Conduction in the relaxation regime

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The conduction behavior of ideal relaxation semiconductors is studied in the general case with trapping included. The investigation of the range of validity of the relaxation concept leads to a more restricted defining condition for the relaxation regime. With increasing deviations from the ideal relaxation regime, the conduction approaches gradually the well-known behavior of conventional (lifetime) semiconductors. In particular, it is shown that the trapping conditions required for "recombinative space-charge injection" are incompatible with thermodynamic steady-state requirements.

I. INTRODUCTION

Though there has been extensive experimental and theoretical work on the transport properties of semiconductors during the past decades, most of the efforts have been concentrated on the highly conducting materials which are of the highest technical importance. Only recently, semiinsulating materials have drawn wider attention, especially due to the increased interest in amorphous and organic semiconductors and in connection with switching phenomena.

Whether the bulk conductivity of a semiconductor is low or high depends only on the concentration and on the mobility of the mobile electrons and holes. However, additional information about the free-carrier recombination lifetime is needed for solving conduction problems whenever there are local deviations from the bulk equilibrium carrier concentration. The finite lifetime of the free carriers is of crucial importance for the conduction behavior of semiconductors, since it opens up a second channel for the change of the local free-carrier concentration. The decay of a small local disturbance, say an excess electron concentration Δn, is characterized by two time constants (see Fig. 1): (a) Due to their mobility, the excess carriers move away into regions of lower concentration. The time constant for this process is the dielectric relaxation time τ_D, defined by the ratio of the dielectric constant κ and the conductivity σ,

$$\tau_D = \kappa/4\pi\sigma \tag{1.1}$$

(provided that diffusion may be neglected). (b) The other process is recombination with carriers of the other type, characterized by the recombination lifetime τ_0. This process tends to establish local thermal equilibrium.

The conduction behavior is expected to be quite different depending on which one of both decay processes is dominant. van Roosbroeck[1] therefore introduced the distinction between "lifetime semiconductors," characterized by a minority carrier lifetime exceeding the dielectric relaxation time, i.e.,

$$\tau_0 > \tau_D\ , \tag{1.2}$$

and "relaxation semiconductors," where recombination-generation processes are dominant due to a carrier lifetime, short compared with the dielectric relaxation time, i.e.,

$$\tau_0 < \tau_D\ . \tag{1.3}$$

In conventional semiconductors, the relation $\tau_0 \gg \tau_D$ is fulfilled and is usually assumed in theoretical treatments. This means that a carrier lives very long as compared to the time needed to level off a disturbance. It is evident that the requirements for $\tau_0 \gg \tau_D$ to hold is that the minority carriers have little chance of recombination and that the carrier mobility must be large. In amorphous and in semiinsulating crystalline semiconductors with short free-carrier lifetimes, these conditions will usually not be fulfilled. The relation may even be reversed so that case (1.3) may be satisfied.

In Sec. II, we suggest that the defining condition

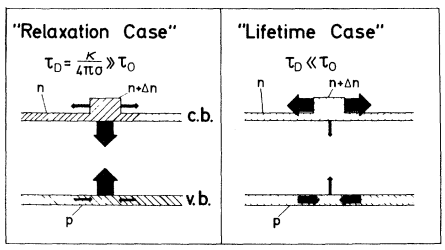

FIG. 1. Reaction of a "relaxation" and a "lifetime" semiconductor to a local disturbance.

(1.3) for "relaxation semiconductors" be replaced by a new one defining the "relaxation regime." This definition stands in closer connection to the characteristic appearance of the particular conduction behavior. In Secs. III–VI, we treat the conduction problem in the ideal relaxation regime $\tau_0 \to 0$ and $\mu \to 0$ for an arbitrary trap distribution. The solutions differ in essential points from those given by van Roosbroeck *et al.*[1] In Sec. VII, the limitations of the relaxation regime are worked out. Finally, in Sec. VIII, it is shown that any solution for finite lifetime and mobility lies in between those for the idealized relaxation regime and those for the idealized lifetime case $\tau_0 \to \infty$ and $\mu \to \infty$.

II. RELAXATION REGIME

The relaxation case was originally defined by van Roosbroeck[1] by the relation (1.3). In subsequent work, various authors[2–5] interpreted and used this relation in rather different ways which led to the fact that there are now several distinct definitions of the relaxation case. The reason for this is that (1.3) alone does not define a particular regime of conduction behavior independently of temperature and applied external field. In order to make this point more specific, it seems, therefore, worthwhile to give here a more exact definition.

If there is a gradient in the equilibrium carrier concentration (e.g., due to carriers injected by the electrodes or due to a doping gradient in a p-n junction), then there will always be a splitting of the Fermi level ϕ into quasi Fermi levels ϕ_n and ϕ_p for electrons and holes, respectively, whenever an external field is applied. The magnitude of this splitting depends on the ratio τ_D/τ_0 and on the electric field. If τ_D/τ_0 is very small as in a normal semiconductor, this splitting will be rather large even for small fields. If, however, τ_D/τ_0 is large—which is the case in our context—then the low-carrier mobility and their short lifetime both cooperate in maintaining local equilibrium, and the splitting is much smaller for the same field. We now define the "relaxation regime" by the condition that the splitting be much smaller than the thermal energy throughout the specimen, i.e.,

$$|\phi_n(x) - \phi_p(x)| \ll kT \quad \text{for all } x. \tag{2.1}$$

This means that the deviations from local thermal equilibrium are negligible in the relaxation regime. As a consequence, one has

$$n(x)\,p(x) \simeq n_i^2 \quad \text{for all } x. \tag{2.2}$$

We will use the approximation

$$\phi_n(x) = \phi_p(x) = \phi(x) \tag{2.3}$$

throughout the specimen as the basic relation in discussing the ideal relaxation regime.

