

Some calculations regarding the characteristic length for superfluidity in liquid helium*

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A superfluid characteristic length has consistently emerged from a variety of independent experiments on helium thin films. Both the magnitude and the temperature dependence of this quantity are now quite well defined within relatively narrow limits. This length appears to consist of two parts: the thickness of solid blanketing the substrate underneath the liquid film plus a pseudolength deriving from supplementary excitations in the system. The first contribution is directly calculated, and the results appear to fit experiment quite well. For the remaining pseudolength we estimate the parameters necessary for a surface-roton-excitation hypothesis to fit experiment. We find, using the analysis presented, that the quantitative features of a surface-excitation spectrum may be deduced with considerable precision from presently available experimental data.

I. INTRODUCTION

Recent experiments on the disappearance of superfluidity in thin films of liquid helium have produced a wealth of detailed data on the subject.¹⁻⁴ In what follows a theoretical picture is offered which has the following features. The experimental results can be understood in a relatively elementary and traditional way, and quantitative conclusions are possible. In particular, hypothesizing surface excitations as a contributing factor, and demanding that theory fit experiment allows us to determine some detailed features of the hypothesized surface-excitation spectrum.

II. NATURE OF THE PROBLEM

In bulk, liquid helium exhibits its superfluid properties below a characteristic temperature—known as the λ temperature T_λ . The pressure dependence of this temperature constitutes the λ line on a pressure versus temperature ($P-T$) phase diagram (cf. Fig. 1). In the two-fluid picture the “extent of superfluidity” present in the system is measured by ρ_s —the superfluid density (effective mass of superfluid per unit volume). The remainder of the fluid is “normal.” Hence if the total density of the liquid is ρ , then the density ρ_n of the normal fluid is determined by

$$\rho_n + \rho_s = \rho . \tag{1}$$

If, now, our system consists of a very thin film of helium (of thickness ranging between a submonolayer to about 20 “layers”) covering the surface of a substrate, then an appropriate measure of its superfluid content is σ_s , the effective mass of superfluid per unit area. Again, if the total mass of film per unit area is σ , then the nonsuperfluid content is accounted for by $\sigma - \sigma_s$.

The essential experimental results to which we address ourselves are the following: For all film

thicknesses d greater than about twice the characteristic length L , it has been demonstrated experimentally that the variation of σ_s with d follows the form^{1,2}

$$\sigma_s = \rho_s (d - L) . \tag{2}$$

For all d less than L it is experimentally found that $\sigma_s = 0$. If the helium in the film exhibited just its bulk properties then we should ideally expect that

$$\sigma_s^{\text{ideal}} = \rho_s (d - H^*) , \tag{3}$$

where H^* represents the depth of nonsuperfluid helium which must necessarily underlay any film of liquid. (cf. Sec. IV following). Hence the departure from bulk ideal behavior is given by

$$\sigma_s - \sigma_s^{\text{ideal}} \equiv \delta\sigma_s = -\rho_s (L - H^*) = -\rho_s l . \tag{4}$$

The behavior with temperature of the characteristic length L has been carefully studied experimentally, and reasonable agreement among the measurements has been obtained employing a variety of distinct and independent techniques over an extended temperature range. The results are nice-

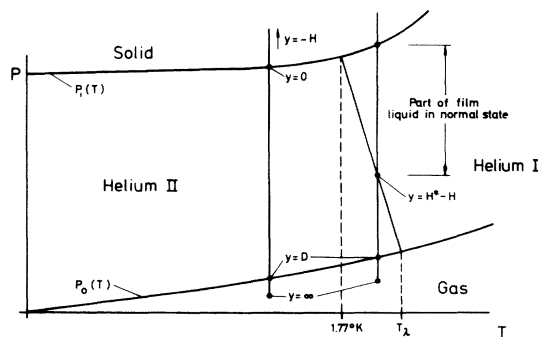


FIG. 1. Liquid-⁴He phase diagram: idealized schematic

ly summarized in Ref. 1.

Our purpose in what follows is to construct a theory which will yield just the observed behavior exhibited in Eq. (2) and will, in addition, reproduce the magnitude and the temperature dependence of both L and l over the range of temperatures explored.

