[4]

Thus the temperature corrections to the spectrum have a fairly minor effect on the thermodynamic functions. However, this may not have been anticipated as inspection of our expression[4] for $\omega(q,T)$ (where $\hbar\omega$ is the energy and q is the wave number of the quasiparticle) shows that our first-order term in a small-q expansion is a q^2 term, whereas Foldy[3] has a q^4 leading term. However, the temperature-dependent coefficient of our q^2 term is such that this term does not play as great a part in shaping the thermodynamic functions near $T \gtrsim 0$ as does the q^4 term of Foldy.

Thus the Foldy ground-state ($T=0$) excitation spectrum gives the correct leading-order terms in the low-temperature ($T \gtrsim 0$) thermodynamic behavior of the charged Bose gas. Low-temperature corrections to the spectrum produce third- and higher-order corrections to the thermodynamic functions at low temperatures.

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