III. DEFINITIONS AND BASIC RELATIONS

As the relaxation regime pertains to amorphous as well as to crystalline semiconductors, we will try to keep the model as general as possible.

We assume the existence of a mobility gap, i.e., the mobility is taken to be μ_n above the energy E_c, μ_p below E_v, and zero between the mobility edges E_c and E_v, according to Fig. 2(b). The density-of-state curve $N(\epsilon)$, in principle, may have any arbitrary shape, as shown in Fig. 2(a). The Fermi level in the unmodulated bulk is ϕ_0. E_c, E_v, and ϕ_0 depend on the coordinate x along the specimen if contacts are applied.

The concentration of mobile electrons and holes at temperature T is given by

$$n(x) = \int_{E_c}^{\infty} N(\epsilon) f(\epsilon - \phi(x))\, d\epsilon = n_i\, e^{\beta[\phi(x) - \psi(x)]} \tag{3.1}$$

and

$$p(x) = \int_{E_c}^{\infty} N(\epsilon)[1 - f(\epsilon - \phi(x))\, d\epsilon = n_i e^{-\beta[\phi(x) - \psi(x)]}, \tag{3.2}$$

respectively, where $\phi(x)$ is the actual local-Fermi level, $f(\epsilon - \phi)$ is the Fermi distribution function

$$f(\epsilon - \phi) = (e^{\beta(\epsilon - \phi)} + 1)^{-1}, \tag{3.3}$$

and $\beta = 1/kT$. The intrinsic free-carrier concen-

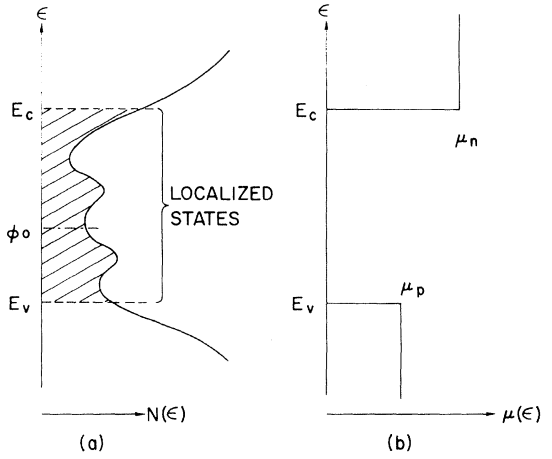

FIG. 2. Generalized model for a relaxation semiconductor. Density-of-states curve $N(\epsilon)$ may have any arbitrary shape. ϕ_0 is the Fermi level in the unmodulated bulk. Mobilities for electrons above E_c and for holes below E_v are assumed to be constant.

tration n_i and the intrinsic Fermi level $\psi(x)$ are defined by

$$n_i = (N_c N_v)^{1/2} e^{-\beta(E_c - E_v)/2} , \tag{3.4}$$

and

$$\psi(x) = \tfrac{1}{2}[E_v(x) + E_c(x)] - \tfrac{1}{2}\beta^{-1} \ln(N_c/N_v) , \tag{3.5}$$

with

$$N_c = \int_{E_c}^{\infty} N(\epsilon)\, e^{\beta(\epsilon - E_c)}\, d\epsilon , \tag{3.6}$$

and

$$N_v = \int_{-\infty}^{E_v} N(\epsilon)\, e^{\beta(E_v - \epsilon)}\, d\epsilon . \tag{3.7}$$

IV. CURRENT AND SPACE CHARGES

If thermal equilibrium is assumed at every point, then both the conductivity $\sigma(x)$ and the space charge $\rho(x)$ are uniquely determined by the local value of $\phi(x) - \psi(x)$.

The current consists of a drift and a diffusion term

$$\begin{aligned} j_n &= e\mu_n nE + eD_n \operatorname{grad} n , \\ j_p &= e\mu_p pE - eD_p \operatorname{grad} p . \end{aligned} \tag{4.1}$$

Using (3.1), and (3.2), and the Einstein relation $D_n = \mu_n/\beta e$, and $D_p = \mu_p/\beta e$ for the diffusion coefficients, one obtains for the total current $j = j_n + j_p$

$$\begin{aligned} j(x) &= n_i(\mu_n e^{\beta(\phi-\psi)} + \mu_p e^{-\beta(\phi-\psi)}) \frac{\partial \phi}{\partial x} \\ &= \frac{1}{e}\sigma(x) \frac{\partial \phi}{\partial x} . \end{aligned} \tag{4.2}$$

It is clear from Eq. (4.2) that the driving force is $(1/e)\ (\partial\phi/\partial x)$, the slope of the Fermi level instead of the electric field $E = -(1/e)\ (\partial\psi/\partial x)$ if diffusion is included.