III. BASIS FOR THE SOLUTION

Both its conceptual simplicity and its striking successes have produced universal acceptance of the Landau theory^{5,6} as the basis for understanding superfluidity in bulk helium. Although the foundation for this theory has been questioned by some, we accept its quantitative triumphs as the motivation for extending its application to the case of thin films. We feel, in view of its accomplishments, that to abandon this theory when dealing with thin film helium is philosophically unsatisfying. (Those with less confidence in the bulk theory would naturally disagree with this viewpoint.) On this basis we redirect our attention away from the characteristic length l . Rather, we view the experiments as indicating an alteration in the excitation level in the system. In the Landau theory ρ_n reflects the density of elementary excitations in the system. The superfluid density ρ_s is simply the difference between the total physical density ρ and ρ_n . In just this spirit we interpret the experimental observations summarized in Eqs. (2)–(4) as follows: The “superfluid content,” as seen in thin-film-helium experiments, is apparently reduced from its expected bulk value ($\delta\sigma_s < 0$). Hence, in such a system, there must be an increased density of excitations $\delta\sigma_n$ because

$$\delta\sigma_n + \delta\sigma_s = 0. \quad (5)$$

Any estimate of this increased density of excitations ($\delta\sigma_n$) immediately yields a characteristic length l through Eqs. (4) and (5).

$$l = \frac{-\delta\sigma_s}{\rho_s} = \frac{\delta\sigma_n}{\rho_s}. \quad (6)$$

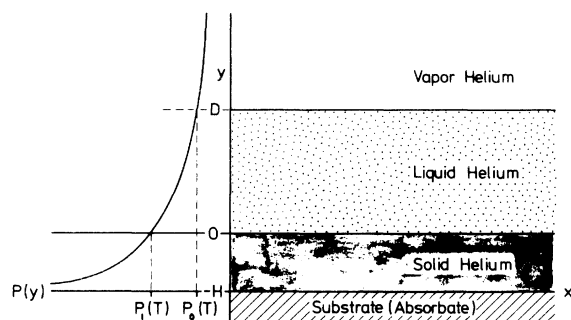


FIG. 2. Schematic view and coordinates to describe the adsorbed helium film.

It then remains only to consider the various possibilities for such an increased excitation density and to calculate $\delta\sigma_n$ for each of these. In conjunction with reasonable estimates of the thickness H^* of nonsuperfluid helium in the film, one can employ l as derived from Eq. (6), to deduce a theoretical value of L . The latter may be compared directly with experiment.

Essentially none of the elements of the program outlined above are new. In disjointed form they have appeared in the literature^{7–10} at least since 1958. However, in no case have theoretical values of L ever approached the magnitude of the experimental ones. Nor has H^* been directly calculated. In many cases even the temperature dependence of l (usually calculated rather than L) is manifestly incorrect. In others, the temperature dependence has been judged correct even though the magnitude was too small. Theory has never produced too large a result, only too small a one. This fact suggested that the fundamental notion embodied in Eq. (6) may indeed be valid, but that the dominant contribution to the increased σ_n had not been identified. Before proceeding on to consider the possible mechanisms to account for l we turn our attention to the calculation of H .

IV. THICKNESS (H, H^*) OF THE NONSUPERFLUID FILM COMPONENT

In order to estimate the thickness of solid helium under the liquid we employ precisely the same principles by which the total film thickness is estimated. The idea incorporates two admittedly tenuous assumptions. The first is that the interfaces are ideally flat (cf. Fig. 2) and the second is that one can describe the state of the helium film in terms of local thermodynamic variables even on a length scale comparable to the average interparticle spacing. We proceed in the face of these weak links on the following bases. First, the experimental results (estimates of d) summarized in Eq. (2) are themselves already predicated on these assumptions (cf. Ref. 1 and the citations listed there). Secondly, we accept the conclusions of Ref. 11 where strong evidence is offered on the empirical confirmation of these assumptions. Ultimately, however, the validity of the analysis must rest upon the success of its predictions.

The argument is as follows: Owing to van der Waals forces, the substrate produces a potential energy field $u(y)$ in which the helium adsorbed on the substrate finds itself (Fig. 2). A fundamental principle of thermodynamics applies when, in a single component system (${}^4\text{He}$), particles are interchangeable among the phases. This is that the total chemical potential μ^* is independent of position y . The total chemical potential is the sum of the bulk potential $\mu(P, T)$ as obtained from a “bulk”

phase diagram (Fig. 1) plus the external energy. This principle yields the equilibrium pressure increase $P(y)$ as the substrate is approached.

$$\mu(P(y), T) + u(y) = \mu^* \quad (\mu^* \text{ independent of } y). \quad (7)$$

Next we note that at $y = D$, the bulk chemical potential $\mu = \mu(P_0, T) = \mu_0(T)$ corresponds to that on the vapor pressure curve. The vapor pressure is $P_0(T)$, and at $y = 0$ the chemical potential must be that of the solidification curve $\mu_1(T)$. Hence we find from (7) that

$$\mu_1(T) + u(0) = \mu_0(T) + u(D), \quad (8)$$

but the difference $\mu_1(T) - \mu_0(T)$ is directly deducible from the thermodynamic data on a substance. In particular, in the case of relatively incompressible fluids, i. e., $\rho_1 - \rho_0 \ll \rho_0$ as for helium, this chemical-potential difference is easily estimated as

$$\mu_1 - \mu_0 \approx \frac{1}{\rho} [P_1(T) - P_0(T)] \approx \frac{2P_1(T)}{\rho_0 + \rho_1}, \quad (9)$$

where $P_1(T)$ represents the solidification-liquefaction pressure and where the average density is taken as the mean of the vapor-pressure liquid density ρ_0 and the solidification liquid density ρ_1 . For helium, $\mu_1 - \mu_0$ rises slowly from about 8.3°K per atom at $T = 1.2^\circ\text{K}$ to a value of about 11.2°K per helium atom near the λ point.

We are concerned here with the limit of large D . In this limit, Eq. (8) is just the statement that the solid-liquid boundary locates itself where the van der Waals attractive energy is just equal to the chemical-potential difference between solidification and vaporization of the liquid.

Turning now to the van der Waals potential $u(y)$, we note that customarily it is assumed to be of the form

$$u(y) = \frac{K\theta}{m} \left(\frac{D_0}{y+H} \right)^3, \quad (10)$$

where m is the mass of one helium atom and traditionally the value D_0 is taken as 3.6 \AA . This corresponds to the mean distance between helium atoms under normal vapor-pressure conditions and is hence used as the yardstick by which statistical layer thicknesses are measured. The parameter θ is a constant characterizing the substrate-helium interaction strength. In the literature (cf. citations and discussion in Ref. 1) values of θ vary between 15 to about 90°K . The exponent 3 seems to provide an adequate fit to data in many cases but values up to 4 and down to 2 have been discussed¹¹ as well as a continuous variation of the exponent with position y .

Anticipating that we will be comparing our theoretical result with the data offered in Ref. 1 we may estimate the thickness H of the solid layer

under an infinitely thick film as follows: We utilize Eq. (9) from which $\mu_1 - \mu_0 \approx 9^\circ\text{K}$ per atom in conjunction with the value $\theta = 27^\circ\text{K}$ employed in Ref.

1. The theoretical expectation for these experiments is then found to be $H/D_0 = (\frac{27}{9})^{1/3} = 1.44$. This is to be compared with the "bottoming" value of 1.47 exhibited in the data.

It should be noted that the length scale in Ref. 1 depends upon the choice of θ in just exactly the same way as our estimate of H . Hence, should the value of θ be revised both our value for H and the scale of Ref. 1 will be altered in the same way. The agreement between our H and the bottoming value of Ref. 1 is independent of the choice of θ .

We take the foregoing as strong evidence that at low temperatures the observed characteristic length L is essentially entirely due to the solid thickness of helium under the liquid film, i. e., $l(T) \approx 0$, $T < 0.8^\circ\text{K}$. This conclusion gives yet further weight to our resolution of L into two parts, one of which reflects excitations [Eq. (6)]. This is because all calculations of l , via Eq. (6) and Eq. (18) to follow, yield an l which vanishes at sufficiently low temperatures, i. e., supplementary excitations fail to be excited at low temperatures.

The idea that the measured L at low temperatures is entirely attributable to the solid undercoat has been assumed by some but certainly not by all. Without a clear theory of how to subtract out the solid layer, however, the estimates of the interesting excitation part l of the measurements is not reliable. For example, the traditional assumption that the solid layer is exactly D_0 thick leads one to conclude that there is a residual l at low temperatures. To deduce l by subtracting out the "constant low-temperature bottoming-out" value at all temperatures, is also not valid. In fact H varies with temperature. In Fig. 3, the actual variation of the expected solid layer thickness H with temperature is exhibited in detail. This is obtained by utilizing the known values of the parameters in Eq. (9) in conjunction with Eqs. (10) and (8) in the thick-film limit (D large). For the case of thinner films (D not large), the solid thickness varies somewhat with total film thickness. This variation is easily estimated from Eq. (8).