From the continuity condition for the steady state

$$j(x) = j = \text{const} , \tag{4.3}$$

it follows that this slope is inversely proportional to the local conductivity for a given external current j, that is,

$$\frac{\partial \phi}{\partial x} = j[\, n_i(\mu_n e^{\beta(\phi-\psi)} + \mu_p e^{-\beta(\phi-\psi)})]^{-1} . \tag{4.4}$$

If $\phi(x) - \psi(x)$ differs from the unperturbed value $(\phi - \psi)_0$, then the local space-charge density

$$\begin{aligned} \rho(x) = -e \int_{-\infty}^{\infty} d\epsilon\, N(\epsilon) \\ \times [f(\epsilon - \phi(x)) - f(\epsilon - \phi_0(x))] \end{aligned} \tag{4.5}$$

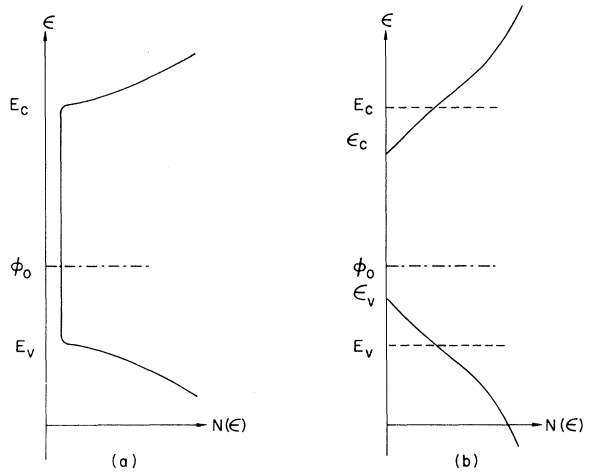

FIG. 3. Density of states for special models. (a) Constant density of states in the mobility gap, (b) linear band tails.

appears. Equation (4.5) shows that $\rho(x)$ is a monotonic function of $\phi(x) - \psi(x)$. The space-charge density is negative if $\phi(x) - \psi(x) > (\phi - \psi)_0$, and positive if $\phi(x) - \psi(x) < (\phi - \psi)_0$.

From Poissons equation

$$\partial^2\psi/\partial x^2 = 4\pi e\rho(x)/\kappa , \tag{4.6}$$

it follows that the bands are curved downwards for $\phi(x) - \psi(x) > (\phi - \psi)_0$ and upwards for $\phi(x) - \psi(x) < (\phi - \psi)_0$.

The space-charge density $\rho(x)$ depends on the shape of $N(\epsilon)$, on $\phi(x)$, $\phi_0(x)$, and on temperature. For materials with a high density of states in the region between ϕ_0 and ϕ, the Expression (4.5) reduces to

$$\rho(x) \simeq -e \int_{\phi_0(x)}^{\phi(x)} N(\epsilon)\, d\epsilon , \tag{4.7}$$

for low temperatures. In this case, ρ is practically determined by the change in the number of carriers in deep localized states and it is nearly temperature independent up to relatively high temperatures. For a model with constant density of states N_0 in the mobility gap, as shown in Fig. 3(a), one gets

$$\phi(x) \simeq -e N_0[\phi(x) - \phi_0(x)] . \tag{4.8}$$

If, however, $N(E)$ is nonzero only in the band tails and outside the mobility gap, then ρ depends on the number of thermally excited carriers and therefore shows a strong temperature dependence for a given value of $\phi - \phi_0$. For a model with linear band tails, as shown in Fig. 3(b), the space-charge density becomes

$$\rho(x) \simeq -e n_t \{\sinh[\beta(\phi - \psi)] - \sinh[\beta(\phi - \psi)_0]\}, \tag{4.9}$$

for $\epsilon_v < (\phi, \phi_0) < \epsilon_c$, where $(\phi - \psi)_0$ and n_t are temperature dependent. Therefore, the width of the space-charge region near an electrode also can be strongly temperature dependent in this case, while it is nearly temperature independent in the above mentioned case.

V. CURRENT-VOLTAGE CHARACTERISTICS

Equations (4.4) and (4.6) form two coupled differential equations

$$\frac{\partial \phi}{\partial x} = \frac{j}{\sigma(\phi - \psi)} \tag{5.1}$$

and

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi e \kappa^{-1} \rho(\phi - \psi) , \tag{5.2}$$

determining $\phi(x)$ and $\psi(x)$, if proper boundary conditions are imposed. The natural boundary conditions are determined by the position of the Fermi level in the electrode material relative to the band edges of the bulk, and by the externally applied voltage. Equations (5.1) and (5.2) then form an eigenvalue equation for the current j. For numerical calculation, however, it is more convenient to define j and the position of the Fermi level at the contacts and to calculate the appropriate external voltage by a step-integration method. Any arbitrary density of states distribution $N(\epsilon)$ and different electron and hole mobilities may be treated in numerical calculations without problems. However, from (4.2) we see that $\sigma(\phi - \psi)$ is always cosh shaped with its minimum at $\phi - \psi = (\frac{1}{2} kT) \times \ln(\mu_p/\mu_n)$, and from (4.5) it follows that $\rho(\phi - \psi)$ must be a monotonic function of $(\phi - \psi)$ with its zero at $\phi - \psi = (\phi - \psi)_0$, independent of the shape of $N(\epsilon)$. Therefore, the qualitative behavior of the solutions does not depend very much on either the ratio of the mobilities or on the actual shape of $N(\epsilon)$. For this reason, we will restrict our discussion to the case

$$\mu_n = \mu_p = \mu , \tag{5.3}$$

and to the two limiting cases of either a high and constant density of states over the whole mobility gap, as in our example in Fig. 3(a), or a negligible density of states over a wide range of the mobility gap, as in our example from Fig. 3(b).

For discussing the properties of the solutions of Eqs. (5.1) and (5.2), we have to consider the appropriate boundary conditions for the different contact-bulk combinations separately: (i) The most interesting case is the one where one contact injects majority—and the other one minority carriers. For this combination, there always exists exactly one conversion point x_c (see Figs. 4 and 5), where $\phi(x_c) = \psi(x_c)$. At this point the conductivity changes from p type to n type and attains its minimum value

$$\sigma_{\min} = 2e\mu n_i . \tag{5.4}$$

In most of the rest of the specimen, the conductivity is exponentially larger and close to the bulk conductivity

$$\sigma_{\text{bulk}} = \sigma_{\min} \cosh[\beta(\phi - \psi)_0] . \tag{5.5}$$

The main contribution to the total resistance

$$S^{-1} \int_0^L \sigma^{-1}(x) dx \tag{5.6}$$

(S being the cross section of the sample), therefore comes from the region around the conversion point x_c, as long as $\cosh[\beta(\phi - \psi)_0] \gg 1$ (which always holds at sufficiently low temperatures), and if the length of the sample is not too large compared to the width of this nearly intrinsic region.