Because the λ line is not vertical (Fig. 1) it is not always true that the nonsuperfluid helium is just equal to the underlying solid. Above 1.77°K , where the λ line meets the solidification curve, a thin region of He I must also be expected. If we denote by H^* the thickness of the entire quantity of nonsuperfluid helium in the adsorbed film, then it is H^* that ought to be subtracted from L to arrive at l , rather than H .

$$l = L - H^*. \quad (11)$$

The thickness of both the solid plus the He I to-

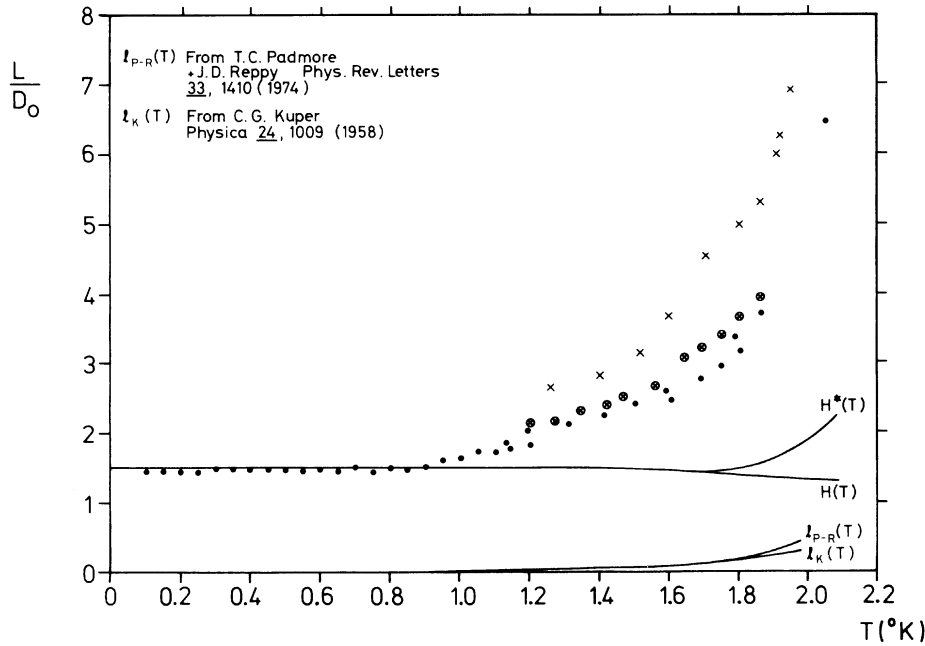


FIG. 3. Some characteristic lengths as functions of temperature. The points are experimental determinations of L as reproduced from Ref. 1. The curves are theoretical determinations of the thickness of solid (H), the thickness of nonsuperfluid (H^*), and two theoretical estimates of $l(T)$ plotted from data in the literature cited. Everything in units of $D_0 = 3.6 \text{ \AA}$.

gether constitutes H^* . (Below 1.77°K , of course, $H=H^*$.) This quantity is obtainable in just the same way as H . One employs Eqs. (8)–(10) again but, between 1.77°K and T_λ , the values ρ_1 and P_1 are replaced by the corresponding quantities characterizing the λ line in the phase diagram (Fig. 1). The plot of H^* vs T is also exhibited in Fig. 3.

Though not appreciated at the time of publication, confirmation for the value of H comes from an entirely independent source. In Ref. 11 is exhibited (Fig. 6 of this reference) an experimental determination of the van der Waals energy as a function of adsorbed mass per unit area. From this figure the energy of 9°K per atom corresponds to an adsorbed mass of about $1.2 \times 10^{-8} \text{ g/cm}^2 = 2.3 \rho_0 D_0$.

In the same series of experiments which yielded this result an estimate was obtained of the mass density of the bottom layer of adsorbed atoms. As explained in Ref. 2 this estimate was founded on the notion that Brunauer-Emmett-Teller isotherms^{12–14} yield the amount of mass in the first two statistical layers of helium. Simply put Brunauer-Emmett-Teller isotherm plots yield the quantity σ_2 where

$$\sigma = \sigma_2 + \rho_0(d - 2D_0). \quad (12)$$

In Ref. 2, rather than σ_2 , the quantity $\sigma_1/\rho_0 D_0$ derived from it is given where

$$\sigma_1 \equiv \sigma_2 - \rho_0 D_0 \quad (13)$$

because, with the conventional wisdom of the time, just exactly the first layer was held to be solid. Now if we retain the Brunauer-Emmett-Teller re-

sult but refrain from insisting that of the first two layers one be solid and one be liquid then we may proceed as follows to find the extent of solid. Commensurate with Eq. (12) we may also express the total adsorbed mass per unit area as

$$\sigma = \sigma_H + \rho_0(d - H), \quad (14)$$

where σ_H represents the adsorbed mass of solid (experimentally $\sigma_H = 2.3 \rho_0 D_0$) and the density beyond the solid is taken to be roughly that of the vapor pressure liquid. Combining Eqs. (12) and (14) it follows immediately that H is given by

$$\frac{H}{D_0} = \frac{\sigma_H}{\rho_0 D_0} - \frac{\sigma_1}{\rho_0 D_0} + 1 = \frac{\sigma_H}{\rho_0 D_0} - \frac{\sigma_2}{\rho_0 D_0} + 2. \quad (15)$$

Since $\sigma_1/\rho_0 D_0$ is, from Ref. 2, approximately 1.8, we arrive at the same value $H = 1.5 D_0$. We note that this result arises independent of any assumption regarding θ or the form of the van der Waals energy [Eq. (10)].

We now turn to the discussion of the physically more interesting quantity l and its interpretation [Eq. (6)] as arising from supplementary excitations not evident in the bulk.

V. SOME SUPPLEMENTARY EXCITATION MECHANISMS

Mechanisms which could conceivably alter the effective normal fluid content are the following:

(i) *Film local-pressure effect.* It is well known that the local pressure rises steadily with depth proceeding inward from the free surface to the substrate (cf. Sec. IV). Hence between this surface and the solid-layer depth the normal-fluid content must exhibit an enhancement commensurate

with the range of high pressures encountered. This notion may be summarized by noting that $\rho_n = \rho_n(y) = \rho_n(P(y))$ so that

$$\delta\sigma_n = \int_0^D [\rho_n(P(y)) - \rho_n(P_0)] dy. \quad (16)$$

The vapor pressure of the liquid at the temperature T is $P_0 = P_0(T)$, and of course $P(y)$ refers to the pressure at position y as shown in Fig. 2. As is clear in this figure, the liquid extent of the film is represented by $D = d - H$. This mechanism, besides contributing too little, can never produce the observed effect that $\sigma_s = 0$ for a finite range of $D > 0$ (Eq. 2).

(ii) *Roton image attraction.* It has been suggested that rotons see their images in the substrate "mirror" and that they are attracted by these images.⁷ This would increase the "bulk" roton (excitation) density near the substrate boundary giving rise to a $\delta\sigma_n$. This view amounts to a position variation of the roton energy gap. It is equivalent to a potential energy of roton interaction $V(y)$ (roton-image attraction energy) added to the constant gap. In the case of $D \rightarrow \infty$ this is readily shown to imply that

$$\rho_{\text{roton}}(y) = \rho_{\text{roton}}(\infty) \exp[-V(y)/KT]. \quad (17)$$

Since $V(y)$ is independent of T and decreases in magnitude with increasing y , unless something rather exotic is envisioned for $V(y)$, Eq. (17) yields a characteristic length which decreases with temperature! In addition, if one envisions rotons as microscopic vortex rings,⁶ a negative $\delta\sigma_n$ is obtained because such rings repel—not attract—their wall images.

(iii) *Discrete nature of the excitation wave vector k_y .* It is evident that in sufficiently restricted geometries the use of a continuous spectrum of wave vectors is invalid. Periodic or rigid boundary conditions at $y = 0$ and at $y = D$, for example (cf. Fig. 2), modify the values of k_y available to the system. This and related points of view have been discussed extensively by Padmore.⁸ A direct plot of the results obtained with this viewpoint as presented in Ref. 8, is exhibited in Fig. 3 together with the experimental points. The magnitude is clearly too small.

(iv) *Surface excitations: Liquid-vapor hydrodynamic regime.* The liquid helium in a thin film is characterized by two surfaces—the liquid-vapor "free" surface and the liquid-solid "bottom" surface. As is well-known classically, surface waves may propagate along these surfaces. Much has been calculated and measured with regard to the surface tension waves propagating along the "free" liquid-vapor boundary. And, in fact, just the contribution of this mechanism to $\delta\sigma_n$ was presented by Kuper in 1958 in his Eqs. (3.13) and (3.14) of

Ref. 7. (His ρ_{surf} is exactly our $\delta\sigma_n$.) This contribution is exhibited graphically in Fig. 3. As noted by Kuper himself it is clearly too small.