For external voltage of less than kT/e, the conductivity $\sigma(x)$ in (5.6) does not deviate appreciably from the zero-bias case $\phi(x) = \text{const}$. For the contribution of the high-resistivity region around x_c, we obtain

$$\simeq \pi \sigma_{\min}^{-1} kT \left| \frac{\partial \psi}{\partial x} \right|_{x_c}^{-1} S^{-1} , \tag{5.7}$$

with

$$\left| \frac{\partial \psi}{\partial x} \right|_{x_c} \simeq \begin{cases} (4\pi e^2 N_0/\kappa)^{1/2} \, |(\phi - \psi)_0| & \text{case (4.8)} \quad [5.8(a)] \\ 2(2\pi e^2 n_t \, |\sinh[\beta(\phi - \psi)_0]| / \kappa)^{1/2} \, |(\phi - \psi)_0|^{1/2} & \text{case (4.9).} \quad [5.8(b)] \end{cases}$$

The fact that the main contribution to the resistance comes from the nearly intrinsic region has several important consequences: (a) The apparent resistance reflects the "intrinsic" temperature dependence of $\sigma_{\min}$

$$R \propto \sigma_{\min}^{-1}(T) \propto e^{\beta(E_c - E_v)/2} , \tag{5.9}$$

in accordance with experimental results found in amorphous systems.[6] (b) The work function of the contacts has little influence on the total resistance as this is largely determined by the mere presence of the conversion point. This is also in accordance with experimental results on amorphous sytsems.[6] (c) As shown in part C of Fig. 4, the width of the

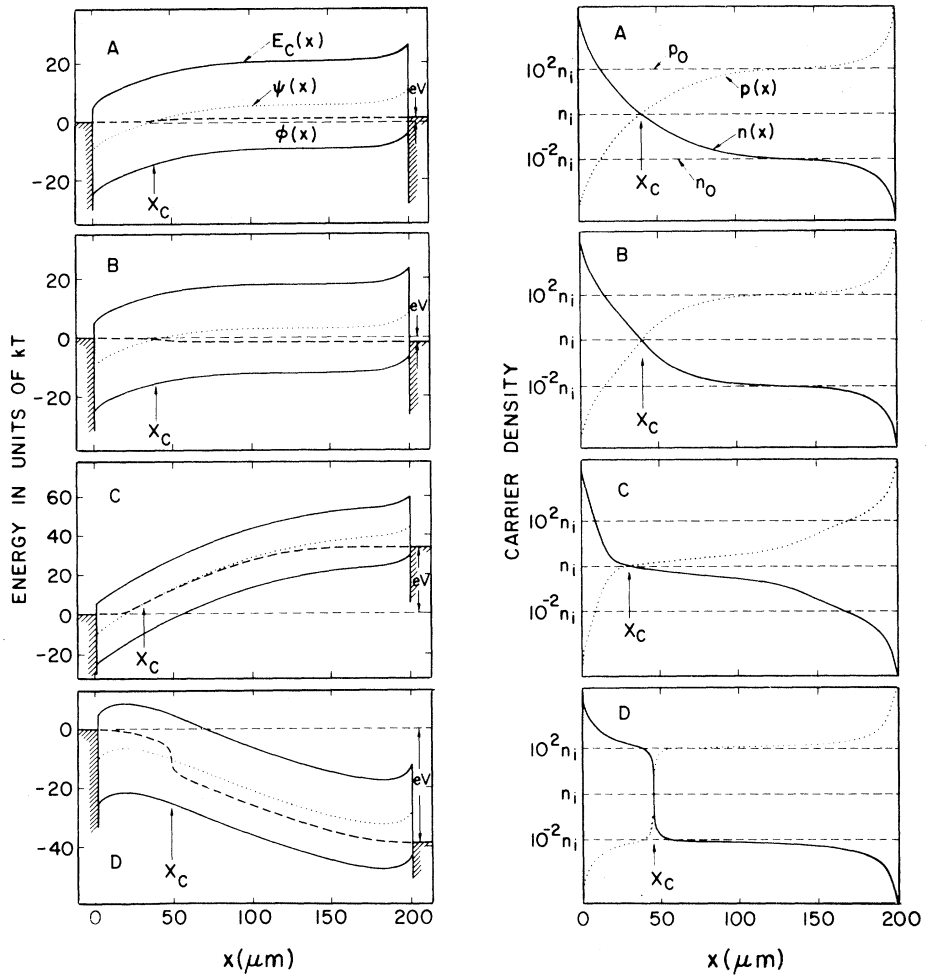

FIG. 4. Potential profiles and carrier distribution in an ideal p-type relaxation semiconductor with an electron- and a hole-injecting contact for different values of reverse and forward bias. A relatively small ratio $\sigma_{bulk}/\sigma_{min} = 50$ and a length of only a few times the space-charge region has been chosen in order to get a more detailed picture of the behavior near the conversion point x_c.

low-conductivity zone increases if a reverse bias is applied, thus leading to a sublinear current-voltage characteristic. For very high external bias, when the low-conductivity zone goes over the entire length, this again turns into a linear relation with $\sigma = \sigma_{min}$, as shown in Fig. 6. This behavior is in good agreement with experiments of Queisser *et al.*[7] For forward bias, the low-conductivity zone contracts according to part D of Fig. 4, and one obtains a superlinear behavior. van Roosbroeck *et al.*[1] also, for this case, obtain a sublinear behavior by introducing the concept of a "recombinative space-charge injection." It has, however, been shown[2,4,5] that this theory leads to qualitatively incorrect results by neglecting diffusion in the vicinity of x_c. Experiments on semi-insulating GaAs (Refs. 7, 8) also, in this case, show a clear sublinear behavior. This feature cannot be explained by the ideal relaxation behavior as defined in Sec. II. Below, we will see that the deviation from the ideal relaxation regime even tends to make the superlinear behavior more pronounced. Therefore, other possibilities must be considered to account for the observed sublinearity such as velocity saturation, contact effects, etc.

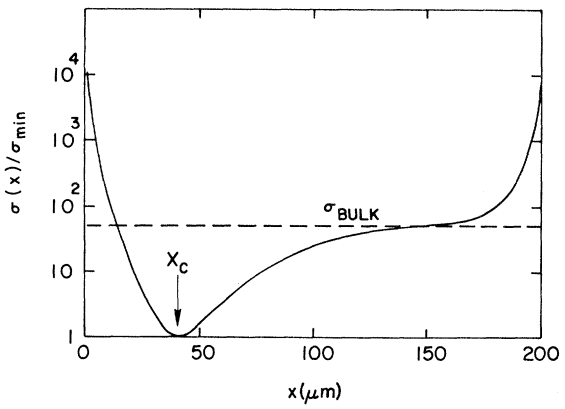

FIG. 5. Local conductivity $\sigma(x)$ at zero bias for the same sample as in Fig. 4.

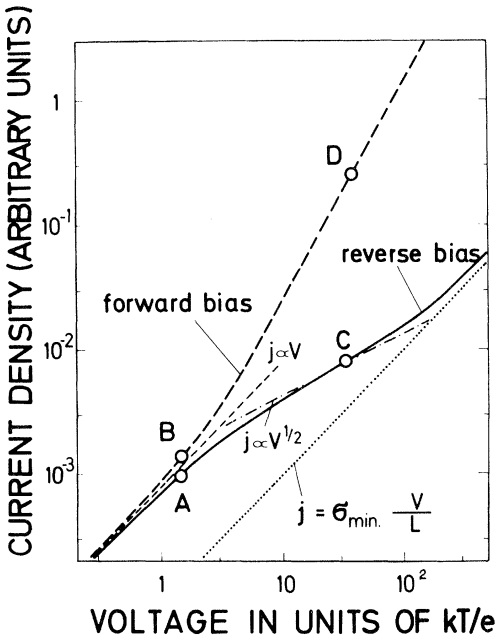

FIG. 6. Current voltage characteristic for the same sample as in Fig. 4. A, B, C, and D. correspond to the external voltages shown in Fig. 4.

Additional consequences of the theory which should be checked experimentally are (d) the resistance (not the resistivity!) is independent of the length of the sample as long as it is dominated by the zone near x_c where $\sigma(x) \simeq \sigma_{min}$. (e) By applying an ac field whose frequency is larger than the reciprocal time constant of establishing the steady state, one measures the bulk conductivity instead of σ_{min}.

(ii) If both contacts are minority carrier injecting, then two conversion points exist if the specimen is longer than the length Λ of the two space-charge regions at the contacts. Λ depends on temperature, as discussed after Eq. (4.9), and therefore, the two conversion points may disappear as the temperature decreases. As long as the two points exist, the I-V characteristic is almost linear for low fields, then becomes sublinear and finally, superlinear. In the linear and sublinear regions, the arguments for length and temperature dependence may be applied as in case (i).

(iii) All combinations which do not produce a conversion point, also do not show a quasiintrinsic behavior nor the other characteristic features mentioned above.

VI. FIELD DEPENDENCE

So far, we have not taken into account the field dependence of the conductivity. There are various high-field phenomena which strongly influence the conductivity, such as the Poole-Frenkel effect, the narrowing of the mobility gap, change of the free-carrier lifetime for hot carriers, velocity saturation, etc. If the main contribution to the total resistance of the sample at low currents comes from the region near x_c as discussed before, then it is essential to know how the field at the conversion point $eF(x_c) = (\partial\psi/\partial x)x_c$ influences the minimum conductivity σ_{min}.

We only want to discuss briefly the Poole-Frenkel effect,[9] which might be the most important enhancement factor. The Poole-Frenkel factor

$$\frac{\sigma(F)}{\sigma(0)} = e^{\beta\Delta(F)} \tag{6.1}$$

takes into account the lowering of the classical ionization energy of Coulomb-trapped carriers, due to a field $eF = \partial\psi/\partial x$.

$$\Delta(F) = 2e\left(\frac{\partial\psi/\partial x}{\kappa}\right)^{1/2}. \tag{6.2}$$

For $\partial\psi/\partial x \simeq 10^4$ eV/cm and $\kappa = 10$, the exponent in (6.1) is of the order of 1 for room temperature. In a material with constant $N(\epsilon)$ in the mobility gap, $(\partial\psi/\partial x)_{x_c}$ is given by [5.8(a)]. $(\partial\psi/\partial x)_{x_c} = 10^4$ eV cm cm^{-1} is obtained for $N_0 = 10^{16}\,\text{cm}^{-3}\,\text{eV}^{-1}$ and $\phi_0 - \phi = 0.25$ eV.