(v) *Surface excitations: Liquid-solid hydrodynamic regime.* One may envision the possibility of surface waves propagating along the liquid-solid interface. In fact such waves should be present. We have calculated the excitation spectrum (ω vs k) for interface waves in the hydrodynamic regime (k^{-1} much greater than the interatomic spacing). Employing the assumption that there is no mass transfer (melting solidification) between the phases and taking account of the presence of shear stresses in the solid and their absence in the liquid we have found that ω rises with k considerably faster than for the liquid-vapor case. This immediately implies that $\delta\sigma_n$ from this cause will be even smaller than what Kuper calculated for the "free" liquid surface. We therefore reject this mechanism.

VI. SURFACE ROTONS

For an isotropic system whose wave vectors k are restricted to only two components (k_x and k_z in our case) a direct application of Landau's original reasoning on ρ_n for bulk, yields for the two-dimensional case.^{7,15}

$$\delta\sigma_n = \frac{\hbar^2}{4\pi KT} \int_0^b k^3 dk \left(-\frac{dn}{d\epsilon} \right), \quad (18)$$

where, with K as Boltzmann's constant,

$$\epsilon \equiv \hbar\omega(k)/KT \quad (19)$$

and n is the Bose-Einstein distribution function for massless particles

$$n = (e^\epsilon - 1)^{-1}. \quad (20)$$

In general the upper limit b represents some appropriate terminal wave vector for the system. On the simple basis that the number of degrees of freedom per unit area in a single layer of surface atoms should equal the number of permissible surface modes per unit area one arrives at $b \approx 1.8 \text{ \AA}^{-1}$. As one might expect this corresponds quite closely to a minimum wavelength of just the interatomic spacing for the high density liquid-solid region.

To estimate $\delta\sigma_n$ from Eq. (18), only the spectrum $\omega = \omega(k)$ for the surface excitations is needed. Mechanisms (iv) and (v) of Sec. V were evaluated employing spectra calculated for the hydrodynamic regime. The extrapolation of the dispersion law (ω vs k) into the short-wavelength regime is, of course, not valid. Generally, however, for spectra obtained in the hydrodynamic regime, the large-wave-number extrapolated frequencies are so high that their contribution to the integral is negligibly small regardless of the value of b . (Us-

ually b is taken to be infinite.) The content, then, of points (iv) and (v) in Sec. V is that the extra excitations produced by the long-wavelength part of the dispersion curve, for either surface of the liquid helium, is insufficient to account for the observed $\delta\sigma_n$.

We now consider the possibility that the dispersion curve does not continually rise monotonically as it does in the hydrodynamic regime. We suppose, instead, that the curve of ω vs k exhibits a dip in the region of some short wavelength corresponding to $k = \kappa \approx 1.8 \text{ \AA}^{-1}$. This is the direct analog of the dip in the bulk-helium spectrum. Because in the bulk this part of the spectrum is called the "roton dip," the analogous two-dimensional region has been termed the "surface-roton" region. However, that this reflects some form of "two dimensional smoke-ring" excitation is not meant, of course, to be implied.

A roton dip in the surface excitation dispersion curve has been proposed previously.^{8,9,16} Reut and Fisher¹⁶ postulated its existence for the free (liquid-vapor) surface. The rotions proposed by them were employed to better adjust the correspondence between the Atkins¹⁷ theory of the liquid-vapor surface tension and the experimental measurements on this quantity. Reut and Fisher decided that the surface roton gap Γ ought to be about 1.95°K and that $\kappa \approx 0.5 \text{ \AA}^{-1}$, from which they conclude that the surface-roton effective mass is of the order $m^* = 0.019m$ (m is the mass of one helium atom).

A recent publication of Padmore¹⁸ examined a two-dimensional array of helium atoms for rotonlike behavior in its excitation spectrum. He found such behavior, but at normal densities the gap obtained (9.3°K) did not differ significantly from the bulk gap. Furthermore, the connection between this model and the fundamental problem of excitations of the interface between two media is yet to be established.

Recently, a very astute publication of Edwards *et al.*¹⁹ has provided us with at least one substantial theoretical basis for expecting roton behavior in a surface excitation spectrum. Simply put, this paper shows that features of the bulk excitation spectrum of any system are necessarily always projected into its surface excitation spectrum. In particular, then, the bulk-helium roton spectrum does find its counterpart in the surface spectrum. Unfortunately it can also be shown, through the same mathematical structure, that the projection of roton bulk behavior onto an inherently monotonic surface excitation cannot produce an energy gap significantly lower than the bulk one—of about 9°K. The calculated curves exhibited by Edwards *et al.* demonstrate just this behavior. Only if the short-wave part of the surface spectrum exhibited its

own inherent rotonlike behavior could a smaller energy gap arise. To the best of our knowledge, no suggestion has been advanced as to what would yield such an inherent true surface-mode roton behavior.