For the given example, $\sigma_{min}(F)$ would be increased by a factor of e as compared to its low-field value, but $\sigma_{min}(F)/\sigma_{bulk}$ still would be $2e^{-9}$. Therefore, the region around x_c would still dominate the resistance of the sample, and the statements made in Sec. V remain valid.

From Eqs. (6.2) and [5.8(a)], we see that $\Delta \propto N_0^{1/4}$. Therefore, σ_{min} is always appreciably smaller than σ_{bulk}, except possibly in cases with very high density of states in the gap. In this case, however, one would expect hopping between localized states to be the dominant conduction mechanism.

VII. RANGE OF VALIDITY FOR THE RELAXATION APPROXIMATION

So far, we have treated the idealized relaxation case assuming local thermal equilibrium at any point of the sample. This not only implies the approximation that the concentration of free electrons and holes (3.1) and (3.2) can be described in a satisfactory way by a common local-Fermi level $\phi_n(x) = \phi_p(x) = \phi(x)$, but we also have assumed that the local-Fermi level $\phi(x)$ describes the occupation probability of any localized state within the mobility gap [see Eq. (4.5)]. The validity of this assumption is of high importance since, in relaxation semiconductors with a high concentration of localized states in the mobility gap, the local charge density $\rho(x)$ and, hence, the band bending strongly depend on the occupation of these localized states.

Our first aim in this section is to show that the occupation probability of all localized states in the

mobility gap is described by Fermi levels $\phi_\epsilon(x)$ which have to lie between the quasi Fermi levels for mobile electrons and holes $\phi_n(x)$ and $\phi_p(x)$, respectively. With this statement proved, it follows directly that a common Fermi level may be used for all states as long as our defining condition for the relaxation regime holds. Subsequently, the limiting conditions for the applicability of the relaxation concept will be investigated.

We start from the general case of nonequilibrium. The continuity equation for the steady state requires that the difference between carrier generation and recombination of particles is equal to the divergence of the particle current. This applies for any energy ϵ,

$$\dot{n}_{\text{gen}}(\epsilon) - \dot{n}_{\text{rec}}(\epsilon) = \operatorname{div}\left(\frac{\sigma(\epsilon)}{-e^2}\ \frac{\partial \phi_\epsilon}{\partial x}\right). \tag{7.1}$$

The generation and recombination rates are given by

$$\dot{n}_{\text{gen}}(\epsilon) = N(\epsilon)[1 - f(\epsilon - \phi_\epsilon)] \times \int_{-\infty}^{\infty} d\epsilon' f(\epsilon' - \phi_{\epsilon'}) N(\epsilon') w_{\epsilon' \to \epsilon} \tag{7.2}$$

and

$$\dot{n}_{\text{rec}}(\epsilon) = N(\epsilon) f(\epsilon - \phi_\epsilon) \times \int_{-\infty}^{\infty} d\epsilon' [1 - f(\epsilon' - \phi_{\epsilon'})] N(\epsilon') w_{\epsilon \to \epsilon'}, \tag{7.3}$$

respectively. We have introduced the generalized Fermi level $\phi_\epsilon(x)$ formally to express the occupation probability of states of energy ϵ in terms of Fermi functions

$$f(\epsilon - \phi_\epsilon) = (e^{\beta(\epsilon - \phi_\epsilon)} + 1)^{-1}. \tag{7.4}$$

The transition probability $w_{\epsilon \to \epsilon'}$ for a phonon assisted transition from a state of energy ϵ to a state of energy ϵ' is related to the inverse process by the detailed balance condition

$$w_{\epsilon \to \epsilon'} = e^{\beta(\epsilon - \epsilon')} w_{\epsilon' \to \epsilon}. \tag{7.5}$$

For energies within the mobility gap, we have by definition $j(\epsilon) \equiv 0$; thus the steady-state condition reduces to

$$\dot{n}_{\text{gen}}(\epsilon) - \dot{n}_{\text{rec}}(\epsilon) = 0 \quad \text{for } E_v < \epsilon < E_c. \tag{7.6}$$

Using the explicit expressions (7.2) and (7.3) for the generation and recombination rate and the detailed balance condition (7.5), we obtain

$$\int_{-\infty}^{\infty} d\epsilon' N(\epsilon') w_{\epsilon \to \epsilon'} [1 - f(\epsilon' - \phi_{\epsilon'})] \times (e^{\beta(\phi_{\epsilon'} - \phi_\epsilon)} - 1) = 0. \tag{7.7}$$

If now the function ϕ_ϵ would have an absolute extremum within the mobility gap, say for $\epsilon = \epsilon_0$, the condition (7.7) could not be satisfied for this energy. That is, the integrand in (7.7) changes sign only, if $\phi_{\epsilon'} - \phi_{\epsilon_0}$ does, and it is positive or negative definite if ϕ_{ϵ_0} represents an absolute extremum of ϕ_ϵ. Therefore, the absolute extrema of ϕ_ϵ for the steady state have to lie outside of the mobility gap where we have $j(\epsilon) \neq 0$ and $\operatorname{div} j(\epsilon) \neq 0$, which allows for a finite value of the left-hand side of Eq. (7.1). Assuming common values for all mobile electrons and for all mobile holes, i.e.,

$$\phi_\epsilon = \phi_n \quad \text{for } \epsilon > E_c$$

and (7.8)

$$\phi_\epsilon = \phi_p \quad \text{for } \epsilon < E_v,$$

respectively, we have

$$\min\{\phi_n, \phi_p\} < \phi_\epsilon < \max\{\phi_n, \phi_p\} \quad \text{for } E_v < \epsilon < E_c, \tag{7.9}$$

which proves our above statement that ϕ_ϵ has to lie between the quasi-Fermi levels of the mobile electrons and holes. This is an important result, since relation (7.9) forces the local space charge within the traps at any point x to lie between the values obtained from Eq. (4.5), if ϕ_n and ϕ_p are inserted for $\phi(x)$. In particular, it is impossible that majority carriers are trapped in excess in a region where the density of free majority carriers is smaller and that of minority carriers is larger than in the unmodulated bulk. Therefore, the region of "compensating majority-carrier space charge" at the end of the "majority-carrier depletion zone", as predicted by van Roosebroeck *et al.*,[1] does not occur. It should be noted that the existence of such a space-charge region is of crucial importance in van Roosbroeck's theory of the forward biased minority-carrier injecting contact and in his theory of switching.[10]