The effect of surface rotions in the present case is easily estimated. The roton part of the spectrum is taken, in analogy with the three-dimensional case, as

$$\hbar\omega = \Gamma + \frac{\hbar^2}{2m^*} (k - \kappa)^2. \quad (21)$$

This spectrum is inserted into the integral (18) and the calculation is performed in exactly the same way as Landau's original one. The result, valid for temperatures $T < \Gamma/K$, is

$$\delta\sigma_n = \hbar\kappa^3 \left(\frac{m^*}{8\pi KT} \right)^{1/2} e^{-\Gamma/KT}. \quad (22)$$

With this result, in conjunction with Eqs. (6) and (11), we may inquire into what parameters Γ and $\kappa^3(m^*)^{1/2}$ will provide a good fit to the experimental data. The entire curve must be fit both in magnitude and form. It happens, in the present case, that the choice for each of the parameters, Γ and $\kappa^3(m^*)^{1/2}$ are independent of each other, i. e., Γ determines the general shape of the curve and the magnitude of the effect is adjustable with $\kappa^3(m^*)^{1/2}$. It is easily demonstrable that a choice of Γ which deviates by more than about $\pm 15\%$ from the value $\Gamma = 4.5^\circ\text{K}$ can never fit the data regardless of the choice of $\kappa^3(m^*)^{1/2}$. Once fixing Γ at 4.5°K , though, the experimental magnitude of the effect then demands that

$$(D_0\kappa)^3 (m^*/m)^{1/2} = 122. \quad (23)$$

Taking κ to be of the order 1.8 \AA^{-1} , the effective mass value is found to be $m^* = 0.20m$. (The corresponding three dimensional parameters are 1.9 \AA^{-1} and $0.16m$.) The resulting theoretical curve is exhibited with the experimental data in Fig. 4. On the same figure (curve *R-F*) the equivalent result is shown when the parameters of Reut and Fisher are employed. Contrary to intuitive expectation, the contribution from their 2°K liquid-vapor surface rotions is much smaller than from our 4.5°K rotions! This results from their very small effective mass (large curvature) and small κ . Although a 2°K dip would, in general, yield more excitations than a 4.5°K one, the amount of phase space (k space) occupied by the dip is quite restricted in their case. Hence the Reut and Fisher rotions produce too small an effect compared with the observed experimental result. In fact, as already pointed out, neither the 2°K of Reut and Fisher nor the 9°K of Edwards *et al.* could ever fit the experimental result regardless of adjustments on m^* and κ . It is interesting to note, how-