We have shown that the error introduced into the calculation of the local space-charge density by assuming a common Fermi level for all states is negligible, if the splitting of the quasi-Fermi levels for electrons and holes is also. Substituting ϕ and $\partial\phi/\partial x$ by ϕ_n and $\partial\phi_n/\partial x$, ϕ_p and $\partial\phi_p/\partial x$, correspondingly, in the expression for the current (4.2), we see that a splitting of less than kT, i.e.,

$$|\phi_n - \phi_p| \ll kT \tag{7.10}$$

has no significant influence on the expression for the current (4.2). If, however, the splitting influences neither of the coupled differential equations for the determination of $\phi(x)$ and $\psi(x)$, the solutions will not be affected if $\phi_\epsilon(x)$, $\phi_n(x)$, and $\phi_p(x)$ are replaced by a common value $\phi(x)$. Whether the condition (7.10) (which is identical with our defining condition (2.1) for the relaxation regime)

is fulfilled can be decided by looking at the continuity equation (7.1). Inserting (7.2) and (7.3) and integrating over the energy from E_c to ∞ gives for the electron current, e.g.,

$$\operatorname{div}\left(\frac{j_n}{-e}\right) = n_i \int_{-\infty}^{E_c} d\epsilon\, N(\epsilon)[1 - f(\epsilon - \phi_\epsilon)] \times w_{n\to\epsilon}\,(e^{\beta(\phi_\epsilon - \psi)} - e^{\beta(\phi_n - \psi)}) \,. \tag{7.11}$$

Using an appropriate mean value of $\langle \phi_\epsilon \rangle_{\mathrm{av}} = \overline{\phi}$, one can define a free-electron lifetime τ by writing (7.11) as

$$\operatorname{div}(j_n/-e) = (n_i/\tau_n)(e^{\beta(\overline{\phi}-\psi)} - e^{\beta(\phi_n - \psi)}). \tag{7.12}$$

Comparing (7.12) with (7.11), we observe that τ_n depends on $\overline{\phi}$. For mobile holes we obtain, correspondingly,

$$\operatorname{div}(j_p/e) = (n_i/\tau_p)(e^{\beta(\psi-\overline{\phi})} - e^{\beta(\psi-\phi_p)}). \tag{7.13}$$

Using $\operatorname{div} j = \operatorname{div} j_n + \operatorname{div} j_p = 0$, one can eliminate $\overline{\phi}$. For sufficiently low splitting $|\phi_n - \phi_p|$, one gets

$$\operatorname{div}(j_n/-e) = (n/\tau_n^{\mathrm{rec}})(1 - e^{\beta(\phi_n - \phi_p)}), \tag{7.14}$$

where

$$n/\tau_n^{\mathrm{rec}} = n_i^2/(n\tau_p + p\tau_n) = p/\tau_p^{\mathrm{rec}} \tag{7.15}$$

is also a function of the local Fermi potential. τ_n^{rec} and τ_p^{rec} have the meaning of the lifetime for free electron-free hole recombination, while the definition of τ_n and τ_p also includes trapping-detrapping processes. With our defining condition for the relaxation regime (2.1), we now get a condition in terms of the divergence of the electron current from (7.14),

$$\left|\frac{\tau_n^{\mathrm{rec}}}{n}\frac{j}{e}\frac{\partial}{\partial x}\left(\frac{n}{n+p}\right)\right| = \beta\,|\phi_n - \phi_p| \ll 1. \tag{7.16}$$

For the conversion point, where the splitting has its maximum value, we obtain

$$\tau_i^{\mathrm{rec}}\frac{\mu}{e}\left(\frac{\partial\phi}{\partial x}\right)_{x_c}\left(\frac{\partial(\phi-\psi)}{\partial x}\right)_{x_c} \ll 2kT\,, \tag{7.17}$$

with

$$\tau_i^{\mathrm{rec}} = \tau_n(\phi = \psi) + \tau_p(\phi = \psi). \tag{7.18}$$

This result can be interpreted in the following way. During one lifetime, a carrier near the conversion point drifts over a distance Λ_i^{dr}, given by

$$\Lambda_i^{\mathrm{dr}} = \tau_i^{\mathrm{rec}}\,\mu\, e^{-1}\left(\frac{\partial\phi}{\partial x}\right)_{x_c}. \tag{7.19}$$

Our condition thus requires that $\phi(x)$ and $\psi(x)$ should diverge from each other by less than kT over the distance Λ_i^{dr} (see Fig. 7).

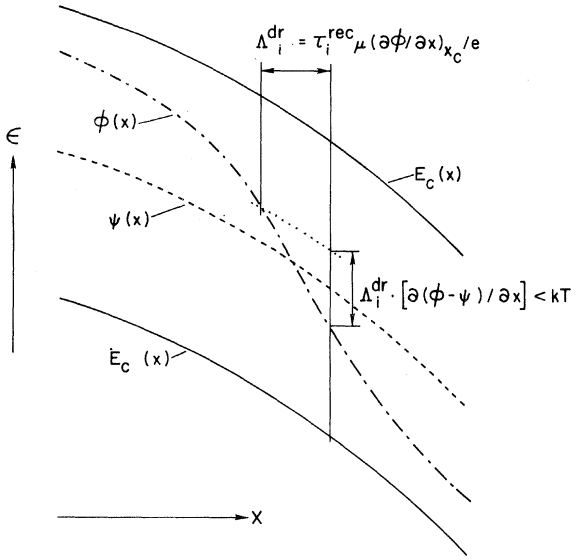

FIG. 7. Illustration of limiting condition for the relaxation regime (see text).

We are now able to justify why we chose a definition for the relaxation regime differing from the one given by van Roosbroek[1]

(i) If (7.17) is fulfilled, we say that the arrangement consisting of a semiconductor with electrodes is working in the relaxation regime. We are not talking merely about a typical bulk property of the semiconducting material as in van Roosbroek's definition (1.2), but rather we are talking about a certain working point which also depends on the electrodes, the temperature, and in particular, on the external voltage. The typical relaxation behavior as described above will be observed as long as the external voltage does not exceed some characteristic upper limit, which follows from (7.17).

(ii) Our requirements on a material for remaining in the relaxation regime up to high-external fields are low mobility and short free-carrier recombination lifetime, as one can see from (7.17). van Roosbroeck's defining condition for relaxation case semiconductors (1.3) also requires low mobilities and short carrier lifetimes, but it also includes the bulk-charge carrier concentration, which does not enter into our more genuine conditions.

VIII. DEVIATIONS FROM THE IDEAL RELAXATION REGIME

For a qualitative discussion of the deviations which are to be expected if the relaxation condition is not fulfilled, it is instructive to consider a $p-n$ junction or a minority-carrier injecting contact. It will turn out that any current-voltage curve for

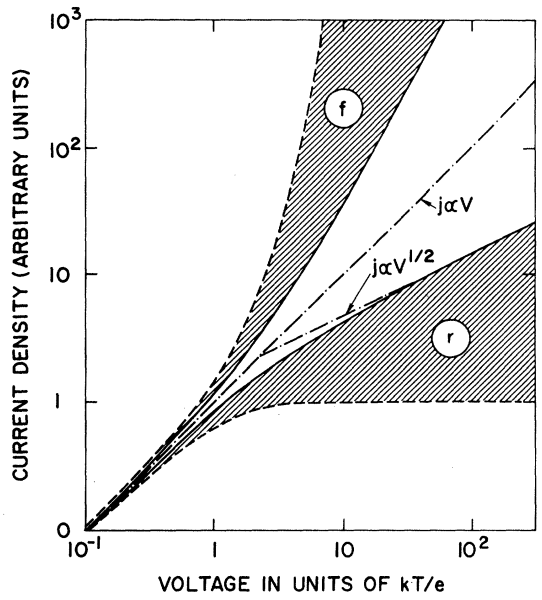

FIG. 8. Current-voltage curves for minority-carrier injecting contact, schematically. Solid and dashed lines refer to the limiting cases of the "ideal relaxation regime" ($\mu\tau^{rec} \to 0$) and of the ideal rectifier," i.e., the "ideal lifetime regime" ($\mu\tau^{rec} \to \infty$), respectively. Real current-voltage curves ($\mu\tau^{rec}$) finite will lie somewhere in the shaded fields r and f for forward and reverse bias depending on the value of $\mu\tau^{rec}$.

finite mobility and lifetime has to lie between those for the ideal rectifier ($\tau^{rec} \to \infty$, $\mu \to \infty$; i.e., ideal lifetime case in van Roosbroeck's terms) and those for the ideal relaxation case ($\tau^{rec} \to 0$, $\mu \to 0$) (see Fig. 8).

In the reverse biased ideal rectifier, there is a region where the splitting of the Fermi levels attains the highest possible negative value $\phi_n(x) - \phi_p(x) = -|eU|$, resulting in a strong carrier depletion given by $n(x)\,p(x) = n_i^2 e^{-\beta|eU|} \ll n_i^2$. In the case of finite lifetime and mobility, however, the carrier generation reduces the value of $\phi_p - \phi_n$ in the depletion region. Due to the additional carriers, the current is higher and does not saturate any more for high-reverse bias. The ideal relaxation case appears as the limiting case. In this case, namely, the generation in the depletion zone is so high as compared to the carriers moving away that there is practically no carrier depletion any more. This means $\phi_n - \phi_p \simeq 0$, and hence, $n(x) \times p(x) \simeq n_i^2$. From (7.14) we see that any deviation from the ideal reverse-relaxation regime can only result in $\phi_n - \phi_p < 0$, and therefore, reduced local conductivity, since the divergence of the particle current is positive. Thus the highest possible reverse current is flowing in junctions with ideal relaxation semiconductors. Any real current-voltage curve for reverse bias will lie in the shaded region r of Fig. 8, whatever the actual lifetimes and mobilities are.

In a similar way, one can show that any real current-voltage curve for forward bias is characterized by $\phi_n - \phi_p > 0$, and therefore, has to lie in the shaded region f between the ideal-rectifier characteristic (no carrier recombination in the space-charge region, and hence, $\phi_n - \phi_p = |eU|$) and the ideal relaxation case (perfect recombination, such that $\phi_n - \phi_p \equiv 0$). In particular, it is clear that deviations from the ideal relaxation regime cannot be the reason for a sublinear current voltage for forward bias.[1,7,8]

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