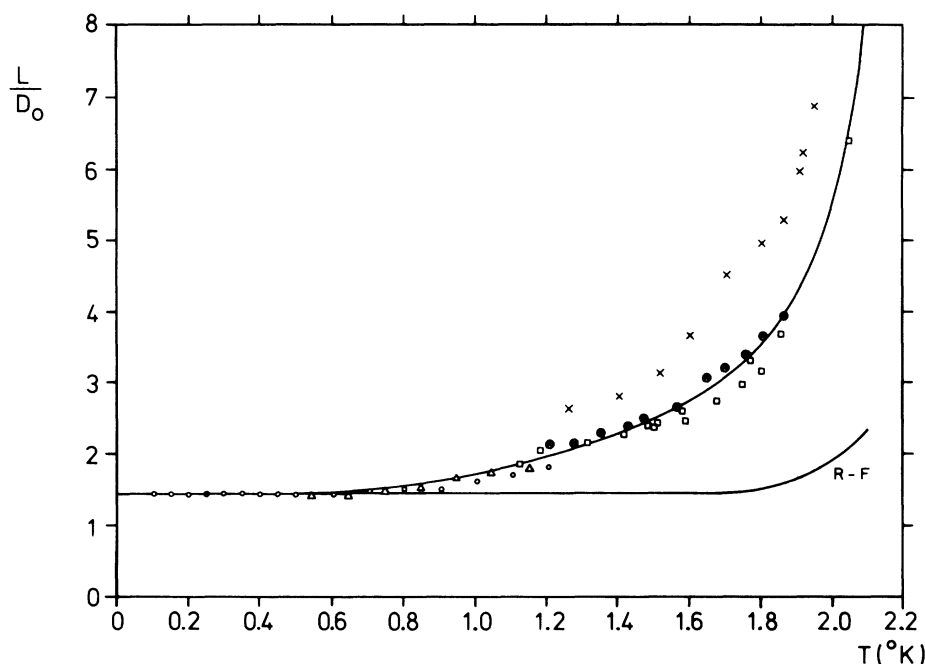


FIG. 4. Points: experimental data reproduced from Ref. 1. Upper curve: $L(T)$ as calculated from surface roton excitations with a gap of 4.5°K . Lower curves (R-F): $L(T)$ as calculated from the surface roton excitations proposed by Reut and Fisher (Ref. 16).

ever, that with the parameters suggested these possibilities produce results which are too small to account for the observed characteristic length l as do all the other mechanisms explored (cf. Sec. V).

VII. CONCLUSION

The characteristic length for the disappearance of superfluidity in helium films is a quantity well explored experimentally. We have outlined a particular point of view regarding a theoretical explanation of the experimental data. The characteristic length is the sum of two parts: the thickness of nonsuperfluid abutting the solid substrate plus a pseudolength arising from supplementary excitations. We have calculated the nonsuperfluid thickness and find that this accounts for the entire effect observed at sufficiently low temperatures. With regard to the pseudolength it appears that all possibilities previously explored produce too small a result. We have estimated the effect that surface rotors might produce. And we find that the experimental data can be fit quantitatively quite well with surface rotors much like the bulk ones except with a roton energy gap only half as large. Whether the surface rotors obtain at the vapor-liquid surface or at the solid-liquid surface (or both) is not clear. If the calculations of Reut and Fisher regarding the roton contributions to the liquid-vapor surface tension are valid, then the present surface rotors must be presumed to be excitations of the solid-liquid surface boundary.

Some physical implications regarding these re-

sults are the following:

(i) The theory presented here suggests that a large part, if not all, of the characteristic length obtained in thin-film experiments is connected only with the substrate side of the helium film. Hence a free (liquid-vapor) surface would have a significantly smaller characteristic length. Therefore appropriate experiments done in floating two-dimensional systems—as in “space,” for example—should exhibit “bulk” behavior down to submonolayer sizes. Similarly, experiments confined to the free surface alone—far removed from a substrate—also ought to yield a far smaller characteristic length. On the other hand, systems totally constrained by solid surfaces and without a “free” surface ought to have an enhanced ($\times 2$) characteristic length.

(ii) If a correspondence is supposed to exist between the present offering and the Ginzburg-Pitaevskii healing length,²⁰ then the foregoing suggests a set of boundary conditions for the order-parameter ψ . Depending upon which surface does exhibit the 4.5°K roton gap, a correspondence might obtain if ψ went to zero only at this surface.

(iii) Unexplored here, but clearly implied, must be implications regarding surface specific heats.

(iv) A definite prediction can be deduced from the proposed dispersion curve with regard to a surface scattering structure factor. Such a quantity could conceivably be measured by neutron scattering in geometrically prepared severely constrained systems.

(v) The theory outlined here suggests a possible

resolution to the problem of Kapitza resistance. The central issue in this problem is to unearth possible mechanisms to explain the relatively efficient heat transfer through the helium-substrate boundary. Phonons are observed to penetrate this boundary better than is theoretically predicted. This is especially so for short-wavelength phonons. The following question is therefore suggested: To what extent does the presence of low-energy (4.5°K) surface excitations enhance the phonon transfer process? One expects that the surface excitations might have the effect of relaxing the momentum and energy constraints limiting the transfer process in their absence thus providing an enhancement mechanism.

Considerable credit for assistance during all of

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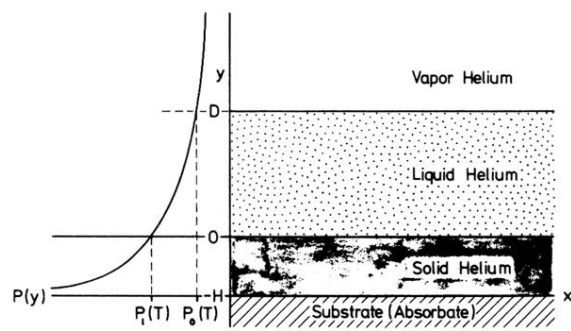


FIG. 2. Schematic view and coordinates to describe the adsorbed helium film.